One-Bit Successive-Cancellation Soft-Output (OSS) Detector for Uplink MU-MIMO Systems With One-Bit Adcs

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ABSTRACT We study an uplink multiuser multiple-input multiple-output (MU-MIMO) system with one-bit analog-to-digital converters (ADCs) in which one base station (BS) with N_r receive antennas serve K users with a single antenna. For this system, the soft-output (SO) detector was recently proposed where a soft-metric (e.g., a log-likelihood ratio (LLR)) is computed from a hard-decision channel output by introducing a novel distance measure in the binary Hamming space. This makes it possible to be naturally incorporated into the state-of-the-art channel codes (e.g., low-density parity-check code or polar code). In this paper, we further improve the performance of the SO detector by exploiting a priori information (e.g., the previously decoded messages), which is called the *one-bit successive-cancellation soft-output* (OSS) detector. The key idea of the proposed OSS detector is that each user k's message is decoded sequentially via the associated channel decoder k in ascending order and a refined search-space is constructed using the previously decoded messages (i.e., the enhanced LLRs are generated). We then present a multiple OSS detector by taking into account a more practical scenario where the BS is equipped with multiple channel decoders. In addition, we propose an efficient way to determine a good decoding order by introducing a novel set-distance measure. Finally, simulation results demonstrate that the proposed OSS detector can significantly improve the existing SO detector for the coded MU-MIMO systems with one-bit ADCs.

INDEX TERMS Massive MIMO, multiuser MIMO detection, one-bit analog-to-digital converter (ADC), soft-output decoding.

I. INTRODUCTION

A massive multiple-input-multiple-output (MIMO) is a promising technique to cope with the predicted wireless data traffic explosion [1]–[4]. In contrast, the massive MIMO considerably increases the hardware cost and the radio-frequency (RF) circuit consumption [5]. Among all the components in the RF chain, a high-resolution analog-to-digital converter (ADC) is particularly power-hungry as the power consumption of an ADC is scaled exponentially with the number of quantization bits and linearly with the baseband bandwidth [6], [7]. To address this challenge, low-resolution ADCs (e.g., 1~3 bits) for massive MIMO systems have been considered as a low-power solution over the past years.

The one-bit ADC is particularly attractive due to its power efficiency and lower hardware complexity.

In recent, there have been extensive researches on MIMO detections and channel estimations for the uplink MU-MIMO systems with low-resolution ADCs [8]–[20]. In [8] and [9], various channel estimation methods were developed such as least-square (LS), maximum-likelihood (ML), zero-forcing (ZF), and Bussgang decomposition based method. Regarding MIMO detectors, the optimal ML detector was introduced and the near-ML detector was also proposed by converting the ML detection problem into a tractable convex optimization problem [8]. Also, low-complexity MIMO detectors were proposed in [10] and [11]. In [12], Studer and Durisi investigated the quantized massive MU-MIMO systems for wideband communications based on orthogonal frequency-division multiplexing (OFDM). A joint

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channel-and-data estimation method was presented in [13] based on Bayes-optimal inference. In [14], an unified framework to construct a family of MIMO detectors for the mixed-ADC receiver architecture was proposed by exploiting a probabilistic Bayesian inference. Also, for wireless sensor networks (WSNs), the advantages of quantized massive MIMO systems were investigated for a decentralized and distributed structure [15]–[18]. A supervised-learning approach, called minimum-center-detection (MCD), was presented in [19]. In addition, by viewing the one-bit ADC MIMO model as an equivalent coding problem, a novel MIMO detector, referred to as *weighted* minimum distance (wMD) decoding, was proposed [20]. It is remarkable that the above hard-decision output detectors cannot generate soft outputs from one-bit quantized observations, which considerably degrades the performance of a following channel code (e.g., Turbo [21], low-density-parity-check (LDPC) [22] or polar code [23]). This problem has been addressed in [24] by efficiently computing *soft* log-likelihood ratios (LLRs) from one-bit quantized (or hard-decision) observations. Thus, it can be naturally incorporated into a state-of-the-art *soft* channel decoder (e.g., belief-propagation decoder [22]). Because of such difference, it was shown in [24] that the soft-output (SO) detector can provide a substantial coded gain (about 10 dB) over the optimal (hard-output) ML detector. Nevertheless, there is still a lot of room to improve the performance of the SO detector since it does not exploit a *priori* information.

In this paper, we propose one-bit successive-cancellation soft-output (OSS) detectors which can exploit the previously decoded messages (i.e., a priori information) conveyed by channel decoders to improve the LLRs. In the SO detector [24], the LLRs are computed via the relative distances among the current observation and all possible *noiseless* channel outputs (say, codewords). Throughout the paper, the set of such codewords is referred to as a code $\mathcal{C}$ (or search-space). The underlying idea of the proposed OSS detector is that the code $\mathcal{C}$ is refined by eliminating unnecessary codewords (having lower probability to be corrected one) with a *priori* information. Specifically, the OSS detector computes the LLRs in a *successive* fashion: each user k's message is sequentially decoded from a channel decoder k for $k = 1, \ldots, K$ in that order, and the soft inputs (e.g., LLRs) of the channel decoder k are computed from the *refined* code (which is constructed by eliminating some unnecessary codewords from $\mathcal{C}$ using the previously decoded messages). Since the codewords in the refined code can create larger distances than those in $\mathcal{C}$, the detection ambiguity can be reduced, thus achieving a better performance. Moreover, we develop a multiple OSS (M-OSS) detector by taking into account a more practical scenario where the BS is capable of operating plural channel decoders simultaneously. The proposed M-OSS detector can remove more unnecessary codewords as the plenty of the decoded messages can be conveyed from the multiple channel decoders concurrently. We further enhance the M-OSS detector by optimizing the decoding order of the

channel decoders. It is remarkable that, due to the use of one-bit ADCs, the conventional ordering based on signal-to-noise ratio (SNR) (e.g., V-BLAST [26]) is not applicable. The proposed ordering method is developed so that the resulting subcodes have a better structure, meaning that the distances of the remaining codewords are as far as possible. Finally, simulation results demonstrate that the proposed *ordered* M-OSS detector attains 2.5 dB coded gain over the existing SO detector [24] for the polar-coded MU-MIMO systems with one-bit ADCs.

The outline of this paper is as follows. In Section II, we describe the system model and briefly review the existing SO detector. In Section III, we propose the OSS detector which can enhance the SO detector by leveraging a priori knowledge on the previously decoded messages. In Section IV, focusing on more practical scenarios where the BS is equipped with multiple channel decoders, we present an *ordered* multiple OSS (M-OSS) detector. Simulation results are provided in Section V. Section VI concludes the paper.

Notation: Lower and upper boldface letters represent the column vectors and matrices, respectively. Let $\langle a : b \rangle \triangleq \{a, a + 1, \ldots, b\}$ for any integers a and $b > a$, and when $a = 1$, it can be further shortened as $\langle b \rangle$. For any vectors $\mathbf{x}$ and $\mathbf{a}$, we let $\mathbf{x_a} = [x_{a_1}, x_{a_2}, \ldots, x_{a_N}]$. For any $\ell = b_0 m^0 + \cdots + b_{L-1} m^{L-1}$, let $g_m(\ell) = [b_0, b_1, \ldots, b_{L-1}]^{\mathsf{T}}$ denote the m-ary expansion of ℓ where $b_i \in \langle 0 : L - 1 \rangle$. We also let $g_m^{-1}(\cdot)$ denote its inverse function. For a vector, $g_m(\cdot)$ is applied in an element-wise manner. $\mathbf{1}_{\{A\}}$ represents an indicator function that equals one if an event A is true and zero otherwise, and $\lceil \cdot \rceil$ denotes the ceiling function.

II. PRELIMINARIES
In this section we describe an uplink MU-MIMO system with one-bit ADCs and review the SO detector proposed in [24].

