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ONE-AND-ONE-HALF-BOUND DICHOTOMOUS CHOICE CONTINGENT VALUATION

by

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The views expressed are the authors' and do not necessarily represent policies or views of their respective institutions.

One-and-One-Half-Bound Dichotomous Choice Contingent Valuation

ABSTRACT

To reduce the potential for response bias on the follow-up bid in multiple-bound discrete choice CVM questions while maintaining much of the efficiency gains of the multiple-bound approach, we introduce the one-and-one-half-bound (OOHB) approach. Despite the fact that the OOHB model uses less information than the double-bound (DB) approach, efficiency gains in moving from single-bound to OOHB capture a large portion of the gain associated with moving from single-bound to DB. In an analysis of survey data, our OOHB estimates demonstrated higher consistency with respect to the follow-up data than the DB estimates and were more efficient as well.

JEL Classification Codes: Q20, Q26, C15, C25

One-and-One-Half-Bound Dichotomous Choice Contingent Valuation

ABSTRACT (long version)

While the double-bound (DB) format for discrete choice contingent valuation method (CVM) has the benefit of higher efficiency in the welfare benefits estimates than single bound discrete choice CVM, it has been subject to criticism due to evidence that some of the responses to the second bid may be inconsistent with the responses to the first bid. As a means to reduce the potential for response bias on the follow-up bid in multiple-bound discrete choice formats such as the DB model while maintaining much of the efficiency gains of the multiple-bound approach, we introduce the one-and-one-half-bound (OOHB) approach and present a real world application. Despite the fact that the OOHB model uses less information than the DB approach, in a laboratory setting, efficiency gains in moving from single-bound to OOHB capture a large portion of the gain associated with moving from SB to DB. Utilizing distribution-free semi-nonparametric estimation techniques on a split survey dataset, our OOHB estimates demonstrated higher consistency with respect to the follow-up data than the DB estimates and were more efficient as well. Hence, OOHB may serve as a viable alternative to the DB format in situations where follow-up response bias may be a concern.

JEL Classification Codes: Q20, Q26, C15, C25

1. Introduction

When measuring respondents' willingness to pay (WTP) for an item, most designers of contingent valuation (CV) studies have switched in recent years from using an open-ended format in which respondents are asked how much they would be willing to pay for the item to a closed-ended format in which they are asked whether or not they would be willing to pay some specified price. The closed-ended format was first introduced by Bishop and Heberlein (1979), who used what is now known as the single-bounded (SB) version in which each subject is presented with a single monetary amount, the amount being varied across respondents. Hanemann, Loomis, and Kanninen (1991) – henceforth, HLK – introduced a variant, the double-bounded (DB) format, in which the subjects are presented with a price as in the SB approach, but after responding they presented with another price and asked whether they would also be willing to pay that amount. The second price is set on the basis of the subject's response to the first price. If the subject responds “yes” the first time, the second price is some amount higher than the first price; if the initial response is “no,” the second price is some amount lower. HLK showed analytically that the extra information gained from the follow-up question makes the DB estimates more efficient than the SB estimates, and they presented an empirical application in which this efficiency gain was quite large – for virtually no extra survey cost there was a significant improvement in the precision of the estimated WTP distribution. Given the estimated distribution, it was apparent ex post that the initial prices in that survey had been chosen poorly and were quite far from optimal; but HLK found that the second prices counteracted this and provided an effective insurance against the poor selection of an initial price.

Because of its statistical efficiency, the DB approach has gained in popularity and is now often favored over the SB approach. At the same time, however, it has aroused controversy because of evidence that responses to the first price may sometimes be inconsistent with the responses to the second, with the latter revealing a lower WTP [Hanemann (1991); McFadden and Leonard (1993); Cameron and Quiggin (1994); Kanninen (1995); Herriges and Shogren (1996), DeShazo (2000)]. Several explanations have been proposed for the anomaly. Carson et al. (1992) suggest an explanation based on cost expectations: a respondent who said “yes” to the initial price sees the second price as a price increase, which he rejects; a respondent who said “no” and is then offered a lower price may suspect that an inferior version of the item will be provided, which he also is disposed to reject. Altaf and DeShazo (1994) suggest that the second bid converts what had seemed to be a straight forward posted-price market into a situation involving bargaining; if this is bargaining, the respondent should say no in order to drive the price down. DeShazo (2000) offers a prospect theory explanation involving loss-aversion and framing on the first price.

Existing applications of the DB approach all use scenarios where the respondent is *not* told ahead of time that she will be confronted with a second price; the interview focuses mainly on the first price, and the second price comes as something of a surprise when introduced at the end. We suspect that this surprise may be the root cause of the discrepancy in the responses to the two prices. To remedy this, we propose an alternative survey design in which the respondent is given two prices up front and told that, while the exact cost of the item is not known for sure, it is known to lie within the range bounded by these two prices.¹ One of the two prices is selected at random, and the respondent is asked whether she would be willing to pay this amount; she is then asked about the other price only if doing so would be consistent with the stated price range. For example, if the

lower of the two prices price was selected initially and she says “yes” to this, she is then asked whether she would be willing to pay the higher price; but, if she says “no” to the lower price, there is no follow-up question because that would go below the stated price range. We believe that eliminating the element of surprise has the potential to remove discrepancies in the responses to the two valuation questions, but it comes at the cost of not always being able to ask the second valuation question: the second question will be appropriate half the time, on average, but not the rest of the time. Hence, we refer to this as the one-and-one-half bound format (OOHB).

Two issues arise in assessing this proposed new survey format: does it actually lessen or remove the discrepancy in survey responses to the two prices, and how large is the cost in terms of reduced statistical efficiency relative to the DB format? The first is an empirical question that can be answered only through actual survey experience. The second one can be answered analytically comparing the statistical properties of the OOHB WTP estimator with those of the DB and SB estimators. Both questions are addressed in this paper. The remainder of the paper is organized as follows. Section 2 formally describes the likelihood functions associated with SB, DB and OOHB formats, analytically characterizes the asymptotic efficiency of OOHB relative to SB and DB, and identifies the optimal design of prices for use in a OOHB survey. At each point, we compare the new results for OOHB with the existing results in the literature for the SB and DB formats. Section 3 presents an empirical comparison based on a split-sample CV survey conducted in Italy using the OOHB and DB formats. Our conclusions are summarized in Section 4.

