One- and Two-Way Decode-and-Forward Relaying for Wireless Multiuser MIMO Networks

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***Abstract*— In this paper, we propose multiuser multiple-input multiple-ouput (MIMO) one- and two-way relaying protocols. Several wireless MIMO node pairs are establishing either unidirectional or bidirectional transmission links via a single decode-and-forward MIMO relay without causing interference to each other. For both half-duplex two-phase relaying protocols, the relay separates messages of different node pairs spatially by using zero-forcing based methods, whereas, in order to cancel the interference within the bidirectional links, XOR precoding followed by self-interference cancellation is employed. Both *sum rate* and *maxmin fairness* optimizations, as well as *quality-of-service* assurance, are considered for the design of the precoders at the relay. The corresponding optimization algorithms for both one- and two-way relaying have been developed using semidefinite programming, which consider the intersection of multiple access and broadcast phases' rate regions.**

***Index Terms*— Multiuser communication, MIMO, decode-and-forward, convex optimization, one- and two-way relaying.**

I. INTRODUCTION

Incorporating the relay channel with multiple-input multiple-output (MIMO) communications, in [1], [2], the capacity of the conventional two-hop MIMO relay channels with precoding has been investigated for a case of a single source/destination (S/D) pair and a relay. MIMO precoding techniques have been investigated in [3] for a scenario with one S/D pair and two relays. In [4], MIMO relaying with a single MIMO source, a MIMO relay, and multiple single-antenna destinations has been considered, where the relay simply amplifies and forwards the received signal to the destinations.

The aforementioned conventional two-hop relaying protocols suffer from spectral efficiency loss due to the two channel uses required for the transmission from source to destination. Two-way relaying (TWR) [5], which also occupies two channel uses, has been proposed to reduce this loss by establishing a bidirectional transmission link between source and destination via a relay. In the first phase, both nodes transmit simultaneously via a multiple access scenario to the relay. In the second phase, the relay broadcasts a common message which is obtained by combining the received messages. Since the nodes know their own transmitted signal, they subtract the back-propagated *self-interference* prior to decoding. Employing XOR precoding and superposition coding, in [6], transmit channel state information (CSI) has been considered at the decode-and-forward (DF) relay, which is motivated by the assumption that the channel stays constant during two phases, and MIMO channels are estimated for decoding in the first phase anyway. The broadcast capacity region of MIMO TWR has been derived in [7]. Moreover, non-regenerative relaying based MIMO TWR has been considered in [8].

In this paper, we propose multiuser MIMO one- and two-way relaying protocols with a single DF relay. e.g., an IEEE 802.11n access point. We assume that all nodes in the network are equipped with multiple antennas, and the transmitting nodes do not have CSIT except the relay, which has perfect global CSI knowledge of the network. In the network, there are multiple wireless node pairs, each of which would like to either create an information flow from one node to other, i.e., one-way relaying (OWR), or exchange information in between two nodes, i.e., TWR, via the same single relay without causing any interference to other node pairs. If a pair prefer to use OWR but also want to exchange information, then two one-way communication links are established one after the other towards opposite directions, i.e., four channel uses. For both schemes, in the downlink from the relay, we employ zero-forcing beamforming to separate private messages of different node pairs. Whereas, as a special case for TWR, on top of spatial separation of pairs, the interference between the exchanged messages of each pair is cancelled through XOR precoding followed by self-interference cancellation. For both relaying schemes, we design the precoding matrices at the relay such that the overall two-phase (hop) sum rate is maximized. A semidefinite programming (SDP) based iterative algorithm is proposed, which exploits the geometrical interpretation of the intersection of the multiple access and the broadcast phases' rate regions. Possible extensions of the optimization to maxmin-fairness and quality-of-service (QoS) requirements are also addressed.[1]

II. SYSTEM AND SIGNAL MODEL

We consider a DF relaying scenario, where K node pairs communicate via a single DF relay, i.e., there are $2K+1$ nodes in total. We define two node sets $\mathcal{U}_1$ and $\mathcal{U}_2$, in which we group one of the members of each pair. Without loss of generality, we assume that each node in the network is equipped with N antennas except the relay, which has M antennas. None of the nodes in $\mathcal{U}_1$ and $\mathcal{U}_2$ has transmit CSI knowledge, and there are no direct links between $\mathcal{U}_1$ and $\mathcal{U}_2$. In the following we

[1]**Notation:** Boldface lowercase and capital letters indicate vectors and matrices, respectively. The superscripts $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$ stand for complex conjugate, matrix transpose, and complex conjugate transpose, respectively. The operators E$\{\cdot\}$, Tr($\mathbf{X}$), rank($\mathbf{X}$), null($\mathbf{X}$), $\mathbf{I}$ and $\succeq$ denote expectation, the trace of the matrix $\mathbf{X}$, the rank of $\mathbf{X}$, the nullspace of $\mathbf{X}$, an identity matrix with a corresponding size, and positive semidefiniteness, respectively. $\mathcal{CN}(0,\sigma^2)$ stands for a zero-mean complex normal distribution with variance σ^2.

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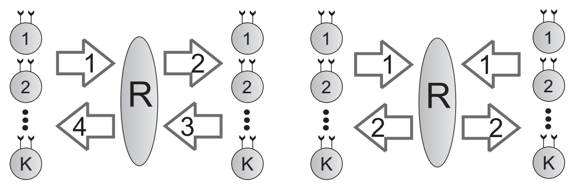

a) One-way b) Two-way

Fig. 1. The transmission protocols for one- and two-way relaying. The numbers on the arrows indicate the indices of channel uses, i.e., time-slots.

present the principles of two half-duplex multiuser relaying protocols: one-way relaying and two-way relaying.

A. Multiuser One-way Relaying

The nodes in $\mathcal{U}_1$ and $\mathcal{U}_2$ are assigned as the sources and the destinations, respectively. The data flow is unidirectional, i.e., from sources to destinations. The one-way protocol is accomplished in two-hops (phases) with equal time share: a concurrent transmission from all sources to the relay and a broadcast transmission from the relay to the destinations. In the first phase, K sources transmit simultaneously to the relay. The source of the kth pair transmits the bit sequence $\mathbf{x}_{\mathrm{s}}^{(k)}$ to its destination. Defining the transmit signals of the kth source as $\mathbf{x}_{\mathrm{s}}^{(k)} \rightarrow \mathbf{a}_{\mathrm{s}}^{(k)} \in \mathbb{C}^N$, the received signal $\mathbf{r} \in \mathbb{C}^M$ at the relay is $\mathbf{r} = \sum_{k=1}^{K} \mathbf{H}_k \mathbf{a}_{\mathrm{s}}^{(k)} + \mathbf{n}_{\mathrm{r}}$, where $\mathbf{H}_k \in \mathbb{C}^{M \times N}$ is the channel matrix between the kth source and the relay with identically and independently distributed (i.i.d.) $\mathcal{CN}(0, \sigma_{\mathrm{H_k}}^2)$ entries, and $\mathbf{n}_{\mathrm{r}} \sim \mathcal{CN}(0, \sigma_{\mathrm{r}}^2 \mathbf{I})$ is the complex additive white Gaussian noise (AWGN). The transmit signals are subject to the power constraint $\mathrm{E}\{(\mathbf{a}_{\mathrm{s}}^{(k)})^H \mathbf{a}_{\mathrm{s}}^{(k)}\} \leq P_{\mathrm{s}}, \forall k$. The relay efficiently decodes the information from all sources assuming that it has sufficient number of antennas, which depends on $\min(2K, M)$ and/or the number of independent spatial streams transmitted from the users.