A. SYSTEM MODEL
We consider a single-cell uplink MU-MIMO system in which there are K users with single-antenna and one base station (BS) that accommodates an array of $N_r \gg K$ antennas. Let $w_k[t] \in \mathcal{W} = \langle 0 : m - 1 \rangle$ denotes an user k's message at time slot t for $k \in \langle K \rangle$, each of which includes $\log m$ information bits. We also denote m-ary constellation set by $\mathcal{S} = \{s_0, \ldots, s_{m-1}\}$. For the ease of expression, it is assumed that $m = 2^p$ for some positive integer p. However, the proposed detectors can be apparently extended to an arbitrary m. At time slot t, the user k transmits the symbol $\tilde{x}_k(w_k[t])$ which is obtained by a modulation function $f(\cdot)$ as

$$\tilde{x}_k(w_k[t]) = f(w_k[t]) \in \mathcal{S}. \quad (1)$$

Let $\tilde{x}(w[t]) = \left[\tilde{x}_1(w_1[t]), \ldots, \tilde{x}_K(w_K[t]) \right]$ denote the transmit vector of all the K users at time slot t. Then, the discrete-time complex-valued baseband received signal at the BS is given by

$$\tilde{\mathbf{r}}[t] = \tilde{\mathbf{H}}\tilde{\mathbf{x}}(\mathbf{w}[t]) + \tilde{\mathbf{z}}[t] \in \mathbb{C}^{N_r}, \quad (2)$$
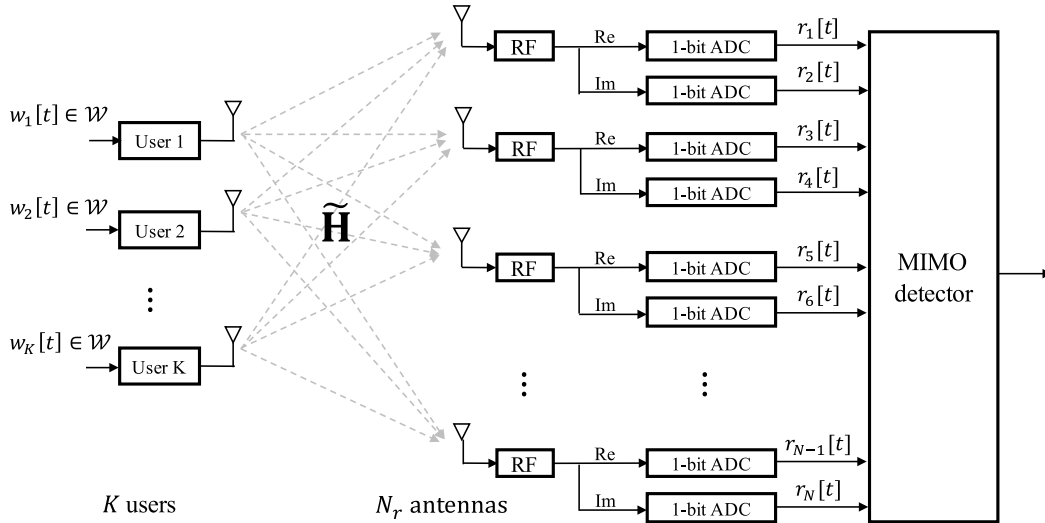
FIGURE 1. Description of an uplink MU-MIMO system with one-bit ADCs.

where $\tilde{\mathbf{H}} \in \mathbb{C}^{N_r \times K}$ denotes the (complex-valued) channel matrix between the BS and the K users, i.e., $\mathbf{h}_i^\mathsf{T} \in \mathbb{C}^{1 \times K}$ represents the channel vector between the K users and the i-th receiver antenna at the BS. Also, $\tilde{\mathbf{z}}[t] = [\tilde{z}_0[t], \ldots, \tilde{z}_{N_r}[t]]^\mathsf{T} \in \mathbb{C}^{N_r}$ denotes the noise vector whose elements are distributed as circularly symmetric complex Gaussian random variables with zero-mean and unit-variance, i.e., $\tilde{z}_i[t] \sim \mathcal{CN}(0, 1)$.

In the MU-MIMO system with one-bit ADCs, each receive antenna of the BS is equipped with RF chain consisting of two one-bit ADCs that separately applied to real and imaginary part (see Fig. 1). Let sign$(\cdot) : \mathbb{R} \rightarrow \{0, 1\}$ denote the one-bit ADC quantizer function with

$$\text{sign}(\tilde{r}[t]) = \begin{cases} 1 & \text{if } \tilde{r}[t] \geq 0 \\ -1 & \text{if } \tilde{r}[t] < 0. \end{cases} \quad (3)$$

Then, the BS observes the quantized received output vector as

$$\begin{aligned} \hat{\mathbf{r}}_R[t] &= \text{sign}(\text{Re}(\tilde{\mathbf{r}}[t])) \in \{1, -1\}^{N_r} \\ \hat{\mathbf{r}}_I[t] &= \text{sign}(\text{Im}(\tilde{\mathbf{r}}[t])) \in \{1, -1\}^{N_r}. \end{aligned} \quad (4)$$

To make an explanation clear, we represent the complex-valued input-output relationship in (2) into the equivalent real-valued one as

$$\mathbf{r}[t] = \text{sign}(\mathbf{H}\mathbf{x}(\mathbf{w}[t]) + \mathbf{z}[t]) \in \{1, -1\}^{2N_r}, \quad (5)$$

where

$$\begin{cases} \mathbf{H} = \begin{bmatrix} \text{Re}(\tilde{\mathbf{H}}) & -\text{Im}(\tilde{\mathbf{H}}) \\ \text{Im}(\tilde{\mathbf{H}}) & \text{Re}(\tilde{\mathbf{H}}) \end{bmatrix} \in \mathbb{R}^{2N_r \times 2K} \\ \mathbf{r}[t] = [\hat{\mathbf{r}}_R[t]^\mathsf{T}, \hat{\mathbf{r}}_I[t]^\mathsf{T}]^\mathsf{T} \\ \mathbf{x}(\mathbf{w}[t]) = [\text{Re}(\tilde{\mathbf{x}}(\mathbf{w}[t]))^\mathsf{T}, \text{Im}(\tilde{\mathbf{x}}(\mathbf{w}[t]))^\mathsf{T}]^\mathsf{T} \\ \mathbf{z}[t] = [\text{Re}(\tilde{\mathbf{z}})^\mathsf{T}, \text{Im}(\tilde{\mathbf{z}})^\mathsf{T}]^\mathsf{T}. \end{cases} \quad (6)$$
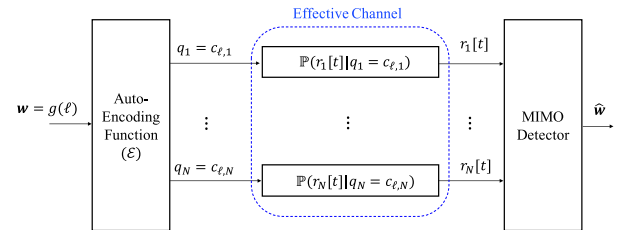

FIGURE 2. The equivalent *N* parallel B-DMCs.

This real system representation will be used in the sequel. A block fading channel is assumed where a channel matrix $\mathbf{H}$ remains flat during n time slots (i.e., coded block length) and changes independently across coded blocks. Also, it is assumed that the channel matrix $\mathbf{H}$ is perfectly known to the BS. It is remarkable that the proposed SO detector can be also performed with an estimated channel matrix $\hat{\mathbf{H}}$ by simply replacing $\mathbf{H}$ with $\hat{\mathbf{H}}$ in the following sections.

B. EQUIVALENT N PARALLEL B-DMCS

In [20], it was shown that a real system representation in (5) can be transformed into an equivalent $N = 2N_r$ parallel binary discrete memoryless channels (B-DMCs). In this section, we define the channel input/output and channel transition probabilities of the N parallel B-DMCs (see Fig. 2). Due to the equivalence, the channel output is clearly equal to $\mathbf{r}[t]$.

