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One-loop corrections to $h \to b\bar{b}$ and $h \to \tau\bar{\tau}$ decays in the Standard Model dimension-6 EFT: four-fermion operators and the large-m_t limit

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Abstract: We calculate a set of one-loop corrections to $h \to b\bar{b}$ and $h \to \tau\bar{\tau}$ decays in the dimension-6 Standard Model effective field theory (SMEFT). In particular, working in the limit of vanishing gauge couplings, we calculate directly in the broken phase of the theory all large logarithmic corrections and in addition the finite corrections in the large-m_t limit. Moreover, we give exact results for one-loop contributions from four-fermion operators. We obtain these corrections within an extension of the widely used on-shell renormalisation scheme appropriate for SMEFT calculations, and show explicitly how UV divergent bare amplitudes from a total of 21 different SMEFT operators are rendered finite within this scheme. As a by-product of the calculation, we also compute to one-loop order the logarithmically enhanced and finite large-m_t corrections to muon decay in the limit of vanishing gauge couplings, which is necessary to implement the G_F input parameter scheme within the SMEFT.

Keywords: Effective field theories, Higgs Physics

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Contents

1 Introduction

One of the main successes of Run-I at the LHC was the discovery [1, 2] of a new particle with a mass of 125 GeV [3]. Early measurements of the various production and decay properties of this particle indicate that it has quantum numbers ($J^{PC} = 0^{++}$) and coupling strengths to fermions and gauge bosons consistent with the Standard Model (SM) Higgs boson [4–7]. As the experimental precision of Higgs measurements improves, comparisons with precise theory calculations will further elucidate the properties of the observed boson and determine whether they are as predicted by the SM.

In this paper we study potential new physics contributions to Higgs boson decays to third generation fermions, namely to $h \to b\bar{b}$ and $h \to \tau\bar{\tau}$ decays. We perform the analysis within the framework of the Standard Model Effective Field Theory (SMEFT), where the effects of new particles at a UV scale $\Lambda_{\rm NP}$ are parameterised through non-vanishing Wilson coefficients of higher-dimensional operators. These operators, which effectively describe the interactions of the new particles with the SM, are built from gauge invariant combinations of SM fields and are added onto the usual dimension-4 SM Lagrangian.

The SMEFT approach is justified as long as the new physics scale $\Lambda_{\rm NP}$ characteristic of the masses of as yet undiscovered particles is much larger than the electroweak scale, a scenario which seems quite likely given the absence of direct evidence for new particles in the Run-I data. The main benefit of such an approach is that no assumptions are made on the nature of new physics, so interpretations of experimental data can be made in a model-independent fashion.[1]

The current precision of Higgs measurements is such that a leading order (LO) analysis within the SMEFT is sufficient. However, as the experimental situation improves (especially at a potential e^+e^- collider, see for example [9]), it will be necessary to carry out next-to-leading order (NLO) calculations within the SMEFT. The main point is the following. When the new physics theory is matched onto the effective one at the scale $\Lambda_{\rm NP}$, the coefficients of the operators which are generated in this matching procedure are defined at the scale $\Lambda_{\rm NP}$. However, measurements of Higgs couplings are performed at the scale of the Higgs mass m_H ($m_H \ll \Lambda_{\rm NP}$). Under such conditions, renormalisation group (RG) improved perturbation theory should be used, and the Wilson coefficients $C_i(\Lambda_{\rm NP})$ should be evolved to the scale m_H according to the solution to the RG equations, determined from an anomalous dimension matrix γ_{ij}. Since γ_{ij} is in general non-diagonal, the RG evolution (RGE) introduces mixing among operators. In other words, a measurement of a process which is sensitive to a particular Wilson coefficient $C_i(m_H)$ in a LO analysis, is in general sensitive to multiple Wilson coefficients at the scale $\Lambda_{\rm NP}$, as implied through the RGE. In addition, one-loop diagrams also generate non-logarithmic finite contributions, and there is no way of knowing if these contributions are large or small without explicitly calculating them. Both of these effects are neglected in an SMEFT LO analysis, and it is therefore important to extend analyses to NLO to consistently interpret experimental data in a robust manner.

From a theoretical point of view, the problem of NLO SMEFT calculations is interesting in its own right, and there have been several recent theoretical advancements in this direction. In [8, 10, 11], the full one-loop anomalous dimension matrix for baryon number conserving dimension-6 operators was calculated, building on partial results given in [12–14]. The corresponding analysis for baryon number violating operators was provided in [15]. Such process-independent results form the basis for RG-improved LO analyses of physical processes, and are also integral to the renormalisation procedure used in process-dependent matrix element calculations such as the one performed in the present work. Various work in such directions can be found in [16–27] — see [22, 23] for detailed discussions on the topic.

In this work, we present results for one-loop corrections to $h \to b\bar{b}$ and $h \to \tau\bar{\tau}$ decays. The main motivation is the eventual phenomenological application of the results, however we take the opportunity to describe in detail how to incorporate the dimension-6 operators into the on-shell renormalisation scheme used in most Standard Model calculations — an

[1]When taking into account baryon number conserving dimension-6 operators, there are 2499 operators and real parameters [8]. A full global fit of data to such a number of degrees of freedom is unrealistic and therefore many simplifying assumptions are made in most analyses, but this is a question of implementation rather than principle.

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excellent review of this procedure is provided in [28]. In order to illustrate this procedure in the context of the SMEFT, we focus on two types of contributions. We first calculate the contributions from four-fermion operators. As this calculation is fairly straightforward, it serves as a useful example to demonstrate how the renormalisation procedure can be more generally applied to SMEFT calculations. After this, we then compute those contributions which arise in the limit of vanishing gauge couplings in the broken phase of the theory, where we identify those terms which are leading in the large-m_t limit. These limits are defined more quantitatively below:

- Vanishing gauge couplings. The QCD corrections, which are present for the case of $h \to b\bar{b}$ decays, are trivially zero. For corrections involving electroweak gauge bosons, vanishing gauge couplings corresponds to neglecting all contributions which do not contain negative powers of $M_{W,Z}^2$, i.e. we calculate terms of $\mathcal{O}(\alpha/M_{W,Z}^2)$. Consequently, it is not necessary to consider real emission diagrams, and the calculation is infrared finite.