In the second phase, the relay re-encodes the received information from the kth source as $\mathbf{a}_{\mathrm{r}}^{(k)}$. Next, it broadcasts the transmit signal $\mathbf{a}_{\mathrm{r}} = \sum_{k=1}^{K} \mathbf{W}_k \mathbf{a}_{\mathrm{r}}^{(k)}$, which is the superposition of $\mathbf{a}_{\mathrm{r}}^{(k)}$'s after precoding with the corresponding matrix $\mathbf{W}_k$. A fixed relay power constraint is imposed such that $\sum_{k=1}^{K} \mathrm{Tr}(\mathbf{W}_k \mathbf{W}_k^H) \leq P_{\mathrm{r}}$. Further details about the structure and the optimization of the precoder matrices $\mathbf{W}_k$ will be given in the following sections. The received signal at the kth destination is then given by

$$\mathbf{r}_{\mathrm{d}}^{(k)} = \mathbf{G}_k \mathbf{W}_k \mathbf{a}_{\mathrm{r}}^{(k)} + \mathbf{G}_k \sum_{\ell=1, \ell \neq k}^{K} \mathbf{W}_\ell \mathbf{a}_{\mathrm{r}}^{(\ell)} + \mathbf{n}_{\mathrm{d}}^{(k)}, \quad (1)$$

where $\mathbf{G}_k \in \mathbb{C}^{N \times M}$ is the channel matrix between the relay and the kth destination with i.i.d. $\mathcal{CN}(0, \sigma_{\mathrm{G_k}}^2)$ entries, and the AWGN noise $\mathbf{n}_{\mathrm{d}}^{(k)} \sim \mathcal{CN}(0, \sigma_{\mathrm{d}}^2 \mathbf{I})$. The one-way transmission protocol is illustrated in Fig. 1.a., through the arrows 1 and 2.

B. Multiuser Two-way Relaying

In contrary with OWR, both members of the kth pair have data to transfer to each other. In other words, we have two sources and two destinations per pair, and hence, a bidirectional link is established via the relay in order to exchange information. The nodes of the kth pair want to exchange the bit sequence duple $(\mathbf{x}_{12}^{(k)}, \mathbf{x}_{21}^{(k)})$, where $\mathbf{x}_{12}^{(k)}$ and $\mathbf{x}_{21}^{(k)}$ represent the bit sequences to transmit from the node in $\mathcal{U}_1$ to the node in $\mathcal{U}_2$ and vice-versa, respectively. In the first phase, all $2K$ nodes transmit concurrently to the relay through the same physical channel. Defining the transmit signals $(\mathbf{x}_{12}^{(k)}, \mathbf{x}_{21}^{(k)}) \rightarrow (\mathbf{a}_{12}^{(k)}, \mathbf{a}_{21}^{(k)}) \in \mathbb{C}^N$, the received signal at the relay is $\tilde{\mathbf{r}} = \sum_{k=1}^{K} \left(\mathbf{H}_k \mathbf{a}_{12}^{(k)} + \mathbf{G}_k^T \mathbf{a}_{21}^{(k)}\right) + \tilde{\mathbf{n}}_{\mathrm{r}}$,where the statistics of the noise term is same as in OWR, and channel reciprocity is assumed, i.e., the channel from the relay to the kth pair's node in $\mathcal{U}_2$ is defined as $\mathbf{G}_k$ in Section II-A, and hence, the channel from the kth pair's node in $\mathcal{U}_2$ to the relay is $\mathbf{G}_k^T$. Next, the relay decodes the information from all nodes.

In the second phase, the relay re-encodes the received information and broadcasts back to all nodes through the same channels as in the first phase, i.e., the channels are assumed to stay constant over two phases. The transmit signal of the relay $\tilde{\mathbf{a}}_{\mathrm{r}}$ is determined by the decoded bit-sequences $\left(\mathbf{x}_{12}^{(1)} \mathbf{x}_{21}^{(1)} \cdots \mathbf{x}_{12}^{(K)} \mathbf{x}_{21}^{(K)}\right)$ from the first phase. The relay needs to separate $2K$ nodes while broadcasting $2K$ independent messages, which is accomplished in two separate levels. Firstly, in order to cancel the interference in between the members of each pair, XOR precoding followed by self-interference cancellation is used. With XOR precoding, two bit sequences are combined on bit-level prior to encoding. Specifically, the relay applies bitwise XOR operation on the both decoded bit-sequences of the kth pair, and codes the resulting bit-sequence $\mathbf{x}_{\mathrm{r}}^{(k)}$, i.e., $\mathbf{x}_{12}^{(k)} \oplus \mathbf{x}_{21}^{(k)} = \mathbf{x}_{\mathrm{r}}^{(k)} \rightarrow \tilde{\mathbf{a}}_{\mathrm{r}}^{(k)}, \forall k \in \{1, \ldots, K\}$. Hence, by after applying self-interference cancellation at the receiver side, $\mathbf{x}_{12}^{(k)}$ and $\mathbf{x}_{21}^{(k)}$ do not anymore cause interference to each other, i.e., separation in bit-level. However, different node pairs and consequently messages, i.e., $\tilde{\mathbf{a}}_{\mathrm{r}}^{(k)}$, still need to be separated through downlink, which is done spatially as will be explained in Section III. Finally, the transmit signal of the relay $\tilde{\mathbf{a}}_{\mathrm{r}}$ is obtained by superposing $\tilde{\mathbf{a}}_{\mathrm{r}}^{(k)}$s after precoding each with the matrix $\tilde{\mathbf{W}}_k$, i.e., $\tilde{\mathbf{a}}_{\mathrm{r}} = \sum_{k=1}^{K} \tilde{\mathbf{W}}_k \tilde{\mathbf{a}}_{\mathrm{r}}^{(k)}$, where $\sum_{k=1}^{K} \mathrm{Tr}(\tilde{\mathbf{W}}_k \tilde{\mathbf{W}}_k^H) \leq P_{\mathrm{r}}$.

