ONE-DIMENSIONAL HEAT CONDUCTION EQUATION OF THE POLAR BEAR HAIR

by

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Original scientific paper
DOI: 10.2298/TSCI15S1S79Z

Hairs of a polar bear (Ursus maritimus) possess special membrane-pore structure. The structure enables the polar bear to survive in the harsh Arctic regions. In this paper, the membrane-pore structure be approximately considered as fractal space, 1-D heat conduction equation of the polar bear hair is established and the solution of the equation is obtained.

Key words: *membrane-pore structure, fractal space, polar bear, heat conduction equation*

Introduction

The polar bear (*Ursus maritimus*) has superior ability to survive in the harsh Arctic regions. Its hairs possess membrane-pore structure, figs. 1 and 2 [1]. The structure plays an important role in keeping body temperature. Each labyrinth cavity of the polar bear hair is a good thermal insulator for keeping warm and the system of labyrinth cavities enable the polar bear to absorb energy from its environment [2].

In the paper we establish 1-D heat conduction equation of the polar bear hair and solve the equation.

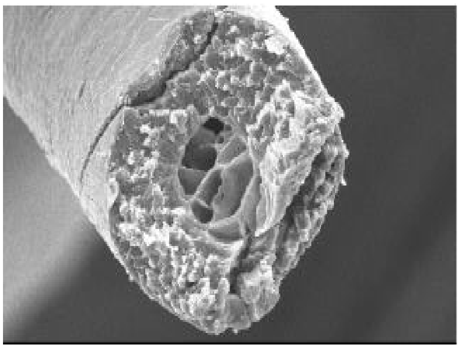

Figure 1. Scanning electron micrographs of polar bear hair, transverse sections

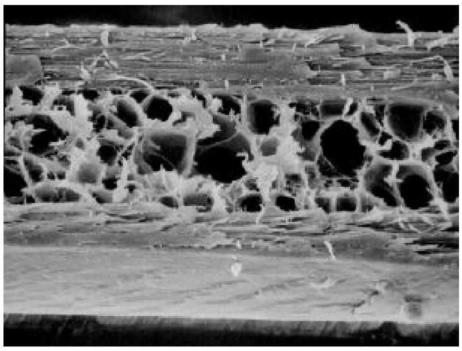

Figure 2. Scanning electron micrographs of polar bear hair, longitudinal sections

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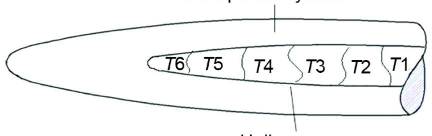

Figure 3. Structure diagram of polar bear hair (*T*1 is the body temperature, and *T*6 – the environment temperature)

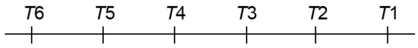

Figure 4. 1-D model diagram of polar bear hair

1-D heat conduction equation of the polar bear hair

The polar bear hair is not a continuous media but a discontinuous media. Its membrane-pore structure can be approximately considered as fractal space. He, J.-H. [3] shows that fractional differential equations can best describe fractal media, and the fractional order is equivalent to its fractional dimensions. Now we consider the polar bear hair as a 1-D heat conductor, figs. 3 and 4.

On the base of Fourier's Law of thermal conduction in a fractal medium [4], 1-D heat conduction equation of the polar bear hair is written as:

$$\frac{\partial T}{\partial t} = \frac{\partial^\alpha}{\partial x^\alpha} D \frac{\partial^\alpha T}{\partial x^\alpha}, \quad x \in (0, L), \quad t \geq 0 \tag{1}$$

where $T(x,t)$ is the temperature at the point *x* and time *t*, D – the thermal diffusivity, *L* – the length of the polar bear hair, α – the fractional dimensions of the polar bear hair, and $\partial^\alpha / \partial x^\alpha$ is He's fractional derivative defined as [5]:

$$\frac{\partial T^\alpha}{\partial x^\alpha} = \frac{1}{\Gamma(n-\alpha)} \frac{\mathrm{d}^n}{\mathrm{d}x^n} \int_{t_0}^{t} (s-x)^{n-\alpha-1} \left[T_0(s) - T(s) \right] \mathrm{d}s \tag{2}$$

where $T_0(x)$ can be the solution of its continuous partner of the problem with the same boundary/initial conditions of the fractal partner.

The polar bear's body temperature is about 37 °C and the environment temperature can be as low as –50 °C, we, therefore, have the boundary conditions:

$$T(0,t) = 37\ ^\circ\mathrm{C}, \qquad T(L,t) = -50\ ^\circ\mathrm{C} \tag{3}$$

The solution of 1-D heat conduction of polar bear hairs

Now we solve eq. (1). By the fractal complex transformation [5-8]:

$$s = \frac{x^\alpha}{\Gamma(1+\alpha)} \tag{4}$$

Equation (1) is converted to a partial differential equation, which reads:

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial s} D \frac{\partial T}{\partial s} \tag{5}$$

In order to search for wave solutions of eq. (5), we can introduce the transformation:

$$T(s,t) = T(\xi), \quad \xi = s - kt = \frac{x^\alpha}{\Gamma(1+\alpha)} - kt \tag{6}$$

Equation (5) becomes:

$$k \frac{\mathrm{d}T}{\mathrm{d}\xi} + \frac{\partial}{\partial \xi} D \frac{\partial T}{\partial \xi} = 0 \tag{7}$$

Solving eq. (7) results in:

$$T(\xi) = c_1 + c_2 \exp\left(-\frac{k\xi}{D}\right) \tag{8}$$

where c_1 and c_2 are integral constants. We, therefore, obtain the general exact solution:

$$T(x,t) = c_1 + c_2 \exp\left[\frac{k^2 t}{D} - \frac{kx^\alpha}{D\Gamma(1+\alpha)}\right] \tag{9}$$

Incorporating the boundary conditions, eq. (3), we finally have:

$$T(x,t) = \left[37 + 13\exp\frac{kL^\alpha}{D\Gamma(\alpha)\alpha} - 50\exp\frac{2kL^\alpha}{D\Gamma(\alpha)\alpha} - 87\exp\frac{kL^\alpha - kx^\alpha}{D\Gamma(\alpha)\alpha} + \right.$$
$$\left. +87\exp\frac{2kL^\alpha - x^\alpha}{D\Gamma(\alpha)\alpha}\right]\left[\exp\frac{kL^\alpha}{D\Gamma(\alpha)\alpha} - 1\right]^{-2} \tag{10}$$

Conclusions

1-D heat conduction equation of the polar bear hair is established and the general exact solution of the equation is obtained.

Acknowledgments

The work is supported by National Natural Science Foundation of China under grant 51463021 and Yunnan province NSF under grant No. 2011FB090.

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Paper submitted: October 10, 2014
Paper revised: January 21, 2015
Paper accepted: February 2, 2015