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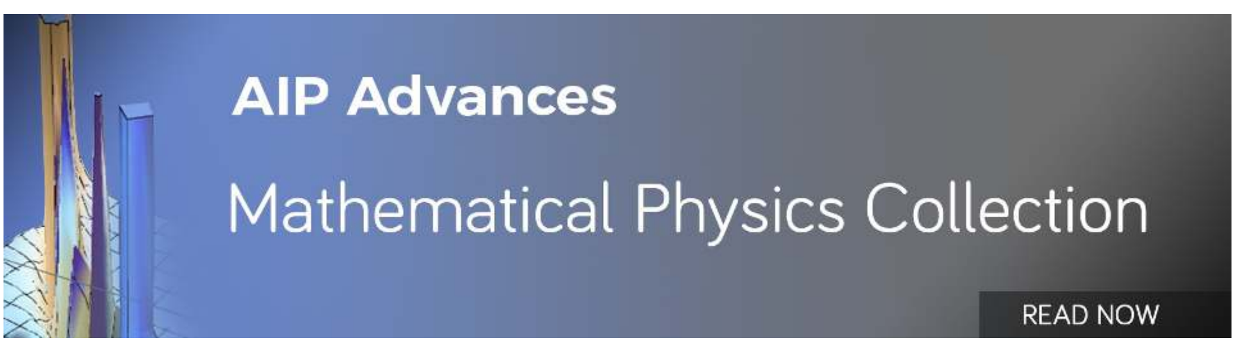

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One-dimensional coupled Burgers' equation and its numerical solution by an implicit logarithmic finite-difference method

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In this paper, an implicit logarithmic finite difference method (I-LFDM) is implemented for the numerical solution of one dimensional coupled nonlinear Burgers' equation. The numerical scheme provides a system of nonlinear difference equations which we linearise using Newton's method. The obtained linear system via Newton's method is solved by Gauss elimination with partial pivoting algorithm. To illustrate the accuracy and reliability of the scheme, three numerical examples are described. The obtained numerical solutions are compared well with the exact solutions and those already available. © *2014 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution 3.0 Unported License.* [http://dx.doi.org/10.1063/1.4869637]

I. INTRODUCTION

Let us consider one dimensional coupled nonlinear Burgers' equation[1,2] in generalized form:

$$\frac{\partial u}{\partial t} + \delta \frac{\partial^2 u}{\partial x^2} + \eta u \frac{\partial u}{\partial x} + \alpha \left(u \frac{\partial v}{\partial x} + v \frac{\partial u}{\partial x} \right) = 0, \tag{1}$$

$$\frac{\partial v}{\partial t} + \mu \frac{\partial^2 v}{\partial x^2} + \xi v \frac{\partial v}{\partial x} + \beta \left(u \frac{\partial v}{\partial x} + v \frac{\partial u}{\partial x} \right) = 0, \tag{2}$$

subject to the initial conditions

$$\left. \begin{array}{l} u(x,0) = a_1(x), \\ v(x,0) = a_2(x), \end{array} \right\} x \in \Omega, \tag{3}$$

and the Dirichlet boundary conditions

$$\left. \begin{array}{l} u(x,t) = b_1(x,t), \\ v(x,t) = b_2(x,t), \end{array} \right\} x \in \Omega, t > 0, \tag{4}$$

where $\Omega = \{x : c \le x \le d\}$ is the computational domain; δ, μ, η and ξ are real constants, α and β are arbitrary constants depending on the system parameters such as Peclet number, stokes velocity of particles due to gravity and the Brownian diffusivity,[3] $u(x,t)$ and $v(x,t)$ are the velocity components

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TABLE I. Comparison of errors for $u(x, t)$ with $\Delta t = 0.001$ for the test case 1.

	I-*LFDM*				Mittal [12]					
	Number of partition = 200		Number of partition = 400		Number of partition = 200		Number of partition = 400		Rashid [11]	
t	L_2	L_∞	L_2	L_∞	L_2	L_∞	L_2	L_∞	L_2	L_∞
0.1	9.95e-04	9.02e-04	9.87e-04	8.95e-04	8.21e-06	7.45e-06	2.05e-06	1.86e-06	—	—
0.5	9.91e-04	6.04e-04	9.84e-04	6.01e-04	2.49e-05	4.10e-05	1.02e-05	6.22e-06	—	—
1.0	9.83e-04	3.65e-04	9.70e-04	3.64e-04	3.00e-05	8.21e-05	2.04e-05	7.56e-06	2.88e-05	1.16e-05

to be determined, a_1, a_2, b_1 and b_2 are the known functions, $\frac{\partial u}{\partial t}$ is unsteady term, $u\frac{\partial u}{\partial x}$ is the nonlinear convection term, $\frac{\partial^2 u}{\partial x^2}$ is the diffusion term.