The received signals at the members of the kth pair in $\mathcal{U}_1$ and $\mathcal{U}_2$ are given respectively by

$$\begin{aligned} \tilde{\mathbf{r}}_{\mathcal{U}_1}^{(k)} &= \mathbf{H}_k^T \tilde{\mathbf{W}}_k \tilde{\mathbf{a}}_{\mathrm{r}}^{(k)} + \mathbf{H}_k^T \sum_{\ell=1, \ell \neq k}^{K} \tilde{\mathbf{W}}_\ell \tilde{\mathbf{a}}_{\mathrm{r}}^{(\ell)} + \tilde{\mathbf{n}}_{\mathcal{U}_1}^{(k)}, \\ \tilde{\mathbf{r}}_{\mathcal{U}_2}^{(k)} &= \mathbf{G}_k \tilde{\mathbf{W}}_k \tilde{\mathbf{a}}_{\mathrm{r}}^{(k)} + \mathbf{G}_k \sum_{\ell=1, \ell \neq k}^{K} \tilde{\mathbf{W}}_\ell \tilde{\mathbf{a}}_{\mathrm{r}}^{(\ell)} + \tilde{\mathbf{n}}_{\mathcal{U}_2}^{(k)}, \quad (2) \end{aligned}$$

where the noise terms $\tilde{\mathbf{n}}_{\mathrm{m}}^{(k)} \sim \mathcal{CN}\left(0, \sigma_{\mathrm{d}}^2 \mathbf{I}\right)$ for $\mathrm{m} \in \{\mathcal{U}_1, \mathcal{U}_2\}$. After each node decodes the intended message, the self-interference cancellation is done by applying a simple XOR operation, i.e., $\mathbf{x}_{\mathrm{r}}^{(k)} \oplus \mathbf{x}_{12}^{(k)} = \mathbf{x}_{21}^{(k)}$, and $\mathbf{x}_{\mathrm{r}}^{(k)} \oplus \mathbf{x}_{21}^{(k)} = \mathbf{x}_{12}^{(k)}$.

III. Achievable Rates and Precoding at the Relay

A. Uplink to the Relay - Multiple Access Phase

Assuming that the relay has enough antennas for perfect decoding, the relay decodes the information from all nodes based on $\mathbf{r}$ and $\tilde{\mathbf{r}}$ for one- and two-way relaying, respectively. Using Gaussian codebooks, the achievable rates of transmitting nodes in the first phase are described by the MIMO multiple access channel (MAC) rate region. The MIMO MAC rate regions without CSIT for multiuser one- and two-way relaying are given respectively by the following set of inequalities

$$R_{\mathcal{S}}^{\mathrm{one}} \leq \log_2 \left| \mathbf{I} + \gamma \sum_{i \in \tilde{\mathcal{S}}_1} \mathbf{H}_i \mathbf{H}_i^H \right|, \forall \mathcal{S} \subseteq \mathcal{U}_1 \quad (3)$$

$$R_{\mathcal{S}}^{\mathrm{two}} \leq \log_2 \Big| \mathbf{I} + \gamma \Big(\sum_{i \in \tilde{\mathcal{S}}_1} \mathbf{H}_i \mathbf{H}_i^H + \sum_{j \in \tilde{\mathcal{S}}_2} \mathbf{G}_j^T \mathbf{G}_j^* \Big) \Big|, \forall \mathcal{S} \subseteq \mathcal{U}_1 \cup \mathcal{U}_2 \quad (4)$$

which constitute in total $2^K - 1$ and $2^{2K} - 1$ inequalities, respectively, and $\gamma := \frac{P_\mathrm{s}}{N\sigma_\mathrm{r}^2}$. $\tilde{\mathcal{S}}$, $\tilde{\mathcal{S}}_1$, $\tilde{\mathcal{S}}_2$ represent the index sets of all nodes in $\mathcal{S}$ for OWR, all $\mathcal{U}_1$ nodes in $\mathcal{S}$ for TWR, and all $\mathcal{U}_2$ nodes in $\mathcal{S}$ for TWR, respectively. Because of no CSIT, we use diagonal covariance matrices with equal weights.

B. Downlink from the Relay - Broadcast Phase

For both relaying schemes, there are K messages that should be separated spatially. For OWR, it is a broadcast channel with K destinations, and for TWR, it is a *modified* broadcast channel with $2K$ destinations, where the members of each pair is demanding the same message. Assuming CSI at the relay, several MIMO broadcasting schemes have been proposed in the literature for the design of the precoders. Although dirty paper coding (DPC) is the optimal in terms of maximizing the sum rate of the conventional broadcast channel, it is not practical because of its nonlinearity. Instead, we focus on low-complexity solutions, e.g., block diagonalization based zero-forcing beamforming (ZFB) [9], [10].

1) One-Way Relaying: We apply zero-forcing between the destinations, and choose the precoding matrices, $\mathbf{W}_k^{\mathrm{one}}$ such that $\mathbf{W}_k^{\mathrm{one}}$ satisfies the zero-forcing condition $\mathbf{G}_\ell \mathbf{W}_k^{\mathrm{one}} = 0$ for $\ell \neq k, k = 1, \ldots, K$. In other words, defining

$$\mathbf{F}_k^{\mathrm{one}} = [\mathbf{G}_1^T \cdots \mathbf{G}_{k-1}^T \; \mathbf{G}_{k+1}^T \cdots \mathbf{G}_K^T]^T,$$

$\mathbf{W}_k^{\mathrm{one}}$ is forced to lie in the nullspace of $\mathbf{F}_k^{\mathrm{one}}$. Since $M > \mathrm{rank}(\mathbf{F}_k^{\mathrm{one}})$ is the condition for nonempty nullspace, $M > \max_{1 \leq k \leq K} \{\mathrm{rank}(\mathbf{F}_k^{\mathrm{one}})\}$ must be satisfied to zero-force and transmit data concurrently to all destinations.

Using the zero-forcing $\mathbf{W}_k^{\mathrm{one}}$ in (1), the received signal at the kth destination simplifies to $\mathbf{r}_\mathrm{d}^{(k)} = \mathbf{G}_k \mathbf{W}_k^{\mathrm{one}} \mathbf{a}_\mathrm{r}^{(k)} + \mathbf{n}_\mathrm{d}^{(k)}$. Hence, the mutual information between the relay and the kth destination can be expressed as

$$\tilde{I}_k^{\mathrm{one}} = \log_2 \left| \mathbf{I} + \frac{1}{\sigma_\mathrm{d}^2} \mathbf{G}_k \mathbf{V}_k^{\mathrm{one}} \mathbf{\Lambda}_k^{\mathrm{one}} (\mathbf{V}_k^{\mathrm{one}})^H \mathbf{G}_k^H \right|,$$