1) CHANNEL INPUT

Given $\mathbf{H}$, we construct a spatial-domain code $\mathcal{C} = \{\mathbf{c}_0, \ldots, \mathbf{c}_{m^K-1}\}$ where each codeword $\mathbf{c}_\ell$ is determined as a function of $\mathbf{H}$ as

$$\mathbf{c}_\ell = [\text{sign}(\mathbf{h}_1^\mathsf{T}\mathbf{x}(g_m(\ell))), \ldots, \mathbf{h}_N^\mathsf{T}\mathbf{x}(g_m(\ell))]^\mathsf{T} \in \mathcal{C}, \quad (7)$$
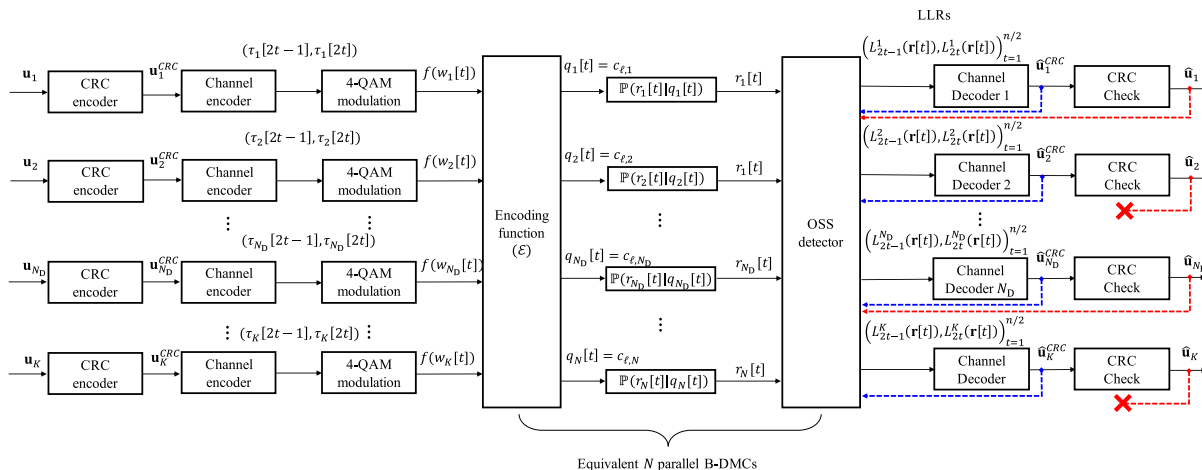
FIGURE 3. Description of the proposed soft-output MIMO detectors for uplink MU-MIMO systems with one-bit ADCs for 4-QAM.

and it can be considered as a perfect channel output without additive noises in (5). Then, in Fig. 2, the output of auto-encoding function $\mathcal{E}$ is obtained as

$$\mathbf{q} = \mathcal{E}(\mathbf{w}, \mathbf{H}) = \mathbf{c}_\ell, \tag{8}$$

where $\ell = g^{-1}(\mathbf{w}) \in \langle 0 : m^K - 1 \rangle$.

2) TRANSITION PROBABILITY

As shown in Fig. 2, the effective channel consists of N unequal parallel BSCs with input $\mathbf{q}$ and output $\mathbf{r}$. Also, for the i-th subchannel, the transition probabilities, depending on user's message $\mathbf{w} = g(\ell)$ and corresponding codeword $\mathbf{c}_\ell$, are defined as

$$\mathbb{P}(r_i[t] \mid q_i = c_{\ell,i}) = \begin{cases} \epsilon_{\ell,i} & \text{if } r_i[t] \neq c_{\ell,i} \\ 1 - \epsilon_{\ell,i} & \text{if } r_i[t] = c_{\ell,i}, \end{cases} \tag{9}$$

where the error probability of the i-th channel is computed as

$$\epsilon_{\ell,i} = Q(\|\mathbf{h}_i^\mathsf{T} \mathbf{x}(g(\ell))\|), \tag{10}$$

and where

$$Q(x) = \frac{1}{2\pi} \int_x^\infty \exp\left(-u^2/2\right) du. \tag{11}$$

C. OVERVIEW OF THE SO DETECTOR

We review the SO detector proposed in [24]. The wMD decoding in [20] produces the *hard-decision* outputs and thus, it is not appropriate to be used with state-of-the-art channel codes (e.g., Turbo, LDPC, and Polar codes). This problem was addressed in [24] by generating soft LLRs from one-bit quantized measurements. We first define a new distance measure that will be used throughout the paper.

Definition 1 (Distance Measure): For any two vectors $\mathbf{x}$ and $\mathbf{y}$ of the same length N, we define a *weighted* Hamming distance $d_{wh}(\mathbf{x}, \mathbf{y}; \{\alpha_i\})$ with the non-negative weights $\{\alpha_i\}_{i=1}^N$ as

$$d_{wh}(\mathbf{x}, \mathbf{y}; \{\alpha_i\}) \triangleq \sum_{i=1}^N \alpha_i \mathbf{1}_{\{x_i \neq y_i\}}. \tag{12}$$

We remark that the conventional Hamming distance is the special case with $\alpha_i = 1$ for all i. ∎

In [20], the wMD decoding finds a decoded message $\hat{\ell}$ such that

$$\hat{\ell} = \underset{\ell \in \langle 0:m^K-1 \rangle}{\arg\min} \; d_{wh}(\mathbf{r}, \mathbf{c}_\ell; \{\alpha_{\ell,i}\}), \tag{13}$$

where $\alpha_{\ell,i} = -\log(\epsilon_{\ell,i})$.

Assuming the above wMD decoding, we will explain how to compute the soft outputs (e.g., LLRs). For the simplicity of explanation, we focus on a 2^p-QAM constellation (e.g., $\mathcal{W} = \langle 0 : 2^p - 1 \rangle$) for some positive p. However, the extension to an arbitrary m-ary constellation is straightforward. Fig. 3 describes the proposed coded system for $p = 2$ (i.e., 4-QAM) where the blue and red lines represent the code-refinement processes for the OSS and M-OSS detectors, respectively. For the OSS detector, one blue-line is only used at a time regardless of the CRC-check. For the M-OSS detector, due to the usage of multiple decoders at the BS, at most N_D lines are used simultaneously. As seen from the second decoder, a message declined by CRC-check is not handled in refining the code-space (see the second channel decoder). The process contains an iteration loop based on a message that passes the CRC-check and should be halted at the stopping condition. Let $(u_k[1], \ldots, u_k[M])$ denote the M-bit information sequence of the user k and let $(\tau_k[1], \ldots, \tau_k[n])$ denote the corresponding coded output. To simplify the notation, we define the

$$[\mathbf{b}]_p \overset{\triangle}{=} \sum_{i=1}^p b_i 2^{p-i}, \tag{14}$$

where $\mathbf{b} = (b_1, \ldots, b_p)$. Then, the user k's channel input message at time slot t is obtained as

$$w_k[t] = [(\tau_k[pt], \tau_k[pt-1], \ldots, \tau_k[pt-p+1])]_p \in \mathcal{W} \tag{15}$$

for $t \in \langle n/p \rangle$, where n is assumed to be a multiple of p. Each user k transmits the $\{w_k[t] : t \in \langle n/p \rangle\}$ to the BS over the n/p time slots.

Definition 2 (Subcode): For any fixed $\{w_k[t] = j\}$, the subcode of $\mathcal{C} = \{\mathbf{c} = \mathcal{E}(\mathbf{w}, \mathbf{H}) : \mathbf{w} \in \mathcal{W}^K\}$ is defined as

$$\mathcal{C}_{|\{w_k[t]=j\}} \triangleq \{\mathbf{c} = \mathcal{E}(\mathbf{w}, \mathbf{H}) : w \in \mathcal{W}^K, w_k[t] = j\}.$$

■

With the above definition, we can efficiently compute the LLRs from the channel observation $\{r[t] : t \in \langle n/p \rangle\}$ as

$$L_{pt-(i-1)}^k(\mathbf{r}[t]) = \min_{\mathbf{c}_\ell \in \mathcal{B}_{(i,1)}^k} d_{wh}(\mathbf{r}[t], \mathbf{c}_\ell; \{\log \epsilon_{\ell,i}^{-1}\})$$
$$- \min_{\mathbf{c}_\ell \in \mathcal{B}_{(i,0)}^k} d_{wh}(\mathbf{r}[t], \mathbf{c}_\ell; \{\log \epsilon_{\ell,i}^{-1}\}) \quad (16)$$

for $i \in \langle p \rangle$ and $t \in \langle n/p \rangle$, where $\epsilon_{\ell,i}$ is given in (10) and

$$\mathcal{B}_{(i,j)}^k = \bigcup_{b \in \{0,1\}^p : b_i = j} \mathcal{C}_{|\{w_k[t]=[b]_p\}}, \quad (17)$$

for $j \in \{0, 1\}$. During the n/p time slots, the BS collects the LLRs $\{L_{pt-(i-1)}^k(\mathbf{r}[t]) : i \in [p], t \in [n/p]\}$ for $k = 1, \ldots, K$ and they are embedded into the corresponding channel decoder k, for $k = 1, \ldots, K$.