The one dimensional coupled Burgers' equation can be taken as a simple model of sedimentation and evolution of scaled volume concentrations of two kinds of particles in fluid suspensions and colloids under the effect of gravity. Various researchers have proposed analytical solution to one dimensional coupled Burgers' equation, e.g. Kaya[4] used Adomian decomposition method, Soliman[5] applied a modified extended tanh-function method, whereas numerical solutions to this system of equation have been attempted by many researchers. Esipov[6] had given numerical solutions and compared the obtained results with those given by the experiment. Abdou[7] used variational iteration method to solve coupled Burgers' equation, whereas Wei[8] used a conjugate filter approach, Khater[9] applied the Chebyshev spectral collocation method, Dehghan[10] gave numerical solutions of coupled viscous Burgers equations by applying the Adomian- Pade technique; Rashid[11] applied Fourier pseudo-spectral method. Mittal[12] has applied a cubic B-spline collocation scheme while Mokhtari[13] used a generalized differential quadrature method. Recently, Srivastava *et al.*[1,2] used a fully implicit scheme and Crank-Nicolson scheme for solving this system of coupled Burgers' equation. Further, Srivastava *et al.*[14,15] proposed two new finite difference schemes, namely an implicit exponential finite-difference and an implicit logarithmic finite-difference method for solving the two dimensional coupled viscous Burgers' equation. One can refer[16–21] for various numerical schemes for two dimensional coupled Burgers' equations whereas the exact solution of two, three and (n + 1)-dimensional Burgers' equation can be seen in.[22–24]

In this article, an implicit logarithmic finite-difference method (I-LFDM) has been applied for the numerical solution of one dimensional coupled Burgers' equation, proposed by Srivastava *et al.*[15] The obtained results are compared well with the exact solutions and those already available in the literature. The accuracy and computational reliability of the I-LFDM scheme are demonstrated in terms of error norms by considering the following three test cases.

Test case 1: Consider the coupled Burgers' equation

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} - 2u\frac{\partial u}{\partial x} + \left(u\frac{\partial v}{\partial x} + v\frac{\partial u}{\partial x}\right) = 0, \tag{5}$$

$$\frac{\partial v}{\partial t} - \frac{\partial^2 v}{\partial x^2} - 2v\frac{\partial v}{\partial x} + \left(u\frac{\partial v}{\partial x} + v\frac{\partial u}{\partial x}\right) = 0. \tag{6}$$

The initial and boundary conditions are taken from the exact solution. Exact solutions to Eqs. (5) and (6) can be expressed as[3]

$$\left.\begin{aligned} u(x,t) &= \exp(-t)\sin(x) \\ v(x,t) &= \exp(-t)\sin(x) \end{aligned}\right\}, x \in [-\pi, \pi], t > 0. \tag{7}$$