- Large-m_t limit. To identify the leading-m_t corrections, we neglect all fermion masses in the one-loop corrections with the exception of the top-quark, and assume $m_t \gg m_H$. However, as a number interesting features of the renormalisation procedure are subleading in this limit, we choose to keep the full mass dependence in the UV singular contributions and also in the coefficients of μ-dependent logarithms.

The corrections defined in this way are a well defined subset of the complete NLO calculation,[2] and extend the analogous SM calculation performed in [30] to include dimension-6 contributions.

The layout of the paper is as follows. First, the ingredients of the SMEFT necessary to compute the tree-level contributions to $h \to b\bar{b}$ and $h \to \tau\bar{\tau}$ are provided in section 2. In section 3, we discuss some details of the on-shell renormalisation scheme in the SMEFT as applied to $h \to f\bar{f}$ decays, and also comment on how the Fermi constant can be incorporated as an input parameter. In section 4, the contribution from four-fermion operators is computed. In section 5, we provide the contributions in the limit of vanishing gauge couplings, applying the large-m_t limit to these corrections. We discuss the phenomenological implications of our results on the interpretation of future data on $h \to b\bar{b}$ and $h \to \tau\bar{\tau}$ decays in section 6. Finally, we give some details of our procedure for calculating the decay amplitudes in the large m_t-limit in appendix A.

2 Tree-level contributions in the SMEFT

In this section we introduce the elements of the SMEFT which are necessary to describe the tree-level $h \to b\bar{b}$ and $h \to \tau\bar{\tau}$ decay amplitudes. We start with the Lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}^{(6)}\,; \qquad \mathcal{L}^{(6)} = \sum_i C_i(\mu) Q_i(\mu)\,, \tag{2.1}$$

[2] The full results, including the dependence on gauge couplings, will be presented in future work [29].

which is decomposed into the Standard Model Lagrangian $\mathcal{L}^{\rm SM}$ and dimension-6 Lagrangian $\mathcal{L}^{(6)}$. The operators appearing in the Lagrangian are naturally defined in the unbroken phase of the gauge theory, where the vacuum expectation value of the Higgs field vanishes. A complete set of 59 gauge-invariant dimension-6 operators was first established in [31] (a refinement of the over-complete basis originally proposed in [32]), and is listed in table 1. The Wilson coefficients C_i of the dimension-6 operators implicitly contain two inverse powers of $\Lambda_{\rm NP}$, and are therefore dimensionful. Additionally, the labeling convention of the operators appearing in table 1 is also applied to the corresponding Wilson coefficient. For example, the Wilson coefficient of the operator Q_{dH} is C_{dH}. This notation will be used throughout.

2.1 Yukawa sector

The effective Yukawa couplings and mass matrices in the broken phase of the theory, where the vacuum expectation value of the Higgs field is non-vanishing, arise from the following terms in the unbroken one:

$$\begin{aligned}\mathcal{L} = &- \left[[Y_u]_{rs}\tilde{H}^{\dagger j}\overline{u}_r\, Q_{sj} + [Y_d]_{rs}H^{\dagger j}\overline{d}_r\, Q_{sj} + [Y_e]_{rs}H^{\dagger j}\overline{e}_r\, L_{sj} + h.c.\right] \\ &+ \left[C^*_{\substack{uH\\ sr}}(H^\dagger H)\tilde{H}^{\dagger j}\overline{u}_r\, Q_{sj} + C^*_{\substack{dH\\ sr}}(H^\dagger H)H^{\dagger j}\overline{d}_r\, Q_{sj} + C^*_{\substack{eH\\ sr}}(H^\dagger H)H^{\dagger j}\overline{e}_r\, L_{sj} + h.c.\right] \\ &- V(H)\,, \end{aligned} \tag{2.2}$$

where

$$V(H) = \lambda\left(H^\dagger H - \frac{1}{2}v^2\right)^2 - C_H(H^\dagger H)^3\,. \tag{2.3}$$

The dimension-6 operators alter the tree level-relations between parameters in the broken and unbroken phase of the theory compared to the SM. We now summarise the modifications relevant for $h \to b\bar{b}$ and $h \to \tau\overline{\tau}$ decay amplitudes, following closely the discussion and notation from [8], which contains all necessary elements.

We write the Higgs doublet in a general R_ξ gauge in the broken phase of the theory as

$$H(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} -\sqrt{2}i\phi^+(x) \\ [1+C_{H,\rm kin}]\,h(x) + i\left[1-\frac{v^2}{4}C_{HD}\right]\phi^0(x) + v_T\end{pmatrix}, \tag{2.4}$$

where ϕ^0 and ϕ^+ are Goldstone boson modes, and the following relations have been introduced

$$C_{H,\rm kin} \equiv \left(C_{H\Box} - \frac{1}{4}C_{HD}\right)v^2\,, \qquad v_T \equiv \left(1 + \frac{3C_H v^2}{8\lambda}\right)v\,. \tag{2.5}$$

The prefactors of the $h(x)$ and $\phi^0(x)$ fields are determined by the requirement that the kinetic terms in the broken phase of the theory are canonically normalised. We have distinguished the quantities v and v_T above, but since the difference between them is a dimension-6 effect, they can be interchanged freely when multiplying a dimension-6 Wilson coefficient, and we will always refer to this quantity as v_T under such circumstances.