III. THE PROPOSED OSS DETECTOR

In this section, we enhance the performance of the existing SO detector by presenting an one-bit successive-cancellation soft-output (OSS) detector. The overall structure of the OSS detector is illustrated in Fig. 3. As explained before, in the SO detector [24], LLRs are computed by searching all the codewords in the code $\mathcal{C}$ separately for each user. Whereas, in the proposed OSS detector, LLRs are computed from the so-called *refined* code $\mathcal{C}_r \subseteq \mathcal{C}$ where the refined code (or subcode) is constructed by exploiting the previously decoded messages (see Fig. 4). In this way, the detection ambiguity can be alleviated since the distances among the remaining codewords tend to be larger. As a consequence, the OSS detector can generate more reliable LLRs than the SO detector in [24].

In the rest of this section, we will explain the OSS detector by focusing on the LLR computations for the channel decoder k, based on the $\langle k - 1 \rangle$ users' decoded messages $\hat{w}_{\langle k-1 \rangle}[t] = \{\hat{w}_1[t], \ldots, \hat{w}_{k-1}[t]\}$:

- From the decoded messages $\hat{\mathbf{w}}_{\langle k-1 \rangle}[t]$, we can obtain the *refined* subcode of the $\mathcal{C}$ as

$$\mathcal{C}_{|\{\mathbf{W}_{\langle k-1 \rangle}[t]=\hat{\mathbf{w}}_{\langle k-1 \rangle}\}} \triangleq \{\mathbf{c} = \mathcal{E}(\mathbf{w}, \mathbf{H}) : \mathbf{w} \in \mathcal{W}, $$
$$\mathbf{w}_{\langle k \rangle}[t] = \hat{\mathbf{w}}_{\langle k-1 \rangle}[t]\}, \quad (18)$$

where $|\mathcal{C}_{|\{\mathbf{W}_{\langle k-1 \rangle}[t]=\hat{\mathbf{w}}_{\langle k-1 \rangle}\}}| = |\mathcal{C}|/2^{k-1}$. Since the distances among the remaining codewords in the subcode can be larger as k grows, the OSS detector can generate more reliable LLRs as the cancellation step proceeds. Fig. 4 describes the LLR comparisons of the SO, OSS, and M-OSS detectors for the detection of the second
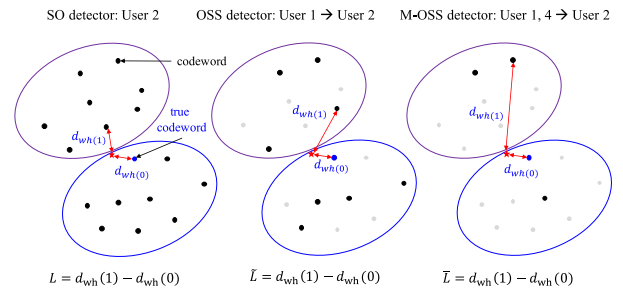

FIGURE 4. The LLR comparisons of the SO, OSS, and M-OSS detectors.

user. For the OSS detector, because of using the previously detected message $\hat{w}_1[t]$, some codewords (represented by grey circles) are eliminated from the $\mathcal{C}$, and the LLR is computed by subtracting distances to each subcode: one is associated with $b_i = 1$ and the other is $b_i = 0$. For the M-OSS detector, we assume the second iteration, and the messages of user 1 and 4 are concurrently employed in refining the $\mathcal{C}$. Then, the magnitude of the LLR becomes even larger thus leading to more reliable decoding.

- Using the above refined subcode and from (16), the enhanced LLRs are computed as

$$\tilde{L}_{pt-(i-1)}^k(\mathbf{r}[t], \hat{\mathbf{w}}_{\langle k-1 \rangle}[t])$$
$$= \min_{\mathbf{c}_\ell \in \mathcal{B}_{(i,1|\hat{\mathbf{W}}_{\langle k-1 \rangle}[t])}^k} d_{wh}(\mathbf{r}[t], \mathbf{c}_\ell; \{\log \epsilon_{\ell,i}^{-1}\})$$
$$- \min_{\mathbf{c}_\ell \in \mathcal{B}_{(i,0|\hat{\mathbf{W}}_{\langle k-1 \rangle}[t])}^k} d_{wh}(\mathbf{r}[t], \mathbf{c}_\ell; \{\log \epsilon_{\ell,i}^{-1}\}), \quad (19)$$

where

$$\mathcal{B}_{(i,j|\hat{\mathbf{W}}_{\langle k-1 \rangle}[t])}^k$$
$$= \bigcup_{\mathbf{b} \in \{0,1\}^p : b_i = j} \mathcal{C}_{|\{w_k[t]=[b]_p, \mathbf{W}_{\langle k-1 \rangle}[t]=\hat{\mathbf{w}}_{\langle k-1 \rangle}[t]\}}. \quad (20)$$

- By embedding the above LLRs $\{\tilde{L}_{pt-1}^k(r[t], \hat{w}_1^{k-1}[t]) : i \in \langle p \rangle, t \in \langle n/p \rangle\}$ into the channel decoder, the user k's message is decoded as the bit-stream $\hat{\mathbf{u}}_k$. Also, the $\{\hat{w}_k[t] : t \in \langle n/p \rangle\}$ are obtained from $\hat{\mathbf{u}}_k$, using the channel encoder and modulation function.
- Then, leveraging the decoded messages $\hat{\mathbf{w}}_1^{k-1}[t]$ and $\hat{w}_k[t]$, the enhanced LLRs for the channel decoder $k + 1$ are computed.

The above process is repeatedly applied until all the K users' messages are decoded (see Algorithm 1 below). It is noticeable that the size of code-space for a next user is reduced to half of the current size.

IV. THE PROPOSED ORDERED M-OSS DETECTOR

In this section, the proposed OSS detector is further improved by developing a multiple OSS (M-OSS) and by optimizing a decoding order. As in conventional successive cancellation detector, the decoding order plays a crucial role in the performance of the OSS detector. Unfortunately, it is too complex

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Algorithm 1 The Proposed OSS Detector

▷ Choose K and N_r for MU-MIMO system
▷ Choose p for $m(= 2^p)$-QAM constellation
▷ Choose blocklength n and code rate R for channel coding

Define the code $\mathcal{C} = \{\mathbf{c}_\ell : \ell = 0, \dots, m^K - 1\}$ in (7)
One-bit quantized observation $r[t]$ for $t \in \langle n/p \rangle$
for $k = 1, \dots, K$ **do**
    if $k = 1$ **then**   ▷ No previous message
        Compute LLRs for the first user using (16)-(17)
and the code $\mathcal{C}$
    else   ▷ Using previously detected message
        Compute LLRs for the i-th user using (19)-(20) and
the refined code $\mathcal{C}_{|\{\mathbf{w}_{\langle k-1 \rangle}[t] = \hat{\mathbf{w}}_{\langle k-1 \rangle}[t]\}}$
    end if
    Decode the bit-stream $\hat{\mathbf{u}}_k[t]$ using the above LLRs
    Re-encode the decoded bit-stream to get $\hat{\mathbf{w}}_k[t]$
    Define the refined code $\mathcal{C}_{|\{\mathbf{w}_{\langle k \rangle}[t] = \hat{\mathbf{w}}_{\langle k \rangle}[t]\}}$
end for

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to find an optimal decoding order with respect to a performance. Instead, we develop an efficient method to designate a good decoding order by introducing a novel set-distance measure among the sets of high-dimensional binary vectors. We remark that the proposed ordering is completely different from the conventional SNR-based ordering and is more suitable for the proposed LLR computations. Next, focusing on a more practical scenario having multiple channel decoders at the BS, we propose the M-OSS detector where the plenty of (correctly) decoded messages are used simultaneously to build a refined search-space (see Fig. 3).

A. THE PROPOSED DECODING ORDER
We propose an effective *advance* ordering method which can further ameliorate the computations of LLRs with considerably lower overhead compared with our priori work in [27]. In the proposed method, a decoding order is chosen by evaluating the set-distance between two disjoint sets (e.g., subcodes) where one is associated with $\tau_k[t] = 0$ and the other is $\tau_k[t] = 1$. This is motivated by the fact that the LLRs with higher reliability are more likely to be attained from far-off subcodes. Based on this, the proposed decoding order is determined as follows.