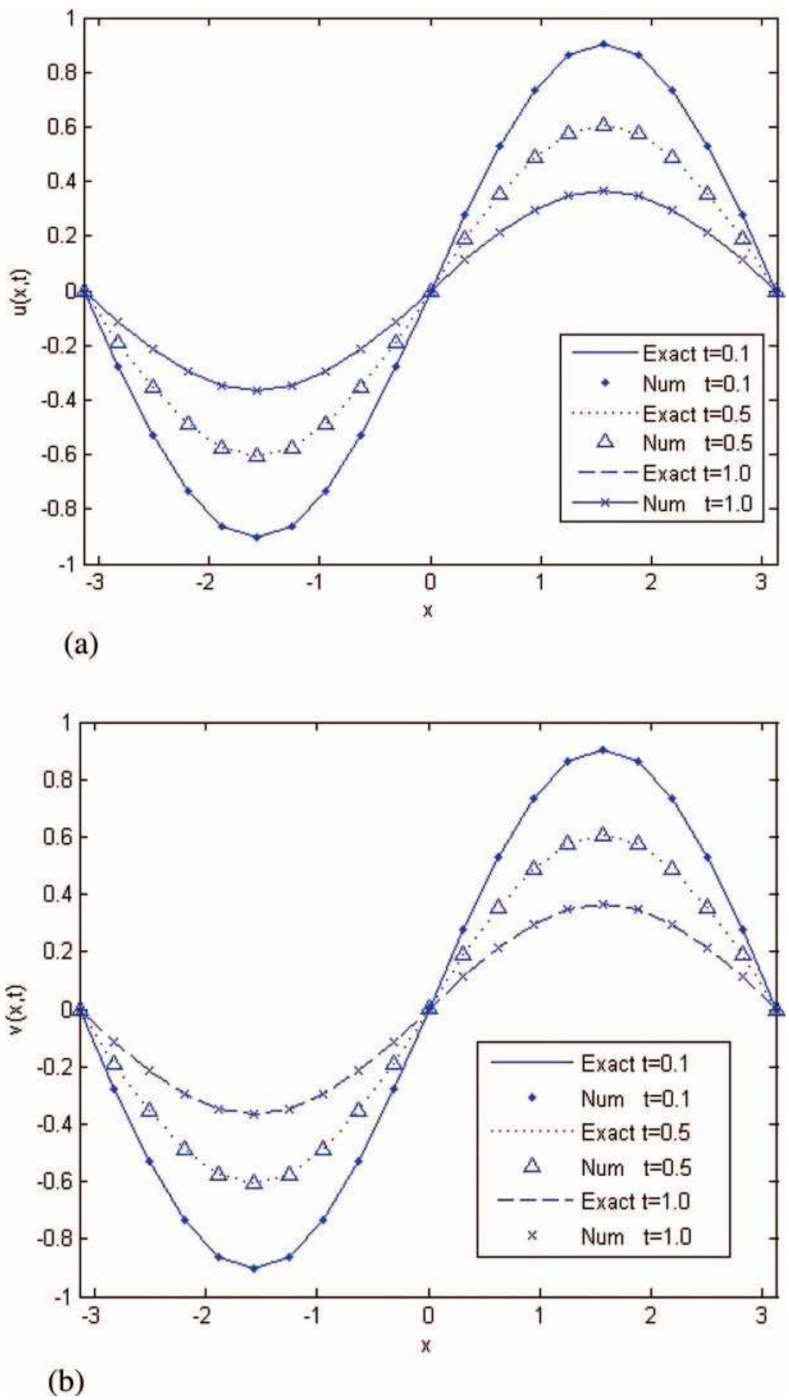

FIG. 1. Comparison between numerical and exact solutions (a) $u(x, t)$ and (b) $v(x, t)$ for the test case 1.

Test case 2: Consider the coupled equation

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} + 2u\frac{\partial u}{\partial x} + \alpha\left(u\frac{\partial v}{\partial x} + v\frac{\partial u}{\partial x}\right) = 0, \tag{8}$$

$$\frac{\partial v}{\partial t} - \frac{\partial^2 v}{\partial x^2} + 2v\frac{\partial v}{\partial x} + \beta\left(u\frac{\partial v}{\partial x} + v\frac{\partial u}{\partial x}\right) = 0. \tag{9}$$

TABLE II. Comparison of errors for $u(x, t)$for the test case 2.

			Khater [9]		Rashid [11]		Mittal [12]		Mokhtari[13]		I-*LFDM*	
t	α	β	L_2	L_∞	L_2	L_∞	L_2	L_∞	L_2	L_∞	L_2	L_∞
0.5	0.1	0.3	1.44e-03	4.38e-05	3.245e-05	9.619e-04	6.736e-04	4.167e-05	2.02e-03	1.00e-04	4.02852e-04	2.64018e-05
	0.1	0.03	6.68e-04	4.58e-05	2.733e-05	4.310e-04	7.326e-04	4.590e-05	5.07e-03	2.52e-04	3.91814e-04	2.62182e-05
1.0	0.1	0.3	1.27e-03	8.66e-05	2.405e-05	1.153e-03	1.325e-03	8.258e-05	4.03e-03	2.01e-04	7.93158e-04	5.20361e-05
	0.1	0.03	1.30e-03	9.16e-05	2.832e-05	1.268e-03	1.452e-03	9.182e-05	1.00e-02	5.04e-04	7.71339e-04	5.16796e-05

TABLE III. Comparison of errors for $v(x, t)$ for the test case 2.

			Khater [9]		Rashid [11]		Mittal [12]		Mokhtari [13]		I-*LFDM*	
t	α	β	L_2	L_∞	L_2	L_∞	L_2	L_∞	L_2	L_∞	L_2	L_∞
0.5	0.1	0.3	5.42e-04	4.99e-05	2.746e-05	3.332e-04	9.057e-04	1.480e-04	1.56e-03	3.80e-05	2.21424e-04	1.04139e-05
	0.1	0.03	1.20e-03	1.81e-04	2.454e-04	1.148e-03	1.591e-04	5.729e-04	1.59e-03	1.85e-04	4.25241e-04	3.0721e-05
1.0	0.1	0.3	1.29e-03	9.92e-05	3.745e-05	1.162e-03	1.251e-03	4.770e-05	3.10e-03	7.58e-05	4.24639e-04	1.97588e-05
	0.1	0.03	2.35e-03	3.62e-04	4.525e-04	1.638e-03	2.250e-03	3.617e-04	3.15e-03	3.67e-04	8.37842e-04	6.09557e-05

