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A One - Dimensional Thermoelastic Problem due to Laser Pulse under Fractional Order Equation of Motion

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Abstract

In this work, we study the thermoelastic properties of an isotropic and homogeneous one dimensional semi-infinite elastic medium subjected to a laser short-pulse heating of time exponentially decaying pulse type in the light of the new theory of fractional order strain thermoelasticity. The solution for temperature, stress and strain distribution functions has been obtained in the Laplace domain. To obtain the different inverse field functions numerically we used a complex inversion formula of Laplace transform based on a Fourier expansion. The effects of different parameters; namely; the pulse intensity, time, fractional order and relaxation time on the thermodynamical temperature, stress and on the strain distribution, are presented graphically.

Key–Words: Fractional Order Strain, Laser Pulse, Generalized Thermoelasticity, Laplace Transform, Fractional Order Equation of Motion, Thermal Loadings.

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1 Introduction

Fourier heat conduction equation is not able to explain the correct rise in temperature in the irradiated regions of metallic surfaces due to short-pulse laser heating and assumption of infinite speed. These anomalies are overcome while considering the hyperbolic heat equation, since its analytical solution help in assessing the thermal response of the irradiated surface. Many research works have been undertaken in this area and the approximate solution of hyperbolic heat conduction equation due to mode locked pulses was reported by Hector et al.[1]. They proved the failure of classical Fourier heat equation in predicting the resulting temperature distribution in substrate material. Later Lin et al. [2] discussed the unsteady form of a unified heat conduction equation and formulated non-dimensional hyperbolic conduction equation and obtained the solution using separation of variables. Subsequently, Duhamel [3] gave the finite integral transform method to analyze the wave model of conduction. The results obtained by this method were similar to those of solving hyperbolic conduction equation using standard methods. Ali and Hany [4] derived directly the hyperbolic heat conduction equation from the relativity theory which was a direct consequence of space time duality without any consideration of microstructure of the heat conduction medium, which explained the relativistic heat conduction incorporating the wave nature of heat transfer. Wang [5] found the solution structure of hyperbolic heat conduction equation and reported the impact of both initial temperature distribution and the source disturbance in the temperature field in those equations. Ottoman [6] introduced the method of direct integration by means of the matrix exponential in the field of generalized thermoelasticity with two relaxation times to study the problem of a thick plate subjected to heating on the upper and lower surfaces of the plate varying exponentially with time. Further, Orodnez-Miranda and Alvarado-Gil [7] investigated the thermal wave oscillations and thermal relaxation time in a hyperbolic heat transport model by showing that the frequency range of strong oscillations in temperature when the thermal relaxation time of finite layer was close to its thermalization time. Christov [8] introduced the Maxwell Cattaneo finite speed heat conduction. The material-invariant version of the Maxwell Cattaneo law was proposed in which the relaxation rate of the heat flux was given by Oldroyds upper-converted derivative [9]. The approximate solution of the hyperbolic heat conduction equation was obtained by Yilbas and Pakdemir [10] by using the perturbation method and reported that the temperature was limited to certain range of time and space variables. Later, Al-Theeb and Yilbas [11] gave the closed form solution for the hyperbolic heat equation by reducing the hyperbolic equation from the electron kinetic theory approach. Al-Qahtani and Yilbas [12] also obtained the closed form solutions for thermal stress fields for the short heating duration and explained the wave nature of the stress behaviour. The earlier analytical solutions of hyperbolic heat conduction equation had limitations due to space and time domains owing to the technique used in perturbation method [10]. For instance, Ready [13] gave the analytical solution for constant intensity laser pulse heating while Blackwell [14] presented the closed-form solution for temperature field considering a convective boundary at the surface. Later, analytical solutions for temperature rise due to laser pulses similar to those used in real life laser heating was given by Yilbas [15] and Yilbas and Kalyon [16].

The above solutions were just based on solid heating and not on surface evaporation due to the complexity of its nature. Lu [17] analyzed the effect at the work piece surface by a laser beam with Gaussian intensity. He investigated the square-shaped temperature distribution there on but the phase change process was omitted. Modest and Abaikans[18] omitted the absorption of the laser beam and phase changes in the semi-infinite work piece during the process of heating. They analyzed analytically the rise in temperature in a moving semi-infinite substrate material due to conduction of heat from the laser. Shi et al.[19] studied the laser heating and phase change process and inferred that the rise in temperature and the rate of melting of surface is proportional to laser power intensity. Morozov et al. [20] presented an analytical model for inverse pulse laser heating. They predicted the thickness of molten material and compared their results with the experimental results and the comparison was satisfactory. Later, Gurasov and Smurov [21] studied the laser vapor plume interaction by proposing a model under estimated amount of energy absorbed by the ablated surface. Yilbas and Kalyon [16] introduced a closed form solution for laser evaporative heating process for pulses varying exponentially and introduced an expression for the evaporation front velocity in their analysis. The above analysis gave good results with experimental findings using the temporal variations of laser pulse being limited with exponential form.