where $\mathbf{V}_k^{\mathrm{one}} = \mathrm{null}(\mathbf{F}_k^{\mathrm{one}})$ and can be computed through the singular value decomposition (SVD) of $\mathbf{F}_k^{\mathrm{one}}$, $\mathbf{\Lambda}_k^{\mathrm{one}}$ is the covariance matrix of the kth transmit signal $\mathbf{a}_\mathrm{r}^{(k)}$ for OWR, and $\mathbf{V}_k^{\mathrm{one}} \mathbf{\Lambda}_k^{\mathrm{one}} (\mathbf{V}_k^{\mathrm{one}})^H := \mathbf{W}_k^{\mathrm{one}} (\mathbf{W}_k^{\mathrm{one}})^H$. The optimal power loading coefficients of $\mathbf{\Lambda}_k^{\mathrm{one}}$ can be found by applying water-filling on $\mathbf{\Sigma}_k$, which is obtained through SVD of $\mathbf{G}_k \mathbf{V}_k^{\mathrm{one}} = \mathbf{U}_k \mathbf{\Sigma}_k \tilde{\mathbf{U}}_k$ [10]. We also assume that $\mathrm{Tr}(\mathbf{\Lambda}_k^{\mathrm{one}}) \leq P_k$, where $\sum_{k=1}^K P_k = P_\mathrm{r}$. Hence, denoting the optimal water filling solution for $\mathrm{Tr}(\mathbf{\Lambda}_k^{\mathrm{one}}) \leq 1$ with $\check{\mathbf{\Lambda}}_k^{\mathrm{one}}$, $\tilde{I}_k^{\mathrm{one}}$ becomes

$$\tilde{I}_k^{\mathrm{one}} = \log_2 \left| \mathbf{I} + (P_k / \sigma_\mathrm{d}^2) \mathbf{\Sigma}_k^2 \check{\mathbf{\Lambda}}_k^{\mathrm{one}} \right|. \quad (5)$$

2) Two-Way Relaying: The zero-forcing condition for TWR differs from the previous case to $\mathbf{G}_\ell \mathbf{W}_k^{\mathrm{two}} = 0$, $\mathbf{H}_\ell^T \mathbf{W}_k^{\mathrm{two}} = 0$ for $\ell \neq k, k = 1, \ldots, K$. Hence, $\mathbf{W}_k^{\mathrm{two}}$ is forced to lie in the nullspace of $\mathbf{F}_k^{\mathrm{two}}$, which is defined as

$$\mathbf{F}_k^{\mathrm{two}} = [\mathbf{H}_1 \; \mathbf{G}_1^T \cdots \mathbf{H}_{k-1} \; \mathbf{G}_{k-1}^T \; \mathbf{H}_{k+1} \; \mathbf{G}_{k+1}^T \cdots \mathbf{H}_K \; \mathbf{G}_K^T]^T.$$

Just like OWR, $M > \max_{1 \leq k \leq K} \{\mathrm{rank}(\mathbf{F}_k^{\mathrm{two}})\}$ must hold to zero-force and transmit data concurrently to all nodes in $\mathcal{U}_1 \cup \mathcal{U}_2$ with TWR.

Substituting $\mathbf{W}_k^{\mathrm{two}}$ in (2), the received signals at the members of the kth pair in $\mathcal{U}_1$ and $\mathcal{U}_2$ for TWR become

$$\tilde{\mathbf{r}}_{\mathcal{U}_1}^{(k)} = \mathbf{H}_k^T \mathbf{W}_k^{\mathrm{two}} \tilde{\mathbf{a}}_\mathrm{r}^{(k)} + \tilde{\mathbf{n}}_{\mathcal{U}_1}^{(k)} \text{ and } \tilde{\mathbf{r}}_{\mathcal{U}_2}^{(k)} = \mathbf{G}_k \mathbf{W}_k^{\mathrm{two}} \tilde{\mathbf{a}}_\mathrm{r}^{(k)} + \tilde{\mathbf{n}}_{\mathcal{U}_2}^{(k)}.$$

Hence, the mutual information between the relay and the members of the kth pair in $\mathcal{U}_1$ and $\mathcal{U}_2$ are written as

$$\tilde{I}_{\mathcal{U}_1,k}^{\mathrm{two}} = \log_2 \left| \mathbf{I} + \frac{1}{\sigma_\mathrm{d}^2} \mathbf{H}_k^T \mathbf{V}_k^{\mathrm{two}} \mathbf{\Lambda}_k^{\mathrm{two}} (\mathbf{V}_k^{\mathrm{two}})^H \mathbf{H}_k^* \right|,$$

$$\tilde{I}_{\mathcal{U}_2,k}^{\mathrm{two}} = \log_2 \left| \mathbf{I} + \frac{1}{\sigma_\mathrm{d}^2} \mathbf{G}_k \mathbf{V}_k^{\mathrm{two}} \mathbf{\Lambda}_k^{\mathrm{two}} (\mathbf{V}_k^{\mathrm{two}})^H \mathbf{G}_k^H \right|,$$

where $\mathbf{V}_k^{\mathrm{two}} = \mathrm{null}(\mathbf{F}_k^{\mathrm{two}})$, $\mathbf{\Lambda}_k^{\mathrm{two}}$ is the covariance matrix of the kth signal $\tilde{\mathbf{a}}_\mathrm{r}^{(k)}$ for TWR, and $\mathbf{V}_k^{\mathrm{two}} \mathbf{\Lambda}_k^{\mathrm{two}} (\mathbf{V}_k^{\mathrm{two}})^H := \mathbf{W}_k^{\mathrm{two}} (\mathbf{W}_k^{\mathrm{two}})^H$. A water-filling solution can not be directly applied here in order to find the maximal rates for both $\tilde{I}_{\mathcal{U}_1,k}^{\mathrm{two}}$ and $\tilde{I}_{\mathcal{U}_2,k}^{\mathrm{two}}$, because $\mathbf{\Lambda}_k^{\mathrm{two}}$ is not beamforming to a single node as in OWR but two, i.e., both nodes of the kth pair.

Achievable Downlink Rates: In the broadcast phase of TWR, we are multicasting the kth common message $\mathbf{a}_\mathrm{r}^{(k)}$ to the nodes of the kth pair, where the rate should be adjusted with respect to the weakest user according to the general multicast channel. However, employing XOR precoding, we differ from the conventional multicast problem such that we can support unbalanced rates for $\tilde{\mathbf{a}}_\mathrm{r}^{(k)}$ by padding zeros to the lower-rate bit sequence before XORing $\mathbf{x}_{12}^{(k)}$ and $\mathbf{x}_{21}^{(k)}$ at the relay. We explain this feature with an intuitive and conceptual example:

Assuming $\mathbf{x}_{12}^{(k)}$ and $\mathbf{x}_{21}^{(k)}$ are perfectly decoded in the first MAC phase at the relay, we denote the length of these bit sequences with $L_{12}^{(k)}$ and $L_{21}^{(k)}$, respectively. Now, say that $\tilde{I}_{\mathcal{U}_1,k}^{\mathrm{two}} > \tilde{I}_{\mathcal{U}_2,k}^{\mathrm{two}}$, i.e., $L_{21}^{(k)} > L_{12}^{(k)}$, and the transmission rates through the first phase are enough to support these broadcast rates. Interpreting s_{12} (s_{21}) consecutive bits of $\mathbf{x}_{12}^{(k)}$ ($\mathbf{x}_{21}^{(k)}$) as a symbol to be encoded by a Gaussian codebook, the relay pads $s_{21} - s_{12}$ zeros after each s_{12} consecutive bits in $\mathbf{x}_{12}^{(k)}$, such that the length of $\mathbf{x}_{12}^{(k)}$ is extended to be $L_{21}^{(k)}$. Obtaining $\mathbf{x}_\mathrm{r}^{(k)}$ by XORing $\mathbf{x}_{21}^{(k)}$ and the *extended* $\mathbf{x}_{12}^{(k)}$, we encode it with a codebook of rate $\max\{\tilde{I}_{\mathcal{U}_1,k}^{\mathrm{two}}, \tilde{I}_{\mathcal{U}_2,k}^{\mathrm{two}}\} = \tilde{I}_{\mathcal{U}_1,k}^{\mathrm{two}}$.