2. Analytical Comparison of the Survey Formats

In the SB format, the i^{th} respondent is asked if she would be willing to pay some given amount B_i^* (henceforth we refer to this as the “bid”) to obtain, say, a given improvement in

environmental quality. The probability of a “yes” response, or a “no” response, $\pi_i^Y(B_i^*)$, can be cast in terms of a random utility maximizing choice by the respondent. Let C_i be the individual's true maximum WTP for the item that is the subject of the survey. This can be a function of economic variables, such as the respondent's income and the prices of commodities that are complements or substitutes for the item in question; demographic and attitudinal variables, such as the respondent's age or sex, or whether or not the respondent is an environmentalist; and possibly other variables relating to the item being valued. We denote all such variables by the vector X_i . Also, by virtue of the random utility framework the individual's WTP is a random variable from the point of view of the econometric observer, reflecting individual variation in preferences and unobserved variables or measurement error in the observed variables. Thus, while the individual knows her own WTP, C_i , to the observer it is a random variable with a given cumulative distribution function (cdf) denoted $G(C_i; \theta)$ where θ represents the parameters of this distribution, which are to be estimated on the basis of the responses to the CV survey. The parameters will be functions of the variables in X_i , but this is left implicit in $G(C_i; \theta)$. For example, there can be a mean of the WTP distribution which depends on covariates, $\mu = X\beta$, and a variance, σ^2 . In this case, $\theta = (\beta, \sigma^2)$. Then, as noted by Hanemann (1984), the response probabilities are related to the underlying WTP distribution by

$$(1a) \quad \pi_i^N \equiv \Pr\{No\ to\ B_i^*\} \equiv \Pr\{B_i^* > C_i\} = G(B_i^*; \theta)$$

$$(1b) \quad \pi_i^Y \equiv \Pr\{Yes\ to\ B_i^*\} \equiv \Pr\{B_i^* \leq C_i\} = 1 - G(B_i^*; \theta)$$

The resulting log-likelihood function for the responses to a CV survey using the SB format is²

$$(2) \quad \ln L^{SB}(\theta) = \sum_{i=1}^N \{d_i^Y \ln[1 - G(B_i^*; \theta)] + d_i^N \ln G(B_i^*; \theta)\}$$

where $d_i^Y = 1$ if the i^{th} response is Yes and 0 otherwise, while $d_i^N = 1$ if the i^{th} response is No and 0 otherwise, The maximum likelihood estimator (MLE), denoted $\hat{\theta}^{SB}$, is the solution to the equation $\partial \ln L^{SB}(\hat{\theta})^{SB} / \partial \theta = 0$. This estimator is consistent (though it may be biased in small samples) and asymptotically efficient. Thus, the asymptotic variance-covariance matrix of $\hat{\theta}^{SB}$ is given by the Cramer-Rao lower bound

$$(3) \quad V^{SB}(\hat{\theta}^{SB}) = \left[-E \frac{\partial^2 \ln L^{SB}(\hat{\theta}^{SB})}{\partial \theta \partial \theta'} \right]^{-1} \equiv I^{SB}(\hat{\theta}^{SB})^{-1},$$

where $I^{SB}(\hat{\theta}^{SB})$ is the information matrix associated with the SB format.

The DB format starts with an initial bid B_i^0 . If the respondent answers Yes, she receives a follow-up bid $B_i^U > B_i^0$; if she answers No, she receives a follow-up bid $B_i^D < B_i^0$. Thus, there are four possible outcomes: (Yes, Yes), (Yes, No), (No, Yes), and (No, No). In terms of the random utility maximizing model given above, the corresponding response probabilities are

$$(4a) \quad \pi^{YY} \equiv \Pr\{B_i^U \leq C_i\} \equiv 1 - G(B_i^U; \theta)$$

$$(4b) \quad \pi^{YN} \equiv \Pr\{B_i^0 \leq C_i \leq B_i^U\} \equiv G(B_i^U; \theta) - G(B_i^0; \theta)$$

$$(4c) \quad \pi^{NY} \equiv \Pr\{B_i^D \leq C_i \leq B_i^0\} \equiv G(B_i^0; \theta) - G(B_i^D; \theta)$$

$$(4d) \quad \pi^{NN} \equiv \Pr\{C_i \leq B_i^D\} \equiv 1 - G(B_i^D; \theta)$$

The log-likelihood function for the responses to a CV survey using the DB format is:

$$(5) \quad \ln L^{DB}(\theta) = \sum_{i=1}^N \{d_i^{YY} \ln[1 - G(B_i^U; \theta)] + d_i^{YN} \ln[G(B_i^U; \theta) - G(B_i^0; \theta)]$$

$$(6) \quad + d_i^{NY} \ln[G(B_i^0; \theta) - G(B_i^D; \theta)] + d_i^{NN} \ln G(B_i^D; \theta)\}$$

where $d_i^{YY} = 1$ if the i^{th} response is (Yes, Yes) and 0 otherwise, $d_i^{YX} = 1$ if the i^{th} response is (Yes, No) and 0 otherwise, $d_i^{NY} = 1$ if the i^{th} response is (No, Yes) and 0 otherwise, $d_i^{NN} = 1$ if the i^{th} response is (No, No) and 0 otherwise. Denote the resulting MLE by $\hat{\theta}^{DB}$; the associated information matrix, $I^{DB}(\hat{\theta}^{DB})$, is equal to minus the expectation of the Hessian of the maximized log-likelihood function in (5).