Given a channel matrix $\mathbf{H}$, we construct a search-space (e.g., the set of noiseless outputs (or codewords)) from (7) and divide them into subcodes on the basis of each user's message $w_k[t]$ with respect to a single-bit location. In the example of the user 1's first-bit location, we obtain the following two disjoint subcodes from (17) as

$$\mathcal{B}^1_{(1,0)} = \bigcup_{\mathbf{b} \in \{0,1\}^p : b_1 = 0} \mathcal{C}_{|w_k[t] = [\mathbf{b}]_p}$$

$$\mathcal{B}^1_{(1,1)} = \bigcup_{\mathbf{b} \in \{0,1\}^p : b_1 = 1} \mathcal{C}_{|w_k[t] = [\mathbf{b}]_p}. \tag{21}$$

In order to evaluate the distance of the above two sets, it is required to define a set-distance measure such as $d_s(\mathcal{B}^1_{(1,0)}, \mathcal{B}^1_{(1,1)})$. We remark that it is not obvious to find an optimal set-distance measure $d_s(\cdot, \cdot)$. Instead, we resort to using the mean distance (i.e., the distance between the centroids of the two sets) such as

$$d_s(\mathcal{A}, \mathcal{B}) \triangleq \left| \frac{1}{|\mathcal{A}|} \sum_{\mathbf{a} \in \mathcal{A}} \mathbf{a} - \frac{1}{|\mathcal{B}|} \sum_{\mathbf{b} \in \mathcal{B}} \mathbf{b} \right|^2. \tag{22}$$

Using this distance measure, we can compute the distance of each i-th bit location for $i \in \langle p \rangle$ and then define the representative distance for the user 1 as

$$d^1 = \sum_{i \in \langle p \rangle} d_s(\mathcal{B}^1_{(i,0)}, \mathcal{B}^1_{(i,1)}). \tag{23}$$

Likewise, we can compute the representative distance for the user k as

$$d^k = \sum_{i \in \langle p \rangle} d_s(\mathcal{B}^k_{(i,0)}, \mathcal{B}^k_{(i,1)}), \tag{24}$$

for $k \in \langle K \rangle$. Now, we are ready to determine the decoding order such as the user index k_i for the i-th decoding is determined as

$$k_i = \operatorname*{argmax}_{k \in \langle K \rangle \setminus \{k_1, \dots, k_{i-1}\}} d^k. \tag{25}$$

B. THE PROPOSED M-OSS DETECTOR
We present a multiple OSS (M-OSS) detector in which the plenty of decoded messages (provided by multiple channel decoders) are exploited simultaneously in the subcode refinement stage. Also, this process is repeated until there is no newly decoded message (called stopping condition). Before stating the proposed detector, we briefly explain how the multiple channel decoders together with CRC codes are performed.

1) MULTIPLE CHANNEL DECODERS
In the OSS detector, we assume the scenario that a single channel decoder is performed K times in a sequential way. However, in practical systems, BS is able to be equipped with multiple channel decoders (say, N_D channel decoders). In this section, we assume that the N_D channel decoders operate simultaneously and then the decoded messages from those decoders are provided to the OSS detector at the same time (see Fig. 3).

2) SC DECODING WITH CRC-CHECK
In the OSS detector, the decoded messages are used regardless of their correctness and thus an unsatisfactory performance can be made especially at lower SNR regimes. This drawback can be addressed by only using CRC-pass decoded messages, namely, a search-space refinement is performed only when the decoded messages pass the
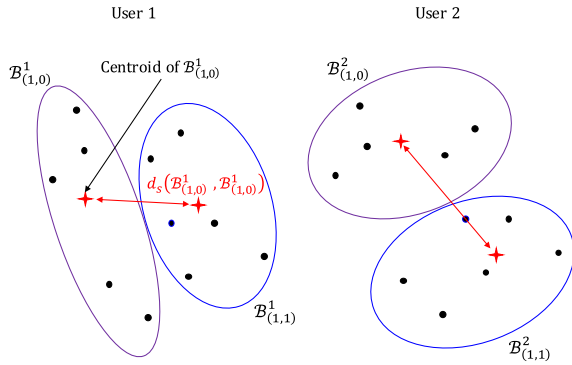
User 1   User 2

FIGURE 5. The mean-distance comparison for two disjoint subcodes.

CRC-check. We first define a CRC-appended information sequence as

$$
\begin{aligned}
\mathbf{u}_k^{CRC} &= (\mathbf{u}_k, \mathbf{a}_\ell(\mathbf{u}_k)) \\
&= (u_k[1], \dots, u_k[M-\ell], a_k[M-\ell+1], \dots, a_k[M]),
\end{aligned}
\tag{26}
$$

where $\mathbf{u}_k$ denotes the information bits of the user k and $\mathbf{a}_\ell(\mathbf{u}_k)$ denotes the redundant CRC bits computed using $\mathbf{u}_k$ and a polynomial for ℓ-length CRC sequence (e.g., $x^{16} + x^{12} + x^5 + 1$ and $\ell = 17$ for CRC-16). Note that M-bit information sequence before channel encoding is composed of $(M - \ell)$ information bits and an ℓ-bit CRC sequence. As in Section III, the information sequence is subsequently encoded as $(\tau_k[1], \dots, \tau_k[n])$ and then, the corresponding modulated symbols are transmitted as $w_k[t]$ for $t \in \langle n/p \rangle$. Also, the enhanced LLRs can be computed from (19). By embedding the LLRs into the channel decoder, the user k's information sequence is decoded as $\hat{\mathbf{u}}_k^{CRC}$. In the OSS detector, the previously decoded messages of all the $\langle k-1 \rangle$ users are used to compute the enhanced LLRs without any exception. Since the CRC-failed decoded messages can degrade the performances of the subsequent decodings, it would be better to use the CRC-pass decoded messages only for the refinement of the search-space. Let $\mathbf{s} = [s_1, \dots, s_S]^T$ be the CRC-check vector where s_i stores the index of user who succeeds in CRC-check among $\langle k-1 \rangle$ users. Then, the enhanced LLRs are computed only using the CRC-pass decoded messages as

$$
\begin{aligned}
&\bar{L}_{pt-(i-1)}^k(\mathbf{r}[t], \ \hat{\mathbf{w}}_{\mathbf{S}}[t]) \\
&= \min_{\mathbf{c}_\ell \in \mathcal{B}_{(i,1|\hat{\mathbf{w}}_{\mathbf{S}}[t])}^k} d_{wh}\left(\mathbf{r}[t], \mathbf{c}_\ell; \left\{\log \epsilon_{\ell,i}^{-1}\right\}\right) \\
&\quad - \min_{\mathbf{c}_\ell \in \mathcal{B}_{(i,0|\hat{\mathbf{w}}_{\mathbf{S}}[t])}^k} d_{wh}\left(\mathbf{r}[t], \mathbf{c}_\ell; \left\{\log \epsilon_{\ell,i}^{-1}\right\}\right),
\end{aligned}
\tag{27}
$$

where $\hat{\mathbf{w}}_{\mathbf{S}}[t]$ is the collection of the decoded message of the CRC-pass users and the refined subcodes are obtained as

$$
\mathcal{B}_{(i,j|\hat{\mathbf{w}}_{\mathbf{S}}[t])}^k = \bigcup_{\mathbf{b} \in \{0,1\}^p : b_i = j} \mathcal{C}_{|\{w_k[t] = [\mathbf{b}]_p, \ \mathbf{w}_{\mathbf{S}}[t] = \hat{\mathbf{w}}_{\mathbf{S}}[t]\}}.
\tag{28}
$$

M-OSS detector: Suppose that the BS is equipped with the multiple channel decoders (say, N_D channel decoders) and CRC-check. Based on this, we present an iterative algorithm to exert the best-effort for a given channel observation. The overall procedures of the M-OSS detector are provided in Algorithm 2. We first describe the procedures at the first iteration (e.g., $T = 1$):

- Before starting the detection procedure, the advance ordering in (25) is determined by considering an average strength of each subcode, which is to say $\mathbf{k} = [k_1, k_2, \dots, k_K]$. As an initialization, $\mathbf{s} = [\ ]$ denotes an indicator vector that contains the indices of correctly decoded users' messages.
- A cluster of channel decoders performs for $\lceil K_{max}^T / N_D \rceil$ steps, where K_{max}^T is the number of undetected users at the first step of T-th iteration and $K_{max}^1 = K$, in general it is same as the number of elements in $\mathbf{k} \setminus \mathbf{s}$.
- At the first step of the first iteration ($T = 1$), the BS defines the undetermined users as $\mathbf{k}^1 = \mathbf{k}$, and then computes the LLRs of $\mathbf{k}_{\langle 1:N_D \rangle}^1$ via (16) as there are no previously detected messages. Then, the LLRs are embedded to the corresponding channel decoders. Thereafter, the BS extracts the indices of the users who successfully pass the CRC-check among $\mathbf{k}_{\langle 1:N_D \rangle}^1$, and then adds them to $\mathbf{s}$.
- Likewise, until $\lceil K_{max}^1 / N_D \rceil - 1$ step, all channel decoders concurrently compute the *enhanced* LLRs of $\mathbf{k}_{\langle (i-1)N_D+1:iN_D \rangle}^1$, where $i \in \{1, 2, \dots, \lceil K_{max}^1 / N_D \rceil - 1\}$, and $\mathbf{s}$ is updated afterward.
- At the last step of the first iteration, i.e., $\lceil K_{max}^1 / N_D \rceil$ step, the number of remaining users can be less than N_D. In this case, the corresponding number of channel decoders is only used.