At the receive sides, the node in $\mathcal{U}_1$ employs a codebook of size proportional to $\tilde{I}_{\mathcal{U}_1,k}^{\mathrm{two}}$ to decode and obtain $\mathbf{x}_\mathrm{r}^{(k)}$ perfectly. On the other hand, although the node in $\mathcal{U}_2$ can only decode $\tilde{I}_{\mathcal{U}_2,k}^{\mathrm{two}}$ bps/Hz, and the common message has been encoded with a codebook of size $\tilde{I}_{\mathcal{U}_1,k}^{\mathrm{two}}$, it has an *apriori* knowledge for decoding, i.e., there are some intentional and redundant zeros padded per symbol to its intended bit sequence. Instead of searching through $2^{n\tilde{I}_{\mathcal{U}_1,k}^{\mathrm{two}}}$ possible codewords, it can omit $2^{n\tilde{I}_{\mathcal{U}_1,k}^{\mathrm{two}}} - 2^{n\tilde{I}_{\mathcal{U}_2,k}^{\mathrm{two}}}$ of these, which stand for the additional $s_{21} - s_{12}$ bits per symbol, and do not bear information for the node in $\mathcal{U}_2$. Hence, although the node in $\mathcal{U}_2$ can not fully decode $\mathbf{x}_\mathrm{r}^{(k)}$, it has perfect access to $\mathbf{x}_{12}^{(k)}$-related part of it. In other words, shrinking the size of its codebook, the number of symbols that can be transmitted (correspondingly the transmission rate) is reduced. To sum up, by using appropriate zero padding and *apriori* knowledge at corresponding nodes, two-way relaying can support unbalanced downlink rates for each

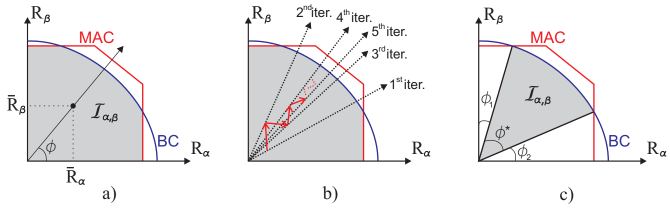

Fig. 2. a) The achievable rate region for any pair of rate tuple. b) The illustration of the iterations for $\phi_{\alpha,\beta}$. c) Search space reduction: For example, in the figure, exclude ϕ_1 and ϕ_2 from the $[0, \pi/2]$ search interval.

common message. Hence, the bidirectional broadcast capacity region derived in [7] is achieved with our zero-padding based proposal. In [3], equal downlink rates were assumed, which achieves the broadcast capacity region at only one point [7].

Remark 1: The aforementioned formulations can be trivially extended to the successive ZFDPC proposed in [9].

C. The Overall Two-Phase Rate Region

We define R_k^{one} as the overall two-phase rate of the transmission between the members of the kth pair for OWR, and $R_{\mathcal{U}_1,k}^{\text{two}}$, $R_{\mathcal{U}_2,k}^{\text{two}}$ as the overall two-phase rates of nodes in $\mathcal{U}_1$ and $\mathcal{U}_2$, respectively, belonging to the kth pair for TWR. Hence, the overall two-phase rate region of both one- and two-way relaying schemes can be defined by combining the multiple access and broadcast phase rate constraints, i.e.,

$$\{R_1^{\text{one}}, \cdots, R_K^{\text{one}}\} \in \mathcal{C}_{\text{MAC}}^{\text{one}} \cap \mathcal{C}_{\text{BC}}^{\text{one}} \quad \text{for OWR},$$
$$\{R_{\mathcal{U}_1,1}^{\text{two}}, R_{\mathcal{U}_2,1}^{\text{two}} \cdots, R_{\mathcal{U}_1,K}^{\text{two}}, R_{\mathcal{U}_2,K}^{\text{two}}\} \in \mathcal{C}_{\text{MAC}}^{\text{two}} \cap \mathcal{C}_{\text{BC}}^{\text{two}} \quad \text{for TWR},$$

where $\mathcal{C}_{\text{MAC}}^{\text{one}}$ and $\mathcal{C}_{\text{MAC}}^{\text{two}}$ are the union set of all MAC related inequalities represented by (3) and (4), respectively; $\mathcal{C}_{\text{BC}}^{\text{one}} := \{R_k^{\text{one}} \leq \tilde{I}_k^{\text{one}},\ \forall k \in \{1, ..., K\}\}$ and $\mathcal{C}_{\text{BC}}^{\text{two}} := \{R_{\mathcal{U}_1,k}^{\text{two}} \leq \tilde{I}_{\mathcal{U}_1,k}^{\text{two}},\ R_{\mathcal{U}_2,k}^{\text{two}} \leq \tilde{I}_{\mathcal{U}_2,k}^{\text{two}}, \forall k \in \{1, \ldots, K\}\}$. The final overall transmission rates should be multiplied with 1/2 to introduce the effect of two channel uses needed for both traffic patterns.

IV. Optimization of the Precoders

A suboptimal but trivial choice of optimizing precoders is to decouple multiple access and broadcast phases. Since the maximal supportable rates for the first phase is readily available (see (3) and (4)), the sum rate for the broadcast phase can be maximized through the well-known schemes and optimization techniques proposed in the literature for the general MIMO broadcast channel [9], [10]. Then, in order to find the overall two-phase rates, the resultant rates from the broadcast phases must be crosschecked with the corresponding multiple access phase's rate constraints. Nevertheless, this decoupling approach is suboptimal since resources can not be fully utilized, i.e., the bottleneck phase can be different for each node/link. Thus, although more complex, we aim to find the overall two-phase optimal rates avoiding decoupling, which differs the optimization procedure from the general broadcast channel optimizations.

While optimizing precoders, we consider two common figures of merit: sum rate and fairness. In the following, we introduce the overall sum rate maximization problem and present the corresponding optimization algorithm in details. Besides, extensions of the optimization to QoS assurance and maxmin fairness are addressed, but the related algorithms are only sketched for the sake of brevity.