We now propose the one-and-one-half bound format (OOHB) in which the respondent is presented with a range, $[B_i^-, B_i^+]$, where $B_i^- < B_i^+$. One of these two prices is selected at random and the respondent is asked whether she would be willing to pay that amount. She is asked about the second price only if that is compatible with her response to the first price. If the lower price, B_i^- , is randomly drawn as the starting bid, the three possible response outcomes are (No), (Yes, No) and (Yes, Yes); we denote the corresponding response probabilities $\pi_i^N, \pi_i^{YN}, \pi_i^{YY}$. If the higher price, B_i^+ , is randomly drawn as the starting bid, the possible response outcomes are (Yes), (No, Yes) and (No, No). We denote the corresponding response probabilities $\pi_i^Y, \pi_i^{NY}, \pi_i^{NN}$. Observe that

$$(6a) \quad \pi_i^N = \pi_i^{NN} = \Pr\{C_i \leq B_i^-\} = G(B_i^-; \theta)$$

$$(6b) \quad \pi_i^{YN} = \pi_i^{NY} = \Pr\{B_i^- \leq C_i \leq B_i^+\} = G(B_i^+; \theta) - G(B_i^-; \theta)$$

$$(6c) \quad \pi_i^{YY} = \pi_i^Y = \Pr\{C_i \geq B_i^+\} = 1 - G(B_i^+; \theta)$$

Let $d_i^N = 1$ if either the starting bid is B_i^- and the response is (No) or the starting bid is B_i^+ and the response is (No, No), and 0 otherwise; let $d_i^{YN} = 1$ if either the starting bid is B_i^- and

the response is (Yes, No) or the starting bid is B_i^+ and the response is (No, Yes), and 0 otherwise; and let $d_i^{YY} = 1$ if either the starting bid is B_i^- and the response is (Yes, Yes) or the starting bid is B_i^+ and the response as (Yes), and 0 otherwise. Then, the log-likelihood function for the responses to a CV survey using the OOHB format is

$$(7) \quad \ln L^{OOHB}(\theta) = \sum_{i=1}^N \{ d_i^Y \ln[1 - G(B_i^+; \theta)] + d_i^{YN} \ln[G(B_i^+; \theta) - G(B_i^-; \theta)] + d_i^N \ln[G(B_i^-; \theta)] \}$$

We denote the resulting MLE by $\hat{\theta}^{OOHB}$; the associated information matrix, $I^{OOHB}(\hat{\theta}^{OOHB})$ is equal to minus the expectation of the Hessian of the maximized log-likelihood function in (7).

With the OOHB survey format, since the respondent is told about the possible range of costs at the beginning of the survey we believe she is less likely to form false cost expectations, enter into bargaining mindset, or experience loss-aversion when responding to the follow-up bid. Consequently, we hypothesize that there is less likely to be a discrepancy between the responses to the first and second bids with the OOHB format than with the DB format. This is tested in an empirical application to be presented in Section 3. However, as noted above, the OOHB format gathers less information per respondent than the DB format, and consequently entails some loss of statistical efficiency relative to the DB format. We address the efficiency impact analytically in the remainder of this section.

The analytical comparison of efficiency is based on the information matrices. HLK assessed the efficiency of the DB format relative to the SB format using the difference in the information matrices

$$(8) \quad \Delta(DB / SB) \equiv I^{DB}(\hat{\theta}^{DB}) - I^{SB}(\hat{\theta}^{SB}).$$

They note that the comparison of efficiency depends inevitably on the specific bids used with each format. If the bids are different, one cannot generally determine which format is the more efficient; for example, it could happen that the SB format with an good choice of bid B_i^* is more efficient than the DB format with a bad choice of initial bid B_i^0 . However, if the initial bid in the

DB format is the same as the SB bid ($B_i^* = B_i^0$), HLK show that $\Delta(DB/SB) = \sum_i^N \Delta_i$, where

$$(9) \quad \Delta_i \equiv AA' / \gamma + WW' / \delta,$$

and where $\gamma \equiv [1 - G(B_i^U; \theta)] \cdot [1 - G(B_i^0; \theta)] \cdot [G(B_i^U; \theta) - G(B_i^0; \theta)]$ and $\delta \equiv [G(B_i^0; \theta) - G(B_i^D; \theta)] \cdot$

$G(B_i^0; \theta) \cdot G(B_i^D; \theta)$ are positive scalars and A and W are vectors given by $A \equiv$

$$[G_0(B_i^0; \theta) \cdot (1 - G(B_i^U; \theta)) - G_\theta(B_i^U; \theta) \cdot (1 - G(B_i^0; \theta))]$$

$$[G_0(B_i^D; \theta) \cdot G(B_i^0; \theta) - G_\theta(B_i^0; \theta) \cdot G(B_i^D; \theta)].$$

Because both AA' and WW' are positive semidefinite matrices, it follows that $I^{DB}(\hat{\theta}^{DB}) \geq I^{SB}(\hat{\theta}^{SB})$ and $V^{DB}(\hat{\theta}^{DB}) \leq V^{SB}(\hat{\theta}^{SB})$: $\hat{\theta}^{DB}$ is asymptotically more efficient than $\hat{\theta}^{SB}$.

In the case of OOHB, there are two efficiency comparisons – a comparison of OOHB with SB, and a comparison of DB with OOHB. Define

$$(10a) \quad \Delta(OOHB/SB) \equiv I^{OOHB}(\hat{\theta}^{OOHB}) - I^{SB}(\hat{\theta}^{SB}) \quad \text{and}$$

$$(10b) \quad \Delta(DB/OOHB) \equiv I^{DB}(\hat{\theta}^{DB}) - I^{OOHB}(\hat{\theta}^{OOHB}),$$

where $\Delta(OOHB/SB) = \sum_i^N \Delta'_i$, say, and $\Delta(DB/OOHB) = \sum_i^N \Delta''_i$. The overall efficiency