The exact solutions to Eqs. (8) and (9) are expressed as[4]

$$\left.\begin{aligned} u(x,t) &= a_0\left(1 - 2A\left(\frac{2\alpha-1}{4\alpha\beta-1}\right)\tanh\left(A\left(x-2At\right)\right)\right),\\ v(x,t) &= a_0\left(\left(\frac{2\beta-1}{2\alpha-1}\right) - 2A\left(\frac{2\alpha-1}{4\alpha\beta-1}\right)\tanh\left(A\left(x-2At\right)\right)\right), \end{aligned}\right\} x\in[-10,10],\, t>0, \tag{10}$$

where $A = a_0\frac{(4\alpha\beta-1)}{(4\alpha-2)}$ and a_0, α, β are arbitrary constants. The initial and boundary conditions are taken from the exact solution.

Test case 3: Consider the following coupled Burgers' equation

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} + \eta u\frac{\partial u}{\partial x} + \alpha\left(u\frac{\partial v}{\partial x} + v\frac{\partial u}{\partial x}\right) = 0, \tag{11}$$

$$\frac{\partial v}{\partial t} - \frac{\partial^2 v}{\partial x^2} + \xi v\frac{\partial v}{\partial x} + \beta\left(u\frac{\partial v}{\partial x} + v\frac{\partial u}{\partial x}\right) = 0. \tag{12}$$

where η, ξ, a_0, α, β are arbitrary constants.

With the initial conditions[12]

$$u(x,0) = \begin{cases} \sin(2\pi x), & x\in[0,0.5],\\ 0, & x\in(0.5,1], \end{cases} \tag{13}$$