Example 1: Consider the case of $N_r = 16$ and $K = 8$. Suppose that the ordered user index is denoted as $\mathbf{k} = \mathbf{k}^1 = [k_1, k_2, \dots, k_8]$ and the BS is equipped with 5 channel decoders (i.e., $N_D = 5$). In this case, the M-OSS detector performs by $\lceil K/N_D \rceil = 2$ times. At the first step, the BS decodes the messages of $\mathbf{k}_{\langle 1:5 \rangle}^1$ such as $\mathbf{w}_{\mathbf{k}_{\langle 1:5 \rangle}^1}$, each of which is followed by CRC-check. We assume that the messages of user 2 and 3 are correctly decoded (i.e., $\mathbf{s} = [k_1, k_2]$). In the second step, the previously decoded messages of user $k_1 = 2$ and $k_2 = 3$ are used to reduce the search-space. Then, using the refined search-space, the BS can recover the messages of $\mathbf{k}_{\langle 6:8 \rangle}^1$. If the user 4's message passes the CRC-check, then $\mathbf{s}$ is updated as $\mathbf{s} = [k_1, k_2, k_4]$. ∎

We next focus on the second iteration (e.g., $T = 2$), the BS has the knowledge on the correctly decoded messages from $\mathbf{s}$ and thus, it can exclude $\mathbf{s}$ from $\mathbf{k}^1 = \mathbf{k}$, i.e, we have $\mathbf{k}^2 = \mathbf{k}^1 \setminus \mathbf{s}$. Before proceeding to the detection, the BS first checks the *stopping criterion*−whether the cardinality of $\mathbf{k}^2$ decreases over $\mathbf{k}^1$− on the ground that there is no means for the BS to further enhance the computation of LLRs if no message is detected at the previous iteration. If the stopping criterion is not satisfied, the second iteration proceeds, which

almost follows the procedures at the first iteration, with minor changes: i) K_{max}^2 is clarified as the cardinality of $\mathbf{k}^2$ and used to designate how many times the BS works; ii) because of $\mathbf{s}$, the BS computes the enhanced LLRs at the first step using (27) instead of (16). Similarly, the T-th iteration can proceed until the stopping criterion is satisfied.

Example 2: Following the Example 1, we focus on the second iteration. In the first iteration (stated at the Example 1), the messages of users whose indices belong to $\mathbf{s} = [k_1, k_2, k_4]$ are stored as the indices of correctly detected messages. The BS can get the *reduced* user set as $\mathbf{k}^2 = \mathbf{k}^1 \setminus s = [k_3, k_5, k_6]$. Then, the BS is able to compute the enhanced LLRs for the users in $\mathbf{k}_{\langle 1:3 \rangle}^2$ by searching the search-space refined by $\mathbf{s}$. ∎

Algorithm 2 The Proposed M-OSS Detector

▷Choose K and N_r for MU-MIMO system
▷Choose p for $2^p(= m)$ QAM constellation
▷Choose blocklength n and code rate R for channel coding
▷Choose the number of decoder N_D, CRC length ℓ and polynomial for the M-OSS

Define the code $\mathcal{C} = \{\mathbf{c}_\ell : \ell = 0, \ldots, m^K - 1\}$ in (7)
Design the order as $\mathbf{k} = \mathbf{k}^1 = [k_1, k_2, \ldots, k_K]$ using (25)
Each user generates CRC-added information sequence $\mathbf{u}_k^{CRC} = (\mathbf{u}_k, \mathbf{a}_\ell(\mathbf{u}_k))$ in (26)
One-bit quantized observation $\mathbf{r}[t]$
$T = 0$
while Newly detected information exists **do**
    $T = T + 1$
    for $i = 1, \ldots, \lceil K_{max}^T / N_D \rceil$ **do**
        if No message is detected **then**
            Compute LLRs using (16)-(17) and the code $\mathcal{C}$
        else   ▷ Using a previously detected message
            Compute LLRs using (27)-(28) and the refined code $\mathcal{C}_{|\{\mathbf{w_S} = \hat{\mathbf{w}}_\mathbf{S}\}}$
        end if
        Decode the bit-stream $\mathbf{u}_{\mathbf{k}_{\langle i(N_D-1)+1:iN_D \rangle}^T}^{CRC}$ [t] the LLRs
        Perform the CRC-check of $\mathbf{u}_{\mathbf{k}_{\langle i(N_D-1)+1:iN_D \rangle}^T}^{CRC}$ [t]
        Store successful indices in the $\mathbf{s}$
        Re-encode the decoded sequence to get $\mathbf{w_S}[t]$
        Define the refined code as $\mathcal{C}_{|\{\mathbf{w_S} = \hat{\mathbf{w}}_\mathbf{S}\}}$
        $\mathbf{k}^{T+1} = \mathbf{k}^T \setminus \mathbf{s}$
    end for
end while

C. COMPUTATIONAL COMPLEXITY

For the OS, OSS and M-OSS detectors, the complexities of LLR computations are proportional to the size of an associated search-space (or subcode) as shown in (16), (19), and (27). Since the SO detector in [24] examines all the codewords for each user independently, it requires the $K|\mathcal{C}|$ number of comparisons with $|\mathcal{C}| = M^K$. In contrast, the proposed OSS detector can reduce the size of subcodes by half as the

TABLE 1. Computational Complexity.

SO	OSS	M-OSS
$O(KM^K)$	$O(M^K)$	$O(\frac{K}{N_D}\mathcal{I}_{max}M^K)$

decoding proceeds. Thus, the total number of comparisons is computed as

$$\sum_{i=1}^{K} |\mathcal{C}|/2^{i-1}, \tag{29}$$

which is well-approximated as $\approx 2|\mathcal{C}|$ for a large K. The complexity ratio of the OSS and SO detectors is equal to $2/K$ and thus, the relative complexity decreases as K grows. Regarding the M-OSS detector, it runs $\mathcal{I}_{max}$ times where $\mathcal{I}_{max}$ stands for the number of iterations such that the stopping condition is satisfied. In addition, each iteration performs at most $\lceil \frac{K}{N_D} \rceil$ times to complete a cycle. Thus, the number of comparisons is proportional to $\lceil \frac{K}{N_D} \rceil \mathcal{I}_{max}$. Table 1 shows the summary of computational complexities and the trend of $\mathcal{I}_{max}$ of the M-OSS detector is depicted in Fig 8. We can see that due to the subcode refinement step, the complexity of the OSS detector is in general smaller than the other detectors. We would like to point out that their complexities can be further reduced by using the idea of sphere decoding in [29] and [30].

V. NUMERICAL RESULTS

We evaluate the performances of the proposed detectors for the polar-coded MU-MIMO systems with one-bit ADCs. For simulations, a Rayleigh fading channel is assumed where each element of a channel matrix $\mathbf{H}$ is drawn from an independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian random variable with zero mean and unit variance. Also, we consider both a perfect channel state information (CSI) and an imperfect CSI with a channel estimation error variance $\sigma_h^2 = 0.1$. We adopt a rate-1/2 polar code of length 128 (e.g., $n = 128$) proposed in [23]. Also, SC-list (SCL) decoding with list-size 4 in [28] is used for the decoding method of the polar code. The soft inputs (e.g., LLRs) of the polar decoder are computed from (16), (19) and (27) for the SO, OSS and M-OSS detectors, respectively.

