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One Boson Exchange Model in Proton-Proton Scattering*)

——*An Approach to the Strong Interaction*——

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We propose a phenomenological model for the strong reaction. In the model we assume that the resonance states can be treated as elementary particles as the pion and nucleon and the physical processes can be described by the sum of the *lowest* order matrix elements. As an application of the model we study the proton-proton scattering below~300 Mev. It is shown that the present model can successfully explain the experimental (YLAM) phase shifts if the pion, scalar boson and vector boson dominantly contribute to the nuclear force at this energy region.

§ 1. Introduction

As a result of the progress in the high energy experiments, it has been found that there exist many resonant states (or isobars) of the meson-meson system and the meson-baryon system—ρ, ω, η, $\cdots$, N^*, N^{**}, $\cdots$. These have much similarity in the particle-like behavior to the so called elementary particles except that the former has in general a short lifetime compared with the latter. Theoretical interest will naturally occur whether some or all of these isobars can be regarded as elementary on a level with pion and nucleon.***)

Most of these isobars are usually considered as not elementary. However, until now, the quantum mesodynamics did neither predict nor succeed in explaining these isobars *systematically*. One of the causes may be owing to the difficulties of the approximation method. But another possibility remains, e.g. the quantum mesodynamics is not an appropriate theory for treating these isobars.

In the Sakata model[1] there is some possibility of explaining *all* of these isobar states as irreducible states of the full symmetry theory,[2),3)] and though the analyses so far made are very preliminary ones, the obtained results seem

*) A preliminary report was given at the meeting on the "model of elementary particle" held at Research Institute for Fundamental Physics of Kyoto University in March, 1962.

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***) Some of these new isobars may have a dynamical unstability similar to the neutral pion. In this case, we have no reason not to treat such a particle in a similar way to the pion.

to be promising. For the treatment of such an isobar state, the Sakata model can take a characteristic standpoint that these isobars should be regarded as the composite states on an equal footing with the *stable* nucleon and pion.

If we take this picture of the isobar based on the Sakata model, then comes a question what are the elementary units among which the strong interactions are realized, the fundamental proton, neutron and Λ-particle[4] or the irreducible composite state, nucleon, pion and even ρ-meson, etc.

Here, we take the standpoint that the elementary units in the strong interaction are the composite states, nucleons, pions, ρ-mesons, etc. If we assume this point of view, then it will be convenient for the latter discussion to arrange the problems of the strong interaction as:

I) What is the dynamical law governing the construction of the composite states, and what is the nature of force between the fundamental baryons?

II) What is the dynamical law that governs the strong reaction among the composite particles?

As for the first problem we should like to note that the full symmetry theory seems to grasp the important kinematical aspect of the model. In the course of the study of the full symmetry theory, a semi-empirical mass formula,[3] which could reproduce the masses of the baryons and mesons together with the resonances, was introduced and played an important role.

However the analyses so far made are rather of the kinematical nature and the answer to the first problem is not yet obtained. Through the investigation of the dynamical law, the spin problem of the composite state[3] which has been encountered in the analysis of the full symmetry theory will be solved.

Concerning the second problem, we propose a model for the strong reaction*) in this paper.

Our model consists of the following assumptions:

A) The elementary units in the strong interaction are the baryon and meson, including all the unstable isobars, not the fundamental baryons, proton, neutron and Λ-particle of the Sakata model.

B) We apply the quantum field theory to these particles, and assume that the unstable isobars can be treated as if stable particles such as pions and nucleons.

C) However, we put the condition that the strong reaction can be described by the sum of the matrix elements corresponding to the *lowest order* Feynman diagrams.**) This roughly means that nearly all of the higher order corrections are left ignored as having no physical effects.

D) The only realizable type of the higher order correction is the damping

*) The present model can be applied to the reaction process only, not to the bound state.

**) We define the diagram as the lowest order when it does contain no closed loops consisting of baryons and/or mesons.

effects which arise from the repetition of the lowest order diagrams through intermediate states on the mass shell. This effect is determined completely by the lowest order matrix elements.

The last assumption is necessary from two reasons. One is that there is no logical reason why we consider that the rescattering of the reactions does not occur when the intermediate states are on the mass shell, even if we assume the realizable physical process is the lowest order process. The other reason is more practical one. As is well known, only with the matrix elements of such lowest order diagrams we can not satisfy the unitarity requirements of the theory. The unitarity is satisfied if we take into account the damping effect.

One may feel that the above model may be too drastic, and destroy the characteristic feature of the field theory, since the higher order correction in the quantum electrodynamics has been so much persuasive to transfix us to the present field theory.

However we note that the reality of the higher order correction in mesodynamics is by no means established as in the case of the Lamb shift, the anormalous magnetic moment of electron,*) etc., of the photon-electron system. If the quantum field theory is not the *final* theory for the baryon and meson system (this will really be the case, if the direction along the Sakata model correctly points to the reality), and still there is some *region* where the description by the quantum field theory may have validity (which is seen in the nuclear force of the one-pion-exchange region[6]), it will be an attempt of significance to use the quantum field theory under the restrictive conditions which contradict with the logics of the theory itself.

In this connection it will be interesting to remind of the *old quantum theory* of N. Bohr where the Newtonian mechanics is imposed on with the quantum condition which is by no means explainable in the concept of the Newtonian mechanics.

Our aim is not to turn the present stage back to some thirty years ago.[7] Our opinion is that the meson and baryon have a new kind of structure revealed as the presence of isobar which has not been observed for the electron-photon system and such difference should be reflected in the theory in some way or other.

On the other hand if one wishes to give a reasoning of the present model from the conservative point of view, where the *really* elementary particles may be the pion and nucleon in one case or the fundamental proton, neutron and Λ-particle, one may consider that the contents of the higher order correction are such ones as can be approximated by the contribution of the isobars in the present model.

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*) In the case of electron, the anormalous magnetic moment can be successfully explained by the higher order correction. But in the present model it is necessary to introduce *primary* anormalous magnetic moment to the nucleons.

The model was originally motivated from the following situation. We previously assumed that all the observed resonance states are attributable to the composite states of the Sakata model and examined the correspondence between the experimental data and the states of the full symmetry theory by a semi-empirical mass formula.[3] If this picture is correct, rises a question what will be the relation between these composite states of the Sakata model and the quantum field theoretical resonances such as 3-3 resonance in Chew's theory.[8] Unfortunately the present mesodynamics has not succeeded in giving the explanation of all the observed resonances and has no guarantees for not producing other unwanted resonances. So in order to make a consistent interpretation by the Sakata model, one way will be to give up to explain resonances in terms of higher order corrections in the field theory and to look for what dynamical model is possible in this direction. This is one of the underlying ideas of the present investigation.

Any way, as the first application of the model, we shall study the proton-proton scattering below ~300 Mev in this paper. We call the present model of nucleon-nucleon scattering as the OBEC (one-boson-exchange-contribution) model.

§ 2. Problem of proton-proton scattering and OBEC model

Since Taketani, Nakamura and Sasaki[5] proposed a method for the pion field theoretical approach to the problem of nuclear force, great efforts have been made by many people[6),9] along this line. It was clarified that the pion field theory is valid in the outer region of the nuclear force range and some complication exists in the inner region which is rather questionable to be comprehended by the pion field theory.

One of the main discrepancies between the experimental data of proton-proton scattering and the pion theory of nuclear force is observed in the large splitting among triplet odd phase shifts which is phenomenologically analysed in terms of the strong attractive spin-orbit force.[10]

The full recoil pion theoretical potential calculated by Hoshizaki and Machida[11] which involves the contribution of two pion exchange in addition to one pion exchange gives too small a spin-orbit potential to explain the experimental data.

New substances besides pion might be needed for the explanation of it. In fact recent experiments have reported on some evidences of dipion resonances, tripion resonances—ρ, ω, η-mesons, etc.[12)~16] These bosons will be appropriate candidates for the substances. Some people[17] have already discussed the effect of the new bosons on the nuclear forces, especially in connection with the spin-orbit force. But the unsatisfactory situations are more or less observed in other forces and it seems necessary to have some systematic view for the introduction of the *new* bosons.

If the Sakata model is taken as the philosophical background, it will be a natural idea to introduce the bosons into the nuclear forces. For in the Sakata model the so called elementary particles and resonance levels are constructed of fundamental particles, proton, neutron and Λ-particle and their antiparticles, and both are treated on the same footing. That is, the bosons will be expected to take part in the strong interactions with the same qualification as the pion.

In this point of view Hoshizaki, Otsuki and two of the present authors (W. W. and M. Y.)[18] attempted to explain the nuclear force by the pion potential added by one boson exchange potentials (this is refered to as H-O-W-Y). They showed that one of the solutions which will reproduce Hamada's phenomenological potential[10] in triplet odd and singlet even states is the one pion exchange potential plus one boson exchange potential (hereafter we call this one-boson-exchange-potential model) where a scalar boson and a vector boson*) with the mass $4m_\pi$ are taken.

The one particle exchange potential, however, ought to give the multi-particle exchange contribution expressed by the ladder type Feynmann diagram, if the Schrödinger equation is solved exactly for that potential. In this sense the one particle exchange picture is somewhat smeared out in the H-O-W-Y.**)

We are interested in describing the nuclear force purely by the one particle exchange contributions of pions and bosons as far as possible.

Thus we set up the following one-boson-exchange-contribution model of nucleon-nucleon scattering which will be just derived from the assumptions A) ~D).

1) As the origin of the nuclear force some bosons are taken into account besides pion. These bosons have strangeness zero and isospin zero or one. We have ρ-meson,[12] ω-meson[13] and η-meson[14] with the confirmed evidences for their existence. There are further many evidences of bosons named as, A–B–C,[15] ζ,[16] etc.

2) We calculate only the lowest order Born term contribution of bosons and decompose the matrix elements of Born term into the partial amplitude without construction of nuclear potential. And then we derive the phase shifts by taking into account only the damping effect.

In the following sections we calculate the phase-shifts of the proton-proton scattering and compare with the experiments.

§ 3. Calculation of phase shifts of proton-proton scattering

In this paper, we consider the boson with spin zero and one. Expressions are shown in the case of the isoscalar boson alone. That of the isovector boson is simply given by multiplying the expression for isoscalar boson by $\tau_1\cdot\tau_2$. But the iso-spin of the boson is not entered into the discussion of proton-proton

*) Strictly speaking, two vector bosons are required—one is of isoscalar, and the other is of isovector—in order to explain neutron-proton scattering simultaneously with proton-proton scattering. But if we confine ourselves to the proton-proton scattering only, we can find the solution with one vector boson.

**) In addition to the fact that the potential treatment does involve the contribution from the energy off shell intermediate state, the non-static effects were not fully taken into consideration in H-O-W-Y.

scattering since $\tau_1\cdot\tau_2=1$ for the proton-proton system. Then possible interactions between boson and nucleon are given by

$$\begin{aligned}
&g_S\bar{\psi}\psi\phi \text{ or } \frac{f_S}{m_N}\bar{\psi}i\gamma_\mu\psi\partial_\mu\phi && \text{for scalar boson,}\\
&g_P\bar{\psi}i\gamma_5\psi\phi+\frac{f_P}{m_N}\bar{\psi}i\gamma_5\gamma_\mu\psi\partial_\mu\phi && \text{for pseudoscalar boson,}\\
&g_V\bar{\psi}i\gamma_\mu\psi\phi_\mu+\frac{f_V}{2m_N}\bar{\psi}\sigma_{\mu\nu}\psi\phi_{\mu\nu} && \text{for vector boson,}\\
&g_A\bar{\psi}i\gamma_5\gamma_\mu\psi\phi_\mu \text{ or } \frac{f_V}{2m_N}\bar{\psi}i\gamma_5\sigma_{\mu\nu}\psi\phi_{\mu\nu} && \text{for axialvector boson,}
\end{aligned}\tag{1}$$

where ψ is a nucleon wave function, ϕ or ϕ_μ is a boson wave function, $\phi_{\mu\nu}=\partial_\mu\phi_\nu-\partial_\nu\phi_\mu$, $\sigma_{\mu\nu}=\frac{1}{2i}(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)$ and m_N is nucleon mass.

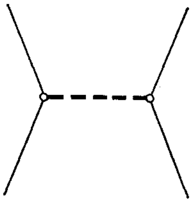

Fig. 1. The one-boson-exchange diagram of nucleon-nucleon scattering. The solid line denotes the nucleon and the bold broken line denotes the boson which includes the pion.