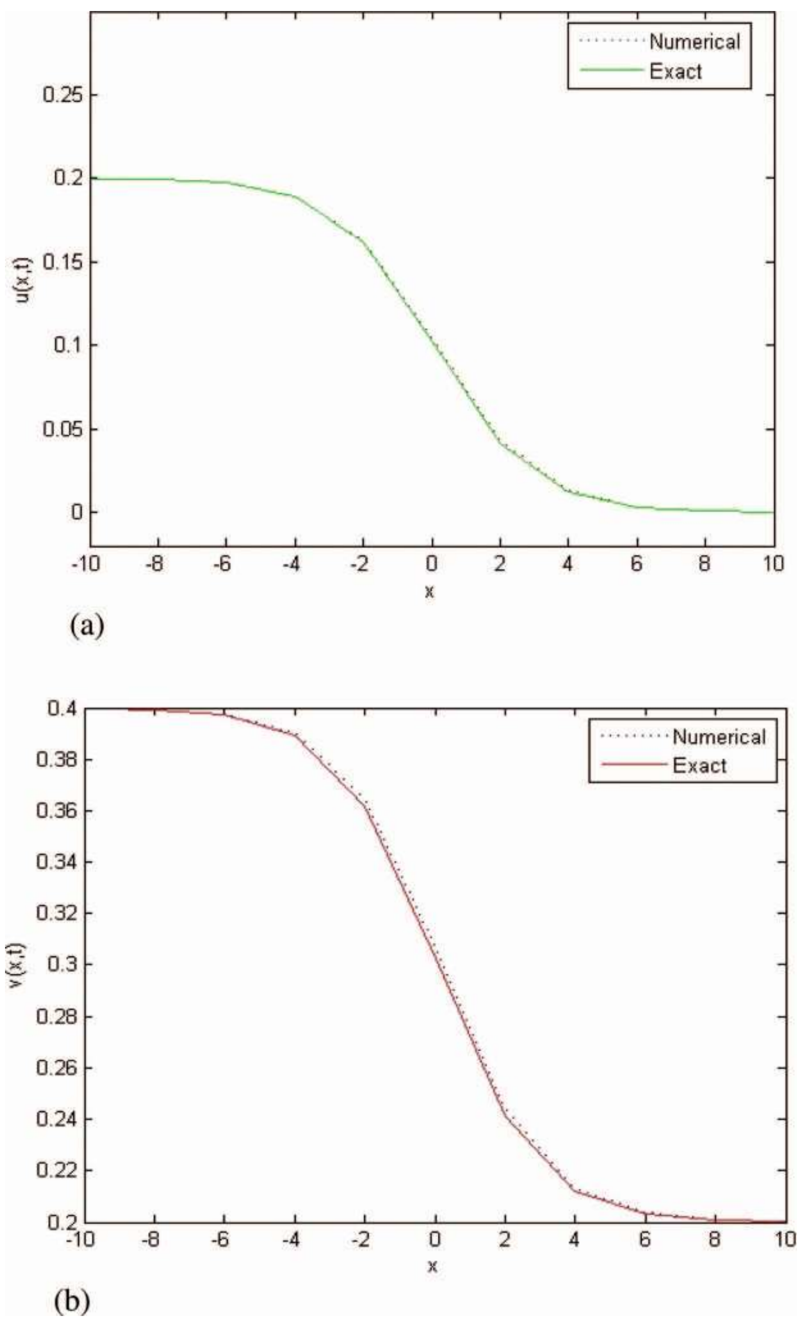

FIG. 2. Comparison between numerical and analytical solutions (a) $u(x, t)$ and (b) $v(x, t)$ at number of partitions = 10, $\Delta t = 0.1$, $t = 1$, $\alpha = 1$, $\beta = 2$ for the test case 2.

$$v(x,0) = \begin{cases} 0, & x \in [0, 0.5], \\ -\sin(2\pi x), & x \in (0.5, 1], \end{cases} \tag{14}$$

and zero boundary conditions.

The rest of the paper is organized as follows: in section II, the I-LFDM scheme is described; in section III, results and discussions are illustrated, and a concluding discussion is presented in section IV.

TABLE IV. Maximum values of u and v for $\alpha = \beta = 10$ for the test case 3.

	I-*LFDM*				Mittal [9]			
t	Max value of u	At point (x)	Max value of v	At point (x)	Max value of u	At point (x)	Max value of v	At point (x)
0.1	0.14047	0.54	0.15058	0.66	0.14456	0.58	0.14306	0.66
0.2	0.054552	0.52	0.050969	0.58	0.05237	0.54	0.04697	0.56
0.3	0.021168	0.52	0.019627	0.52	0.01932	0.52	0.01725	0.52
0.4	0.0082883	0.48	0.0077130	0.50	0.00718	0.50	0.00641	0.50

II. NUMERICAL SCHEME (I-LFDM)

In this section, we illustrate the implicit logarithmic finite-difference method (I-LFDM).

Assume $X(u)$ and $Y(v)$ be any two continuously differentiable functions. Multiplying Eqs. (1) and (2) by the derivatives of X and Y, respectively, yielding

$$\frac{\partial X}{\partial t} = -X'(u)\left[\delta\frac{\partial^2 u}{\partial x^2} + \eta u\frac{\partial u}{\partial x} + \alpha\left(u\frac{\partial v}{\partial x} + v\frac{\partial u}{\partial x}\right)\right], \tag{15}$$

$$\frac{\partial Y}{\partial t} = -Y'(v)\left[\mu\frac{\partial^2 v}{\partial x^2} + \xi v\frac{\partial v}{\partial x} + \beta\left(u\frac{\partial v}{\partial x} + v\frac{\partial u}{\partial x}\right)\right]. \tag{16}$$

Using the forward differences for $\left(\frac{\partial X}{\partial t}, \frac{\partial Y}{\partial t}\right)$, and the usual central finite-differences for the convection and diffusion terms of Eqs. (15) and (16), one can get the following implicit finite-difference scheme

$$X\left(u_i^{n+1}\right) = X\left(u_i^n\right) - \Delta t X'\left(u_i^n\right)\left[\begin{array}{l}\delta\left(\dfrac{u_{i+1}^{n+1} - 2u_i^{n+1} + u_{i-1}^{n+1}}{(\Delta x)^2}\right) + \eta u_i^{n+1}\left(\dfrac{u_{i+1}^{n+1} - u_{i-1}^{n+1}}{2\Delta x}\right)\\ +\alpha u_i^{n+1}\left(\dfrac{v_{i+1}^{n+1} - v_{i-1}^{n+1}}{2\Delta x}\right) + \alpha v_i^{n+1}\left(\dfrac{u_{i+1}^{n+1} - u_{i-1}^{n+1}}{2\Delta x}\right)\end{array}\right], \tag{17}$$

$$Y\left(v_i^{n+1}\right) = Y\left(v_i^n\right) - \Delta t Y'\left(v_i^n\right)\left[\begin{array}{l}\mu\left(\dfrac{v_{i+1}^{n+1} - 2v_i^{n+1} + v_{i-1}^{n+1}}{(\Delta x)^2}\right) + \xi v_i^{n+1}\left(\dfrac{v_{i+1}^{n+1} - v_{i-1}^{n+1}}{2\Delta x}\right)\\ +\beta u_i^{n+1}\left(\dfrac{v_{i+1}^{n+1} - v_{i-1}^{n+1}}{2\Delta x}\right) + \beta v_i^{n+1}\left(\dfrac{u_{i+1}^{n+1} - u_{i-1}^{n+1}}{2\Delta x}\right)\end{array}\right]. \tag{18}$$