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The Higgs boson mass is found by expanding (2.3), and leads to

$$m_H^2 = 2\lambda v_T^2 \left(1 - \frac{3C_H v^2}{2\lambda} + 2C_{H,\text{kin}}\right). \tag{2.6}$$

Similarly, the effective mass and Yukawa matrices for fermions are

$$[M_f]_{rs} = \frac{v_T}{\sqrt{2}}\left([Y_f]_{rs} - \frac{1}{2} v_T^2 C^*_{\underset{sr}{fH}}\right), \tag{2.7}$$

$$\begin{aligned} [\mathcal{Y}_f]_{rs} &= \frac{1}{\sqrt{2}}\left([Y_f]_{rs}\left[1 + C_{H,\text{kin}}\right] - \frac{3}{2} v_T^2 C^*_{\underset{sr}{fH}}\right) \\ &= \frac{1}{v_T}\left[M_f\right]_{rs}\left[1 + C_{H,\text{kin}}\right] - \frac{v_T^2}{\sqrt{2}} C^*_{\underset{sr}{fH}}. \end{aligned} \tag{2.8}$$

The Yukawa and mass matrices depend on distinct linear combinations of the SM Yukawa matrix and the dimension-6 terms C^*_{fH}. Therefore, after transforming from the gauge to mass eigenstates by performing field redefinitions on the fermion fields, the operators in the mass basis contain a myriad of flavour violating effects beyond those in the CKM matrix. While such flavour violating effects beyond those present in the SM are interesting phenomenologically (see for example [33]), particularly in light of the excess observed in $h \to \tau\mu$ events by the CMS [34] collaboration, the main focus of the present work is on one-loop corrections rather than questions of flavour. Therefore, we ignore such flavour-violating couplings in this work. This can be made more rigorous by imposing minimal flavour violation (MFV) [35, 36], an assumption which ensures that the mass and Yukawa matrices are simultaneously diagonalizable at all scales, a feature preserved by the RG running [8]. The transition from the gauge to mass eigenstates then proceeds much as in the SM, and in fact can be rendered trivial by considering only the third generation in the calculation of one-loop effects. We will use this set up throughout the paper, i.e. consider one generation of fermions and set the CKM element V_{tb} to unity.

With these simplifications in place, the Yukawa couplings to third generation fermions, defined as the coefficients of the $hf\bar{f}$ coupling in the mass basis of the broken theory, are related to the physical masses according to

$$y_f = \sqrt{2}\frac{m_f}{v_T} + \frac{v_T^2}{2} C^*_{fH}, \tag{2.9}$$

and it is a simple matter to calculate the tree-level decay amplitude for the process $h \to f\bar{f}$:

$$i\mathcal{M}^{(0)}(h \to f\bar{f}) = -i\bar{u}(p_f)\left(\mathcal{M}^{(0)}_{f,L} P_L + \mathcal{M}^{(0)*}_{f,L} P_R\right) v(p_{\bar{f}}), \tag{2.10}$$

where

$$\mathcal{M}^{(0)}_{f,L} = \frac{m_f}{v_T}\left[1 + C_{H,\text{kin}}\right] - \frac{v_T^2}{\sqrt{2}} C^*_{fH}. \tag{2.11}$$

2.2 Input parameters

We have expressed the result (2.11) in terms of v_T, but in practice this parameter should be eliminated in terms of observables of the broken phase of the theory. In the renormalisation procedure discussed in the next section, we choose to work with the following set of

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independent, physical parameters:

$$\bar{e}, m_H, M_W, M_Z, m_f, C_i \,. \tag{2.12}$$

Using the expressions from [8], one has

$$\begin{aligned}
M_W^2 &= \frac{\bar{g}_2^2 v_T^2}{4} \,, \\
M_Z^2 &= \frac{v_T^2}{4}(\bar{g}_1^2 + \bar{g}_2^2) + \frac{1}{8} v_T^4 C_{HD} (\bar{g}_1^2 + \bar{g}_2^2) + \frac{1}{2} v_T^4 \bar{g}_1 \bar{g}_2 C_{HWB} \,, \\
\bar{e} &= \bar{g}_2 \bar{s}_w - \frac{1}{2} \bar{c}_w \bar{g}_2 v_T^2 C_{HWB} \,, \\
\bar{s}_w^2 &= \frac{\bar{g}_1^2}{\bar{g}_1^2 + \bar{g}_2^2} + \frac{\bar{g}_1 \bar{g}_2 (\bar{g}_2^2 - \bar{g}_1^2)}{(\bar{g}_1^2 + \bar{g}_2^2)^2} v_T^2 C_{HWB} \,.
\end{aligned} \tag{2.13}$$

The barred quantities appear as couplings in the covariant derivative in the broken phase of the theory after rotation to the mass eigenbasis; in particular $\bar{g}_2$ governs the charged current couplings while $\bar{e}$ is the electric charge. Manipulating the above expressions we can write

$$\frac{1}{v_T} = \frac{\bar{e}}{2 M_W \bar{s}_w} \left(1 + \frac{\hat{c}_w}{2 \hat{s}_w} C_{HWB} \hat{v}_T^2 \right) , \tag{2.14}$$

where we have defined

$$\hat{v}_T \equiv \frac{2 M_W \hat{s}_w}{\bar{e}} \,; \qquad \hat{s}_w^2 \equiv 1 - \frac{M_W^2}{M_Z^2} \,, \quad \hat{c}_w^2 \equiv 1 - \hat{s}_w^2 \,, \tag{2.15}$$

such that the hatted quantities are the usual definitions in the SM. In expression (2.14), we denote parameters multiplying the Wilson coefficients by the hatted quantities. This is consistent to $\mathcal{O}(1/\Lambda_{NP}^2)$, and is the notation which will be adopted throughout this work. It is possible to re-express v_T and $\bar{s}_w$ in terms of the gauge boson masses, and quantities derived from them. In particular, the quantity $\bar{s}_w$ can be expressed as

$$\bar{s}_w^2 = \hat{s}_w^2 - \frac{\hat{c}_w^2 v_T^2}{2} \left(C_{HD} + 2 \frac{\hat{s}_w}{\hat{c}_w} C_{HWB} \right) , \tag{2.16}$$

and inserting this into (2.14) leads to

$$\frac{1}{v_T} = \frac{1}{\hat{v}_T} + \frac{\hat{c}_W}{\hat{s}_W} \left(C_{HWB} + \frac{\hat{c}_W}{4 \hat{s}_W} C_{HD} \right) \hat{v}_T \,. \tag{2.17}$$