comparison in (8) can be decomposed into the sum of these two comparisons

$$(11) \quad \Delta(DB/SB) \equiv \Delta(DB/OOHB) + \Delta(OOHB/SB).$$

As with (8), the efficiency comparisons in (10a) and (10b) depend on the specific bids used with each format, and are generally indeterminate if the bids are noncomparable across formats. However, if the SB bid is the same as either of the two OOHB bids ($B_i^* = B_i^-$ or $B_i^* = B_i^+$), then Δ'_i can be shown to be positive semi-definite, so that $\hat{\theta}^{OOHB}$ is asymptotically more efficient than $\hat{\theta}^{SB}$. Similarly, if the OOHB bids are the same as two of the DB bids ($B_i^- = B_i^0$ and $B_i^+ = B_i^U$, or $B_i^- = B_i^D$ and $B_i^+ = B_i^0$), then Δ''_i can be shown to be positive semi-definite, so that $\hat{\theta}^{DB}$ is asymptotically more efficient than $\hat{\theta}^{OOHB}$. Specifically, it can be shown that if $B_i^* = B_i^-$,

$$(12) \quad \Delta'_i \equiv AA' / \gamma,$$

while, if $B_i^- = B_i^0$ and $B_i^+ = B_i^U$,

$$(13) \quad \Delta''_i \equiv WW' / \delta. \quad ^3$$

Hence, although the OOHB format was unknown at the time, the two positive semi-definite matrices in HLK's formula (9) for the efficiency gain of DB over SB turn out to measure, respectively, the efficiency gain of OOHB over SB and the efficiency gain of DB over OOHB.

Which of these gain matrices is larger – the gain from OOHB over SB or that from DB over OOHB – cannot be determined in general. However, some specific results emerge when the formats are compared in the context of optimal bid design. The existing literature focuses mainly on the criterion of locally D-optimal design, based on maximizing the determinant of the information matrix, and deals with the special case where the WTP distribution takes the form of a two-parameter logistic distribution

$$(14) \quad G(C; \theta) = [1 + e^{\alpha - \beta C}]^{-1};$$

in this case, $\theta \equiv (\alpha, \beta)$ and $E\{C\} = \text{median}\{C\} = \alpha/\beta$. For the SB format, Minkin (1987) shows that, when there is an even number of observations N , the determinant of the information matrix $I^{SB}(\hat{\theta}^{SB})$ corresponding to the logistic model (14) is maximized when half of the bid values satisfy $-\alpha + \beta\bar{B} = 1.5434$ and the other half satisfy $-\alpha + \beta\bar{B} = -1.5434$. Thus, given a preliminary estimate of α and β , the optimal SB design is a two-point design, $\bar{B} = (\alpha \pm 1.5434)/\beta$, which is symmetric about the median of the WTP distribution. With this optimal design, Minkin shows that the resulting value of the determinant of the information matrix is

$$(15) \quad |\bar{I}^{SB}| = N^2(0.051)/\beta^2.$$

For the DB format, Kanninen (1995) shows that the determinant of the information matrix $I^{DB}(\hat{\theta}^{DB})$ corresponding to the two-parameter logistic model (14) is maximized with a three-point design where the first bid is the median of the WTP distribution, α/β , and the two follow-up bids are $\bar{B} = (\alpha \pm 1.5434)/\beta$. With this optimal design, Kanninen shows that the resulting value of the determinant of the information matrix is

$$(16) \quad |\bar{I}^{DB}| = N^2(0.2870)/\beta^2,$$

approximately a five-fold improvement over its value with the optimal SB bid in (15).

For the OOHB format with the two-parameter logistic WTP distribution in (14), when the bids B_i^- and B_i^+ are spaced symmetrically about the median of the WTP distribution with $B_i^- = (\alpha - w)/\beta$ and $B_i^+ = (\alpha + w)/\beta$, the determinant of the information matrix is ⁴

$$(17) \quad |I^{OOHB}(w)| = \frac{N^2 w^2}{\beta^2 (1 + e^{-w})^4 (1 + e^w)(e^w - 1)}.$$

This is maximized numerically, leading to an optimal value of $w = 1.46745$. The resulting value of the determinant of the information matrix is

$$(18) \quad \left| \bar{I}^{OOHB} \right| = N^2(0.21084) / \beta^2.$$

Comparing (15), (16), and (18), when one uses D-optimal bids the OOHB format captures the majority share (68%) of the gain in efficiency associated with the DB format; the gain in switching from SB to OOHB significantly outweighs the gain in switching from OOHB to DB.⁵

By construction, this analytic analysis has focused on the statistical implications of alternative CV elicitation procedures with regard to the additional information gained from further questioning. The other consideration is the cognitive implications: can the sequence of presenting information to survey respondents create cost expectations, convey an impression of bargaining, or induce a framing that influences the survey responses? To investigate these issues, we turn to an empirical field experiment. The analytic analysis suggests that the loss of statistical efficiency from using OOHB instead of DB may be small or negligible. What remains to be determined is whether, in the field, OOHB succeeds in reducing or eliminating the discrepancy in the survey responses to the follow-up valuation question.

3. A Field Test of the OOHB Format

We present here the results of a CV survey conducted in Italy to value Cava Grande del Cassibile, a Regional Nature Reserve run by the Italian Forest Service in southeast Sicily, near Syracuse. The survey was conducted by the Università degli Studi di Catania in June-September 1995 and 1996 and took the form of on-site interviews of adult visitors (aged 18 or over) as they left the Reserve. Access to the Reserve is currently free; in the CV surveys, respondents were asked whether they would be willing to pay a charge for admission. The survey involved a split

sample experiment between the DB and OOHB elicitation formats, with random assignment between formats and $N = 400$ for each format.⁶ In the DB version, respondents were asked “if the price of an admission to the Reserve were B^0 , would you purchase it?” with the subsequent follow up “And, if the price of an admission was B^U , would you still buy it?” or “And if the price was B^D instead, would you buy it?” In the OOHB version, respondents were first told that “the price of admission to the Reserve will be somewhere in the range of B^- to B^+ lire.” One of the prices was selected at random, and the respondent was asked “If the price of this admission was [selected price], would you buy it?” with a follow up question using the other price where this was logical. Different prices were randomly assigned across subjects.⁷ These prices were derived on the basis of a pretest of 130 open-ended DB surveys, using the bid design approach in Cooper (1993).⁸

To analyze the responses to the DB and OOHB surveys, we used both a parametric approach, based on the logistic and log-logistic WTP distributions in (14) and (20), and a semi-nonparametric distribution-free (SNPDF) approach, first applied to SB data by Creel and Loomis (1997) and extended here to DB and OOHB data.⁹ The reason for the SNPDF approach is to reduce the sensitivity of our econometric analysis to specific parametric assumptions regarding the form of the WTP distribution. In the event, both approaches produced similar results. For brevity, only the SNPDF results are presented here; the parametric results are available from the authors.