A. Sum Rate Optimization

The general sum rate maximization problems for OWR and TWR are formulated respectively as

$$\mathcal{P}_{\text{OWR}}: \max_{\{P_k\}_{k=1}^K, \{R_i\}_{i=1}^K} \sum_{i=1}^K R_i$$
$$\text{subject to } \{R_1, \cdots, R_K\} \in \mathcal{C}_{\text{MAC}}^{\text{one}} \cap \mathcal{C}_{\text{BC}}^{\text{one}},$$
$$\sum_{k=1}^K P_k \leq P_{\text{R}}, R_i \geq 0 \forall i, P_k \geq 0, \forall k$$

$$\mathcal{P}_{\text{TWR}}: \max_{\{\mathbf{\Lambda}_k^{\text{two}}\}_{k=1}^K, \{R_i\}_{i=1}^{2K}} \sum_{i=1}^{2K} R_i$$
$$\text{subject to } \{R_1, \cdots, R_{2K}\} \in \mathcal{C}_{\text{MAC}}^{\text{two}} \cap \mathcal{C}_{\text{BC}}^{\text{two}},$$
$$\sum_{k=1}^K \text{Tr}(\mathbf{\Lambda}_k^{\text{two}}) \leq P_{\text{R}}, R_i \geq 0 \forall i, \mathbf{\Lambda}_k^{\text{two}} \succeq 0, \forall k,$$

where $\{R_1, \cdots, R_K\} := \{R_1^{\text{one}}, \cdots, R_K^{\text{one}}\}$ for OWR and $\{R_1, \cdots, R_{2K}\} := \{R_{\mathcal{U}_1,1}^{\text{two}}, R_{\mathcal{U}_2,1}^{\text{two}}, \cdots, R_{\mathcal{U}_1,K}^{\text{two}}, R_{\mathcal{U}_2,K}^{\text{two}}\}$ for TWR. All constraints within both $\mathcal{P}_{\text{OWR}}$ and $\mathcal{P}_{\text{TWR}}$ can be modeled as SDP constraints independently from either optimizing for power ($\mathcal{P}_{\text{OWR}}$) or covariance matrices ($\mathcal{P}_{\text{TWR}}$).

Although both $\mathcal{P}_{\text{OWR}}$ and $\mathcal{P}_{\text{TWR}}$ are convex problems, they can not be solved through a trivial water-filling solution or a simple SDP. Both compromise several `logdet` consisting upper bound constraints on the variables building the objective function, which can not be directly and trivially solved by the available numerical SDP tools. In the following, we propose an iterative algorithm which exploits the geometry of the intersection of the uplink and the downlink rate regions, and is independent of the value of K and different relaying schemes. In the sequel, we consider sum rate optimization only for TWR in details, and drop the expressions for OWR for the sake of brevity. However, the derivations for TWR can be directly applied for OWR by just changing the corresponding constraints and optimization variables, which does not affect neither the solution method, nor the structure of the algorithm.

Having $\tilde{K} = 2K$ nodes for information flow in TWR, we search for the sum-rate optimal $\tilde{K}$ rate tuple inside a space with $\tilde{K}$ dimensions, defined by the constraints in multiple access and broadcast phases' rate regions. Moreover, this intersection region is convex by definition, i.e., it is the convex hull of all achievable $\tilde{K}$ rate tuple. In the following, we firstly give an intuitive simple motivating example for $\tilde{K} = 2$ case, and then, building on this framework, the general sum rate maximization algorithm for arbitrary $\tilde{K}$ is presented.

1) 2-Dimensional Case: Each rate pair within $\tilde{K}$ rate tuple, i.e., $(\bar{R}_\alpha, \bar{R}_\beta)$, where $\alpha, \beta \in \{1, \ldots, \tilde{K}\}$, $\alpha \neq \beta$, is defined in a 2-dimensional convex rate region $\mathcal{I}_{\alpha,\beta}$ as depicted in Fig. 2.a. with the shaded area. The relation between any tuple within this region can be expressed through an angle $\phi_{\alpha,\beta} \in [0, \pi/2]$ as $\bar{R}_\beta = \bar{R}_\alpha \tan(\phi_{\alpha,\beta})$. Hence, the maximum possible sum rate of this pair, i.e., $\bar{R}_\alpha + \bar{R}_\beta = \bar{R}_\alpha(1 + \tan(\phi_{\alpha,\beta}))$, for a given $\phi_{\alpha,\beta}$ is found by the quasi-convex problem $\mathcal{P}_{\phi_{\alpha,\beta}}$:

$$\max \ \tau \quad \text{subject to } \left(\tau, \tau\tan(\phi_{\alpha,\beta})\right) \in \mathcal{C}_{\text{MAC}}^{(\alpha,\beta)} \cap \mathcal{C}_{\text{BC}}^{(\alpha,\beta)}, \tau \geq 0$$

where $\tau := \bar{R}_\alpha$, $\mathcal{C}_{\text{MAC}}^{(\alpha,\beta)}$ and $\mathcal{C}_{\text{BC}}^{(\alpha,\beta)}$ represent the convex MAC and BC region constraints associated with the rate pair (α, β), respectively; and we omit the power and semidefiniteness constraints for notational simplicity. Modeling the mutual information expressions of BC region as SDP constraints, the

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problem $\mathcal{P}_{\phi_{\alpha,\beta}}$ can be efficiently and optimally solved by a bisection method combined with SDP feasibility checks [11], [12], but we drop the explicit expressions for the sake of brevity. Computing the optimal $\tau^\star$, we find the optimal rate pair $(\bar{R}^\phi_\alpha, \bar{R}^\phi_\beta) = (\tau^\star, \tau^\star \tan(\phi_{\alpha,\beta}))$ which maximizes the sum rate of the pair on the direction of $\phi_{\alpha,\beta}$. Note that the change of optimization coordinates from cartesian to polar gives us the opportunity to use the efficient bisection algorithm and reduce the computational complexity significantly as $\tilde{K}$ increases. Moreover, as it will be explained in the next section, it provides the flexibility to reduce the optimization search space.

Next, since the rate region $\mathcal{I}_{\alpha,\beta}$ is convex in terms of $\phi_{\alpha,\beta}$ (independently from the chosen pair or the employed relaying scheme), we can search over the optimal $\phi_{\alpha,\beta}$ that maximizes $\bar{R}_\alpha + \bar{R}_\beta$, using an unconstrained minimization method, e.g., the steepest descent method. To sum up, for each iteration of the descent algorithm, i.e., for each chosen $\phi_{\alpha,\beta}$, we solve $\mathcal{P}_{\phi_{\alpha,\beta}}$, and iterate until iterating $\phi_{\alpha,\beta}$ further does not induce significant change in sum rate (see Fig. 2.b. for an illustration.).