The matrix elements of the scattering amplitude corresponding to the various initial and final spin states can be expressed, taking only the lowest order Born terms corresponding to Fig. 1:[19]

For scalar boson with the scalar coupling (the vector coupling does not give the contribution)

$$\begin{aligned}
M^S_{11}&=\sqrt{\pi}\,g_S^2K[(1-\varepsilon x_0)^2\mathfrak{F}^0-\frac{1}{\sqrt{3}}\varepsilon^2Y_1^0],\\
M^S_{00}&=\sqrt{\pi}\,g_S^2K[(1-\varepsilon^2-2\varepsilon x_0+2\varepsilon^2x_0^2)\mathfrak{F}^0-\frac{1}{\sqrt{3}}\varepsilon^2Y_1^0],\\
M^S_{01}&=\sqrt{\pi}\sqrt{2}\,g_S^2K[-(1-\varepsilon x_0)\varepsilon\mathfrak{F}^1+\sqrt{\frac{2}{3}}\varepsilon^2Y_1^1],\\
M^S_{10}&=\sqrt{2\pi}\,g_S^2K[-(1-\varepsilon x_0)\varepsilon\mathfrak{F}^{-1}+\sqrt{\frac{2}{3}}\varepsilon^2Y_1^{-1}],\\
M^S_{1-1}&=\sqrt{\pi}\,g_S^2K\varepsilon^2\mathfrak{F}^{-2},\\
M^S_{ss}&=\sqrt{\pi}\,g_S^2K[(1+\varepsilon^2-2\varepsilon x_0)\mathfrak{F}^S+2\varepsilon Y_0^0].
\end{aligned}\tag{2}$$

For pseudoscalar boson

$$M^P_{11}=\sqrt{\pi}\,G_P^2K\varepsilon[(1-x_0)^2\mathfrak{F}^0-\frac{1}{\sqrt{3}}Y_1^0],$$

$$M^P_{00}=2\sqrt{\pi}\,G_P^2K\varepsilon[-(x_0-1)x_0\mathfrak{F}^0+\frac{1}{\sqrt{3}}Y_1^0],\tag{3}$$

$$M^P_{01}=\sqrt{2\pi}\,G_P{}^2K\varepsilon[(x_0-1)\mathfrak{Y}^1+\sqrt{\frac{2}{3}}\,Y_1{}^1],$$

$$M^P_{10}=-\sqrt{2\pi}\,G_P{}^2\,K\varepsilon[(x_0-1)\mathfrak{Y}^{-1}+\sqrt{\frac{2}{3}}Y_1{}^{-1}],$$

$$M^P_{1-1}=\sqrt{\pi}\,G_P{}^2\,K\varepsilon\mathfrak{Y}^{-2},$$

$$M^P_{ss}=2\sqrt{\pi}\,G_P{}^2\,K\varepsilon[(x_0-1)\mathfrak{Y}^s-Y_0{}^0],$$

where

$$G_P=g_P+2f_P, \tag{4}$$

For vector boson by the use of the equation of motion of the nucleon we can rewrite (1) as

$$G_V\,\bar{\psi}i\gamma_\mu\psi\,\phi_\mu+\frac{if_V}{m_N}[\bar{\psi}\partial_\mu\psi-\partial_\mu\bar{\psi}\cdot\psi]\phi_\mu, \tag{5}$$

where

$$G_V=g_V+2f_V \tag{6}$$

We divide M for vector boson into 3 parts: M^V, M^{VT} and M^T which are proportional to $G_V{}^2$, G_Vf_V and $f_V{}^2$ respectively. These are given by

$$\begin{aligned}
M^V_{11}&=-\sqrt{\pi}\,G_V{}^2K[\{1+\varepsilon+4\varepsilon x_0+\varepsilon(1+\varepsilon)x_0{}^2\}\mathfrak{Y}^0-\frac{1}{\sqrt{3}}\varepsilon(1+\varepsilon)\,Y_1{}^0],\\
M^V_{00}&=-\sqrt{\pi}\,G_V^2\,K[\{1-\varepsilon^2+8\varepsilon x_0-2\varepsilon(1-\varepsilon)x_0{}^2\}\mathfrak{Y}^0+\frac{2}{\sqrt{3}}\,\varepsilon(1-\varepsilon)\,Y_1{}^0],\\
M^V_{01}&=-\sqrt{2\pi}\,G_V^2\,K[\{2\varepsilon+\varepsilon(1+\varepsilon)x_0\}\mathfrak{Y}^1+\sqrt{\frac{2}{3}}\,\varepsilon(1+\varepsilon)\,Y_1{}^1],\\
M^V_{10}&=-\sqrt{2\pi}\,G_V^2K[\{4\varepsilon-\varepsilon(1-\varepsilon)x_0\}\mathfrak{Y}^{-1}-\sqrt{\frac{2}{3}}\,\varepsilon(1-\varepsilon)\,Y_1{}^{-1}],\\
M^V_{1-1}&=-\sqrt{\pi}\,G_V^2K\varepsilon(1+\varepsilon)\mathfrak{Y}^{-2},\\
M^V_{ss}&=-\sqrt{\pi}\,G_V^2\,K(1+6\varepsilon+\varepsilon^2)\mathfrak{Y}^s;
\end{aligned} \tag{7}$$

$$\begin{aligned}
M^{VT}_{11}&=4\sqrt{\pi}\,G_Vf_V\,K[(A+2\varepsilon x_0-B\varepsilon^2x_0{}^2)\mathfrak{Y}^0+\frac{1}{\sqrt{3}}\,B\varepsilon^2Y_1{}^0],\\
M^{VT}_{00}&=4\sqrt{\pi}\,G_Vf_V\,K[(A+B\varepsilon^2+2\varepsilon x_0-2B\,\varepsilon^2x_0{}^2)\mathfrak{Y}^0+\frac{2}{\sqrt{3}}B\varepsilon^2\,Y_1{}^0],\\
M^{VT}_{01}&=4\sqrt{2\pi}\,G_Vf_VK[(\varepsilon-B\varepsilon^2x_0)\mathfrak{Y}^1-\sqrt{\frac{2}{3}}B\varepsilon^2Y_1{}^1],
\end{aligned} \tag{8}$$

$$M_{10}^{VT}=4\sqrt{2\pi}\,G_V f_V\,K[(\varepsilon-B\varepsilon^2x_0)\mathfrak{F}^{-1}-\sqrt{\frac{2}{3}}B\varepsilon^2Y_1^{-1}],$$

$$M_{1-1}^{VT}=-4\sqrt{\pi}\,G_V f_V\,KB\varepsilon^2\mathfrak{F}^{-2},$$

$$M_{ss}^{VT}=4\sqrt{\pi}\,G_V f_V\,K[(A+2\varepsilon x_0-B\varepsilon^2)\mathfrak{F}^s-2\varepsilon Y_0^0]\ ;$$

$$M_{11}^{T}=-\frac{2\sqrt{\pi}\,f_V^2K}{m_N^2}[(\kappa+p^2x_0)(1-\varepsilon x_0)^2\mathfrak{F}^0-\frac{1}{\sqrt{3}}\{(\kappa+p^2x_0)\varepsilon^2-2\varepsilon p^2\}Y_1^0],$$

$$M_{00}^{T}=-\frac{2\sqrt{\pi}\,f_V^2K}{m_N^2}[(\kappa+p^2x_0)(1-\varepsilon^2-2\varepsilon x_0+2\varepsilon^2x_0^2)\mathfrak{F}^0$$
$$-\frac{1}{\sqrt{3}}\{2(\kappa+p^2x_0)\varepsilon^2-2\varepsilon p^2\}Y_1^0],$$

$$M_{01}^{T}=-\frac{2\sqrt{2\pi}\,f_V^2K}{m_N^2}[-(\kappa+p^2x_0)(\varepsilon-\varepsilon^2x_0)\mathfrak{F}^1+\sqrt{\frac{2}{3}}\{(\kappa+p^2x_0)\varepsilon^2-\varepsilon p^2\}Y_1^1],$$

$$M_{10}^{T}=-\frac{2\sqrt{2\pi}\,f_V^2K}{m_N^2}[-(\kappa+p^2x_0)(\varepsilon-\varepsilon^2x_0)\mathfrak{F}^{-1}+\sqrt{\frac{2}{3}}\{(\kappa+p^2x_0)\varepsilon^2-\varepsilon p^2\}Y_1^{-1}],\tag{9}$$

$$M_{1-1}^{T}=-\frac{2\sqrt{\pi}\,f_V^2K}{m_N^2}\varepsilon^2(\kappa+p^2x_0)\mathfrak{F}^{-2},$$

$$M_{ss}^{T}=-\frac{2\sqrt{\pi}\,f_V^2K}{m_N^2}[(\kappa+p^2x_0)(1+\varepsilon^2-2\varepsilon x_0)\mathfrak{F}^s$$
$$+\{2(\kappa+p^2x_0)\varepsilon-(1+\varepsilon^2)p^2\}Y_0^0].$$

For axialvector boson with axialvector coupling M are given by

$$M_{11}^{A}=\sqrt{\pi}\,g_A^2\,K[\{1+\varepsilon+4\varepsilon x_0+(1+\varepsilon)\varepsilon x_0^2-\left(\frac{2m_N}{m_B}\right)^2\varepsilon(x_0-1)^2\}\mathfrak{F}^0$$
$$-\frac{1}{\sqrt{3}}\{(1+\varepsilon)\varepsilon-\left(\frac{2m_N}{m_B}\right)^2\varepsilon\}Y_1^0],$$

$$M_{00}^{A}=\sqrt{\pi}\,g_A^2K[\{1-\varepsilon^2-8\varepsilon x_0-2(1-\varepsilon)\varepsilon x_0^2+2\left(\frac{2m_N}{m_B}\right)^2\varepsilon(x_0^2-x_0)\}\mathfrak{F}^0$$
$$+\frac{2}{\sqrt{3}}\{(1-\varepsilon)\varepsilon-\left(\frac{2m_N}{m_B}\right)^2\varepsilon\}Y_1^0],$$

$$M_{01}^{A}=\sqrt{2\pi}\,g_A^2K[\{2\varepsilon+(1+\varepsilon)\varepsilon x_0-\left(\frac{2m_N}{m_B}\right)^2\varepsilon(x_0-1)\}\mathfrak{F}^1$$
$$+\sqrt{\frac{2}{3}}\{(1+\varepsilon)\varepsilon-\left(\frac{2m_N}{m_B}\right)^2\varepsilon\}Y_1^1],$$

$$M_{10}^{A}=-\sqrt{2\pi}\,g_A^2\,K[\{4\varepsilon+(1-\varepsilon)\varepsilon x_0-\left(\frac{2m_N}{m_B}\right)^2\varepsilon(x_0-1)\}\mathfrak{F}^{-1}$$

$$+\sqrt{\frac{2}{3}}\{(1-\varepsilon)\varepsilon-\left(\frac{2m_N}{m_B}\right)^2\varepsilon\}Y_1^{-1}], \tag{10}$$

$$M^A_{1-1}=\sqrt{\pi}\,g_A^2K\{(1+\varepsilon)\varepsilon-\left(\frac{2m_N}{m_B}\right)^2\varepsilon\}\mathfrak{F}^{-2},$$

$$M^A_{ss}=-\sqrt{\pi}\,g_A{}^2K[\{(3+2\varepsilon+3\varepsilon^2)+2\left(\frac{2m_N}{m_B}\right)^2\varepsilon(x_0-1)\}\mathfrak{F}^s$$

$$-2\left(\frac{2m_N}{m_B}\right)^2\varepsilon\, Y_0^0];$$

and for axialvector boson with pseudotensor coupling

$$M^{PT}_{11}=-\frac{2\sqrt{\pi}f_A{}^2K\varepsilon}{m_N{}^2}[(\kappa+p^2x_0)(x_0-1)^2\mathfrak{F}^0-\frac{1}{\sqrt{3}}\{\kappa+p^2x_0-2p^2\}Y_1^0],$$

$$M^{PT}_{00}=-\frac{4\sqrt{\pi}f_A{}^2K\varepsilon}{m_N{}^2}[-(\kappa+p^2x_0)(x_0-1)x_0\mathfrak{F}^0+\frac{1}{\sqrt{3}}\{\kappa+p^2x_0-p^2\}Y_1^0],$$

$$M^{PT}_{01}=-\frac{2\sqrt{2\pi}f_A{}^2K\varepsilon}{m_N{}^2}[(\kappa+p^2x_0)(x_0-1)\mathfrak{F}^1+\sqrt{\frac{2}{3}}\{\kappa+p^2x_0-p^2\}Y_1^1],$$

$$M^{PT}_{10}=\frac{2\sqrt{2\pi}f_A{}^2K\varepsilon}{m_N{}^2}[(\kappa+p^2x_0)(x_0-1)\mathfrak{F}^{-1}+\sqrt{\frac{2}{3}}\{\kappa+p^2x_0-p^2\}Y_1^{-1}],$$

$$M^{PT}_{1-1}=-\frac{2\sqrt{\pi}f_A{}^2K\varepsilon}{m_N{}^2}(\kappa+p^2x_0)\mathfrak{F}^{-2}, \tag{11}$$

$$M^{PT}_{ss}=-\frac{4\sqrt{\pi}f_A{}^2K\varepsilon}{m_N{}^2}[(\kappa+p^2x_0)(x_0-1)\mathfrak{F}^s-\{\kappa+p^2x_0-p^2\}Y_0^0].$$