A simple way to motivate the SNPDF approach is to observe that, with the logistic WTP distribution (14), the CV response probabilities corresponding to, say, (1a), (4b) and (6b) take the form

$$(1a') \quad \pi_i^N = G(B_i^*; \theta) \equiv F[\Delta V(B_i^*)]$$

$$(4b') \quad \pi_i^{YN} = G(B_i^U; \theta) - G(B_i^0; \theta) \equiv F[\Delta V(B_i^U)] - F[\Delta V(B_i^0)]$$

$$(6b') \quad \pi_i^{YN} = G(B_i^+; \theta) - G(B_i^-; \theta) \equiv F[\Delta V(B_i^+)] - F[\Delta V(B_i^-)]$$

where $F(z) = [1 + e^{-z}]^{-1}$ is the standard logistic cdf and

$$(19) \quad \Delta V(\beta) \equiv -\alpha + \beta B$$

is what Hanemann (1984) calls a utility difference function, which is increasing in the bid price, B . The SNPDF approach retains the logistic cdf in the response probabilities such as (1a'), (4b') and (6b'), but replaces the linear utility difference with a Fourier flexible form (e.g. Gallant., 1982). where (omitting quadratic term as in Loomis and Creel)

$$(20) \quad \Delta V(\mathbf{x}, \theta_k) = \mathbf{x}\beta + \sum_{\alpha=1}^A \sum_{j=1}^J (v_{j\alpha} \cos[j\mathbf{k}'_{\alpha} s(\mathbf{x})] - w_{j\alpha} \sin[j\mathbf{k}'_{\alpha} s(\mathbf{x})])$$

where the vector \mathbf{x} contains all arguments of the utility difference model, A and J are positive integers, and \mathbf{k}_{α} are vectors of positive and negative integers that form indices in the conditioning variables, after shifting and scaling of \mathbf{x} by $s(\mathbf{x})$.¹⁰ There exists a coefficient vector such that, as the sample size becomes large, $\Delta V(\mathbf{x})$ in (20) can be made arbitrarily close to a continuous unknown utility difference function for any value of \mathbf{x} . In our particular specification, the bid price is the only explanatory variable, so that \mathbf{k}_{α} is a (1x1) unit vector and $\max(A)$ equals 1. We choose the same value for integer J as do Creel and Loomis, leading to

$$(21) \quad \Delta V(B) = \gamma + \delta B + \delta_v \cos s(B) + \delta_w \sin s(B)$$

where $s(B)$ prevents periodicity in the model and is a function that shifts and scales the variable to lie in an interval less than 2π (Gallant).¹¹ Specifically, the variable is scaled by subtracting its minimum value, then dividing by the maximum value, and then multiply the resulting value by $2\pi - 0.00001$, which produces a final scaled variable in the interval $[0, 2\pi - 0.00001]$. When $\delta_v =$

$\delta_w = 0$, (21) reduces to (19) with $\delta = \beta$ and $\gamma = -\alpha$: the logistic WTP model is nested within the SPNDF model. The four coefficients in the utility difference function (22) are estimated by maximum likelihood, using the log-likelihood function in (5) for the DB data and response probabilities consisting of (4b) and the analogs to (4a,c,d), and the log-likelihood function in (7) for the OOHB data and response probabilities consisting of (6b') and the analogs to (6a,c).¹² Given the coefficient estimates, the median of the implied SPNDF WTP distribution is the quantity C^* that satisfies:

$$(22) \quad 0.5 = G(C^*; \theta) \equiv F[\Delta V(C^*)]$$

Since the standard logistic has a median of zero, C^* solves

$$(23) \quad 0 = \Delta V(C^*) = \gamma + \delta C^* + \delta_v \cos s(C^*) + \delta_w \sin s(C^*).$$

The coefficient estimates from the Cava Grande surveys are presented in Table 1; the coefficient estimates from the DB data are shown in the second column, while those from the OOHB are shown in the sixth column. Also shown are the coefficient estimates obtained when one takes the response to the first valuation question in the DB or OOHB surveys and fits an SB model, using the utility difference function in (21), the log-likelihood function in (2), and the response probabilities consisting of (1a') and the analog to (1b). The SB coefficient estimates from the DB data are shown in the first column of Table 1; those from the OOHB data are shown in the fifth column of Table 1. The remaining columns in Table 1 show the results when there is a selective discarding of the second responses in the DB and OOHB surveys – discarding the second responses whenever they involve either a *higher* follow-up bid (third and seventh columns) or a lower follow-up bid (fourth and eighth columns).