Recently, Othman et al [22]-[25] studied the behavior of a thermoelastic , linear isotropic medium with voids subjected to laser pulse heating in the presence of different parameters; such as, the magnetic field, initial stress, time, rotation and gravity. Their studies were made in the context of G-N theory of types II and III. They found that these parameters are of great influence on the behavior of the physical quantities; temperature, stress and the component of displacement. They also reported that the amplitudes of these physical quantities depend strongly on the initial stress and rotation. They also deduced that the nature of the applied forces and the type of thermal loading have great effects on the distribution of these physical quantities. They carried out comparison between their studies and different theories of thermoelasticity, such as, Lord and Shulman theory and Chandrasekharaiah and Tzou model.

It is found that there are two types of common pulse shapes which depend mainly on the type and operation of the laser used; it may be in the form of step input pulses or time exponential decaying pulses. The mathematical analysis, therefore, will depend on the appropriate pulse type under consideration.

In recent times, fractional calculus has been proved useful in improving various models of physical process including those of polymer materials. The fractional order differential operator is non-local , this leads to the dependency of the current state of system to its previous state. Various definitions and applications of fractional derivatives, have been studied by many researchers [26]-[39]. The famous Riemann-Liouville definition of derivative of fractional order $\beta \in (n-1, n)$ of the function $f(t)$; plays an important role in the theory of fractional derivatives and integrals, defined as follows [26]:

$$_{RL}D_t^{\beta} f(t) = \frac{d^n}{dt^n}[\frac{1}{\Gamma(n-\beta)} \int_0^t (t-\tau)^{n-\beta-1} f(\tau) d\tau]$$
$$n-1 < \beta < n \tag{1}$$

Another definition was proposed by Caputo [27]:

$$_{C}D_t^{\beta} f(t) = \frac{1}{\Gamma(n-\beta)} \int_0^t (t-\tau)^{n-\beta-1} \frac{d^n f(\tau)}{d\tau^n} d\tau \tag{2}$$
$$n-1 < \beta < n$$

where $\Gamma(\beta)$ is the Gamma function. There are two major advantages of Caputo's definition; the first one is the initial conditions required when using the fractional differential operator. In case of Caputo fractional differentiation operator standard initial conditions in terms of integer order derivatives are required, whereas in Riemann - Liouville definition initial conditions of fractional derivatives are required. The second difference which is most impressing in conformities between the two operators is the differentiation of the constant function. According to Caputo's definition it holds:

$$_{C}D_t^{\beta} c = 0, \qquad c = constant \tag{3}$$

whereas according to the definition of Riemann-Liouville it holds:

$$_{RL}D_t^{\beta} c = \frac{c}{\Gamma(1-\beta)} t^{-\beta} \neq 0, \qquad c = constant \tag{4}$$

However, if $f(0) = 0$ then, the fractional derivative defined by Caputos and Riemann - Liouville is the same as [28]. The case $0 < \beta < 1$ represents the weak diffusion, while $\beta = 1$ represents normal diffusion, $1 < \beta < 2$ is strong diffusion whereas $\beta = 2$ represents the ballistic diffusion. Povestenko [26] and [29] studied the thermal stress theories using state space time telegraphic equation. Ezzat and Karamany [30] gave a new model using expansion of time given by Taylors fractional order expansion. Using the linear fractional order two temperature theory, they analysed anisotropic elastic non-homogeneous solid. They also build a new electro-thermo-elasticity model with references to the fractional order with heat conduction. By considering derivatives of order $0 < \beta \leq 2$, Sherief et al [31] derived the following heat conduction equation:

$$q_i + \tau_o \frac{\partial q_i}{\partial t^{\beta}} = -\kappa \ \frac{\partial T}{\partial t} \qquad 0 < \beta \leq 2 \tag{5}$$

where κ is the thermal diffusivity coefficient.