In (2) ∼ (11), the following abbreviations are used

$$\mathfrak{F}^0=\sum_{l=1,\text{ odd}}^{\infty}(2l+1)^{1/2}Q_l(x_0)Y_l^0(\theta,\varphi),$$

$$\mathfrak{F}^{\pm1}=(x_0{}^2-1)^{1/2}\sum_{l=1,\text{odd}}^{\infty}\left[\frac{2l+1}{l(l+1)}\right]^{1/2}Q_l^1(x_0)Y_l^{\pm1}(\theta,\varphi),$$

$$\mathfrak{F}^{\pm2}=(x_0{}^2-1)\sum_{l=3,\text{odd}}\left[\frac{2l+1}{(l-1)l(l+1)(l+2)}\right]^{1/2}Q_l^2(x_0)Y_l^{\pm2}(\theta,\varphi),$$

$$\mathfrak{F}^s=\sum_{l=0,\text{even}}^{\infty}(2l+1)^{1/2}Q_l(x_0)Y_l^0(\theta,\varphi), \tag{12}$$

$$K=(E+m_N)^2/2p^2E,$$

$$\varepsilon=(E-m_N)/(E+m_N),$$

$$A=(2E-m_N)/m_N,$$

$$B=(2E+m_N)/m_N,$$

$$\kappa=3p^2+2m_N{}^2,$$

where E and p are energy and momentum of nucleon in the barycentic system respectively. Suffixes of M matrices correspond to transitions between states with z-component of spin j and i in the case of triplet state and $i=s$, $j=s$ in the singlet case. $Q_l^m(x_0)$ is the associate Legendre function of the second kind and the argument x_0 is defined as

$$x_0=1+m_B^2/m_N T=1+m_B^2/2p^2.$$

T is the incident laboratory kinetic energy of nucleon. Here we note that

$$\frac{1}{f_V^2}M^T=-\frac{4}{g_S^2}M^S+O(p^2/m_N^2) \tag{13}$$

and

$$\frac{1}{f_A^2}M^{PT}=-\frac{4}{G_P^2}M^P+O(p^2/m_N^2).$$

From M_{ij} we can obtain the transition amplitudes between the initial and final states specified by the possible total and orbital angular momenta J and l, according to the procedure given by Stapp, Ypsilantis and Metropolis.[20] The notations used in the following are the same as in reference 20);

α_l: transition amplitude between singlet states with orbital angular momentum l,

α_{lJ}: transition amplitude between triplet states with total angular momentum J and orbital angular momentum l,

α^J: transition amplitude between triplet states with total angular momentum J and $\Delta l=\pm 2\,(l_i=J\pm 1\rightarrow l_f=J\mp 1)$.

For scalar boson

$$\alpha^S_{l,l+1}=\frac{ig_S^2\,Kp}{2(2l+3)^2}[4(l+1)(l+2)\varepsilon^2 Q_{l+2}(x_0)-2(2l+3)^2\varepsilon Q_{l+1}(x_0)$$
$$+\{(2l+3)^2+\varepsilon^2\}Q_l(x_0)],$$

$$\alpha^S_{l,l}=\frac{ig_S^2Kp}{2(2l+1)}[-2l\varepsilon Q_{l+1}(x_0)+(2l+1)(1+\varepsilon^2)Q_l(x_0)-2(l+1)\varepsilon Q_{l-1}(x_0)],$$

$$\alpha^S_{l,l-1}=\frac{ig_S^2Kp}{2(2l-1)^2}[\{(2l-1)^2+\varepsilon^2\}Q_l(x_0)-2(2l-1)^2\varepsilon Q_{l-1}(x_0)$$
$$+4l(l-1)\varepsilon^2 Q_{l-2}(x_0)], \tag{14}$$

$$\alpha^{SJ}=\frac{ig_S^2Kp}{(2J+1)^2}[J(J+1)]^{1/2}\varepsilon^2[Q_{J+1}(x_0)-Q_{J-1}(x_0)],$$

$$\alpha_l^S=\frac{ig_S^2Kp}{2}[(1+\varepsilon^2-2\varepsilon x_0)Q_l(x_0)+2\varepsilon\delta_{l0}].$$

For pseudoscalar boson

$$\alpha^P_{l,l+1} = -\frac{iG_P{}^2Kp\varepsilon}{2l+3}[Q_{l+1}(x_0) - Q_l(x_0)],$$

$$\alpha^P_{l,l} = -\frac{iG_P{}^2Kp\varepsilon}{2l+1}[lQ_{l+1}(x_0) - (2l+1)Q_l(x_0) + (l+1)Q_{l-1}(x_0)],$$

$$\alpha^P_{l,l-1} = -\frac{iG_P{}^2Kp\varepsilon}{2l-1}[Q_l(x_0) - Q_{l-1}(x_0)],$$

$$\alpha^{PJ} = -\frac{iG_P{}^2Kp\varepsilon}{2J+1}[J(J+1)]^{1/2}[Q_{J+1}(x_0) - 2Q_J(x_0) + Q_{J-1}(x_0)], \tag{15}$$

$$\alpha_l{}^P = iG_P{}^2Kp\varepsilon[(x_0-1)Q_l(x_0) - \delta_{l0}].$$

For vector boson

$$\alpha^V_{l,l+1} = -\frac{iG_V{}^2Kp}{2(2l+3)^2}[4(l+1)(l+2)\varepsilon^2Q_{l+2}(x_0) + 4(2l+3)(3l+4)\varepsilon Q_{l+1}(x_0) + \{(2l+3)+\varepsilon\}^2Q_l(x_0)],$$

$$\alpha^V_{l,l} = -\frac{iG_V{}^2Kp}{2(2l+1)}[4l\varepsilon Q_{l+1}(x_0) + (2l+1)(1+\varepsilon)^2Q_l(x_0) + 4(l+1)\varepsilon Q_{l-1}(x_0)],$$

$$\alpha^V_{l,l-1} = -\frac{iG_V{}^2Kp}{2(2l-1)^2}[\{(2l-1)-\varepsilon\}^2Q_l(x_0) + 4(2l-1)(3l-1)\varepsilon Q_{l-1}(x_0) + 4l(l-1)\varepsilon^2Q_{l-2}(x_0)], \tag{16}$$

$$\alpha^{VJ} = \frac{iG_V{}^2Kp}{(2J+1)^2}[J(J+1)]^{1/2}[\{(2J+1)-\varepsilon\}\varepsilon Q_{J+1}(x_0) - 2(2J+1)\varepsilon Q_J(x_0) + \{(2J+1)+\varepsilon\}\varepsilon Q_{J-1}(x_0)],$$

$$\alpha_l{}^V = -\frac{iG_V{}^2Kp}{2}(1+6\varepsilon+\varepsilon^2)Q_l(x_0);$$

$$\alpha^{VT}_{l,l+1} = \frac{2iG_Vf_VKp}{(2l+3)^2}[-4(l+1)(l+2)B\varepsilon^2Q_{l+2}(x_0) + 2(2l+3)^2\varepsilon Q_{l+1}(x_0) + \{(2l+3)^2A - B\varepsilon^2\}Q_l(x_0)],$$

$$\alpha^{VT}_{l,l} = \frac{2iG_Vf_VKp}{2l+1}[2l\varepsilon Q_{l+1}(x_0) + (2l+1)(A-B\varepsilon^2)Q_l(x_0) + 2(l+1)\varepsilon Q_{l-1}(x_0)],$$

$$\alpha^{VT}_{l,l-1} = \frac{2iG_Vf_VKp}{(2l-1)^2}[\{(2l-1)^2A - B\varepsilon^2\}Q_l(x_0) + 2(2l-1)^2\varepsilon Q_{l-1}(x_0) - 4l(l-1)B\varepsilon^2Q_{l-2}(x_0)], \tag{17}$$

$$\alpha^{VTJ} = -\frac{4iG_Vf_VKp}{(2J+1)^2}[J(J+1)]^{1/2}B\varepsilon^2[Q_{J+1}(x_0) - Q_{J-1}(x_0)],$$

$$\alpha_l^{VT}=2iG_Vf_VKp[(A+2\varepsilon x_0-B\varepsilon^2)Q_l(x_0)-2\varepsilon\delta_{l0}];$$

$$\alpha^T_{l,l+1}=-\frac{if_V^2Kp}{m_N^2(2l+3)^2}\Big[\frac{4(l+1)(l+2)(l+3)}{2l+5}\varepsilon^2p^2Q_{l+3}(x_0)$$
$$+\{4(l+1)(l+2)\kappa\varepsilon^2-2(l+2)(2l+3)\varepsilon^2p^2\}Q_{l+2}(x_0)$$
$$+\Big\{\frac{(l+1)(2l+3)[(2l+3)^2-2]}{(2l+1)(2l+5)}\varepsilon^2p^2+\frac{(l+1)(2l+3)^2}{2l+1}p^2$$
$$-2(2l+3)^2\kappa\varepsilon\Big\}Q_{l+1}(x_0)$$
$$+\{(2l+3)^2\kappa+\varepsilon^2\kappa-2(l+1)(2l+3)\varepsilon p^2\}Q_l(x_0)$$
$$+\frac{l}{2l+1}\{(2l+3)^2+\varepsilon^2\}p^2Q_{l-1}(x_0)\Big],$$

$$\alpha^T_{l,l}=-\frac{if_V^2Kp}{m_N^2(2l+1)}\Big[-\frac{2l(l+2)}{2l+3}\varepsilon p^2Q_{l+2}(x_0)+\{(l+1)(1+\varepsilon^2)p^2-2l\kappa\varepsilon\}Q_{l+1}(x_0)$$
$$+(2l+1)\{\kappa(1+\varepsilon^2)-\frac{4l(l+1)}{(2l-1)(2l+3)}\varepsilon p^2\}Q_l(x_0)$$
$$-\{2(l+1)\varepsilon\kappa-l(1+\varepsilon^2)p^2\}Q_{l-1}(x_0)-\frac{2(l-1)(l+1)}{2l-1}\varepsilon p^2Q_{l-2}(x_0)\Big],$$

$$\alpha^T_{l,l-1}=-\frac{if_V^2Kp}{m_N^2(2l-1)^2}\Big[\frac{l+1}{2l+1}\{(2l-1)^2+\varepsilon^2\}p^2Q_{l+1}(x_0)$$
$$+\{(2l-1)^2\kappa+\varepsilon^2\kappa-2l(2l-1)\varepsilon p^2\}Q_l(x_0)$$
$$+\Big\{\frac{l[(2l-1)^2-2]}{(2l+1)(2l-3)}\varepsilon^2p^2+\frac{l(2l-1)^2}{2l+1}p^2-2(2l-1)^2\kappa\varepsilon\Big\}Q_{l-1}(x_0)$$
$$+\{4(l-1)l\kappa\varepsilon^2-2(l-1)(2l-1)\varepsilon p^2\}Q_{l-2}(x_0)$$
$$+\frac{4(l-2)(l-1)l}{2l-3}\varepsilon^2p^2Q_{l-3}(x_0)\Big],\qquad(18)$$

$$\alpha^{TJ}=-\frac{if_V^2Kp}{m_N^2(2J+1)^2}[J(J+1)]^{1/2}\Big[\frac{2(J+2)}{2J+3}\varepsilon^2p^2Q_{J+2}(x_0)+2\kappa\varepsilon^2Q_{J+1}(x_0)$$
$$-\frac{2(2J+1)}{(2J-1)(2J+3)}\varepsilon^2p^2Q_J(x_0)-2\varepsilon^2\kappa^2Q_{J-1}(x_0)-\frac{2(J-1)}{2J-1}\varepsilon^2p^2Q_{J-2}(x_0)\Big],$$

$$\alpha_l^T=-\frac{if_V^2Kp}{m_N^2}[(\kappa+p^2x_0)(1+\varepsilon^2-2\varepsilon x_0)Q_l(x_0)$$
$$+\{2(\kappa+p^2x_0)\varepsilon-(1+\varepsilon^2)p^2\}\delta_{l0}].$$

For axialvector boson with axialvector coupling,

$$\alpha^A_{l,l+1}=\frac{ig_A^2Kp}{2(2l+3)^2}[4(l+1)(l+2)\varepsilon^2Q_{l+2}(x_0)-\{4l(2l+3)$$

$$-2(2l+3)\left(\frac{2m_N}{m_B}\right)^2\}\varepsilon Q_{l+1}(x_0)+\{(2l+3+\varepsilon)^2-2(2l+3)\left(\frac{2m_N}{m_B}\right)^2\varepsilon\}Q_l(x_0)],$$

$$\alpha^A_{l,l}=\frac{ig_A^2Kp}{2(2l+1)}[2l\{2+\left(\frac{2m_N}{m_B}\right)^2\}\varepsilon Q_{l+1}(x_0)+(2l+1)\{(1+\varepsilon)^2-2\left(\frac{2m_N}{m_B}\right)^2\varepsilon\}Q_l(x_0)$$

$$+2(l+1)\{2+\left(\frac{2m_N}{m_B}\right)^2\}\varepsilon Q_{l-1}(x_0)],$$

$$\alpha^A_{l,l-1}=\frac{ig_A^2Kp}{2(2l-1)^2}[\{(2l-1-\varepsilon)^2+2(2l-1)\left(\frac{2m_N}{m_B}\right)^2\varepsilon\}Q_l(x_0)$$

$$+\{-4(l+1)(2l-1)+2(2l-1)\left(\frac{2m_N}{m_B}\right)^2\}\varepsilon Q_{l-1}(x_0)+4(l-1)l\varepsilon^2Q_{l-2}(x_0)],$$

$$\alpha^{AJ}=-\frac{ig_A^2Kp}{2(2J+1)}[J(J+1)]^{1/2}[2\{1-\frac{\varepsilon}{2J+1}-\left(\frac{2m_N}{m_B}\right)^2\}\varepsilon Q_{J+1}(x_0)$$

$$+4\{3+\left(\frac{2m_N}{m_B}\right)^2\}\varepsilon Q_J(x_0)+2\{1+\frac{\varepsilon}{2J+1}-\left(\frac{2m_N}{m_B}\right)^2\}\varepsilon Q_{J-1}(x_0)],$$

$$\alpha_l^A=-\frac{ig_A^2Kp}{2}[\{3+2\varepsilon+3\varepsilon^2+2\left(\frac{2m_N}{m_B}\right)^2\varepsilon(x_0-1)\}Q_l(x_0)-2\left(\frac{2m_N}{m_B}\right)^2\varepsilon\delta_{l0}]. \tag{19}$$