Now if we consider $X(u) = e^u$ and $Y(v) = e^v$, then one can obtain an implicit logarithmic finite-difference method (I- *LFDM*) as

$$u_i^{n+1} = u_i^n + \log\left[1 - \Delta t\left\{\begin{array}{l}\delta\left(\dfrac{u_{i+1}^{n+1} - 2u_i^{n+1} + u_{i-1}^{n+1}}{(\Delta x)^2}\right) + \eta u_i^{n+1}\left(\dfrac{u_{i+1}^{n+1} - u_{i-1}^{n+1}}{2\Delta x}\right)\\ +\alpha u_i^{n+1}\left(\dfrac{v_{i+1}^{n+1} - v_{i-1}^{n+1}}{2\Delta x}\right) + \alpha v_i^{n+1}\left(\dfrac{u_{i+1}^{n+1} - u_{i-1}^{n+1}}{2\Delta x}\right)\end{array}\right\}\right], \tag{19}$$

$$v_i^{n+1} = v_i^n + \log\left[1 - \Delta t\left\{\begin{array}{l}\mu\left(\dfrac{v_{i+1}^{n+1} - 2v_i^{n+1} + v_{i-1}^{n+1}}{(\Delta x)^2}\right) + \xi v_i^{n+1}\left(\dfrac{v_{i+1}^{n+1} - v_{i-1}^{n+1}}{2\Delta x}\right)\\ +\beta u_i^{n+1}\left(\dfrac{v_{i+1}^{n+1} - v_{i-1}^{n+1}}{2\Delta x}\right) + \beta v_i^{n+1}\left(\dfrac{u_{i+1}^{n+1} - u_{i-1}^{n+1}}{2\Delta x}\right)\end{array}\right\}\right], \tag{20}$$