Fig. 6 shows the FER comparisons of the SO, OSS, and M-OSS detectors for the polar-coded MU-MIMO system with one-bit ADCs where $K = 6$. We observe that the proposed OSS detector can outperform the SO detector by producing more reliable LLRs. Also, the performance gap becomes larger as N_r decreases for a fixed K. This is because, when N_r is smaller, the codewords tend to be located more densely and thus, eliminating some codewords in the OSS detector has more effect on the improvements of LLRs.

Fig. 7 shows the coded FER performances for various soft-output MIMO detectors such as ZF-type detector [8], SO detector [24], and proposed ordered OSS detector, where $K = 6$ and $N_r = 12$. Here, the solid lines are simulated under
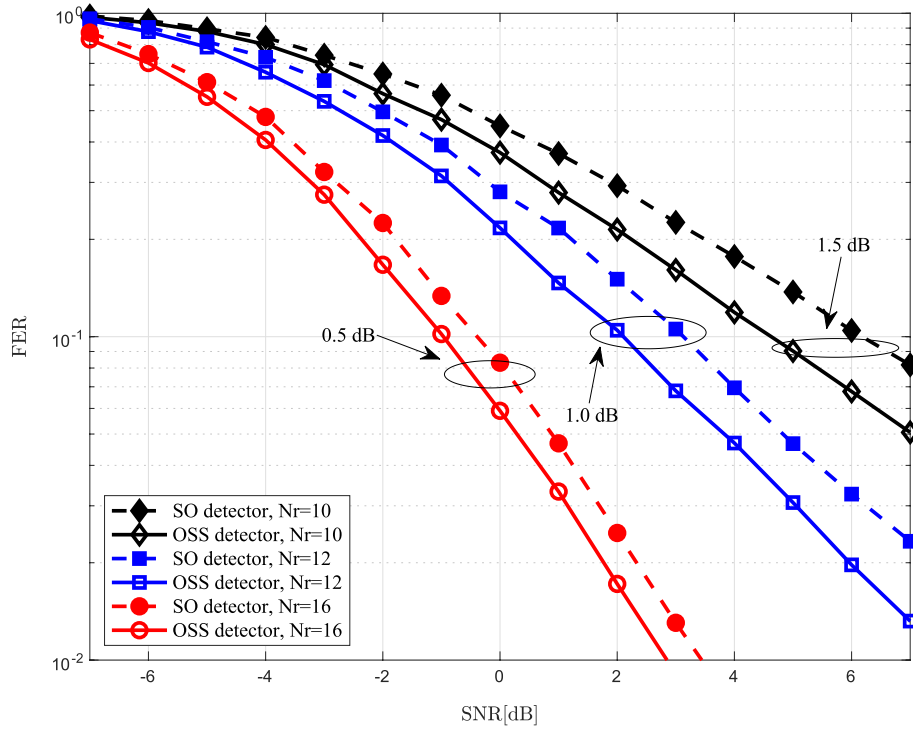
FIGURE 6. Performance comparisons of various MIMO detectors as a function of N_r for the polar-coded MU-MIMO system with one-bit ADCs.
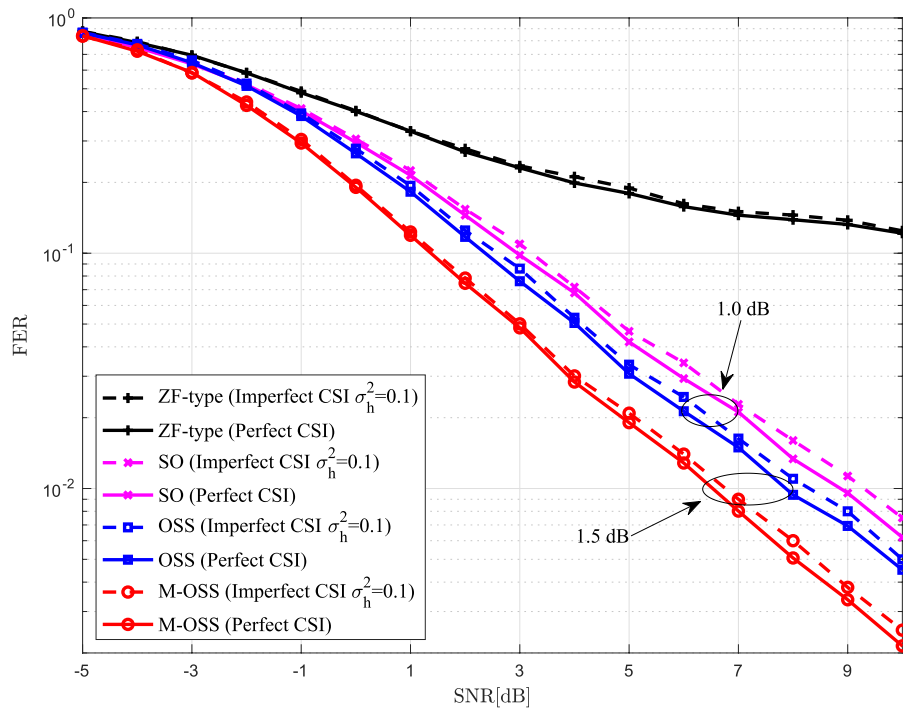

FIGURE 7. FER comparisons of various MIMO detectors for imperfect channel estimations.

perfect CSI, and the dotted lines are experimented under imperfect CSI with $\sigma_h^2 = 0.1$. Note that ML detector [8], near-ML detector [8], and supervised-learning detector [19]

are excluded in the comparison because they cannot generate soft outputs. As already shown in [24], their performances are much worse than that of the SO detector. We observe that
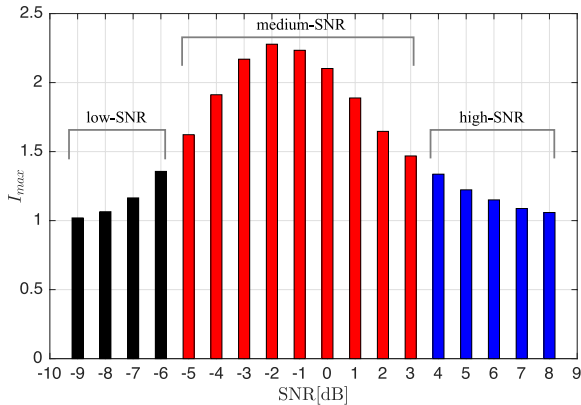
FIGURE 8. $\mathcal{I}_{max}$ versus wide range of SNR for $K = 6$ and $N_r = 12$.

the proposed detector significantly outperforms the ZF-type detector and the gap becomes larger as SNR increases. Furthermore, it can provide the additional coding gain over the SO detector in [24]. Fig. 7 shows that the proposed M-OSS detector has a remarkable performance gain over both the SO and OSS detectors. In the M-OSS detector, it is assumed that BS is equipped with 3 channel decoders (e.g., $N_D = 3$), and CRC-16-CCITT (with the generator polynomial of $x^{16} + x^{15} + x^5 + 1$) and $\ell = 17$ are used. At FER $= 10^{-2}$, the OSS detector attains the 1.0dB gain over the existing SO detector, mainly due to the use of enhanced LLRs. Moreover, the M-OSS detector can obtain additional 1.5dB gain over the OSS detector. We emphasize that such improvement is non-trivial in the coded systems.

Fig. 8 shows the average number of iterations for the M-OSS detector where it is computed using the 2,000 samples for each SNR. It is noticeable that at both low-SNR and high-SNR regimes, $\mathcal{I}_{max}$ is very close to 1. In the low-SNR regime, it is more likely that no message is decoded correctly at the first iteration, which implies that the M-OSS detector cannot proceed to the next iteration due to the stopping condition. In this regime, thus, it is hard to have an additional gain over the OSS detector. In contrast, at the high-SNR range, the M-OSS detector can yield an excellent performance since all the messages are fully detected at the first iteration, namely, the M-OSS detector does not have to proceed to the next iteration. In the medium-SNR regime, the iterative operation actively works since some part of users are correctly decoded (i.e., $\mathcal{I}_{max} > 1$). Also, it is noticeable that $\mathcal{I}_{max}$ is approximately peaked at 2.3, i.e., the iterative algorithm operates at most 2.3 times on average.