For axialvector boson with pseudotensor coupling

$$\alpha^{PT}_{l,l+1}=\frac{2if_A^2Kp\varepsilon}{m_N^2(2l+3)}\left[\frac{l+2}{2l+3}p^2Q_{l+2}(x_0)+\left(\kappa-\frac{l+1}{2l+1}p^2\right)Q_{l+1}(x_0)\right.$$

$$\left.-\left(\kappa-\frac{l+1}{2l+3}p^2\right)Q_l(x_0)-\frac{l}{2l+1}p^2Q_{l-1}(x_0)\right],$$

$$\alpha^{PT}_{l,l}=\frac{2if_A^2Kp\varepsilon}{m_N^2(2l+1)}\left[\frac{l(l+2)}{2l+3}p^2Q_{l+2}(x_0)+\{l\kappa-(l+1)p^2\}Q_{l+1}(x_0)\right.$$

$$-\{(2l+1)\kappa-l(l+1)\left(\frac{1}{2l+3}+\frac{1}{2l-1}\right)p^2\}Q_l(x_0)+\{(l+1)\kappa-lp^2\}$$

$$\left.\times Q_{l-1}(x_0)+\frac{(l-1)(l+1)}{2l-1}p^2Q_{l-2}(x_0)\right],$$

$$\alpha^{PT}_{l,l-1}=\frac{2if_A^2Kp\varepsilon}{m_N^2(2l-1)}\left[\frac{l+1}{2l+1}p^2Q_{l+1}(x_0)+\left(\kappa-\frac{l}{2l-1}p^2\right)Q_l(x_0)\right.$$

$$\left.-\left(\kappa-\frac{l}{2l+1}p^2\right)Q_{l-1}(x_0)-\frac{l-1}{2l-1}p^2Q_{l-2}(x_0)\right], \tag{20}$$

$$\alpha^{PTJ}=\frac{2if_A^2Kp\varepsilon}{m_N^2(2J+1)}[J(J+1)]^{1/2}\left[\frac{J+2}{2J+3}p^2Q_{J+2}(x_0)+\{\kappa-\frac{2(J+1)}{2J+1}p^2\}Q_{J+1}(x_0)\right.$$

$$\left.-\{2\kappa-\left(\frac{J+1}{2J+3}+\frac{J}{2J-1}\right)p^2\}Q_J(x_0)+\{\kappa-\frac{2J}{2J+1}p^2\}Q_{J-1}(x_0)+\frac{J-1}{2J-1}p^2Q_{J-2}(x_0)\right],$$

$$\alpha_l^{PT} = -\frac{2if_A^2 Kp\varepsilon}{m_N^2}[(\kappa+p^2x_0)(x_0-1)Q_l(x_0)-(\kappa+p^2x_0-p^2)\delta_{l0}].$$

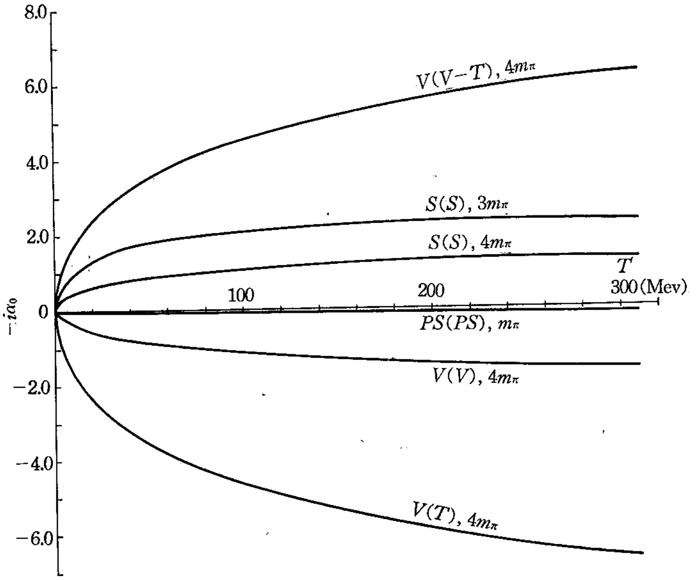

Fig. 2-1.

Fig. 2-1~14. The scattering amplitude $-i\alpha$ calculated for pion, vector boson with mass $4m_\pi$ and scalar boson with mass $3m_\pi$ and $4m_\pi$.

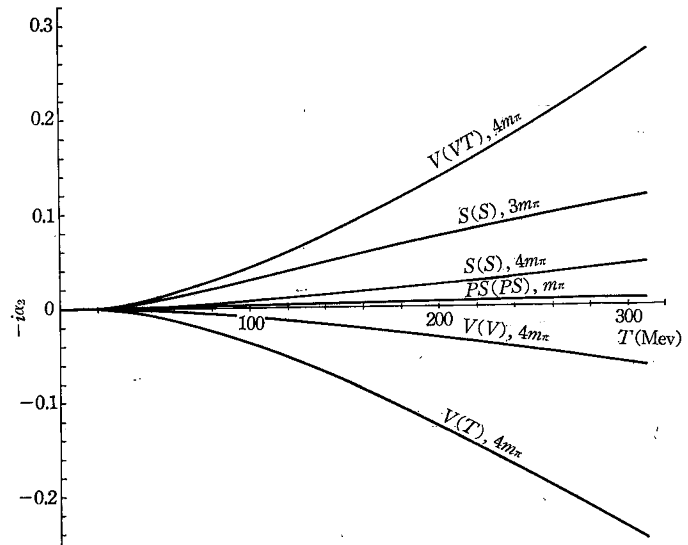

Fig. 2-2.

In Figs. 2–1∼2–14 we show the calculated value of $-i\alpha$ for pion, for scalar boson with $m_s=3m_\pi$ and $4m_\pi$ and for vector boson with $m_V=4m_\pi$ taking $g^2=1$ and $f^2=1$. They are used in the discussion in the next section.

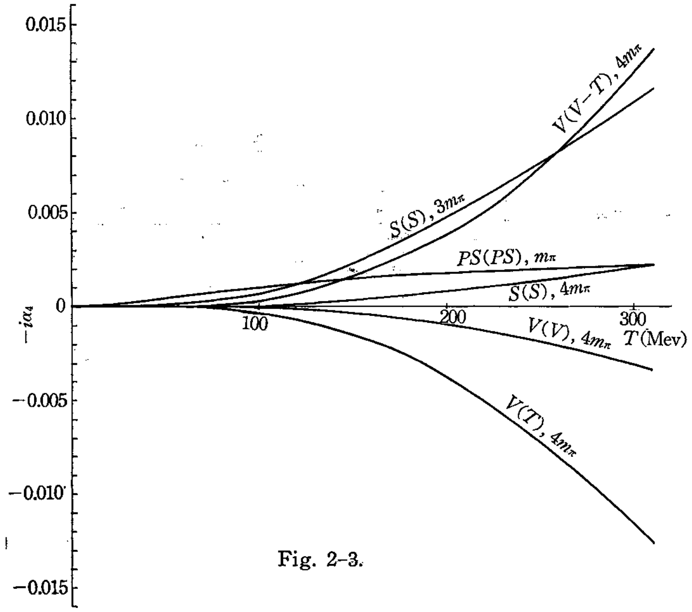

Fig. 2-3.

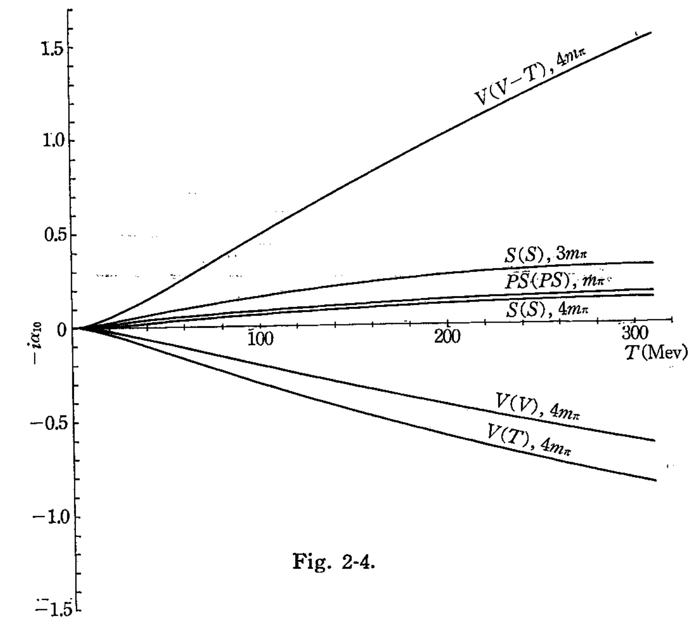

Fig. 2-4.

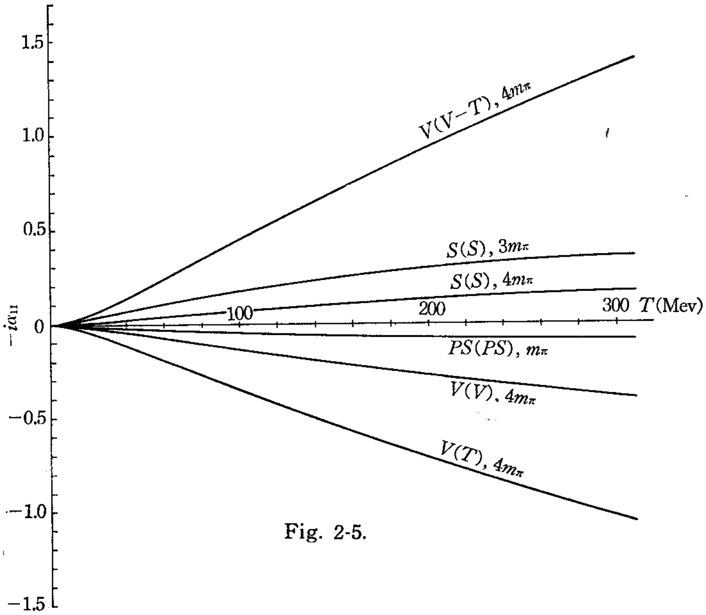
1.5
1.0
0.5
0
−0.5
−1.0
−1.5
$-i\alpha_{11}$
100
200
300
T(Mev)
V(V−T), $4m_\pi$
S(S), $3m_\pi$
S(S), $4m_\pi$
PS(PS), m_π
V(V), $4m_\pi$
V(T), $4m_\pi$

Fig. 2-5.

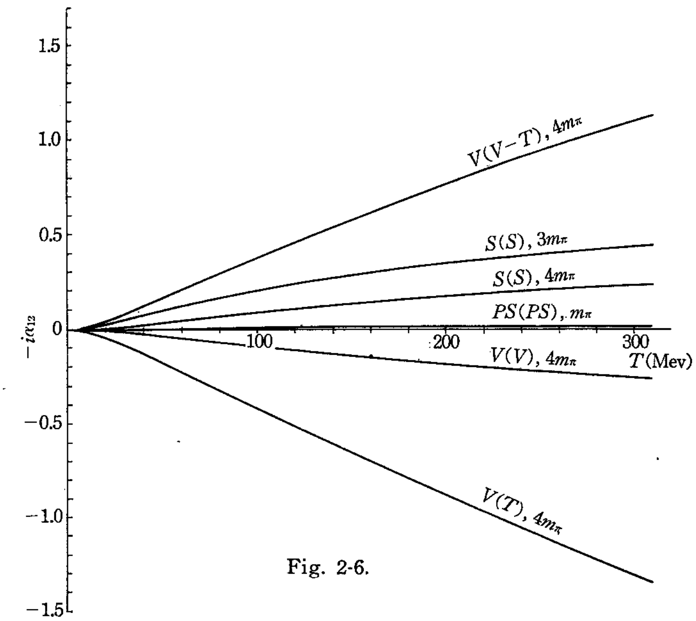
1.5
1.0
0.5
0
−0.5
−1.0
−1.5
$-i\alpha_{12}$
100
200
300
T(Mev)
V(V−T), $4m_\pi$
S(S), $3m_\pi$
S(S), $4m_\pi$
PS(PS), m_π
V(V), $4m_\pi$
V(T), $4m_\pi$

Fig. 2-6.