Equation (2.17) then allows to write v_T in terms of the parameters (2.12), that is, the physical parameters in the broken phase of the theory. While this is a reasonable choice, it is instead customary to eliminate M_W in favour of the Fermi constant G_F, defined and extracted through the muon decay rate [37]. At tree level, and ignoring contributions which do not interfere with the SM, we can write [8]

$$\frac{1}{\sqrt{2}} \frac{1}{v_T^2} = G_F - \frac{1}{\sqrt{2}} \left(C^{(3)}_{\underset{ee}{Hl}} + C^{(3)}_{\underset{\mu\mu}{Hl}} \right) + \frac{1}{2\sqrt{2}} \left(C_{\underset{\mu e e \mu}{ll}} + C_{\underset{e \mu \mu e}{ll}} \right) , \tag{2.18}$$

where the operator $C^{(3)}_{\substack{Hl \\ ll}}$, which alters the W boson coupling to the lepton doublets, and also the four-fermion operators $C_{\substack{ll \\ e\mu\mu e}}$ and $C_{\substack{ll \\ \mu e e\mu}}$ explicitly enter the amplitude for muon decay. One can then insert the above equation into (2.17) and solve for M_W as a function of G_F and the other observables appearing in (2.12).

3 The one-loop renormalisation procedure

From a practical point of view, the calculation of one-loop corrections to $h \to b\bar{b}$ and $h \to \tau\bar{\tau}$ decays in the SMEFT has two components — the bare one-loop matrix elements, and the UV counterterms required to subtract the UV poles (and in some cases finite parts) from these divergent matrix elements. The calculation of the one-loop matrix elements is conceptually straightforward and will be discussed later on. In this section we cover the somewhat more subtle issue of constructing the UV counterterms. In particular, we explain how to adapt the on-shell renormalisation scheme used to calculate electroweak corrections in the SM to the SMEFT case.

To renormalise bare amplitudes we must provide UV counterterms for the set of independent, physical parameters in (2.12), and also perform wavefunction renormalisation on external fields. We choose to renormalise the masses and electric charge in the on-shell scheme, and construct counterterms related to these quantities exactly as in the SM. This requires the computation of a number of two-point functions directly in the broken phase of the theory. On the other hand, we renormalise the Wilson coefficients C_i in the $\overline{\text{MS}}$ scheme, as is standard in EFT calculations. Crucially, the counterterms associated with the Wilson coefficients can be taken from results in the unbroken phase of the theory calculated in [8, 10, 11].

We begin with wavefunction, mass, and electric charge renormalisation, which proceeds as in the SM. We will only discuss those contributions relevant for $h \to b\bar{b}$ and $h \to \tau\bar{\tau}$ decays. Defining the renormalised fields in terms of bare ones, indicated with the superscript (0), we have

$$\begin{aligned}
h^{(0)} &= \sqrt{Z_h}h = \left(1 + \frac{1}{2}\delta Z_h\right) h\,, \\
f_L^{(0)} &= \sqrt{Z_f^L} f_L = \left(1 + \frac{1}{2}\delta Z_f^L\right) f_L\,, \\
f_R^{(0)} &= \sqrt{Z_f^R} f_R = \left(1 + \frac{1}{2}\delta Z_f^R\right) f_R\,,
\end{aligned} \tag{3.1}$$

where the fermion subscript f refers to either b-quarks or τ-leptons. We define renormalised masses and the renormalised electric charge as

$$M^{(0)} = M + \delta M, \qquad \bar{e}_0 = \bar{e} + \delta\bar{e}\,, \tag{3.2}$$

where M is a generic mass. The Higgs mass does not enter our tree-level expression for the decay rate, and it is therefore not renormalised. As a consequence, the contribution from tadpole diagrams are cancelled exactly by those of the corresponding counterterms, and these contributions can therefore be effectively ignored in our calculation.

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To determine wave function renormalisation factors and the counterterms related to mass and electric charge renormalisation, we follow the procedure outlined in [28, 38], which requires the computation of a set of two-point functions in the broken phase of the theory. We write the generic two-point functions as

$$
\begin{aligned}
\Gamma^f(p) &= i(\not{p} - m_f) + i\left[\not{p}\left(P_L\Sigma_f^L(p^2) + P_R\Sigma_f^R(p^2)\right) + m_f\left(\Sigma_f^S(p^2)P_L + \Sigma_f^{S*}(p^2)P_R\right)\right], \\
\Gamma^H(k) &= i(k^2 - m_H^2) + i\Sigma^H(k^2), \\
\Gamma^W_{\mu\nu}(k) &= -ig_{\mu\nu}(k^2 - M_W^2) - i\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\Sigma_T^W(k^2) - i\frac{k_\mu k_\nu}{k^2}\Sigma_L^W(k^2), \\
\Gamma^{ab}_{\mu\nu}(k) &= -ig_{\mu\nu}(k^2 - M_a^2)\delta_{ab} - i\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\Sigma_T^{ab}(k^2) - i\frac{k_\mu k_\nu}{k^2}\Sigma_L^{ab}(k^2),
\end{aligned} \tag{3.3}
$$