VI. CONCLUSION

We proposed the one-bit successive-cancellation soft-output (OSS) detector for the coded MU-MIMO systems with one-bit ADCs. The underlying idea of the proposed detector is that the previously decoded messages (provided by the channel decoders) are exploited to increase the magnitude of the soft outputs (e.g., log-likelihood ratios) for a subsequent channel decoder, and it leads to much more reliable detection performance. In addition, we presented the multiple OSS (M-OSS) detector which can further improve the OSS detector by leveraging the capability of multiple channel decoders, and the proper ordering method has added up an additional advantage as well. Simulation results demonstrated that the proposed detector provides a non-trivial coded gain over the existing SO detector for the polar-coded MU-MIMO systems with one-bit ADCs.

Despite its superior performance, the computational complexities of the proposed detectors are quite expensive especially when the number of user K is large. One important future work, thus, is to develop a proper method which can reduce the size of search-space (i.e., code-space) with an acceptable performance loss. Toward this, it would be an attractive to combine the sphere decoding methods (called one-bit sphere decodings) in [29] and [30] with the proposed detectors. Another interesting future work is to consider a frequency-selective channel, which can improve the practical deployments of the proposed methods.

REFERENCES

[1] T. L. Marzetta, "Noncooperative cellular wireless with unlimited numbers of base station antennas," *IEEE Trans. Wireless Commun.*, vol. 9, no. 11, pp. 3590–3600, Nov. 2010.

[2] E. G. Larsson, O. Edfors, F. Tufvesson, and T. L. Marzetta, "Massive MIMO for next generation wireless systems," *IEEE Commun. Mag.*, vol. 52, no. 2, pp. 186–195, Feb. 2014.

[3] A. Adhikary *et al.*, "Joint spatial division and multiplexing for mm-wave channels," *IEEE J. Sel. Areas Commun.*, vol. 32, no. 6, pp. 1239–1255, Jun. 2014.

[4] L. Lu, G. Y. Li, A. L. Swindlehurst, A. Ashikhmin, and R. Zhang, "An overview of massive MIMO: Benefits and challenges," *IEEE J. Sel. Topics Signal Process.*, vol. 8, no. 5, pp. 742–758, Oct. 2014.

[5] H. Yang and T. L. Marzetta, "Total energy efficiency of cellular large scale antenna system multiple access mobile networks," in *Proc. IEEE Online Conf. Green Commun.*, Piscataway, NJ, USA, Oct. 2013, pp. 27–32.

[6] B. Murmann. *ADC Performance Survey 1997-2015*. [Online]. Available: https://web.stanford.edu/~murmann/adcsurvey.html

[7] A. Mezghani and J. A. Nossek, "Modeling and minimization of transceiver power consumption in wireless networks," in *Proc. IEEE/ITG WSA*, Feb. 2011, pp. 1–8.

[8] J. Choi, J. Mo, and R. W. Heath, "Near maximum-likelihood detector and channel estimator for uplink multiuser massive MIMO systems with one-bit ADCs," *IEEE Trans. Commun.*, vol. 64, no. 5, pp. 2005–2018, Mar. 2016.

[9] Y. Li, C. Tao, G. Seco-Granados, A. Mezghani, A. L. Swindlehurst, and L. Liu, "Channel estimation and performance analysis of one-bit massive MIMO systems," *IEEE Trans. Signal Process.*, vol. 65, no. 15, pp. 4075–4089, Aug. 2017.

[10] C. Mollén, J. Choi, E. G. Larsson, and R. W. Heath, "One-bit ADCs in wideband massive MIMO systems with OFDM transmission," in *Proc. IEEE Int. Conf. Acoust. Speech Signal Process. (ICASSP)*, Mar. 2016, pp. 3386–3390.

[11] C. Mollén, J. Choi, E. G. Larsson, and R. W. Heath, Jr., "Uplink performance of wideband massive MIMO with one-bit ADCs," *IEEE Trans. Wireless Commun.*, vol. 16, no. 1, pp. 87–100, Jan. 2017.

[12] C. Studer and G. Durisi, "Quantized massive MU-MIMO-OFDM uplink," *IEEE Trans. Commun.*, vol. 64, no. 6, pp. 2387–2399, Jun. 2016.

[13] C. K. Wen, C. J. Wang, S. Jin, K. K. Wong, and P. Ting, "Bayes-optimal joint channel-and-data estimation for massive MIMO with low-precision ADCs," *IEEE Trans. Signal Process.*, vol. 64, no. 10, pp. 2541–2556, May 2016.

[14] T. C. Zhang, C. K. Wen, S. Jin, and T. Jiang, "Mixed-ADC massive MIMO detectors: Performance analysis and design optimization," *IEEE Trans. Wireless Commun.*, vol. 15, no. 11, pp. 7738–7752, Sep. 2016.

[15] D. Ciuonzo, P. S. Rossi, and S. Dey, "Massive MIMO Channel-Aware Decision Fusion," *IEEE Trans. Signal Process.*, vol. 63, no. 3, pp. 604–619, Feb. 2015.

[16] A. Shirazinia, S. Dey, D. Ciuonzo, and P. S. Rossi, "Massive MIMO for decentralized estimation of a correlated source," *IEEE Trans. Signal Process.*, vol. 64, no. 10, pp. 2499–2512, May 2016.

[17] F. Jiang, J. Chen, A. L. Swindlehurst, and J. A. López-Salcedo, "Massive MIMO for wireless sensing with a coherent multiple access channel," *IEEE Trans. Signal Process.*, vol. 63, no. 12, pp. 3005–3017, Jun. 2015.

[18] D. Ciuonzo, P. S. Rossi, and S. Dey, "Massive MIMO meets decision fusion: Decode-and-fuse vs. Decode-then-fuse," in *Proc. IEEE 8th Sensor Array Multichannel Signal Process. Workshop (SAM)*, Jun. 2014, pp. 265–268.

[19] Y.-S. Jeon, S.-N. Hong, and N. Lee, "Supervised-learning-aided communication framework for MIMO systems with low-resolution ADCs," *IEEE Trans. Veh. Technol.*, vol. 67, no. 8, pp. 7299–7313, Aug. 2018.

[20] S.-N. Hong, S. Kim, and N. Lee, "A weighted minimum distance decoding for uplink multiuser MIMO systems with low-resolution ADCs," *IEEE Trans. Commun.*, vol. 66, no. 5, pp. 1912–1924, May 2018.

[21] C. Berrou, A. Glavieux, and P. Thitimajshima, "Near Shannon limit error-correcting coding and decoding: Turbo-codes. 1," in *Proc. Int. Conf. Commun.*, Geneva, Switzerland, May 1993, pp. 1064–1070.

[22] T. Richardson and R. Urbanke, *Modern Coding Theory*. Cambridge, U.K.: Cambridge Univ. Press, 2008.

[23] E. Arikan, "Channel polarization: A method for constructing capacity-achieving codes for symmetric binary-input memoryless channels," *IEEE Trans. Inf. Theory*, vol. 55, no. 7, pp. 3051–3073, Jul. 2009.

[24] S.-N. Hong and N. Lee, "Soft-output detector for uplink MU-MIMO systems with one-bit ADCs," *IEEE Commun. Lett.*, vol. 22, no. 5, pp. 930–933, May 2018.

[25] K. D. Rao, *Channel Coding Techniques for Wireless Communications*. New Delhi, India: Springer, 2015.

[26] P. W. Wolniansky, J. G. Foschini, G. D. Golden, and R. A. Valenzuela, "V-BLAST: An architecture for realizing very high data rates over the rich-scattering wireless channel," in *Proc. ISSSE*, Oct. 1998, pp. 295–300.

[27] Y.-S. Cho, S.-H. Kim, and S.-N. Hong, "Successive cancellation soft output detector for uplink MU-MIMO systems with one-bit ADCs," in *Proc. IEEE Int. Conf. Commun. (ICC)*, Kansas City, MO, USA, May 2018, pp. 1–6.

[28] I. Tal and A. Vardy, "List decoding of polar codes," *IEEE Trans. Inf. Theory*, vol. 61, no. 5, pp. 2213–2226, May 2015.

[29] S. Kim, N. Lee, and S.-N. Hong, "Uplink massive MIMO systems with one-bit ADCs: A low-complexity weighted minimum distance decoding," in *Proc. IEEE Global Commun. Conf.*, Singapore, Dec. 2017, pp. 1–6.

[30] Y.-S. Jeon, N. Lee, S.-N. Hong, and R. W. Heath, "One-bit sphere decoding for uplink massive MIMO systems with one-bit ADCs," *IEEE Trans. Wireless Commun.*, vol. 17, no. 7, pp. 4509–4521, Jul. 2018.

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