Youssef [32] introduced a new form of the heat conduction law as follows:

$$q_i + \tau_o \ \dot{q} = -\kappa \ I^{\beta-1} T_i \qquad 0 < \beta \leq 2 \tag{6}$$

and proved the uniqueness of the solution in this case. Considerable research studies and different contributions of the fractional order approaches have been carried out by Bassiouny and Zeinab [33], Bassiouny and Sabry [34], Othman et al [35]-[36] and many other authors [37]-[40]. Within the framework of the new theory of generalized thermo-elasticity with fractional order strain equation given by Youssef [39]; we investigate in the present study, the thermoelastic properties of an isotropic and homogeneous one dimensional semi-infinite elastic medium subjected to a laser short-pulse heating of time conduction exponentially decaying pulses type. Based on the definition introduced by Podlubny [38] given by:

$$D_t^{\beta} f(t) = \frac{1}{\Gamma(1-\beta)} \int_0^t \frac{f'(\tau)}{(t-\tau)^{\beta}} d\tau, \qquad \beta > 0 \quad (7)$$

the solution of the thermodynamic temperature , stress and strain distribution functions are obtained in the domain of Laplace transform and are presented graphically. Discussions are also made in the light of the new theory of generalized thermoelasticity with fractional order strain equation given by Youssef [39].

2 One Dimensional Formulation

In industrial applications of laser, in general, the size of the laser spot at the work piece is small and the absorption depth of the work piece is considerably smaller than the thickness of the work piece. Consequently, one-dimensional heating situation may become appropriate to formulate the heating problem [41]. For this we consider a half-space $(x \geq 0)$ with the x-axis pointing into the medium with initial temperature distribution T_0. This half-space is irradiated uniformly the bounding plane (x = 0) with non-Gaussian laser pulse. We assume that there are no body forces affecting the medium and all the state functions initially are equal to zero. The generalized thermoelastic governing differential equations, in the absence of body force, inner heat sources and free charge can be found in Youssef [39]. For the present one dimensional model, it is convenient to assume the following form of the displacement:

$$u_x = u(x,t), \qquad u_y = u_z = 0 \quad (8)$$

where u_x is the component of the displacement vector and the following one dimensional linearized system of equations given by:

The heat equation:

$$K\,(\frac{\partial^2\theta(x,t)}{\partial x^2}) = (\frac{\partial}{\partial t} + \tau_o\frac{\partial^2}{\partial t^2})(\rho C_E\,\theta(x,t) + T_o\gamma(1+\tau^{\beta}D_t^{\beta})e(x,t)) - (1+\tau_o\frac{\partial}{\partial t})I_1\delta e^{-(\Omega t+\delta x)} \quad (9)$$

where K is the thermal conductivity, τ_o is the relaxation time, ρ is the density, C_E is the specific heat at constant stain, T_0 is the reference temperature, $\gamma = \alpha_T(3\lambda + 2\mu)$, α_T is the thermal linear expansion coefficient, $\theta = T - T_0$ is the temperature increment such that $\theta/T_0 \ll 1$, I_1 is the power intensity , δ is the absorption coefficient, Ω is pulse parameter and e is the cubic dilatation.

Equation of motion:

$$\rho\frac{\partial^2 e(x,t)}{\partial t^2}=(\lambda+2\mu)\,(1+\tau^\beta D_t^\beta)\frac{\partial^2 e(x,t)}{\partial x^2}-\gamma\frac{\partial^2\theta(x,t)}{\partial x^2} \tag{10}$$

and the constitutive equation can be written in the form:

$$\sigma(x,t)=(1+\tau^\beta D_t^\beta)(\lambda+2\mu)e(x,t)-\gamma\theta(x,t) \tag{11}$$

and

$$e(x,t)=\frac{\partial u(x,t)}{\partial x} \tag{12}$$

where λ, μ are Lamé constants and σ is the principal stress component.

The medium is traction free and it is subjected to the following boundary conditions at the near end $x=0$:

$$\theta(0,t)=0,\quad \sigma(0,t)=0, \tag{13}$$

and it satisfies the following conditions at $x=\infty$

$$\theta(\infty,t)=0,\quad ,\quad \sigma(\infty,t)=0,\qquad 0<t<\infty \tag{14}$$

The medium is assumed to be initially at rest and has reference temperature T_0 so that the initial conditions become:

$$\theta(x,0)=0,\ \ e(x,0)=0,\ \ \sigma(x,0)=0,$$
$$\partial\theta(x,0)/\partial t=0,\quad \partial e(x,0)/\partial t=0,\quad \partial\sigma(x,0)/\partial t=0 \tag{15}$$

It is convenient now to introduce the following dimensionless variables Youssef [39]

$$\begin{aligned}
u' &= c_o\eta u,\ \ t'=c_o^2\eta t,\ \eta=\frac{\rho C_E}{\kappa},\quad \sigma'=\frac{\sigma}{\lambda+2\mu},\\
t_o' &= c_o^2\eta t_o,\quad \theta'=\frac{\gamma\,\theta}{\lambda+2\mu},\quad x_o'=c_o\eta x,\\
c_o^2 &= \frac{\lambda+2\mu}{\rho},\ \Omega'=\Omega/c_o^2\eta,\ \tau'=c_o^2\eta\tau,\\
\tau_o' &= c_o^2\eta\tau_o,\ \tau'^{\beta}=c_o^2\eta\tau^\beta,\ \delta'=\delta/c_o\delta\eta
\end{aligned} \tag{16}$$

where c_o is the longitudinal wave speed and η is the thermal viscosity.