Of particular interest in Table 1 is the comparison of the log-likelihoods of the regressions (the “LnL” row in the table) with the “LnL_R” row, which has the log-likelihood values for the regressions with the coefficients δ_v and δ_w restricted to 0, i.e., a standard linear random utility model (RUM). Since the latter is nested in the former, likelihood ratio tests, i.e., $\lambda_{LR} = 2[\ln L - \ln L_R]$ with critical value $\chi^2(2,0.05) = 5.99$, can be used to compare the models. Note in particular that restricted and unrestricted DB.1 and DB.3 regressions are statistically different from each other at the 5% level. While in any single SNPFD regression the impacts of nay-saying or yea-saying in the follow-up response cannot be separately identified from other factors such as sample design or specification of the RUM, analysis of the four DB regressions suggests that the responses to the upper bids are not consistent with the responses to the first bid. Firstly, the null hypothesis that the coefficients δ_v and δ_w equal 0 is not rejected for the DB.SB regression while it is for DB.1, DB.2, and DB.3, suggesting that it is the impact of the follow-up bids, and not necessarily the linear RUM specification, that is driving the difference. Secondly, the two coefficient restrictions are just barely rejected in DB.2, but are strongly rejected in DB.3, suggesting that the lower-bound data remaining in the DB.2 regression is having little impact on the regression results while the upper-bound data remaining in the DB.3 regression is having significant impact on the regression results. For the OOHB data on the other hand, all the likelihood ratio values are less than the critical value, suggesting that the follow-up bids are not introducing bias into the model. As the regression results for the OOHB data demonstrate, the OOHB regressions appear to be notably less sensitive to the inclusion or exclusion of either the lower or upper bids. This result should not be particularly surprising given that the OOHB model utilizes less information on follow-up bids than does DB. For instance, it could be that the DB model is not fitting itself well to the bid design structure imposed on it, regardless of whether the

follow-up responses are biased or not. The OOHB result may also be influenced by the fact that the bid range is announced to the respondent before the CVM question, thereby reducing response bias. These two possibilities are not separately identifiable with the available data sets.¹³

Because it is the welfare measures, and not the coefficient estimates, that are generally of primary interest, it is useful to compare the estimated welfare measures in Table 2 that are derived from each regression. Furthermore, because the welfare measure are nonlinear functions of the coefficients, the observations from the coefficient analysis above may not hold for the welfare estimates. For Table 2.A, we calculate an E(WTP) function that is sometimes referred to as a spike model. Suppose one wants to allow for indifference—with some positive probability, the individual has a zero WTP for the change in q . Indifference is equivalent to a probability mass, or spike, at $C = 0$. A CDF satisfying $C \in [0, \infty]$ with a spike at $B = 0$ is

$$\Pr\{\text{"yes"}\} = \begin{cases} 1 & \text{if } B = 0 \\ P[\Delta V(B)] & \text{if } 0 < B < \infty \end{cases}$$

where $\Delta V(B)$ is from equation (21), and the point estimates of mean WTP (first row of Table 2.A) are calculated by integrating this density function between $B = 0$ and ∞ (Cooper, 2001).¹⁴ The sixth row gives the standard errors associated with these point estimates, derived via the jackknife method with 1,000 repetitions in each case.¹⁵ The empirical 95% confidence intervals for median WTP based on the jackknife output are shown in the fourth and fifth rows. The second and third rows gives Efron's (1987) Bias Corrected Accelerated (BCa) 95% confidence intervals, which adjust the jackknife output for potential nonnormalities. Table 2.B presents the WTP results for the median measure, calculated using (23).¹⁶ Consistent with the observations on

the coefficient estimates, the OOHB welfare point estimates are relatively stable across the regressions while the DB welfare estimates appear to be quite sensitive to the upper bids.

One question is does the new question format (OOHB) change WTP based on the response to the first bid with respect to WTP based on the first bid in the DB format. Using the median WTP and the standard errors in part B of Table 2, we conduct a paired t-test of the median WTP for DB.SB with that for OOHB.SB. Similarity of the BCa confidence intervals with the empirical confidence intervals suggests that SB WTP is distributed approximately normally, and hence, a paired t-test for these independent samples is appropriate. The test statistic for the paired t-test is 0.2255, which does not reject the hypothesis that $WTP_{DB.SB} - WTP_{OOHB.SB} = 0$ at the 5% level of significance. The hypothesis $WTP_{DB.1} - WTP_{OOHB.1} = 0$ for median WTP has a test statistic of -0.458, and equality of the DB and OOHB WTP is not rejected.

The results in Table 2 show the OOHB estimates to be more stable across the alternative OOHB models than the DB estimates are across the alternative DB models. This is especially true of WTP for DB.3, which suggests that the follow-up responses to a Yes to the first bid are highly biased. A comparison of the confidence intervals for the DB model shows that the mean WTP measure masks some of the bias associated with the follow-up responses when compared to the median WTP. Finally, the main motivation for multiple-bound formats is to obtain greater efficiency than the SB estimate. This goal is not achieved with the DB estimates, given that the multiple-bound DB welfare estimates all have higher coefficients of variation and wider confidence intervals than DB.SB. On the other hand, the multiple-bound OOHB estimates all have lower coefficients of variation and confidence intervals than the OOHB.SB estimate.

4. Conclusion

This paper introduces the one-and-one-half-bound model (OOHB) as an alternative to the double-bound (DB) for discrete choice CVM. Aside from differences in how the follow-up bids are handled, the major distinguishing characteristic of OOHB over DB is its prior announcement to the respondent of the uncertainty about the costs of the program whose value is being elicited. We analytically demonstrate that in the move from single-bound (SB) to DB, OOHB captures two-thirds of the gains in efficiency associated with the move from SB to DB. For our real world data sets, OOHB demonstrated efficiency gains (in terms of coefficients of variation) over the SB and DB models. In fact, the DB was less efficient than the SB estimate, in spite of the additional information provided by the follow-up bids. Testing the DB model specifications with and without the follow-up bid information incorporated in the MLE, we find inconsistency imposed by the high follow-up bids, e.g., the median (mean) welfare estimate without the lower bound was 2% (82%) the size of the estimate without the upper bound data. This artefact may also be the cause of the efficiency decrease in the DB over the SB and OOHB models. The OOHB model demonstrated noticeably less sensitivity to the follow-up bids, with the median (mean) welfare estimate without the lower bound 88% (95%) the size of the estimate without the upper bound data. For our split dataset, while null hypothesis that the OOHB and DB welfare measures are the same cannot be rejected, the DB was somewhat pointless given that it did not improve upon the SB estimate in terms of efficiency. Given that our application of OOHB shows it to have no obvious vices, it may serve as a viable alternative to the DB format in situations where follow-up response bias or sample design may be a concern.

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Table 1. Semi-nonparametric coefficient estimates for DB and OOHB surveys (N=400) (coefficient / standard error in parentheses).