where $a, b = A, Z$, and $M_A^2 = 0$. Given results for the two-point functions, one can calculate the counterterms from wavefunction renormalisation in the on-shell scheme according to[3]

$$
\begin{aligned}
\delta Z_f^L &= -\widetilde{\mathrm{Re}}\,\Sigma_f^L(m_f^2) + \Sigma_f^S(m_f^2) - \Sigma_f^{S*}(m_f^2) \\
&\quad - m_f^2\frac{\partial}{\partial p^2}\widetilde{\mathrm{Re}}\left[\Sigma_f^L(p^2) + \Sigma_f^R(p^2) + \Sigma_f^S(p^2) + \Sigma_f^{S*}(p^2)\right]\Big|_{p^2=m_f^2}, \\
\delta Z_f^R &= -\widetilde{\mathrm{Re}}\,\Sigma^{f,R}(m_f^2) \\
&\quad - m_f^2\frac{\partial}{\partial p^2}\widetilde{\mathrm{Re}}\left[\Sigma_f^L(p^2) + \Sigma_f^R(p^2) + \Sigma_f^S(p^2) + \Sigma_f^{S*}(p^2)\right]\Big|_{p^2=m_f^2}, \\
\delta Z_h &= -\,\mathrm{Re}\frac{\partial\Sigma^H(k^2)}{\partial k^2}\Big|_{k^2=m_H^2}.
\end{aligned} \tag{3.4}
$$

The mass counterterms are computed as

$$
\begin{aligned}
\delta m_f &= \frac{m_f}{2}\widetilde{\mathrm{Re}}\left(\Sigma_f^L(m_f^2) + \Sigma_f^R(m_f^2) + \Sigma_f^S(m_f^2) + \Sigma_f^{S*}(m_f^2)\right), \\
\frac{\delta M_W}{M_W} &= \widetilde{\mathrm{Re}}\,\frac{\Sigma_T^W(M_W^2)}{2M_W^2}.
\end{aligned} \tag{3.5}
$$

We have listed relations for the W boson above — those for the Z boson are completely analogous. The symbol $\widetilde{\mathrm{Re}}$ takes the real part of the matrix elements in the two-point functions but not of the CKM matrix elements or the Wilson coefficients themselves. The renormalisation of the electric charge is also computed from two-point functions according to

$$
\frac{\delta\bar{e}}{\bar{e}} = \frac{1}{2}\frac{\partial\Sigma_T^{AA}(k^2)}{\partial k^2}\Big|_{k^2=0} + \frac{(v_f - a_f)}{Q_f}\frac{\Sigma_T^{AZ}(0)}{M_Z^2}, \tag{3.6}
$$

where v_f and a_f are the vector and axial coupling of the Z boson to fermions and Q_f is the fermion electric charge. In the SM the difference between the vector and axial couplings is $v_f - a_f = -Q_f s_w/c_w$, which when inserted into (3.6) leads to the usual relation for electric charge renormalisation [28]. In the SMEFT, the expression for $v_f - a_f$ is altered by

[3]We follow the convention of [38] and choose δZ_f^R to be real.

dimension-6 contributions. However, the quantity $\Sigma_T^{AZ}(0)$ itself is subleading in the limit of vanishing gauge couplings and so the exact form of $v_f - a_f$ is irrelevant to what follows.

We next turn to counterterms related to operator renormalisation. At the level of the Lagrangian, such counterterms have the form $\delta C_i Q_i$, where $\delta C_i = \sum_j \gamma_{ij} C_j$ and thus involves a linear combination of all Wilson coefficients in the basis. We need such counterterms for each operator appearing in the tree-level expression (2.11). To one-loop order in the $\overline{\text{MS}}$ scheme, we can write

$$C_i^{(0)} = C_i(\mu) + \frac{\delta C_i(\mu)}{16\pi^2} = C_i(\mu) + \frac{1}{2\hat{\epsilon}}\frac{1}{16\pi^2}\dot{C}_i(\mu)\,, \tag{3.7}$$

where we have defined

$$\dot{C}_i(\mu) \equiv 16\pi^2\left(\mu\frac{d}{d\mu}C_i(\mu)\right)\,. \tag{3.8}$$

It is understood that we evaluate the right-hand side of the above equation at one-loop order using the results from [8, 10, 11]. We have also introduced the notation

$$\frac{1}{\hat{\epsilon}} \equiv \frac{1}{\epsilon} - \gamma_E + \ln(4\pi)\,, \tag{3.9}$$

where ϵ is the dimensional regulator for integrals evaluated in $d = 4 - 2\epsilon$ dimensions. UV divergences in loop integrals always appear as factors of $1/\hat{\epsilon}$, and rather than clutter notation we shall write such factors as $1/\epsilon$ in the rest of the paper, with the understanding that such poles are accompanied by the universal, finite terms on the right-hand side of (3.9). When the UV poles of the bare and counterterm matrix elements are cancelled, so too are these constant terms.

With these ingredients in place, we can now construct the explicit form of the UV counterterms for the specific case of $h \to f\bar{f}$. We take the tree-level expression (2.11), interpret the quantities in it as bare parameters, and then replace these bare parameters by the renormalised ones. For the vacuum expectation value v_T, this leads us to write

$$\frac{1}{v_T^{(0)}} = \frac{1}{v_T}\left(1 - \frac{\delta v_T}{v_T}\right)\,. \tag{3.10}$$