Using equations (16) after dropping the primes for convenience,into the equations (9) and (10) lead to:

$$\begin{aligned}
\frac{\partial^2\theta(x,t)}{\partial x^2} &= [\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}](\theta(x,t)+\\
&\varepsilon_1\xi(1+\tau^\beta D_t^\beta)e(x,t))-\varepsilon_2 I_1\delta e^{-(\Omega t+\delta x)}
\end{aligned} \tag{17}$$

$$\frac{\partial^2 e(x,t)}{\partial t^2}=(1+\tau^\beta D_t^\beta)\frac{\partial^2 e(x,t)}{\partial x^2}-\omega\frac{\partial^2\theta(x,t)}{\partial x^2} \tag{18}$$

while the constitutive equations (11) and (12) become:

$$\sigma(x,t)=(1+\tau^\beta D_t^\beta)e(x,t)-\omega\theta(x,t) \tag{19}$$

and

$$e(x,t)=\frac{\partial u(x,t)}{\partial x} \tag{20}$$

where $\varepsilon_2=\frac{I_1\delta(1-\tau_o\Omega)}{c_o\eta K}$, $\xi=\frac{\gamma}{\rho C_E}$, $\omega=\frac{\gamma\,T_o}{\lambda+2\mu}$, $\varepsilon_1=\frac{\gamma^2 K}{\varrho C_E(\lambda+2\mu)}$ and $\varepsilon_2=\frac{I_1\delta(1-\tau_o\Omega)}{c_o\eta K}$ are non dimensional constants.

Using the definition of Laplace transform

$$L\{f(t)\}=\int_0^\infty e^{-st}f(t)dt \tag{21}$$

with the Laplace transform for the fractional order derivative according to Podlubny [38]:

$$L\{_C D_t^{\beta}\, f(t)\} = s^{(\beta\, - n)} L\{f^{(n)}(t)\} \tag{22}$$

where s denotes the complex parameter related to Laplace transform. Following Youssef [39], we apply the transformations (21) and (22) to the of equations (17) - (20), we get the following generalized thermoelasticity system of equations based on the fractional order strain equation of generalized thermoelasticity [39]: the heat equation takes the following form:

$$\frac{\partial^2 \overline{\theta}(x,s)}{\partial x^2} = s(s\tau_o + 1)[(\varepsilon_1 \xi(\tau^{\beta} s^{\beta} + 1)\,\overline{e}(x,s) + \overline{\theta}(x,s)] - \varepsilon e^{-\delta x} \tag{23}$$

and the transformed equation of motion assumes the form;

$$\frac{\partial^2 e(x,s)}{\partial x^2} = \frac{s^2 \overline{e}(x,s)}{(1+s^{\beta}\tau^{\beta})} + \frac{(1+s)\omega}{(1+s\beta\tau^{\beta})}\frac{\partial^2 \overline{\theta}(x,s)}{\partial x^2} \tag{24}$$

while the transformed constitutive equations take the form;

$$\overline{\sigma}(x,s) = (1+s^{\beta}\tau^{\beta})\,\overline{e}(x,s) - \omega\,(1+s)\overline{\theta}(x,s) \tag{25}$$

$$\overline{e}(x,s) = \frac{\partial \overline{u}\,(x,s)}{\partial x} \tag{26}$$

Combining equations (24) and (25) gives:

$$\frac{\partial^2 \overline{\sigma}(x,s)}{\partial x^2} = s^2 \overline{e}(x,s) \tag{27}$$

where $\varepsilon = \frac{\varepsilon_2}{s+\Omega}$.