	DB.SB	DB.1	DB.2	DB.3	OOHB.SB	OOHB.1	OOHB.2	OOHB.3
Coefficient			(No Upper)	(No lower)			(No upper)	(No Low)
α	2.983	1.963	2.149	-2.338	3.337	2.646	1.554	2.386
	(5.672)	(11.73)	(8.452)	(-3.151)	(2.305)	(3.582)	(10.35)	(2.65)
δ	-0.319	-0.221	-0.214	0.132	-0.379	-0.293	-0.1789	-0.2572
	(-5.045)	(-12.49)	(-6.434)	(2.692)	(-1.996)	(-3.20)	(-9.42)	(-2.376)
δ_v	0.1506	0.473	0.07989	0.4126	-0.6044	-0.24968	-0.001049	0.1363
	(1.744)	(13.27)	(1.537)	(4.557)	(-1.126)	(-0.9693)	(-0.02899)	(-0.4499)
δ_w	-0.0656	-0.00926	0.1275	1.8821	-0.1716	-0.20458	0.0101808	-0.2074
	(-0.593)	(-0.6896)	(1.655)	(5.192)	(-1.107)	(-2.302)	(0.3311)	(-1.956)
LnL	-179.89	-333.89	-241.93	-103.96	-219.00	-364.40	-274.69	-312.67
LnL _R	-181.70	-373.70	-244.93	-308.78	-221.00	-367.28	-274.42	-314.41

Table 2. WTP point estimates and associated statistics (Lira x1000)

	DB.SB	DB.1	DB.2 (No Upper)	DB.3 (No lower)	OOHB.SB	OOHB.1	OOHB.2 (No upper)	OOHB.3 (No Low)
<u>A. Mean WTP point estimates ($0 < WTP < \infty$) and associated statistics (Lira x1000)</u>								
E(<i>wtp</i>)	9.167	7.954	9.192	7.591	8.816	8.313	8.903	8.418
95% BCa	8.624 -	6.422 -	7.633 -	5.769 -	7.483 -	7.499 -	8.095 -	7.488 -
c.i. for <i>wtp</i>	9.726	9.534	10.801	9.472	10.193	9.154	9.737	9.378
95% Empir.	8.562 -	6.674 -	8.605 -	6.311 -	8.086 -	7.712 -	8.095 -	7.748 -
c.i. for <i>wtp</i>	9.738	9.355	11.658	9.350	10.815	9.385	9.737	9.699
S. Error	0.281	0.798	0.808	0.945	0.691	0.422	0.419	0.482
Coef. of var.	0.031	0.100	0.086	0.123	0.077	0.050	0.047	0.057
<u>B. Median WTP point estimates ($-\infty < WTP < \infty$) and associated statistics (Lira x1000)</u>								
<i>wtp</i>	8.707	8.183	10.543	0.228	8.778	7.695	8.803	7.713
95% BCa	7.179 -	-10.966 -	-3.627 -	-35.490 -	7.068 -	6.917 -	7.961 -	6.889 -
c.i. for <i>wtp</i>	10.285	28.389	24.795	39.138	10.543	8.498	9.672	8.564
95% Empir.	7.330 -	-8.501 -	-0.978 -	-13.408 -	6.998 -	6.991 -	7.972 -	6.967 -
c.i. for <i>wtp</i>	10.450	25.877	18.253	85.172	10.553	8.528	9.678	8.621
S. Error	0.792	10.035	7.251	19.015	0.886	0.403	0.436	0.427
Coef. of var.	0.091	1.194	0.718	1.572	0.101	0.052	0.050	0.055

Appendix. Facsimiles of the CVM questions and data description

The double bound question is: “4. Consider for a moment that to have access to the Cava Grande Nature Reserve you will be asked to purchase an admission ticket. If the price of this admission ticket was [BID] lira, would you purchase it and thus be able to make use of the Cava Grande? Yes No

4.1 (*For who responds YES to question 4*). And if the ticket price was [BIDU], would you still buy it? Yes No

4.2 (*For who responds NO to question 4*). And if the ticket price was [BIDL] instead, would you buy it? Yes No .

The OOHB question differs based on whether the lower bound or the upper bound bid is (randomly) chosen as the starting value. The first part is common to both:

“4. Consider for a moment that to have access to the Cava Grande Nature Reserve you will be asked a purchase an admission ticket whose price will be somewhere in the range of [BIDL] to [BIDU] lira.” If the lower bound bid is chosen as the starting bid, then follows:

“If the price of this admission ticket was [BIDL] lira, would you purchase it and thus be able to make use of the Cava Grande? Yes (go to question 4.1) No

4.1 (*To only ask to respondent who answered YES to question 4*). And if the ticket price was [BIDU] lira, would you still buy it? Yes No .

If the upper bound bid is chosen as the starting bid, then follows: “If the price of this admission ticket was [BIDU] lira, would you purchase it and thus be able to make use of the Cava Grande? Yes No (go to question 4.1)

4.1 (*To only ask to respondent who answered NO to question 4*). And if the ticket price was [BIDL] lira instead, would you buy it? Yes No .

ENDNOTES

¹ This survey design was originally suggested to us by Paul Ruud.

² The SB model, as well as the DB and OOHB models to be presented below, can readily be modified to incorporate responses of “don't know,” along the lines of Deacon and Shapiro (1975) and Svento (1993).

³ Details of the proof are available from the authors.

⁴ The derivation is available from the authors.

⁵ The analytical comparisons of alternative survey formats presented in Section 3 involve asymptotic results that hold for large samples. Because of the high costs of data collection, researchers often have to work with quite small samples. With these finite samples, the actual experience with the alternative survey formats could turn out to be quite different from what an asymptotic analysis suggests. To investigate this, we performed a Monte Carlo simulation comparing the relative performance of WTP estimates derived from realistic sized samples using the SB, DB, and OOHB formats. The simulation results showed that most efficiency gain came in moving from SB to OOHB and suggested that the increased follow-up questioning of the DB format relative to OOHB can make it more vulnerable to some forms of specification error, to the point where it yields either no performance gain over OOHB or even a slightly worse performance, as measured in terms of MSE. A detailed discussion of the Monte Carlo study is available from the authors.