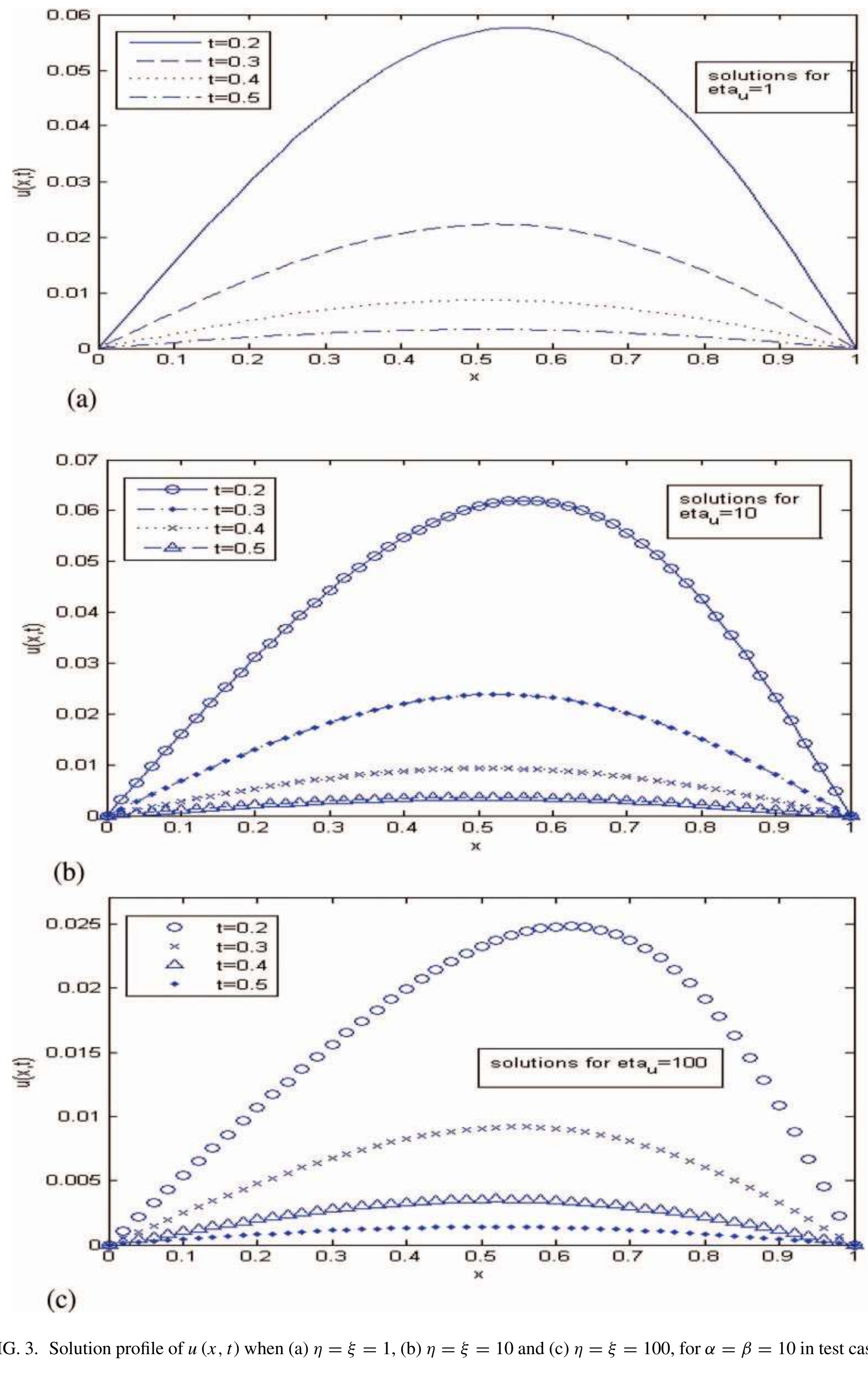

FIG. 3. Solution profile of $u\,(x, t)$ when (a) $\eta = \xi = 1$, (b) $\eta = \xi = 10$ and (c) $\eta = \xi = 100$, for $\alpha = \beta = 10$ in test case 3.