3 Solution in the Laplace Domain

Eliminating $\overline{e}$ between the equations (23) and (24), we get the following fourth order equation;

$$ce^{-\delta x} + b\overline{\theta} - a\frac{\partial^2 \overline{\theta}}{\partial x^2} + \frac{\partial^4 \overline{\theta}}{\partial x^4} = 0 \tag{28}$$

where

$$a = \frac{s^2}{l_2} - L, \quad b = \frac{l_1 s^3}{l_2}, \;\; c = \frac{\varepsilon(l_2\delta^2 - s^2)}{(s+\Omega)l_2} \tag{29}$$

The most general solution of equation (28) is of the form:

$$\overline{\theta}(x,s) = \sum_{i=1}^{2} \theta_i e^{-k_i x} + \psi e^{-\delta x} \tag{30}$$

where $\psi = -c/(b - a\delta^2 + \delta^4)$ and $\pm k_1, \pm k_2$ are the roots of the characteristic equation:

$$b - ak^2 + k^4 = 0 \tag{31}$$

It is worth mentioning that the roots of the characteristic equation (28) are functions of s and given by:

$$\begin{aligned} k_1 &= \pm(\sqrt{a + \sqrt{a^2 - 4b}})/\sqrt{2}, \\ k_2 &= \pm(\sqrt{a - \sqrt{a^2 - 4b}})/\sqrt{2} \end{aligned} \tag{32}$$

Hence the most general solution of the equations (23) and (24) take the following forms:

$$\overline{e} = \sum_{i=1}^{2} \theta_i e^{-k_i x}(k_i^2 - l_1) + f_1(s) e^{-\delta x} \tag{33}$$

$$\overline{\sigma} = \sum_{i=1}^{2} \theta_i e^{-k_i x}(k_i^2 - L) + f_2(s) e^{-\delta x} \tag{34}$$

4 Determination of the Parameters:

Using the dimensionless variables (16) and applying the Laplace transform to the boundary and initial conditions (13), (14) and (15) respectively yield: the boundary conditions in Laplace domain:

$$\overline{\theta}(0,s) = 0, \qquad \overline{\sigma}(0,s)=0$$
$$\overline{\theta}(\infty,s) = 0, \qquad \overline{\sigma}(\infty,s)=0 \qquad (35)$$

while the initial conditions (15) take the following forms:

$$\overline{\theta}(x,0)=0, \quad \overline{e}(x,0)=0, \quad \overline{\sigma}(x,0)=0 \qquad (36)$$

Using these conditions, the parameters L, θ_i and $f_i(s), i = 1,2$ are determined as given below:

$$\theta_1 = -\theta_2 = \frac{\varepsilon + \psi(\delta^2 - sL)(s+\Omega)}{(k_1^2 - k_2^2)(s+\Omega)} \qquad (37)$$

$$f_1(s) = \frac{1}{s\varepsilon_1 \xi l_1 l_2}[\psi(\delta^2 - sl_1) + \frac{\varepsilon}{s+\Omega}] \qquad (38)$$

$$f_2(s) = \frac{1}{s\varepsilon_1 \xi l_1 (s+\Omega)}[\varepsilon + \psi(\delta^2 - sL(s+\Omega))] \qquad (39)$$

where $l_1 = 1 + s\tau_o$, $l_2 = 1 + s^{\beta}\tau^{\beta}$ and $L = l_1(1 + (1+s)\varepsilon_1\xi\omega)$.

Equations (30), (33) and (34) represent the complete solution in the Laplace transform domain.

5 Numerical Inversion of the Laplace Transform

To obtain the solutions of the non-dimensional field functions in the domain of Laplace, we compute numerically the inverse of these field functions by a method based on Fourier expansion technique. In this technique, the original function $f(t)$ of the Laplace transform $\overline{f}(s)$ is approximated by:

$$f(t) = \frac{\exp(ct)}{t_1}[\frac{1}{2}\overline{f}(c) + \Re(\sum_{1}^{N} \overline{f}(c + \frac{ik\pi}{t_1}\exp(\frac{ik\pi}{t_1})] \quad 0 < t_1 < 2t \qquad (40)$$

where $\Re$ is the real part, i is imaginary number unit and N is a sufficiently large integer representing the number of terms in the truncated Fourier series chosen such that:

$$\exp(ct)\Re\left[\overline{f}(c + \frac{iN\pi}{t_1})\exp(\frac{iN\pi t}{t_1})\right] \leq \epsilon_1 \qquad (41)$$

where ϵ_1 is prescribed small positive number that corresponds to the degree of accuracy required. The parameter c is a positive free parameter that must be greater than the real part of all the singularities of $\overline{f}(s)$. The optimal choice of c was obtained according to the criteria described in Honig and Hirdes [42].