We can derive an explicit expression for δv_T as a function of the counterterms for the physical observables (2.12) using (2.17). Defining

$$\begin{aligned}
&\frac{\delta \hat{c}_w}{\hat{c}_w} \equiv \frac{\delta M_W}{M_W} - \frac{\delta M_Z}{M_Z}, \qquad \frac{\delta \hat{s}_w}{\hat{s}_w} \equiv -\frac{\hat{c}_w^2}{\hat{s}_w^2}\frac{\delta \hat{c}_w}{\hat{c}_w}\,,\\
&\frac{\delta \hat{v}_T}{\hat{v}_T} \equiv \frac{\delta M_W}{M_W} + \frac{\delta \hat{s}_w}{\hat{s}_w} - \frac{\delta \bar{e}}{\bar{e}}\,.
\end{aligned} \tag{3.11}$$

we find that

$$\begin{aligned}
\frac{\delta v_T}{v_T} = \frac{\delta M_W}{M_W} + \frac{\delta \bar{s}_w}{\bar{s}_w} - \frac{\delta \bar{e}}{\bar{e}} - \frac{\hat{v}_T^2 \hat{c}_w}{2\hat{s}_w}\delta C_{HWB}&\\
- \frac{\hat{c}_w}{2\hat{s}_w}\hat{v}_T^2\left(\frac{\delta \hat{c}_w}{\hat{c}_w} - \frac{\delta \hat{s}_w}{\hat{s}_w} + 2\frac{\delta \hat{v}_T}{\hat{v}_T}\right) C_{HWB}\,.&
\end{aligned} \tag{3.12}$$

The counterterm for $\delta\bar{s}_w$ can be computed using (2.16). One has

$$\begin{aligned}\frac{\delta\bar{s}_w}{\bar{s}_w} = \frac{\delta\hat{s}_w}{\hat{s}_w} - \frac{\hat{c}_w^2}{2\hat{s}_w^2}\hat{v}_T^2\left(\frac{\delta\hat{c}_w}{\hat{c}_w} - \frac{\delta\hat{s}_w}{\hat{s}_w} + \frac{\delta\hat{v}_T}{\hat{v}_T}\right)C_{HD} - \frac{\hat{c}_w^2\hat{v}_T^2}{4\hat{s}_w^2}\delta C_{HD} \\ - \frac{\hat{v}_T^2\hat{c}_w}{2\hat{s}_w}\delta C_{HWB} - \frac{\hat{c}_w}{2\hat{s}_w}\hat{v}_T^2\left(\frac{\delta\hat{c}_w}{\hat{c}_w} - \frac{\delta\hat{s}_w}{\hat{s}_w} + 2\frac{\delta\hat{v}_T}{\hat{v}_T}\right)C_{HWB}\,. \qquad (3.13)\end{aligned}$$

Note that the two-point functions, and the renormalisation counterterms derived from them as discussed above receive both SM and dimension-6 contributions. We make this explicit by defining expansion coefficients according to

$$\delta Z = \frac{1}{16\pi^2}\left(\delta Z^{(4)} + \delta Z^{(6)}\right) + \ldots\,, \qquad (3.14)$$

and similarly for δM, $\delta\bar{e}$ and δv_T. The superscript (4) then refers to SM contributions, while the superscript (6) refers to dimension-6 contributions. The counterterms δC_i related to operator renormalisation are purely dimension-6, so we do not label them with a (redundant) superscript (6).

The counterterm for the $h \to f\bar{f}$ decay amplitude can now be written as

$$i\mathcal{M}^{\text{C.T.}}(h \to f\bar{f}) = -i\bar{u}(p_f)\left(\delta\mathcal{M}_L P_L + \delta\mathcal{M}_L^* P_R\right)v(p_{\bar{f}})\,, \qquad (3.15)$$

where we distinguish SM and dimension-6 contributions through the notation

$$\delta\mathcal{M}_L = \frac{1}{16\pi^2}\left(\delta\mathcal{M}_L^{(4)} + \delta\mathcal{M}_L^{(6)}\right) + \ldots\,. \qquad (3.16)$$

The SM contributions read

$$\delta\mathcal{M}_L^{(4)} = \frac{m_f}{v_T}\left(\frac{\delta m_f^{(4)}}{m_f} - \frac{\delta v_T^{(4)}}{v_T} + \frac{1}{2}\delta Z_h^{(4)} + \frac{1}{2}\delta Z_f^{(4),L} + \frac{1}{2}\delta Z_f^{(4),R*}\right)\,, \qquad (3.17)$$

and the dimension-6 contributions are

$$\begin{aligned}\delta\mathcal{M}_L^{(6)} = &\left(\frac{m_f}{v_T}C_{H,\text{kin}}\right)\left(\frac{\delta m_f^{(4)}}{m_f} - \frac{\delta v_T^{(4)}}{v_T} + \frac{1}{2}\delta Z_h^{(4)} + \frac{1}{2}\delta Z_f^{(4),L} + \frac{1}{2}\delta Z_f^{(4),R*}\right) \\ &- \frac{v_T^2}{\sqrt{2}}C_{bH}^*\left(2\frac{\delta v_T^{(4)}}{v_T} + \frac{1}{2}\delta Z_h^{(4)} + \frac{1}{2}\delta Z_f^{(4),L} + \frac{1}{2}\delta Z_f^{(4),R*}\right) \\ &+ \frac{m_f}{v_T}\left(\frac{\delta m_f^{(6)}}{m_f} - \frac{\delta v_T^{(6)}}{v_T} + \frac{1}{2}\delta Z_h^{(6)} + \frac{1}{2}\delta Z_f^{(6),L} + \frac{1}{2}\delta Z_f^{(6),R*}\right) \\ &+ \frac{m_f}{v_T}\delta C_{H,\text{kin}} - \frac{v_T^2}{\sqrt{2}}\delta C_{fH}^*\,. \qquad (3.18)\end{aligned}$$

Clearly, for the $h \to b\bar{b}$ matrix element it is necessary to include the b-quark mass ($\delta m_b/m_b$) and wavefunction (δZ_b) renormalisation factors in the counterterm, while for the $h \to \tau\bar{\tau}$ matrix element the corresponding τ-lepton factors should be included. The above results (3.18) are valid in the $\overline{\mathrm{MS}}$ scheme for the Wilson coefficients, and the on-shell scheme (pole scheme) for the masses and the electric charge. One may instead wish to use different definitions for these masses, such as the $\overline{\mathrm{MS}}$ scheme, which shuffles finite contributions between the matrix elements and the masses. We will provide an example on how this can be done when we consider four-fermion contributions in section 4.