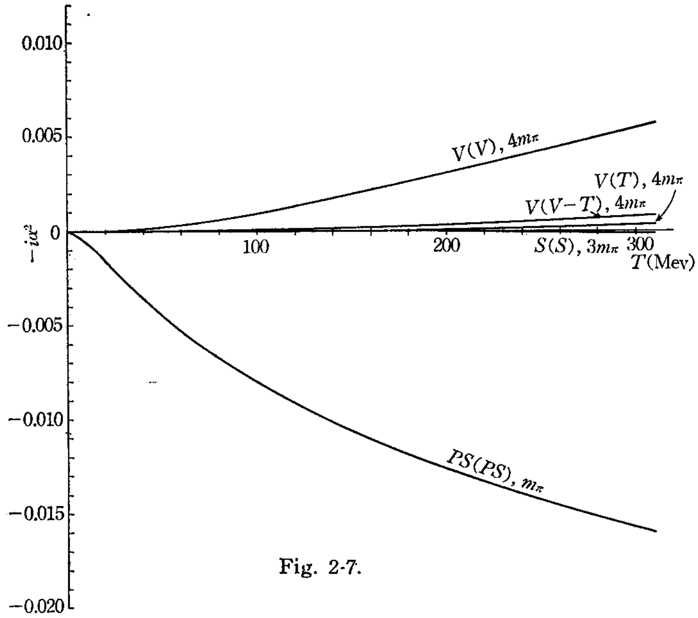
0.010
0.005
0
−0.005
−0.010
−0.015
−0.020
$-i\alpha^2$
100
200
300
T(Mev)
V(V), 4mπ
V(T), 4mπ
V(V−T), 4mπ
S(S), 3mπ
PS(PS), mπ

Fig. 2-7.

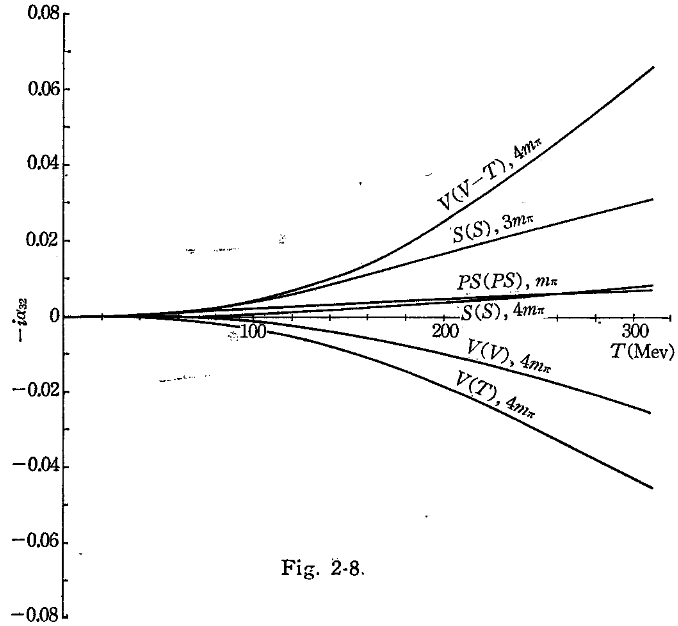
0.08
0.06
0.04
0.02
0
−0.02
−0.04
−0.06
−0.08
$-i\alpha_{32}$
100
200
300
T(Mev)
V(V−T), 4mπ
S(S), 3mπ
PS(PS), mπ
S(S), 4mπ
V(V), 4mπ
V(T), 4mπ

Fig. 2-8.

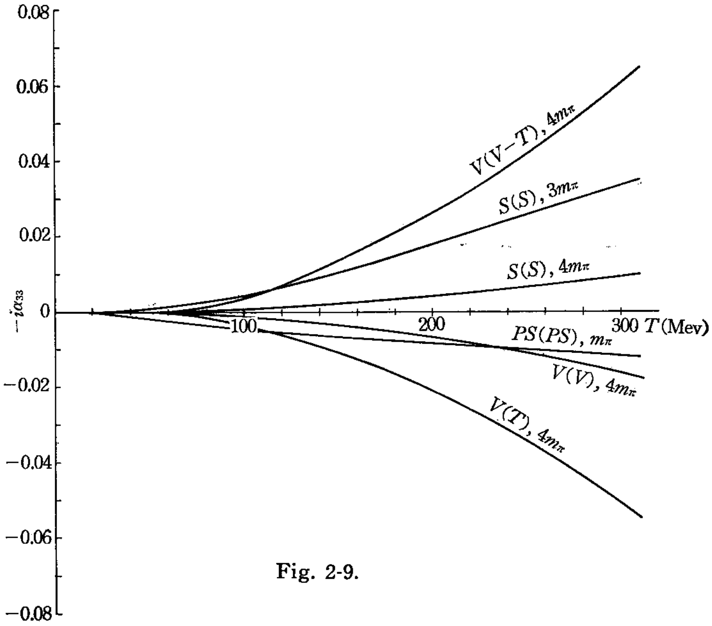

Fig. 2-9.

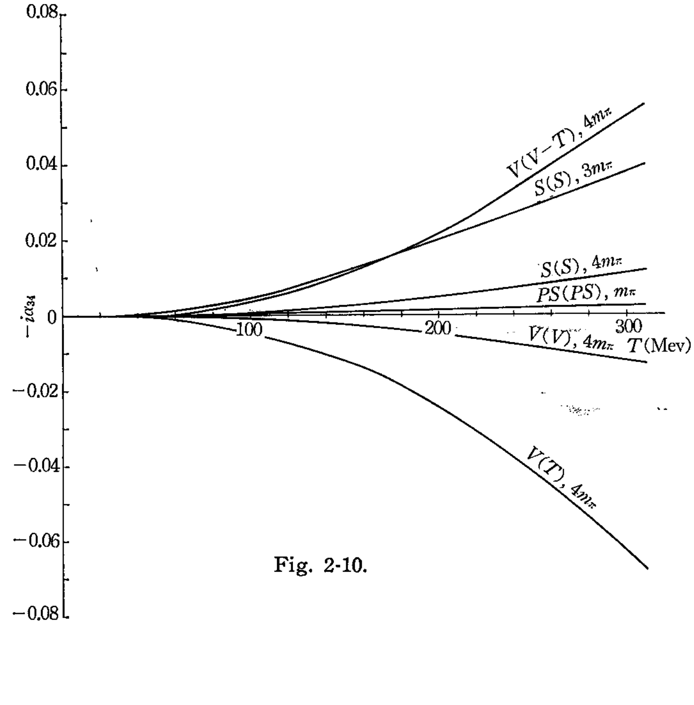

Fig. 2-10.

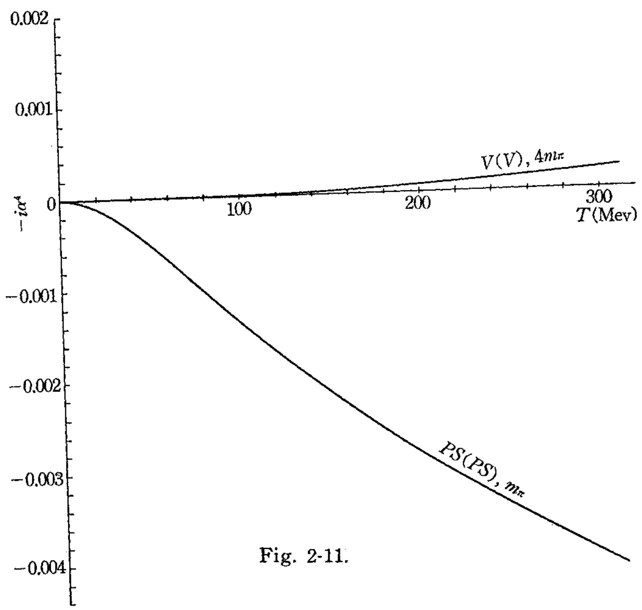

Fig. 2-11.

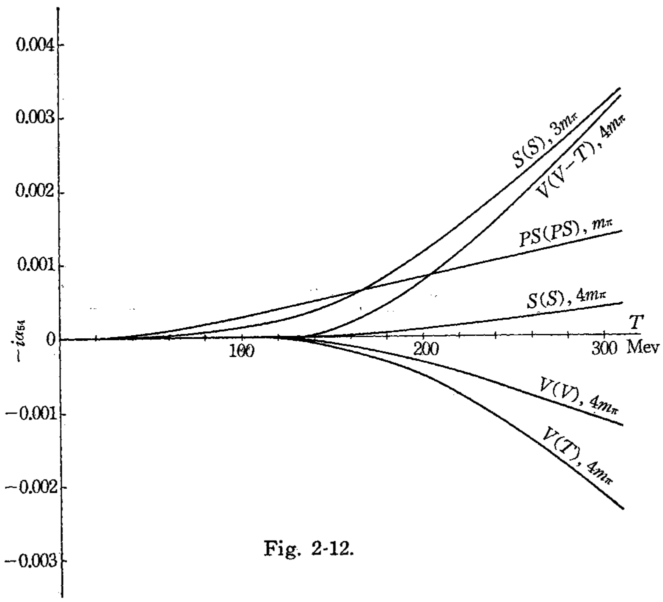

Fig. 2-12.

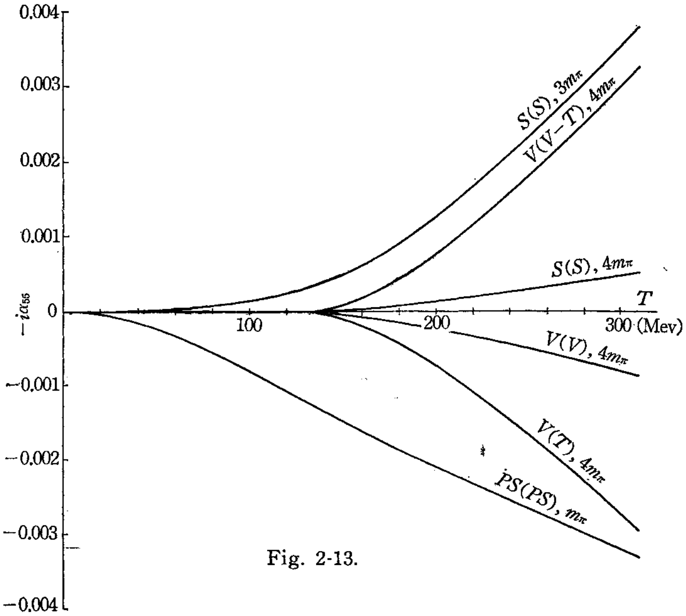

Fig. 2-13.

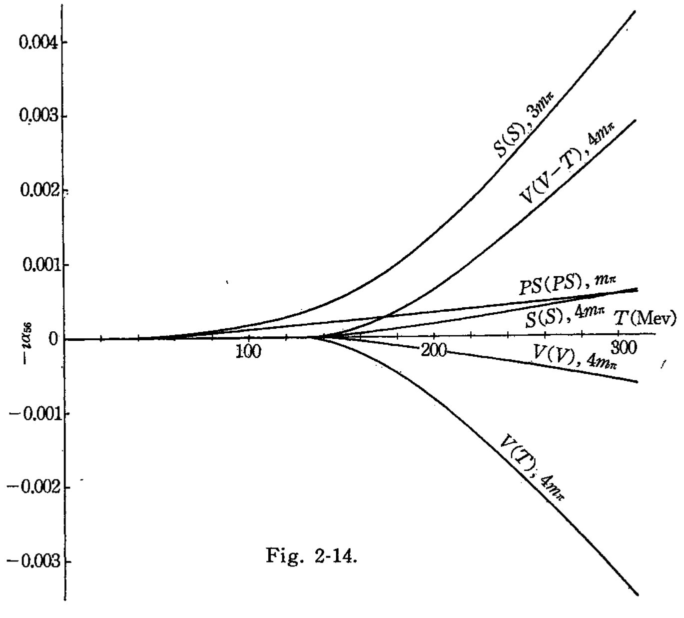

Fig. 2-14.