6 Numerical Results and Discussion

For numerical computation, we use the following physical constants of copper materials extracted from [39]:

$k = 386 N/K sec, \alpha_T = 1.78(10)^5 K^-1, C_E = 383.1\, m^2/K$, $\eta = 8886.73\, m^2/sec, \mu = 3.86(10)^{10}\, N/m^2, \lambda = 7.76(10)^{10}$ N/m^2, $\rho = 8954\ kg/m^3$, $\tau_0 = 0.02\ sec$, $T_o = 293\ K$,

$\xi = 1.60861, \omega = 0.0104, \varepsilon_1 = 1.618, I_1 = 10^{13}$.

In all figures the dotted black lines represent the case when the parameter takes the minimum value; the red dashed lines represent the case when the parameter takes the maximum value and the solid black lines represent the case when the value of the parameter lie between the minimum and the maximum value.

We investigate the distributions of the field functions; the thermodynamic temperature θ, the normal stress component σ and the strain distribution e with different values of the parameters such as the pulse intensity Ω, fractional order β and time t. The results were presented in three groups of figures, each group presents the effect of one of the parameters Ω, β and t. Figures 1 show the dependence of the field functions; the thermodynamical temperature θ, the stress σ and the strain e on the impulse intensity Ω.

Figure 1(a) shows that the rise of the temperature is rapid at the end near to the point of application of heat on the material. The increase in the pulse intensity results in decreasing the amplitude of the temperature. The cooling cycle starts at $x \cong 0.6$ and the temperature decreases as x-coordinates increases. It is also noticed that the temperature decay is gradual, on moving away from the point of application of heat.

Figure 1(b) illustrates that the stress is compressive at the near end of the medium. As value of x- axis increases, the stress becomes tensile and then starts to reduce rapidly at $x \cong 1$. The absolute value of the amplitude of the stress distribution decreases with increasing the impulse intensity. Gradual decay has been noticed as x- axis increases due to the internal energy gain from the irradiated field.

Figure 1(c) shows that the strain distribution resembles the same behaviour like the stress distribution function but at $x = 0$ the magnitude of the strain $e \neq 0$ and the component $\sigma = 0$. It is also noticed that as the impulse intensity increases the magnitude of the strain decreases.

Figures 2 show the effect of the fractional order parameter β on the field functions; θ, σ and e. Figure 2(a) reflects the profile of the thermodynamical temperature along x- axis.It can be seen that the temperature increases rapidly at the near end and at $x \approx 0.6$ it attains its maximum. At $x \gtrapprox 0.6$ the temperature starts decreasing rapidly along x - axis and vanishes at $x \cong 3.5$ It can also be seen from figure 2(a) that the temperature decay inside the material starts gradually, which is associated with internal energy gain from the irradiated field in the region. We further notice that next to the surface vicinity the temperature decreases exponentially and it is towards the far end of the material. Comparison with figure 1(a) indicates that the temperature resembles the same behaviour and profile.

Figure 2(b) shows that the absolute value of the magnitude of the stress component σ decreases as the value of fractional order parameter β increases and it is compressive through the entire domain.

Figure 2(c) shows that the strain distributions function resembles the same behaviour under the variation of the impulse intensity but it vanishes early at $x \cong 2.0$.

Figures 3 illustrate the variation of different distribution field functions with different value of time. Figure 3(a) shows that the magnitude of temperature increases as the time t increases resembling similar behaviour as in figure1(a) and not like in figure2(a). As the absorbed power equals the conducted one inside the material the temperature attains its maximum value at $x \cong 0.6$ and the cooling cycle starts beyond this point.

It is also noticed that the temperature initially increases with increasing the value of the time near the heated end. This may be due to the increased absorbed energy which over compensates the heat loss due to the heat conduction inside the material.

Figure 3(b) illustrates the dependence of the stress distribution on time. The stress is compressive in the entire domain because of the diffusional energy transfer from the surface to the solid bulk which suppresses the internal energy gain from the irradiated field in the surface region.

Figure 3(c) shows that the absolute value of the amplitude of the strain distribution function increases as the time increases. The absolute value of the magnitude starts decreasing at the point $x \cong 0.45$ and vanishes at $x = 3$.

Conclusions

The effect of the parameters Ω, β and t are studied and it is noticed that:

All the field functions θ, σ and e are strongly influenced by the parameters Ω, β and t.

The increase of the value of the parameters Ω and β results in decreasing the absolute value of the amplitude of these field functions, but the increase of the parameter t results in increasing these amplitudes.

The behaviour of the thermally induced stress and strain of the medium resemble the same behaviour except at $x = 0$.

The components of the stress and strain attain the equilibrium state at the same points with the variation of the parameters.

The thermodynamical temperature attains its equilibrium state faster than the stress and strain functions. It attains the equilibrium state at $x = 0$.