The procedure to calculate the one-loop corrections to the $h \to f\bar{f}$ decay rate in a given renormalisation scheme is now clear. Compute

$$\mathcal{M}^{(1)}(h \to f\bar{f}) = \mathcal{M}^{(1),\mathrm{bare}} + \mathcal{M}^{\mathrm{C.T.}}, \tag{3.19}$$

where each of the terms receives both SM and dimension-6 contributions. This procedure is straightforward to implement in the case where the parameters (2.12) are used as input. However, as mentioned in section 2.2, it is customary to eliminate M_W dependence in favour of the Fermi constant G_F as measured from muon decay. In order to do so we must modify the tree-level relation (2.18) to a form appropriate at one-loop. We do this by writing

$$\frac{1}{\sqrt{2}}\frac{1}{v_T^2}\left(1+\Delta r\right) = G_F + \Delta R^{(6)}. \tag{3.20}$$

The expression for Δr, which summarises the finite non-QED radiative corrections to muon decay in terms of two-point functions can be found in [39]. The contributions labelled as $\Delta R^{(6)}$ summarise the finite process specific contributions to muon decay in the SMEFT. Evaluating the expression for Δr in the limit of vanishing gauge-couplings, we find that

$$\Delta r = 2\left(\frac{\delta M_W}{M_W} - \frac{\delta v_T}{v_T}\right). \tag{3.21}$$

To implement this scheme to one-loop order, we first define expansion coefficients as

$$\begin{aligned}\Delta r &= \frac{1}{16\pi^2}\left(\Delta r^{(4,1)} + \Delta r^{(6,1)}\right), \\ \Delta R^{(6)} &= \Delta R^{(6,0)} + \frac{1}{16\pi^2}\Delta R^{(6,1)}.\end{aligned} \tag{3.22}$$

The tree-level piece $\Delta R^{(6,0)}$ is obtained by matching with (2.18). We shall give explicit expressions for the one-loop corrections to Δr and $\Delta R^{(6)}$ in section 5, where we also give explicit results for the renormalised one-loop decay amplitude after eliminating v_T dependence using (3.20). For now, we simply note that the counterterms derived after writing v_T in terms of G_F take a very simple form. They can be obtained from (3.17) and (3.18) by replacing $\delta v_T/v_T$ with $\delta M_W/M_W$, which follows from the definition of Δr, and then in addition adding on the extra dimension-6 pieces contained in $\Delta R^{(6)}$ by hand.

4 The one-loop contribution from four-fermion operators

In this section we compute the one-loop contributions from four-fermion operators to $h \to b\bar{b}$ and $h \to \tau\bar{\tau}$ decays. Not only are these the simplest dimension-6 contributions to compute, we will see in section 6 that they are among the most important numerically. At the same time, their calculation nicely illustrates many aspects of the renormalisation procedure outlined in the previous section.

The list of operators which must be considered are those labelled as Class '8' in table 1. In general, the coefficients of the four-fermion operators carry four flavour indices labeling the fermion generations. In the current study, we consider only b-quark and τ-lepton final states, and only the radiative corrections due the third generation field content are considered, and consequently these flavour indices are redundant and will be dropped in what follows. For example, the scalar operator $(\bar{L}R)(\bar{R}L)$ is labelled as $Q_{l\tau bq} = (\bar{l}^j\tau)(\bar{b}q_j)$.

It is convenient to calculate the one-loop corrections by performing Passarino-Veltmann reduction [40] and writing the results in terms of the standard one-loop scalar integrals. In order to make explicit the UV divergent parts of these integrals, we write the one-loop scalar integrals as

$$A_0\left(s\right) = \frac{s}{\epsilon} + \hat{A}_0(s)\,, \tag{4.1}$$

$$B_0\left(s, m_1^2, m_2^2\right) = \frac{1}{\epsilon} + \hat{B}_0(s, m_1^2, m_2^2)\,, \tag{4.2}$$

where we have defined the finite, μ-dependent integrals

$$\hat{A}_0(s) = s - s\ln\left(\frac{s - i0}{\mu^2}\right)\,, \tag{4.3}$$

$$\hat{B}_0(s, m_1^2, m_2^2) = 2 - \log\left(\frac{s-i0}{\mu^2}\right) + \sum_{i=1}^{2}\left[\lambda_i \ln\left(\frac{\lambda_i - 1}{\lambda_i}\right) - \ln\left(\lambda_i - 1\right)\right]\,, \tag{4.4}$$

and

$$\lambda_i = \frac{s - m_2^2 + m_1^2 \pm \sqrt{(s - m_2^2 + m_1^2)^2 - 4s(m_1^2 - i0)}}{2s}\,. \tag{4.5}$$

In section 5, when we consider the large-m_t limit, we will also use explicit results for special values of the arguments of the triangle integral C_0. These results can be obtained from [30], and are provided in appendix A.