⁶ Due to a missing survey, actual sample size for the OOHB survey is 399.

⁷ The bids sets (Lira x 1000) are, in order $\{B^0, B^D$ and B^-, B^U and $B^+\}$: $\{0.5, 0.25, 2\}, \{2, 0.5, 3\}, \{3, 2, 4\}, \{4, 3, 5\}, \{5, 4, 6\}, \{6, 5, 7\}, \{7, 6, 8\}, \{8, 7, 9\}, \{9, 8, 10\}, \{10, 9, 11\}, \{11, 10, 12\}, \{12, 11, 14\}, \{14, 12, 30\}$, where the OOHB bids are $\{B^-, B^+\}$ and the DB bids are $\{B^0, B^D, B^U\}$. At the time of the survey, $1US\$ \approx 1,600$ lire.

⁸ In the absence of response bias in the follow-up, we would expect that for any bid B , $\text{prob}(\text{yes to } B | \text{yes to } A) > \text{prob}(\text{yes to } B | \text{where } B = \text{first bid})$, where $A < B$, given that $\text{prob}(\text{yes to } B | \text{yes to } A) = \text{prob}(\text{yes to } B) / \text{prob}(\text{yes to } A)$ and $0 < \text{prob}(\text{yes to } B) < \text{prob}(\text{yes to } A) < 1$. However, because respondents may feel exploited when an initial Yes is followed by a higher price, we may see the biased condition $\text{prob}(\text{yes} | \text{yes}) < \text{prob}(\text{yes} | \text{first})$. In a table that is available from the authors, we make nonparametric comparisons of $\text{prob}(\text{yes} | \text{yes})$ and $\text{prob}(\text{yes} | \text{first})$. To calculate these probabilities nonparametrically requires that some respondents' B^U (B^+) equals other respondents' B^0 (B^-). For our data, the follow-up bids were in fact chosen in this manner. The results show that in most instances, $\text{prob}(\text{yes} | \text{yes}) < \text{prob}(\text{yes} | \text{first})$ for both DB and OOHB, but a little less so for the latter. However, the DB results indicate the ratio of $\text{prob}(\text{yes} | \text{yes})$ to $\text{prob}(\text{yes} | \text{first})$ is substantially lower at the upper bid levels, while for the OOHB approach, the correlation between the ratio and the bid size is low. Hence, there seems to be some evidence that respondents to the DB survey are indeed annoyed by being asked a higher bid after saying Yes to the initial bid, particularly at the higher bid values. The sample size requirements are high for testing the equality of nonparametric measures such as these, and ours was insufficient at each bid level to adequately perform statistical comparisons of these probabilities. Hence, we develop the parametric tests in Table 1 and 2 to assess the bias in the follow-up response.

⁹ Chen and Randall (1997) present an alternative model for SB data similar to that of Creel and Loomis; their model could be extended to DB and OOHB data in the same manner.

¹⁰ In addition to appending $X\beta$ to the Fourier series in equation (20), Gallant suggests appending quadratic terms when modeling nonperiodic functions. Our experiments suggest that inclusion of the quadratic terms as well in the regressions had little impact on the WTP estimates. Hence, we leave them out for the sake of efficiency.

¹¹ With 13 unique bid values in our data set, our specification permits a $\max(J) = 5$ to avoid singularity in the regression. For our data, since increasing J to values above 1 yielded little change in the regression results, $J = 1$ appears to provide the best balance in the trade-off between bias and efficiency.

¹² The GAUSS program for performing the maximization is available from the authors.

¹³ To compare the OOHB and DB response structures, we can pool their respective likelihood functions and compare the restricted and unrestricted forms. For comparing the first response in DB with that in OOHB, we use a likelihood ratio test to compare the pooled restricted log-likelihood $LnL_{SB}^R(\mathbf{x}_{DB.SB}, \mathbf{x}_{OOHB.SB}, \boldsymbol{\theta}) = LnL_{DB.SB}(\mathbf{x}_{DB.SB}, \boldsymbol{\theta}) + LnL_{OOHB.SB}(\mathbf{x}_{OOHB.SB}, \boldsymbol{\theta})$ to the unrestricted pooled $LnL_{SB}^{UR}(\mathbf{x}_{DB.SB}, \mathbf{x}_{OOHB.SB}, \theta_{DB.SB}, \theta_{OOHB.SB}) = LnL_{DB.SB}(\mathbf{x}_{DB.SB}, \theta_{DB.SB}) + LnL_{OOHB.SB}(\mathbf{x}_{OOHB.SB}, \theta_{OOHB.SB})$. The test ratio is $2[LnL_{SB} - LnL_{SB}^R] = 2[-398.49 - (-424.880)] = 52.76$, which does not accept the null hypothesis the DB.SB and the OOHB.SB regressions are the same. However, if we do the same test for the pooled full DB and OOHB regression (DB.1 and OOHB.1 in table 1), the test ratio is a much higher $2[LnL - LnL^R] = 2[-728.29 - (-1496.1)] = 1535.62$. A comparison of this test ratio to the single bound one suggests that most of the difference between the OOHB and DB regressions is due to the follow-up.

¹⁴ For practical purposes, the upper limit of this numerical integration is some value that drives $\text{Prob}\{\text{"yes"}\}$ to near zero. In our case, the highest bid value of 30,000 lira produced the desired effect with $\text{Prob}\{\text{"yes"} \text{ to } 30,000 \text{ lira}\} < 0.001\%$ for each of the eight models.

¹⁵ This involves drawing observations from the real data set randomly, with replacement, to produce a simulated data set with the same sample size as the real data set. This was replicated 1,000 times for each model in Table 2.

¹⁶ Nuisance values are a possibility for the good in question, thereby making a strong case for the use of the median estimate, which assumes $-\infty < \text{WTP} < \infty$.