All figures illustrate that the numerical solution of the field functions satisfies the boundary conditions.

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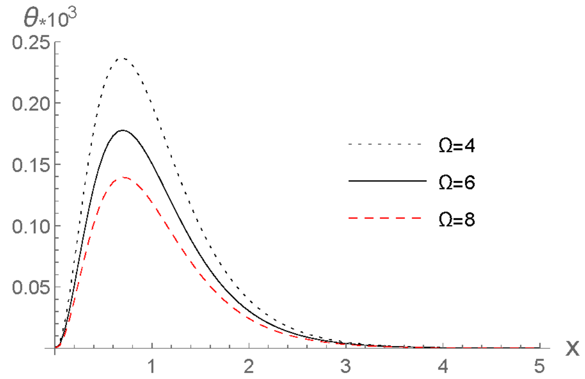
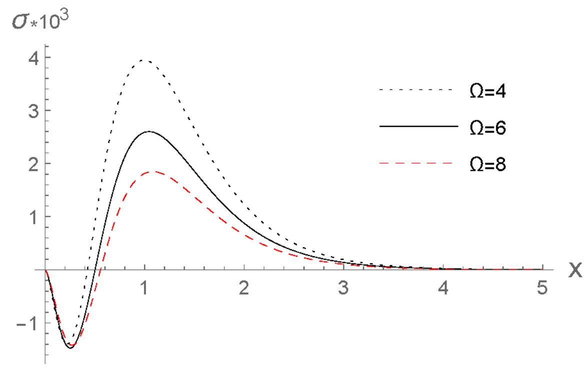
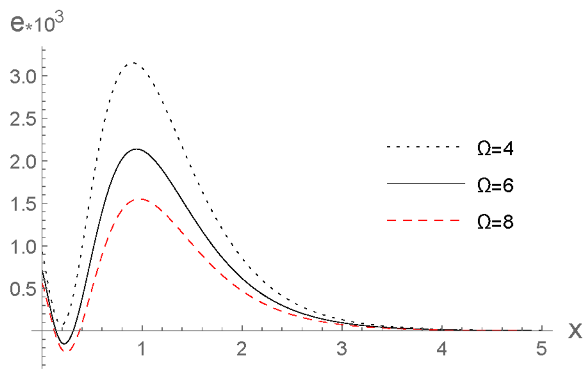


(a) Thermodynamical temperature



(b) Stress distribution



(c) Strain distribution

Figure 1: Effect of Pulse parameter Ω on the Different filed functions at $t = 0.3, \beta = 0.5, \delta = 3.0 \;\; and \;\; \tau_o = 0.02$