The damping effect, which arises from the contribution of repetition of the process on the mass shell, is taken into account if we relate the above to S-matrix as

$$S=\frac{1+(i/2)T}{1-(i/2)T}\,, \tag{21}$$

where

$iT=\alpha_l$ and $\alpha_{l,J}$ for singlet states and for triplet states with $l=J$ respectively, and

$$iT=\begin{pmatrix} \alpha_{J-1,J} & \alpha^J \\ \alpha^J & \alpha_{J+1,J} \end{pmatrix} \text{ for triplet states with } J=l\pm 1.$$

The phase shifts and coupling parameters can be expressed by the S-matrix,

$$\begin{aligned} &S=\exp(2i\ {}^1\delta_J{}^l) \text{ for singlet states} \\ &S=\exp(2i\ {}^3\delta_J{}^l) \text{ for triplet states with } l=J, \end{aligned} \tag{22}$$

which are reduced to

$$\tan {}^1\delta_J{}^l=\alpha_l/2i$$

and (23)

$$\tan {}^3\delta_J{}^l=\alpha_{l,l}/2i$$

and for triplet states with $J=l\pm 1$, we have

$$S=\begin{pmatrix} (1-\rho_J{}^2)^{1/2}\exp(2i\,{}^3\delta_J^{l=J-1}) & i\rho_J\exp[i({}^3\delta_J^{l=J-1}+{}^3\delta_J^{l=J+1})] \\ i\rho_J\exp[i({}^3\delta_J^{l=J-1}+{}^3\delta_J^{l=J+1})] & (1-\rho_J{}^2)^{1/2}\exp(2i\,{}^3\delta_J^{l=J+1}) \end{pmatrix} \tag{24}$$

and S is rewritten as,

$$S=\frac{1}{D}\begin{pmatrix} 1+\frac{1}{4}(\alpha^J)^2-\frac{1}{4}\alpha_{J+1,J}\,\alpha_{J-1,J}-\frac{1}{2}(\alpha_{J+1,J}-\alpha_{J-1,J}), & \alpha^J \\ \alpha^J & 1+\frac{1}{4}(\alpha^J)^2-\frac{1}{4}\alpha_{J+1,J}\,\alpha_{J-1,J}+\frac{1}{2}(\alpha_{J+1,J}-\alpha_{J-1,J}) \end{pmatrix} \tag{25}$$

where

$$D=1-\frac{1}{2}(\alpha_{J+1,J}+\alpha_{J-1,J})+\frac{1}{4}\alpha_{J+1,J}\,\alpha_{J-1,J}-\frac{1}{4}(\alpha^J)^2. \tag{26}$$

${}^1\delta_J{}^l$, ${}^3\delta_J{}^l$ and ρ_J are nuclear bar phase shifts and coupling parameters respectively.[20]

§ 4. Analysis of experimental data

The phase shift analyses of the proton-proton scattering data below 345 Mev have been performed by several groups and now it seems that a reasonable set of phase shift solution have been found. Phase shift analyses are desired to be such as to guarantee that the phase shifts are given by OPEC for very large impact parameters. There have been made two such types of phase shift analyses. One is the "modified phase shift analysis" at each energy by Berkeley group[19),21)] and Perring.[22)] They have obtained several sets of phase shifts at each energy, but a reasonable set could be rather uniquely selected. The other is the "gradient search analysis for phase shift" by Yale group and Berkeley group.[23)] Yale group have found the best one named YLAM. In addition to satisfying several reasonable criterions the YLAM is in good agreement with a reasonable set of phase shifts obtained by Berkeley group and Perring. Therefore we take YLAM as the experimental data and discuss our model with this set of phase shifts.

In H-O-W-Y the set of pion, scalar boson and vector boson*) is the simplest solution that may well reproduce Hamada's phenomenological potential, if the nuclear potential is assumed to be given as a sum of one-boson-exchange-potential. There $4m_\pi$ is taken as the mass of the scalar boson and the vector boson for the sake of avoiding the tiresome numerical arguments. For such set of bosons it has been shown that Hamada's potential is reproducible if we take the following coupling constants

$$\begin{aligned} &g_S^2=5.0,\\ &G_V^2+G_V'^2=42.3 \text{ or } g_V^2+g_V'^2=2.4, \\ &f_V^2+f_V'^2=6.2,\\ &G_V f_V+G_V' f_V'=15.8, \end{aligned} \tag{27}$$

where the primed ones are the coupling constants of isovector boson and the non-primed are those of isoscalar boson respectively.

Our present model may be expected to show a similar nature to the OBEP model of H-O-W-Y in qualitative features.

In this connection and also from the experimental evidence of the scalar and vector bosons with masses $2\sim 6m_\pi$ we consider a set of vector bosons of mass $4m_\pi$ and a scalar boson of mass $3m_\pi$ or $4m_\pi$ in addition to the pion. As discussed later, the set may be regarded to effectively give the combined contributions from many bosons with mass $2m_\pi\sim 6m_\pi$, some of which have been found as ρ-meson, ω-meson, etc. Now the free parameters in the present case are f_V, G_V and f_S. In the following we put simply $\overline{G_V f_V}=\sqrt{G_V^2 f_V^2}$, where $\overline{G_V f_V}$ is the coupling constant for the mixed term of vector coupling and tensor coupling. But if more than one vector boson participate in the nuclear forces (in fact this is really the case), then this relation will not be satisfied since $\overline{G_V f_V}$ are given by

$$\overline{G_V f_V}\equiv\sum_i G_{V_i} f_{V_i} \tag{28}$$

*) cf. footnote on page 995.

which is in general not equal to and not greater than $\sqrt{\sum_i G_{V_i}^2}\cdot\sqrt{\sum_i f_{V_i}^2}$. However we examine rather restricted case of $G_V f_V = \sum_i G_{V_i} f_{V_i}$ for simplicity and the result of H-OW-Y that satisfies this approximately.

From the impact parameter arguments[24] and the success of Taketani's principle for the low energy nucleon-nucleon scattering, it will be natural to consider that the larger the impact parameter is, the more successful the present model is. But it is not easy to say *a priori* to what extent the present model may describe well the phenomenon, since we have no knowledge of the contributions of other higher mass bosons than those taken here. It should be found *a posteriori* from the results.

In view of the status of the present experimental data, we start from the

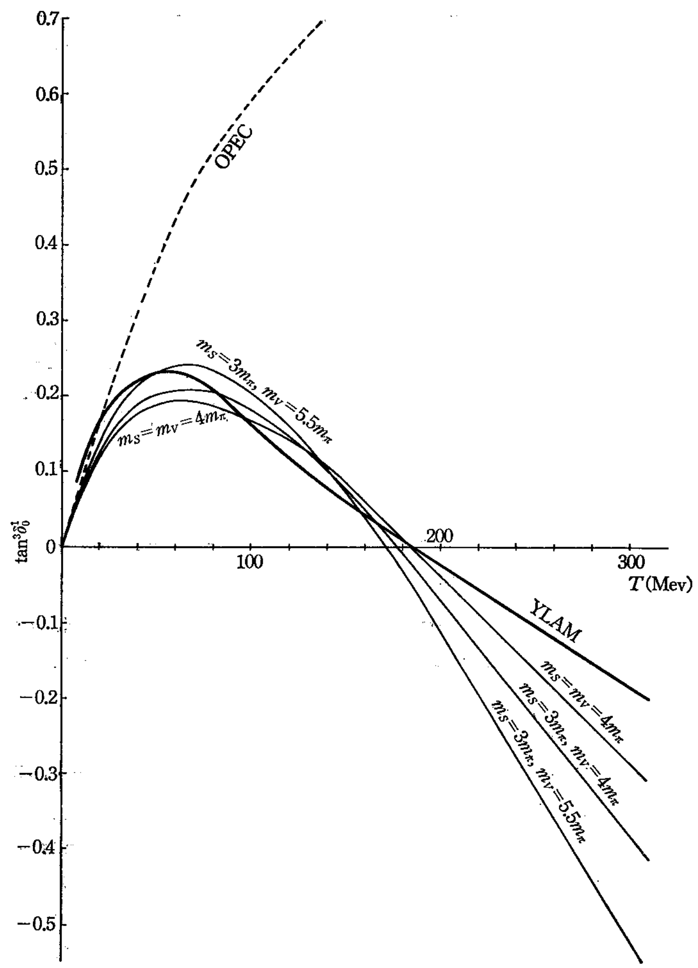

Fig. 3-1. The YLAM and calculated phase shifts for $^3P_0$ state.

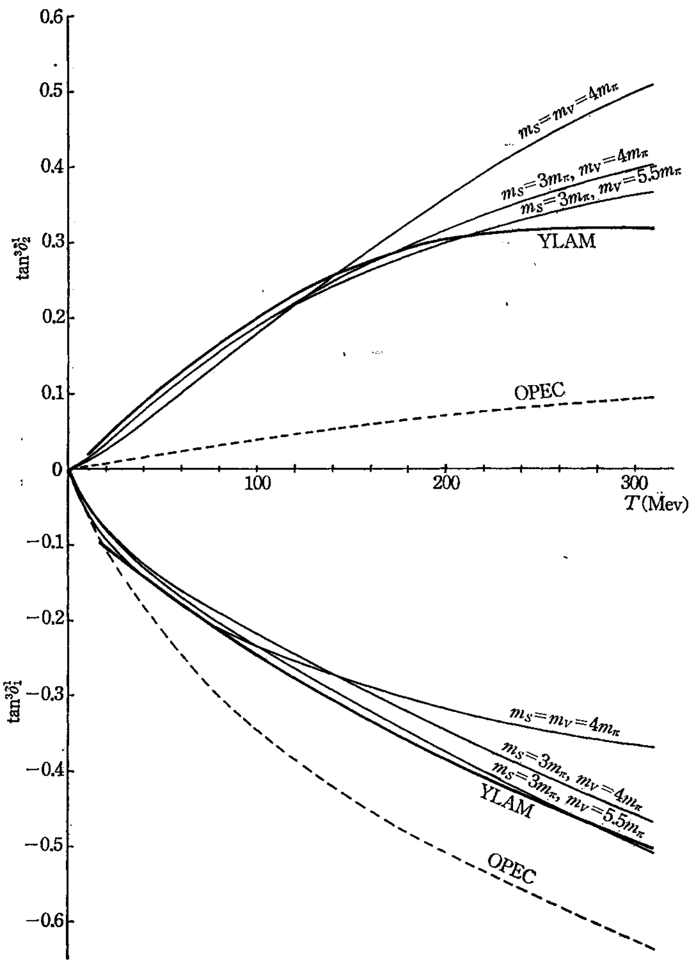

Fig. 3-2. The YLAM and calculated phase shifts for $^3P_1$ and $^3P_2$ states.

discussion of P-wave phase shifts for medium energy (~120 Mev) where the mutual differences among the results from the different phase shift analyses are relatively small.

1) P-wave phase shifts

The YLAM P-wave phase shifts corresponding to three possible total angular momentum states are shown in Figs. 3-1 and 3-2. In order to find what combination of the coupling constants will be allowed to reproduce the experimental data, we assume $0.1<\tan{}^3\delta_0{}^1<0.3$, $-0.3<\tan{}^3\delta_1{}^1<-0.2$ and $0.18<\tan{}^3\delta_2{}^1<0.3$ at 120 Mev. For these the allowed area of the coupling constants is calculated and given in Fig. 4. From this we can take, for example, as a good combination of the coupling constants for $m_S=3m_\pi(4m_\pi)$

$$\begin{aligned} &g_S^2 = 2.4 \ \ (5.2), \\ &G_V^2 = 22.5 \ \ (22.5) \ \text{ or } \ g_V^2 = 1.2 \ \ (1.0) \\ &f_V^2 = 3.3 \ \ (3.5), \\ &G_V f_V = 8.6 \ \ (8.8), \\ &G_\pi^2 = 14.4,^{*)} \end{aligned} \tag{29}$$

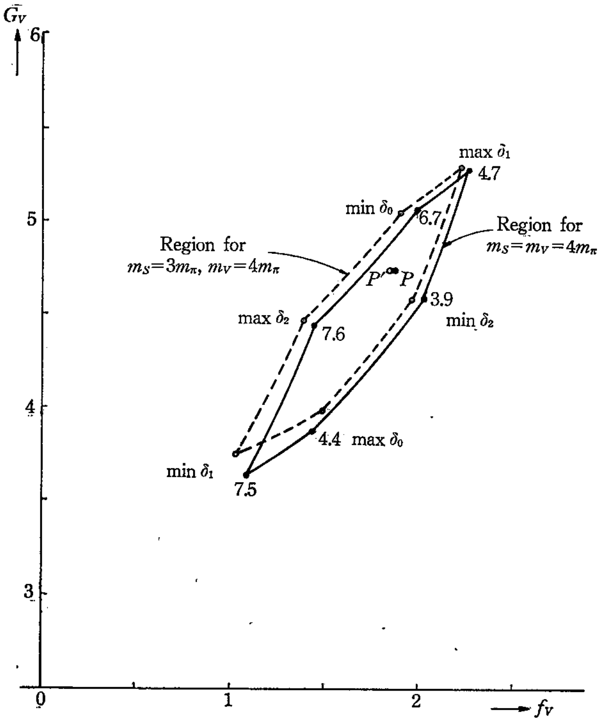

Fig. 4. The allowed region of coupling constants for which the triplet P-wave phase shifts fall in the region $0.1 < \tan {}^3\delta_0{}^1 < 0.3$, $-0.3 < \tan {}^3\delta_1{}^1 < -0.2$ and $0.18 < \tan {}^3\delta_2{}^1 < 0.3$ at 120 Mev. The solid line and dotted line correspond to the case of $m_S = 4m_\pi$ and $m_S = 3m_\pi$ respectively. The meaning of max δ_i and min δ_i attached to each vertex is as follows. Let us consider, for example, the curve connecting the points specified by max δ_1 and min δ_2. For the vector coupling constants G_V and f_V on this curve, $\tan {}^3\delta_1{}^1$ takes the maximum value -0.2 and $\tan {}^3\delta_2{}^1$ takes minimum value 0.18, while $\tan {}^3\delta_0{}^1$ varies between 0.1 and 0.3. The scalar coupling constant g_S^2 is appropriately determined so as to satisfy $\tan {}^3\delta_1{}^1 = -0.2$ and $\tan {}^3\delta_2{}^1 = 0.18$ with the vector coupling constants and takes the attached value for $m_S = 4m_\pi$ at each vertex ($3.9 \leq g_S^2 \leq 4.7$). Points P and P' in the allowed regions are chosen as the well fit cases of which coupling constants are listed in Eq. (29).