2) The General Case: Previously, we disregarded the constraints enforced by the relation of the chosen (α, β) pair with other nodes in the set $\mathcal{N}\backslash\{\alpha, \beta\}$, where $\mathcal{N} = \mathcal{U}_1 \cup \mathcal{U}_2$ is the set of all nodes in the network. Since each pair (α, β) out of $\tilde{K}$ dimensions (representing $\tilde{K}$ rate tuple) can be interpreted through an angle $\phi_{\alpha,\beta}$, it may be conjectured that we need $\tilde{K}!/(2(\tilde{K}-2)!)$ angles to represent all relations between all dimensions, whereas, in essence, we need only $\tilde{K}-1$ angles. This statement can be immediately proven by setting one dimension fixed, say the νth one, and expressing all the rest $\tilde{K}-1$ dimensions in terms of the νth dimension and a corresponding angle. Thus, the rates for all dimensions can be expressed as

$$\mathcal{R}_\nu = \big(R_\nu, R_\nu \tan(\phi_{\nu,\mu_1}), \ldots, R_\nu \tan(\phi_{\nu,\mu_{\tilde{K}-1}})\big),$$

for any $\nu \in \mathcal{N}$, where $\mu_1, \ldots, \mu_{\tilde{K}-1}$ are the members of the set $\mathcal{N}\backslash\{\nu\}$, and $|\mathcal{N}\backslash\{\nu\}| = \tilde{K}-1$. Having the knowledge of the vector $\boldsymbol{\phi} = [\phi_{\nu,\mu_1}, \ldots, \phi_{\nu,\mu_{\tilde{K}-1}}]$, one can derive all other angles through the relation $\tan(\phi_{\mu_i,\mu_j}) = \tan(\phi_{\nu,\mu_i})/\tan(\phi_{\nu,\mu_j})$, $\forall i,j \in \{1,\ldots,\tilde{K}-1\}$, $i \neq j$. Hence, for a given vector $\boldsymbol{\phi}$ and any ν, the maximum sum rate for TWR is obtained through the following quasi-convex problem

$$\mathcal{P}_{\boldsymbol{\phi}}: \max_{R_\nu, \{\boldsymbol{\Lambda}_k^{\rm two}\}_{k=1}^K} R_\nu \text{ subject to } \mathcal{R}_\nu \in \mathcal{C}^{\rm two}_{\rm MAC} \cap \mathcal{C}^{\rm two}_{\rm BC}, R_\nu \geq 0, \quad \boldsymbol{\Lambda}_k^{\rm two} \succeq 0 \forall k,\ \ \textstyle\sum_{k=1}^{\tilde{K}} {\rm Tr}(\boldsymbol{\Lambda}_k^{\rm two}) \leq P_{\rm R},$$

which can be efficiently solved with a bisection method combined with SDP feasibility checks [11], [12]. Since the $\tilde{K}$ dimensioned achievable rate region is convex by definition, we can search over $\boldsymbol{\phi}$, whose elements $\phi_i \in [0, \pi/2]$, $i = 1, \ldots, \tilde{K}-1$, using an unconstrained minimization method.

The overall sum rate maximization algorithm is summarized with $\mathcal{A}_{\rm sum}$. While implementing the bisection part of the $\mathcal{A}_{\rm sum}$, $R_\nu^{\min}$ is set to 0, $R_\nu^{\max}$ is chosen large enough according to the operation mean SNR value, and ϵ is a small positive number indicating the precision of the bisection algorithm. Moreover, the search direction for ϕ is found through a numerical first derivative computation, i.e., $(f(x+\varepsilon)-f(x))/\varepsilon$.

$\mathcal{A}_{\rm sum}$: SUM RATE MAXIMIZATION ALGORITHM

initiate: → $\boldsymbol{\phi} \in [0, \pi/2]$
repeat: → ***solve*** $\mathcal{P}_{\boldsymbol{\phi}}$ ***for given*** $\boldsymbol{\phi}$
  initiate: → $R_\nu^{\min}, R_\nu^{\max}$
  repeat: → $R_\nu := (R_\nu^{\min} + R_\nu^{\max})/2$
    → ***solve the feasibility problem for*** $R_\nu, \boldsymbol{\phi}$
    $\mathcal{R}_\nu \in \mathcal{C}^{\rm one}_{\rm MAC} \cap \mathcal{C}^{\rm one}_{\rm BC}, \sum_{k=1}^{\tilde{K}} {\rm Tr}(\boldsymbol{\Lambda}_k^{\rm two}) \leq P_{\rm R}, \boldsymbol{\Lambda}_k^{\rm two} \succeq 0 \forall k,$
    → ***if feasible*** $R_\nu^{\min} := R_\nu$
    → ***else*** $R_\nu^{\max} := R_\nu$
  until: → $R_\nu^{\max} - R_\nu^{\min} < \epsilon$
  → ***compute the search direction for*** $\boldsymbol{\phi}$: $\Delta\boldsymbol{\phi}$
    numerical first derivative computation for sum rate
  → ***line search for choosing step size:*** $\boldsymbol{t}$
  → ***update:*** $\boldsymbol{\phi} := \boldsymbol{\phi} + \boldsymbol{t}\Delta\boldsymbol{\phi}$
until: → no further significant improvement on sum rate.

The iterations for the descent algorithm continues until the difference at the sum rate becomes negligible for a new iteration. The convergence and the optimality of the algorithm are ensured through the related conditions of the employed methods [11].

Remark 2: The search space for each ϕ_i in $\boldsymbol{\phi}$ can be reduced by some pre-optimizations, i.e., $\phi_i \in [\phi_i^{\min}, \phi_i^{\max}]$ instead of $[0, \pi/2]$. Since each information flow's MAC single bound is known readily (see (3),(4)), the feasibility of the corresponding rate expression at the BC phase can be checked to satisfy the given MAC rate or not, i.e., check if MAC and BC intersect for single user bounds. If they intersect, calculating the angle between this rate and its corresponding pair's rate, this portion can be excluded from the optimization space (see Fig. 2.c.).

B. Extensions to Maxmin Fairness and QoS Assurance

1) Maxmin Fairness: Depending on the instantaneous channel realizations and/or distances of the nodes to the relay, sum rate maximization may lead some nodes to have a very low transmission rate, which may be unfair but chosen so to maximize sum rate over the network. In order to prevent such situations, the R_is can be chosen such that maxmin-fairness is applied over the whole network. Hence, for OWR, we aim to allocate powers to individual messages such that fairness between S/D pairs, i.e., $\min_i R_i^{\rm one}$ is maximized. Likewise, for TWR, the covariance matrices are chosen such that fairness between all nodes, i.e., $\min_i \min\{R^{\rm two}_{\mathcal{U}_1,i}, R^{\rm two}_{\mathcal{U}_2,i}\}$, or between all pairs, i.e., $\min_i(R^{\rm two}_{\mathcal{U}_1,i} + R^{\rm two}_{\mathcal{U}_2,i})$ is maximized. For both OWR and TWR, the problem formulation will be same as $\mathcal{P}_{\rm OWR}$ or $\mathcal{P}_{\rm TWR}$, except that the objective functions are changed with the corresponding ones mentioned previously.