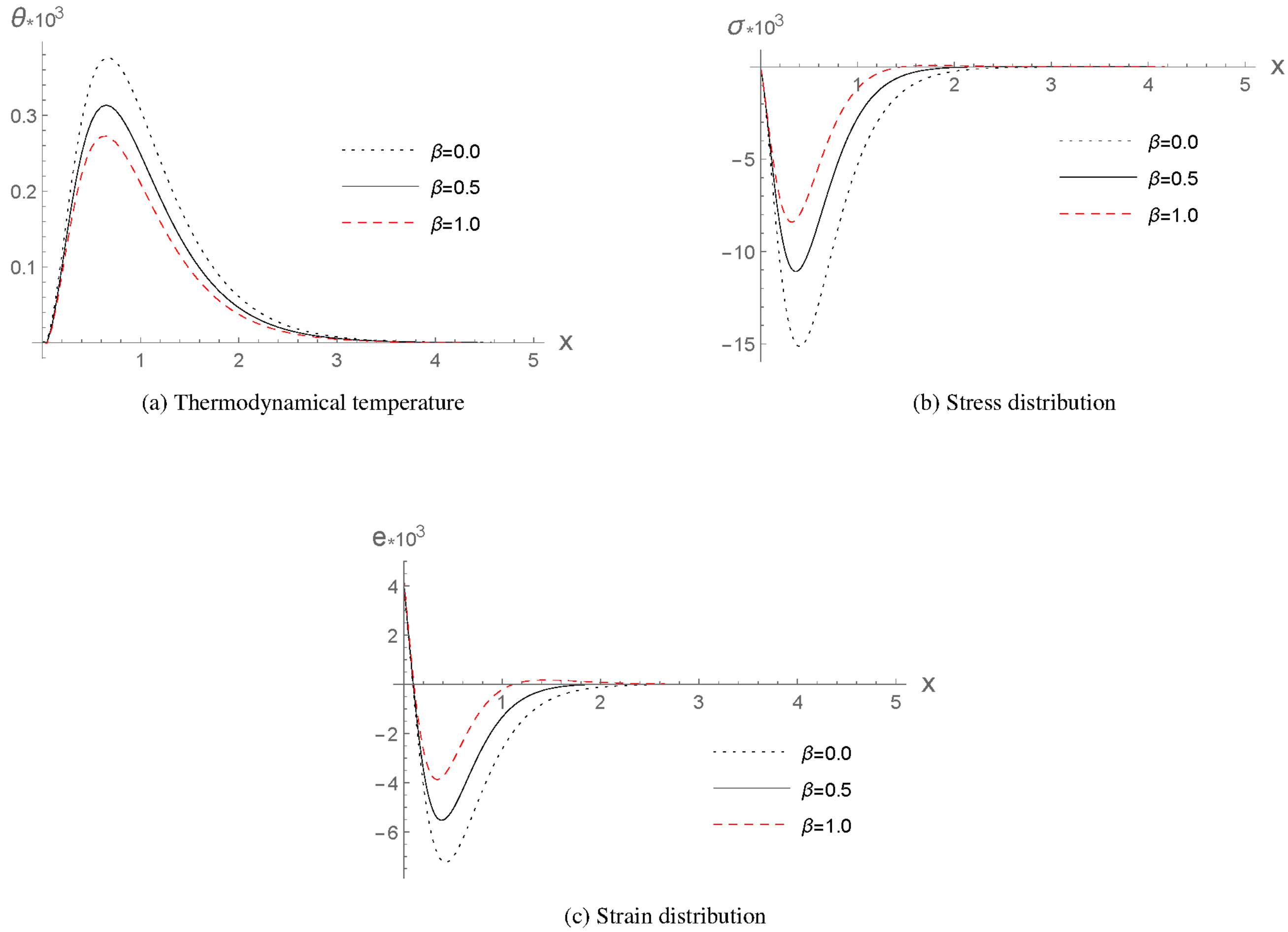
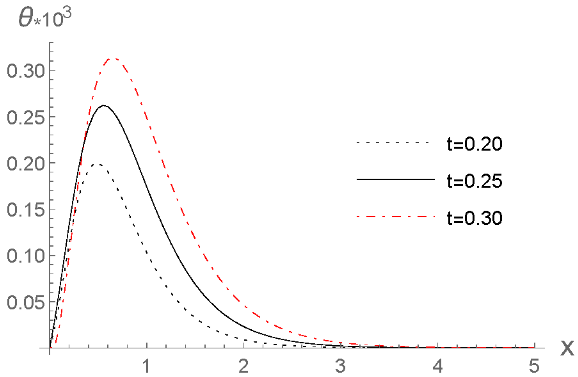
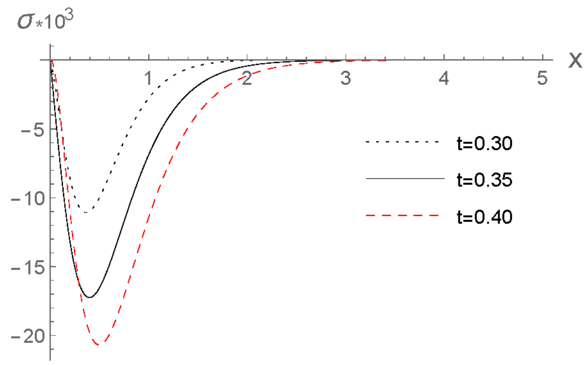


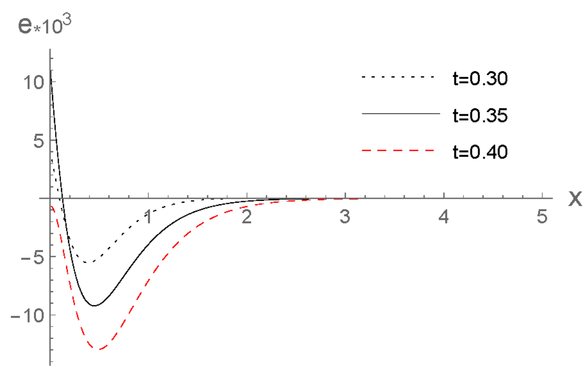
Figure 2: Effect of fractional order parameter β on the Different filed functions at $t = 0.3, \Omega = 6.0, \delta = 3.0 \;\; and \;\; \tau_o = 0.02$



(a) Thermodynamical temperature



(b) Stress distribution



(c) Strain distribution

Figure 3: Effect of time parameter t on the Different filed functions at $\beta = 0.5, \delta = 3.0,\ \Omega = 6.0\ \ and\ \ \tau_o = 0.02$