These results have the same order of magnitude as those of H-O-W-Y and have similar natures. (See Eq. (27).)

The coupling constants of vector boson are somewhat different between OBEP and OBEC while the coupling constants of scalar boson are nearly the same in both cases. This fact will not be unreasonable, since for the vector boson the non-static effects are large compared with the scalar boson and there occurs a large cancellation among the contributions from f_V^2, G_V^2 and $G_V f_V$ terms, so that a small change in each term might induce a relatively large change in the coupling constants.**)

The P-wave phase shifts calculated for these coupling constants (29) are given in Fig. 3–1 and 3–2. The agreement with the YLAM phase shifts up to $\sim$200 Mev is rather satisfactory.

It is noted that

*) The pion coupling constant is taken from the modified phase shift analysis.[19]

**) See footnote on next page.

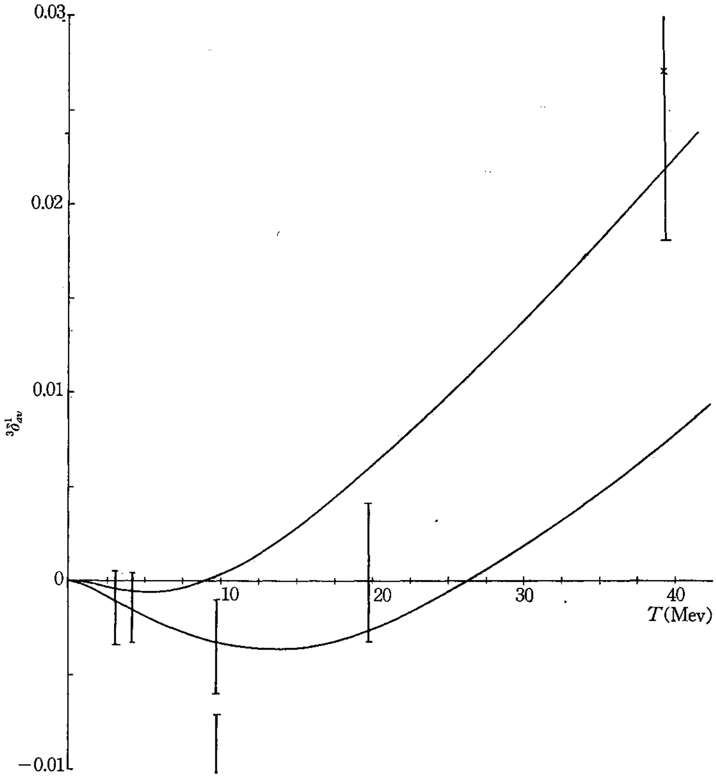

Fig. 5. The experimental and calculated average P-wave phase shift, $^3\delta_{av}{}^1$. For experimental data see reference 25). There is also reported the experimental value 0.027 ± 0.009 at 39.4 Mev.

i) we adjusted the parameter at 120 Mev, but almost the same results will be obtained even if we make the adjustment of the parameter at lower energy (>60 Mev) or at higher energy (<200 Mev) and the allowed area will not move appreciably from the ones in Fig. 4 for both cases.

ii) The effect of the change in the boson mass is able to be absorbed to certain extent in the renormalization of the coupling constants, if we concern only the overall behavior of the P-wave phase shifts. If, however, we examine closer the effects of the change in the boson mass,

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Footnote of the foregoing page :

For the scalar boson the OBEC amplitude for $^3P_0$, $^3P_1$ and $^3P_2$ states can be expressed as $\alpha_c+\boldsymbol{l}\cdot\boldsymbol{s}\alpha_{LS}$ and will have a similar nature to those calculated from OBEP where the static central and nonstatic spin-orbit coupling potentials were taken for the scalar boson. However for the vector boson with vector coupling, of which the potentials were also static central and nonstatic spin-orbit ones, the OBEC amplitudes do not show the splitting like $\alpha_c+\boldsymbol{l}\cdot\boldsymbol{s}\alpha_{LS}$. In this respect, it is interesting to note that experimental P and F-wave phase shifts cannot be explained at the same time, if we confine the non-static force to the spin-orbit force only.[25] This fact may imply that there may exist the nonstatic effects of considerable amount other than those of the spin-orbit type.

the contribution to the P-waves becomes larger (smaller) at low energy as the mass is changed to be lower (higher).

iii) For higher wave phase shifts, the contribution of boson becomes larger, if we renormalize the coupling constants by P-waves, when the mass is lowered and vice versa, as will be expected from the impact parameter consideration.

Now we discuss our results a little more in detail in the low energy region. The most important parameter in the low energy proton-proton angular distribution is a weighted average of three triplet P-wave phase shifts[26)]

$$\sin {}^3\delta^1_{av} = \frac{1}{18}(\sin 2\,{}^3\delta_0{}^1 + 3\sin 2\,{}^3\delta_1{}^1 + 5\sin 2\,{}^3\delta_2{}^1).$$

This quantity can be measured with better accuracy from the angular distribution

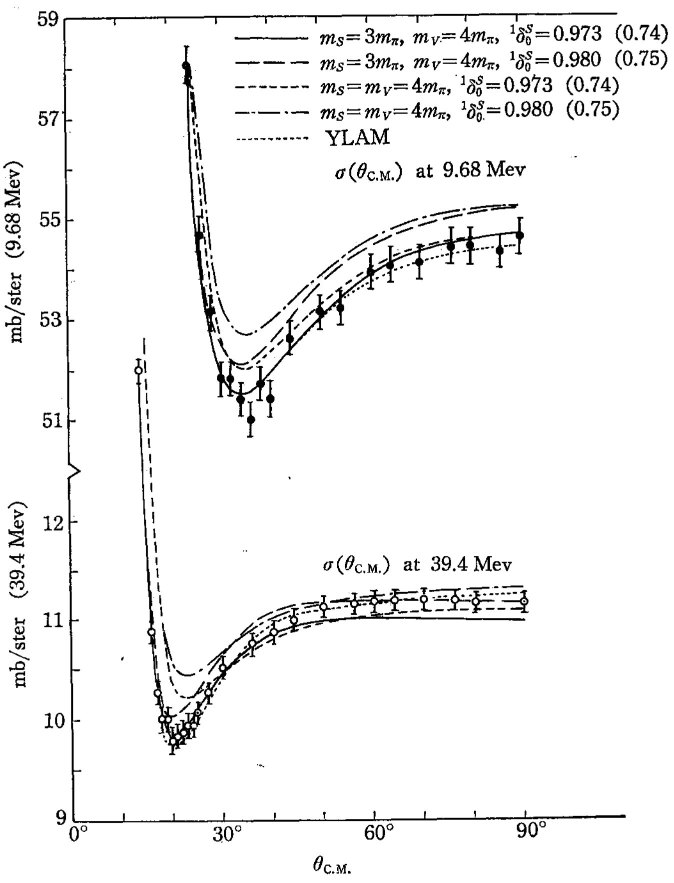

Fig. 6. The experimental and calculated angular distributions at 9.68 Mev and 39.4 Mev. The calculation is made by assuming the S-wave phase shifts given in the Figure (numerals in the brackets are those at 39.4 Mev).

than any other phase shifts (which involve an ambiguity and large error). To reproduce the energy dependence of ${}^3\delta^1_{av}$ is a necessary requirement for any theory of proton-proton scattering.

We calculated this quantity taking the coupling constants (29). The results are given in Fig. 5. The agreement is considered to be satisfactory for the case of $m_S=3m_\pi$. The angular distributions at 9.68 Mev[27] and 39.4 Mev[28] were calculated by assuming reasonable values of ${}^1\delta_0{}^0$ and taking the other phase shifts from the present calculation, and are given in Fig. 6.

From the energy dependence of ${}^3\delta^1_{av}$ and the angular distributions, one may have the impression that the mass $3m_\pi$ is more favourable than $4m_\pi$ for scalar boson. But this will be true only when the present model can be really applied to the energies up to $\sim$200 Mev. As far as we are concerned with the data at low energy, say below 50 Mev, a similar fit to the experiments can be obtained for both $3m_\pi$ and $4m_\pi$ by appropriately choosing the coupling constant, as is suggested from Furuichi's arguments[29] for the contribution from the distant singularity. From the comparison of the several cases of the choice made for m_S and m_V, it seems to us that the energy dependence of ${}^3\delta_0{}^1$ such as in the YLAM or the modified analysis will be obtained when $m_S<m_V$ and somewhat smaller mass such as $3m_\pi$ for m_S.

In the above calculations we used the parameters determined so as to give the P-wave phase shifts at $\sim$120 Mev without any reference to the fit at low energy, and there will possibly be contributions from other bosons that can not be involved by the renormalization of the parameters (coupling constant and mass) in the present simple case of a scalar boson and a vector boson. To obtain a closer fit to the experimental data will not be so meaningful in the present stage.

2) Higher wave phase shifts

The agreement of theoretical P-wave phase shifts with the experiment below 200 Mev will imply that all the higher wave phase shifts up to the energy corresponding to the impact parameter $\sim 0.7\times 1/m_\pi$ will give a satisfactory fit to the experiments, if the theory is correct.

For the coupling constants (29) we calculated all phase shifts up to H-waves. The results are given in Figs. 7–1$\sim$7–6. Our results show satisfactory fits to the experimental data not only qualitatively but also quantitatively. For higher waves than H-waves, the phase shifts are reduced to the one pion exchange contribution approximately.

3) S-wave phase shift

For the S-wave phase shift ${}^1\delta_0{}^0$, we can expect contributions from many other higher mass states and a contact interaction which might also exist in the model (A) $\sim$ (D), even at low energy, in addition to those considered here. Nevertheless it must be stressed that the obtained phase shifts shows the qualitative behavior of the observed data. The fact that the S-wave phase shift changes its sign at

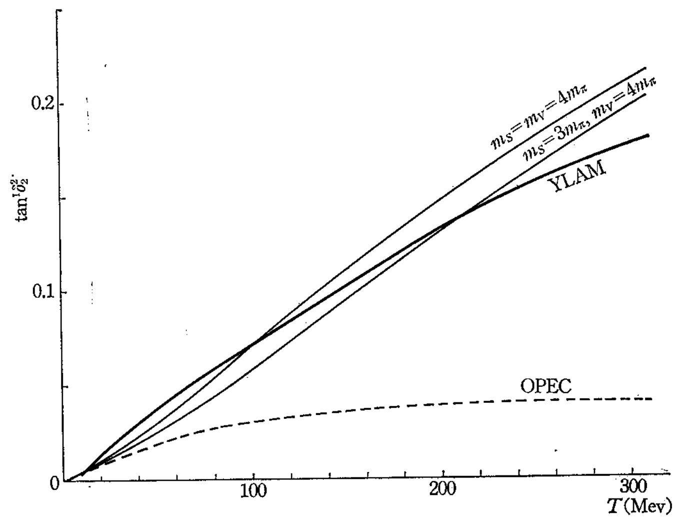

Fig. 7-1. The YLAM and calculated phase shifts for ${}^1D_2$ state.

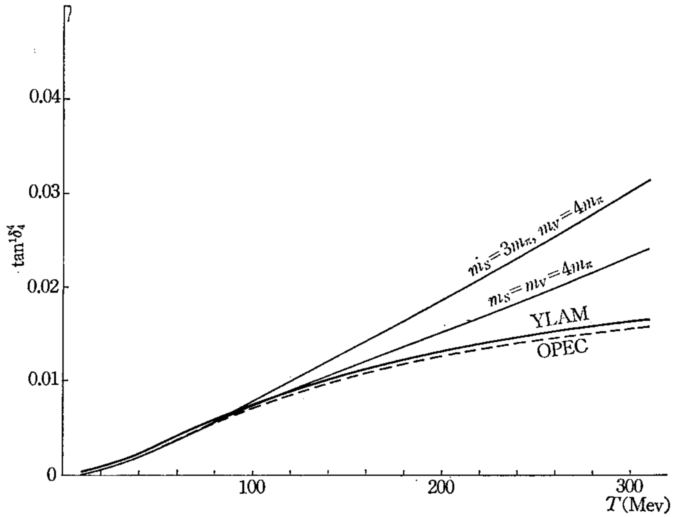

Fig. 7-2. The YLAM and calculated phase shifts for ${}^1G_4$ state.