We outline the maxmin-fairness maximization algorithm basing on $\mathcal{A}_{\rm sum}$ without going into details. Changing the objective function correspondingly and applying $\mathcal{A}_{\rm sum}$ once over the node set $\mathcal{N}$, the maximal minimum rate is found for any one $i \in \{1, ..., \tilde{K}\}$, say $R_i^{\rm maxmin}$. Then, we create a constraint set $\mathcal{C}_{\rm maxmin}$ and add the constraint $\{R_i \geq R_i^{\rm maxmin}\}$ to it. Moreover, we update the set of nodes $\mathcal{T} = \mathcal{N}\backslash\{i\}$ for maxmin optimization. In the next iteration, we add $\mathcal{C}_{\rm maxmin}$ to $\mathcal{A}_{\rm sum}$, and maximize again the minimum of the rates in the set $\mathcal{T}$. In other words, while assigning a rate of at least $R_i^{\rm maxmin}$ to the ith node, we are assigning the highest maxmin rates to the rest of the nodes in $\mathcal{T}$. Continuing with this fashion until $\mathcal{T} = \emptyset$, and reducing the number elements of $\mathcal{N}$ with each iteration, we allocate resources such that the most maxmin-fair rates are allocated to the nodes.

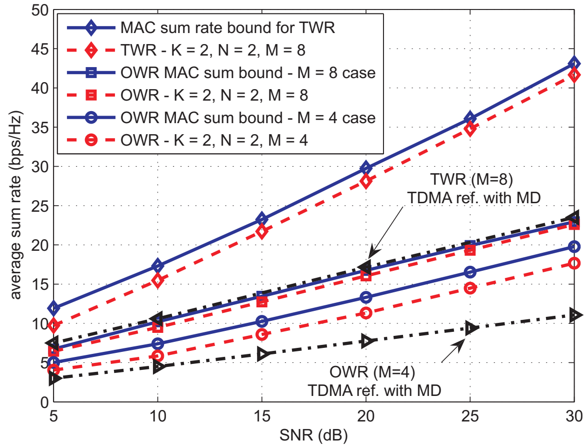

Fig. 3. Average sum rates versus SNR for OWR and TWR ($K = 2, N = 2$).

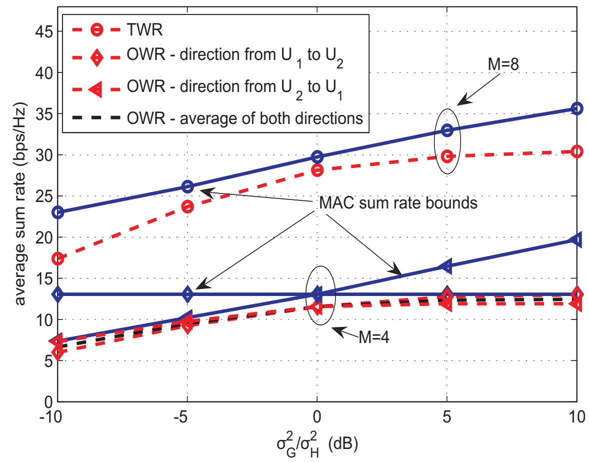

Fig. 4. Average sum rates with unbalanced link qualities ($K = 2, N = 2$).

2) QoS Assurance: In order to provide sufficient and/or required transmission rates to all nodes, QoS assurance can be introduced, while maximizing the sum rate. In other words, each node is supplied with the least transmission rate it requires. Collecting all QoS related constraints on linear combinations of R_i in a constraint set $\mathcal{C}_{\text{QoS}}$, we can add the constraint $\{R_1, \cdots, R_{\tilde{K}}\} \in \mathcal{C}_{\text{QoS}}$ to both $\mathcal{P}_{\text{OWR}}$ and $\mathcal{P}_{\text{TWR}}$, and use $\mathcal{A}_{\text{sum}}$ correspondingly. The QoS constraints will be taken into account during the SDP feasibility checks.

V. Simulations and Conclusions

In this section, we present Monte Carlo simulation results. Throughout the simulations, we use the MATLAB based semidefinite tool *Yalmip* [12] to solve the designed semidefinite problems. The channel matrices are assumed to stay constant over the two phases. All nodes have the same noise variance, i.e., $\sigma_{\text{n}}^2 = \sigma_{\text{r}}^2 = \sigma_{\text{d}}^2$, and the relay is assumed to have a sum transmit power of node pairs, i.e., $P_{\text{R}} = KP_{\text{s}}$ both for OWR and TWR. The average signal-to-noise ratio is defined as $\text{SNR} = P_{\text{s}}/\sigma_{\text{n}}^2$. There are $K = 2$ node pairs, i.e., 4 users, where each node is equipped with $N = 2$ antennas.

In Fig. 3, we compare the sum rates of both multiuser MIMO OWR and TWR schemes vs. SNR. We assume symmetric channel quality for both $\mathcal{U}_1$ and $\mathcal{U}_2$, and set $\sigma_{\text{H}_\text{k}}^2 = \sigma_{\text{G}_\text{k}}^2 = 1, \forall k$. Within each pair, users want to exchange information, which is accomplished in four channel uses with OWR (from $\mathcal{U}_1$ to $\mathcal{U}_2$, then from $\mathcal{U}_2$ to $\mathcal{U}_1$), and two channel uses with TWR. As references for both OWR and TWR, we plot sum rates when the relay serves the node pairs one by one (TDMA manner), and also employs multiuser diversity (MD) such that it chooses the best pair to serve depending on the instantaneous channel conditions. The relay has $M = KN = 4$ (for OWR) or $M = 2KN = 8$ (for TWR) antennas. It is depicted that TWR offers a significant sum rate improvement over OWR and recovers the spectral efficiency loss observed in OWR. Even assigning $2KN = 8$ antennas to OWR scheme can not recover the loss with respect to TWR. The ultimate upper bounds, i.e., the MAC sum rate bound, are also shown in Fig. 3. For OWR, increasing M shrinks the gap to the upper bound. The significant advantage of using multiuser precoding is confirmed by comparing the proposed multiuser schemes' performance with the reference scenarios. Moreover, multiuser OWR with $M = 8$ can only perform as much as the TDMA based reference scenario for TWR does.

The impact of unbalanced link quality is investigated in Fig. 4, where we fix $P_{\text{s}}/\sigma_{\text{n}}^2 = 20$ dB and $\sigma_{\text{H}}^2 = \sigma_{\text{H}_\text{k}}^2 = 1, \forall k$, and vary $\sigma_{\text{G}}^2 = \sigma_{\text{G}_\text{k}}^2 \forall k$. For OWR, the transmission from $\mathcal{U}_1$ to $\mathcal{U}_2$ is limited with multiple access phase for high $\sigma_{\text{G}}^2/\sigma_{\text{H}}^2$, whereas broadcast phase is the bottleneck for the the transmission from $\mathcal{U}_2$ to $\mathcal{U}_1$. Since all $\text{H}_k \forall k$, and $\text{G}_k \forall k$ are consisted in both phases for TWR, the broadcast phase turns out to the performance limiting factor. We note that, for both schemes, the gap between sum rates and upper bounds can be further reduced by using higher complexity precoders, e.g., ZFDPC.

To conclude, multiuser TWR has been shown to supply considerable sum rate improvement over OWR, but requires more antennas at the relay, which may limit the number of users to serve in practice.

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