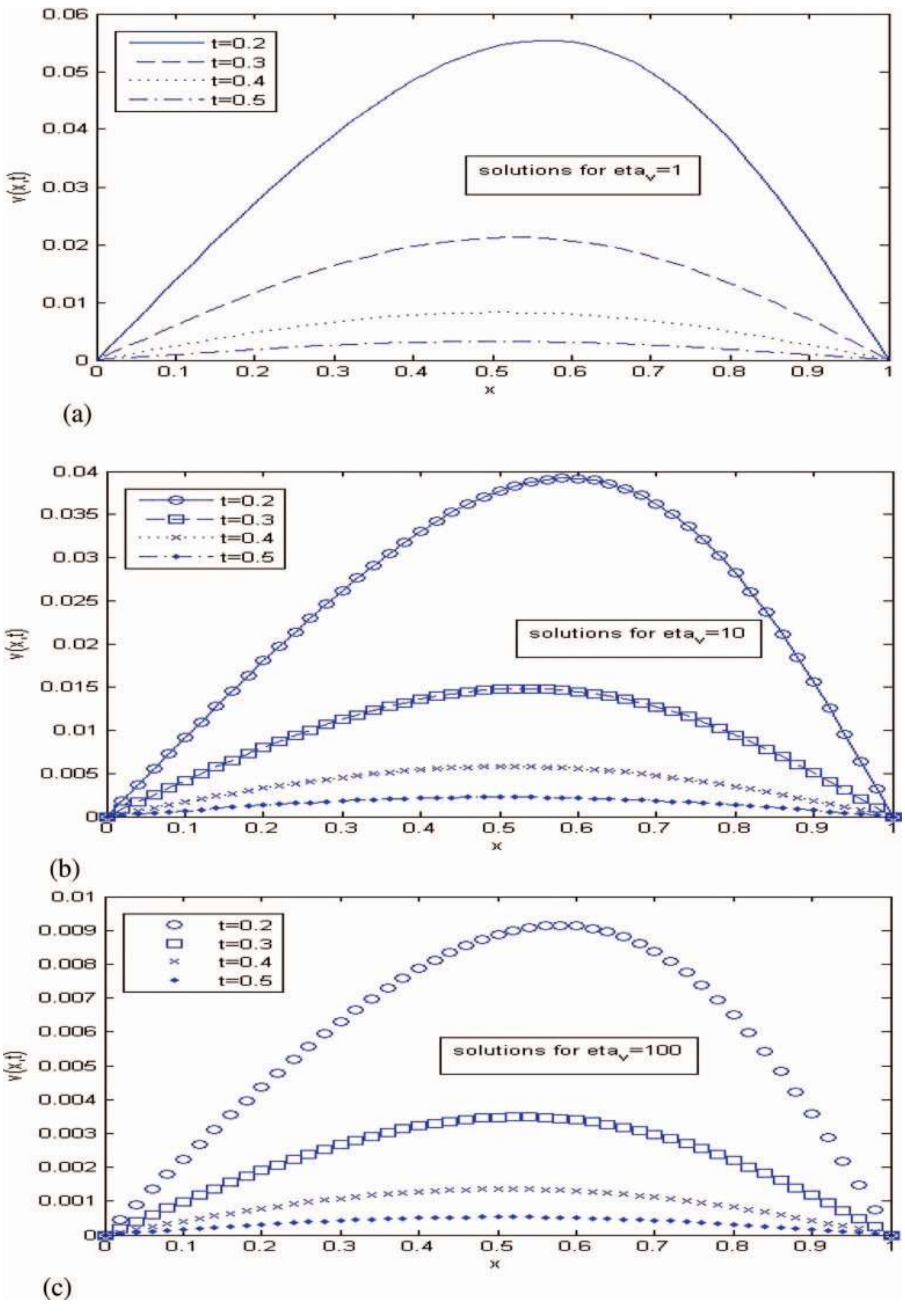

FIG. 4. Solution profile of $v(x, t)$ when (a)$\eta = \xi = 1$, (b) $\eta = \xi = 10$ and (c) $\eta = \xi = 100$, for $\alpha = \beta = 10$ in the test case 3.

where u_i^n and v_i^n denote the discrete approximations of $u(x, t)$ and $v(x, t)$, respectively, at the grid point $(i\Delta x, n\Delta t)$ for $i = 0, 1, 2..., n_x$, n = 0, 1, 2,..., $\Delta x = 1/n_x$ is the grid size in x-direction, and Δt represents the time step.

The nonlinear systems of equations obtained from Eqs. (19) and (20) can be written in the form

$$Q(s) = 0, \tag{21}$$

where

$$Q = (q_1, q_2, ..., q_{2n})^T, \quad s = \left(u_1^{n+1}, v_1^{n+1}, u_2^{n+1}, v_2^{n+1}, ..., u_n^{n+1}, v_n^{n+1}\right)^T, \quad n = n_x - 1,$$

and $q_1, q_2, ..., q_{2n}$ are the nonlinear equations obtained from the nonlinear equations.

The system of equations (21) is solved by Newton's method. The linear system obtained by Newton's iterative method, is solved by Gauss elimination method with partial pivoting.

The accuracy and consistency of the I-*LFDM* scheme is measured in terms of error norms which are defined as[12]

$$\left.\begin{aligned} L_2 &:= \|u^{exact} - u^{computed}\|_2 = \sqrt{\frac{\sum_{j=0}^{n} |u_j{}^{exact} - u_j{}^{computed}|^2}{\sum_{j=0}^{n} |u_j^{exact}|^2}} \\ \mathrm{L}_\infty &:= \|u^{exact} - u^{computed}\|_\infty = \max_j |u_j{}^{exact} - u_j{}^{computed}| \end{aligned}\right\}. \tag{22}$$

where u^{exact} and $u^{computed}$ denote exact and computed solutions, respectively.

III. RESULTS AND DISCUSSIONS

In this section, we describe the numerical computations considering the uniform grid. For the test case 1, the numerical solutions have been carried out in the domain $x \in [-\pi, \pi]$ with Δt =0.001 and are shown in Table I at different $t \in [0, 1]$ and different number of partitions. From Table I, it can be observed that the scheme is consistent since the errors reduce as the number of partitions refines. Fig. 1 depicts the numerical and exact solutions of u and v.

For the problem 2, the numerical computations have been described for $x \in [-10, 10]$, $\Delta t = 0.01$ and number of partitions as 100. Tables II and III depict the comparison of L_2 and L_∞ error norms with those already available in the literature. Computed and exact solutions of u and v are shown in Fig. 2 with number of partitions as 10, and $\Delta t = 0.1$, $t = 1$, $\alpha = 1$, $\beta = 2$.

In the test case 3, the solutions have been carried out on $x \in [0, 1]$ with $\Delta t = 0.01$ and number of partitions as 50. Maximum values of u and v at different time levels for $\alpha = \beta = 10$ have been given in Table IV. Figs. 3 and 4 show the numerical results obtained for different time levels $t \in [0, 1]$ at $\alpha = \beta = 10$ for u and v when $\eta = \xi = 1, 10, 100$, respectively. From the Figs. 3 and 4, it can be seen that the numerical solution decays to zero with increasing time levels and with the increasing η and ξ.

IV. CONCLUSIONS

A numerical approximation is proposed for solving one dimensional coupled Burgers' equation using an implicit logarithmic finite difference scheme. The efficiency and reliability of the I-LFDM scheme is illustrated through three numerical examples. The obtained numerical outputs show that the described scheme performs well in case of one dimensional coupled Burgers' equation.

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