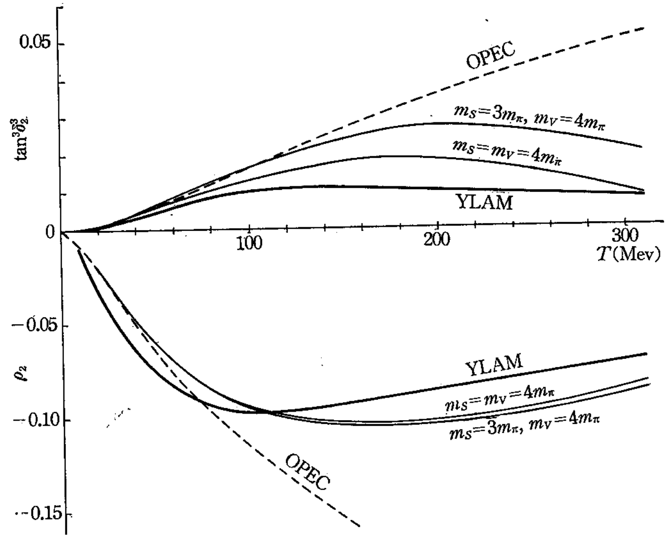

Fig. 7-3. The YLAM and calculated coupling parameter ρ_2 and phase shifts for ${}^3F_2$ state.

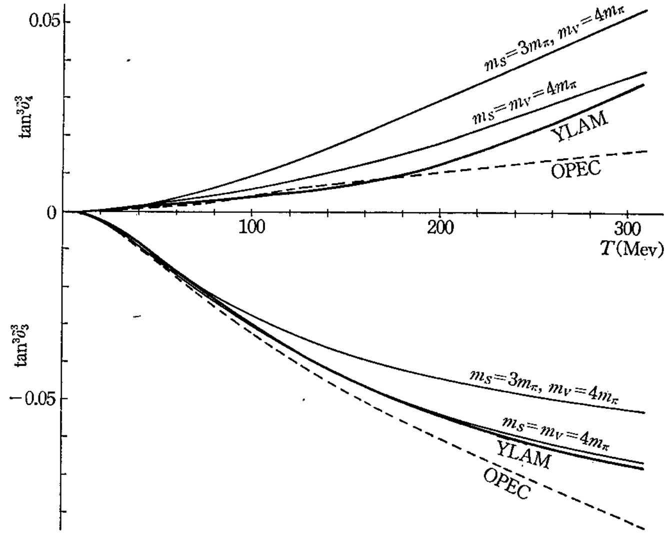

Fig. 7-4. The YLAM and calculated phase shifts for ${}^3F_3$ and ${}^3F_4$ states.

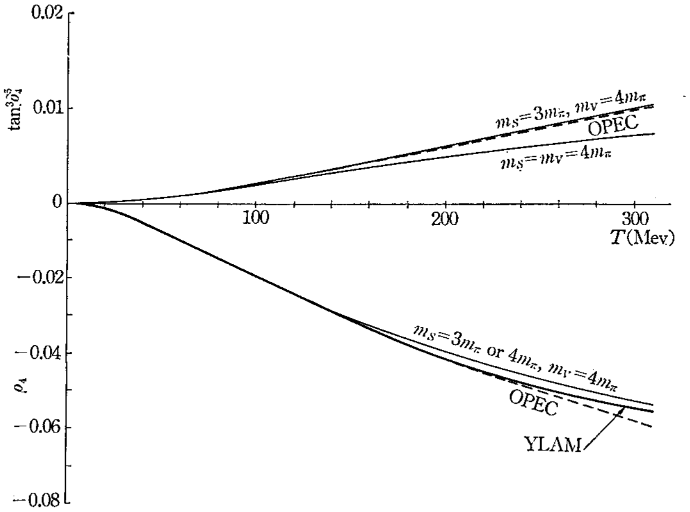

Fig. 7-5. The YLAM and calculated coupling parameters ρ_4 and phase shifts for $^3H_4$ state.

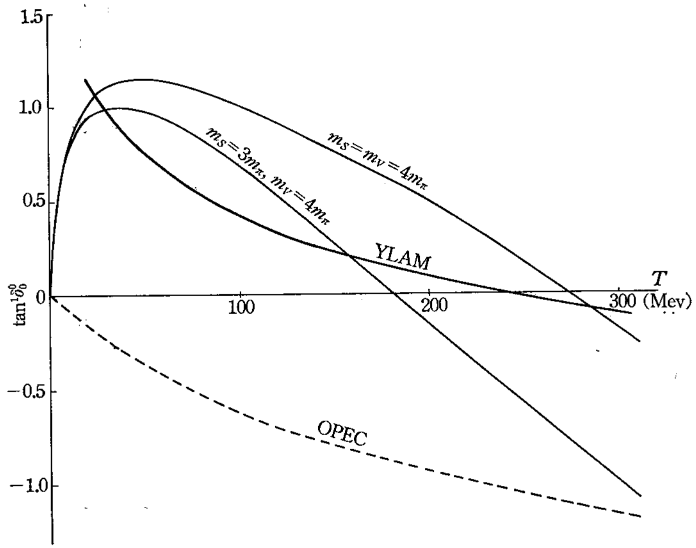

Fig. 8. The YLAM and calculated phase shifts for $^1S_0$.

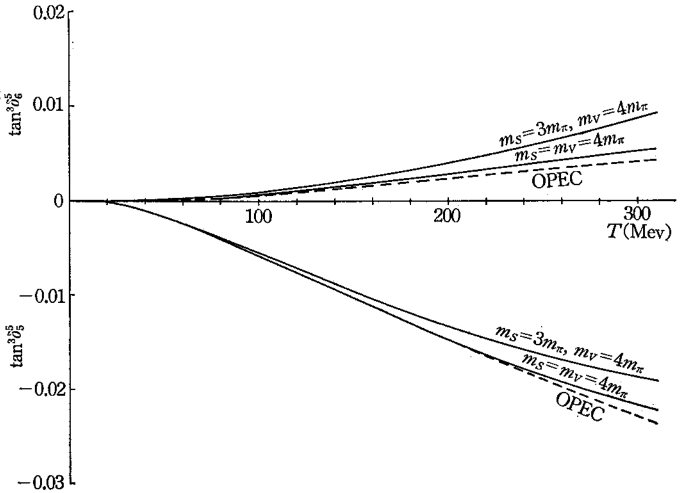

Fig. 7-6. The YLAM and calculated phase shifts for $^3H_5$ and $^3H_6$ states.

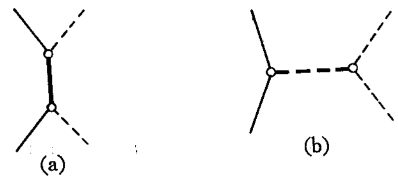

Fig. 9. The types of diagram contributing to pion-nucleon scattering. The broken line denotes the pion and the bold line denotes the fermion which includes the nucleon.

high energy, in spite of not introducing "hard-core" like interaction such as in the potential approach, is partly due to the inclusion of δ_{l0} term in the matrix elements, of which Fourier transforms are $\delta(\boldsymbol{r}), \delta'(\boldsymbol{r}), \cdots$. They are usually neglected in the potential approach.

§ 5. Further discussion and concluding remarks

So far we have proceeded without making reference to the observed evidences of the boson that can be identified with the bosons in the present model. There are isovector vector ρ-meson,[12] isoscalar vector ω-meson,[13] isoscalar η-meson[14] which might be pseudoscalar although vector case is not completely excluded, isoscalar scalar A-B-C particle,[15] isovector ζ-meson[16] which might be 0^+. There are also some evidences of other mesons than these just mentioned. Compared with this experimental situation, our present model seems to be very promising. Although we must investigate the present model by the "real" bosons when their existence and spin-parity are well settled, we should like to make some comments compared with the present experimental evidences of bosons. The first is that the mass of vector bosons should be taken larger than $4m_\pi$, if the ρ and ω mesons will mainly be responsible as the required vector boson in our model. As we have said before, it is expected that such a change in mass can be reduced to a certain extent to the renormalization of the coupling constants. To show this is really the case, we calculated the P-wave phase shift by taking the mass of the vector boson as $5.5m_\pi$. The results are also given in Figs. 3–1 and 3–2 and show our expectation is right. Moreover such a change in mass induces shift of the peak in ${}^3\delta_0{}^1$ to the low energy which will be favorable compared with the experiments.

There is a favorable evidence of pseudoscalar for η-meson and there is no reason why an axialvector boson might not exist. If this is the case, the satisfactory agreement with the experimental phase shifts might be due to the following situations. The first is that the effects of supposed pseudoscalar boson or pseudovector boson can be partly included in the effect of pion, scalar boson and vector boson. For this we pointed out the relations (13). The second is that the contribution of pseudoscalar boson has a tendency of cancellation with that of pseudovector boson with pseudotensor coupling so that the resulting effect may become smaller if both bosons may exist. The third will be that the nuclear matrix elements for pseudoscalar or axialvector boson with pseudotensor coupling will be relatively small owing to the presence of γ_5 matrix in interaction.

We have shown in the preceding analysis that the present model can give an explanation of the proton-proton scattering data which can be successfully given by the pion-field theoretical approach, if scalar boson and vector boson of mass $\sim 4m_\pi$ are assumed to give a dominant contribution to the nuclear force besides the pion. In spite of the very simple character of the assumed mechanism the agreement with the experiments is surprisingly good. It would be certainly believable that the reality exists in our one particle exchange picture for the nuclear forces.

It will be interesting to apply the present model to other processes of strong interaction along the directions (A) $\sim$ (D), and the validity of our model will be finally checked by the systematic theoretical and experimental investigation of pion-nucleon scattering, nucleon-nucleon scattering, pion-pion scattering, etc. In this connection we shall remark the following.

I) We have discussed so far the proton-proton scattering only and not touched on the neutron-proton scattering. The reason is that the experimental data on proton-proton scattering are much more abundant and there is an advantage of checking the model without the discrimination of the contribution from isoscalar and isovector bosons.

But neutron-proton scattering may serve as other important test for the OBEC model and the charge properties of the intermediate bosons will be clarified only by the study of neutron-proton scattering in the correlation of the proton-proton scattering. The analysis of neutron-proton scettering by the OBEC model will be presented in the near future.

II) The model of pion-nucleon scattering given by (A) $\sim$ (D) will show much similarity to the analysis of Bowcock et al.[30] Although their approach is dispersion theoretic, they have inferred the possibility of "isobar model" which will be just the same as our model (see Fig. 9).

III) The application to the single (and immediately generalizable to multiple) meson production processes in nucleon-nucleon collision gives the diagrams in Fig. 10.

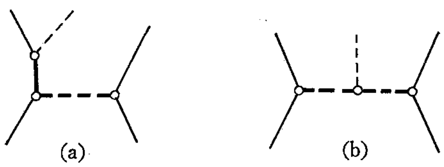

Fig. 10. The types of diagram contributing to the single pion production in nucleon-nucleon collision.

This is the peripheral (one-boson-exchange) collision model.[31] Since in proton-proton scattering at high energy we have a large contribution from bosons other than pions, we must take into account the contribution from other intermediary boson such as ρ meson, etc.

(IV) More isobars may be expected to be participating in the pion production in pion-nucleon collision than in nucleon-nucleon collision. For this reaction we have previously stressed the need of systematic investigation and proposed a generalized isobar model[32] which is a generalization of the Lindenbaum and Sternheimer 3–3 nucleon isobar model[33] from the standpoint of the Sakata model. The situation afterwards have become just as we expected. The previous model is rather of a kinematical nature, and should be refined along the line (A) $\sim$ (D). The model suggests the processes shown in Fig. 11 as the dynamical mechanism for the pion production. The importance of the peripheral collision will be clear, but it will not cover all of the reaction.

We shall give further discussions of the strong interaction based on the present model together with the refinement of the analysis of proton-proton scattering in future.

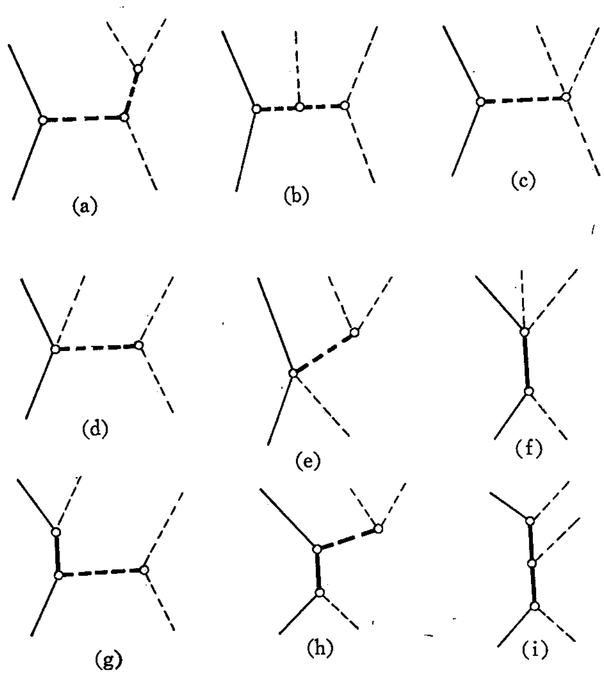

Fig. 11. The types of diagram contributing to the single pion production in pion-nucleon collision.

Finally it should be stressed that the present theory is, as is clear from its construction, a transitional one and will have a limitation for its workability as the old quantum theory was. Its limitation, though not yet observed, will be closely connected with something like the structure in the more inner region and the clarification will be required to built up the more elaborate theory.

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