4.1 Bare matrix element

We begin by computing the contribution from the four-fermion operators to the bare matrix element. The four-fermion operators do not contribute to the tree-level result (2.11), and so it is only necessary to evaluate the one-loop contributions. The relevant diagrams are of the form of that shown in the left-hand side of figure 1. The contributions from the vector operators $(\bar{L}L)(\bar{L}L)$ and $(\bar{R}R)(\bar{R}R)$ vanish due to their Dirac structure. We write the non-vanishing contribution for the sum of all four-fermion diagrams to the bare matrix element as

$$i\mathcal{M}_8^{(1),\text{bare}}(h \to f\bar{f}) = -i\frac{1}{16\pi^2}\bar{u}(p_f)\left(C_{8,f}^{L,(1),\text{bare}}P_L + C_{8,f}^{R,(1),\text{bare}}P_R\right)v(p_{\bar{f}})\,. \tag{4.6}$$

For $h \to b\bar{b}$ decays one finds

$1 : X^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$
$Q_{\widetilde W}$	$\epsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$

$2 : H^6$	
Q_H	$(H^\dagger H)^3$

$3 : H^4 D^2$	
$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
Q_{HD}	$\left(H^\dagger D_\mu H\right)^* \left(H^\dagger D_\mu H\right)$

$5 : \psi^2 H^3 + \text{h.c.}$	
Q_{eH}	$(H^\dagger H)(\bar l_p e_r H)$
Q_{uH}	$(H^\dagger H)(\bar q_p u_r \widetilde H)$
Q_{dH}	$(H^\dagger H)(\bar q_p d_r H)$

$4 : X^2 H^2$	
Q_{HG}	$H^\dagger H\, G^A_{\mu\nu} G^{A\mu\nu}$
$Q_{H\widetilde G}$	$H^\dagger H\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$
Q_{HW}	$H^\dagger H\, W^I_{\mu\nu} W^{I\mu\nu}$
$Q_{H\widetilde W}$	$H^\dagger H\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$
Q_{HB}	$H^\dagger H\, B_{\mu\nu} B^{\mu\nu}$
$Q_{H\widetilde B}$	$H^\dagger H\, \widetilde B_{\mu\nu} B^{\mu\nu}$
Q_{HWB}	$H^\dagger \tau^I H\, W^I_{\mu\nu} B^{\mu\nu}$
$Q_{H\widetilde W B}$	$H^\dagger \tau^I H\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$

$6 : \psi^2 X H + \text{h.c.}$	
Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r) \tau^I H W^I_{\mu\nu}$
Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$
Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r) \widetilde H\, G^A_{\mu\nu}$
Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r) \tau^I \widetilde H\, W^I_{\mu\nu}$
Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r) \widetilde H\, B_{\mu\nu}$
Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r) H\, G^A_{\mu\nu}$
Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r) \tau^I H\, W^I_{\mu\nu}$
Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r) H\, B_{\mu\nu}$

$7 : \psi^2 H^2 D$	
$Q_{Hl}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar l_p \gamma^\mu l_r)$
$Q_{Hl}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar l_p \tau^I \gamma^\mu l_r)$
Q_{He}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar e_p \gamma^\mu e_r)$
$Q_{Hq}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar q_p \gamma^\mu q_r)$
$Q_{Hq}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar q_p \tau^I \gamma^\mu q_r)$
Q_{Hu}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar u_p \gamma^\mu u_r)$
Q_{Hd}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar d_p \gamma^\mu d_r)$
Q_{Hud} + h.c.	$i(\widetilde H^\dagger D_\mu H)(\bar u_p \gamma^\mu d_r)$

$8 : (\bar L L)(\bar L L)$	
Q_{ll}	$(\bar l_p \gamma_\mu l_r)(\bar l_s \gamma^\mu l_t)$
$Q_{qq}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar q_s \gamma^\mu q_t)$
$Q_{qq}^{(3)}$	$(\bar q_p \gamma_\mu \tau^I q_r)(\bar q_s \gamma^\mu \tau^I q_t)$
$Q_{lq}^{(1)}$	$(\bar l_p \gamma_\mu l_r)(\bar q_s \gamma^\mu q_t)$
$Q_{lq}^{(3)}$	$(\bar l_p \gamma_\mu \tau^I l_r)(\bar q_s \gamma^\mu \tau^I q_t)$

$8 : (\bar R R)(\bar R R)$	
Q_{ee}	$(\bar e_p \gamma_\mu e_r)(\bar e_s \gamma^\mu e_t)$
Q_{uu}	$(\bar u_p \gamma_\mu u_r)(\bar u_s \gamma^\mu u_t)$
Q_{dd}	$(\bar d_p \gamma_\mu d_r)(\bar d_s \gamma^\mu d_t)$
Q_{eu}	$(\bar e_p \gamma_\mu e_r)(\bar u_s \gamma^\mu u_t)$
Q_{ed}	$(\bar e_p \gamma_\mu e_r)(\bar d_s \gamma^\mu d_t)$
$Q_{ud}^{(1)}$	$(\bar u_p \gamma_\mu u_r)(\bar d_s \gamma^\mu d_t)$
$Q_{ud}^{(8)}$	$(\bar u_p \gamma_\mu T^A u_r)(\bar d_s \gamma^\mu T^A d_t)$

$8 : (\bar L L)(\bar R R)$	
Q_{le}	$(\bar l_p \gamma_\mu l_r)(\bar e_s \gamma^\mu e_t)$
Q_{lu}	$(\bar l_p \gamma_\mu l_r)(\bar u_s \gamma^\mu u_t)$
Q_{ld}	$(\bar l_p \gamma_\mu l_r)(\bar d_s \gamma^\mu d_t)$
Q_{qe}	$(\bar q_p \gamma_\mu q_r)(\bar e_s \gamma^\mu e_t)$
$Q_{qu}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar u_s \gamma^\mu u_t)$
$Q_{qu}^{(8)}$	$(\bar q_p \gamma_\mu T^A q_r)(\bar u_s \gamma^\mu T^A u_t)$
$Q_{qd}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar d_s \gamma^\mu d_t)$
$Q_{qd}^{(8)}$	$(\bar q_p \gamma_\mu T^A q_r)(\bar d_s \gamma^\mu T^A d_t)$

$8 : (\bar L R)(\bar R L) + \text{h.c.}$	
Q_{ledq}	$(\bar l_p^j e_r)(\bar d_s q_{tj})$

$8 : (\bar L R)(\bar L R) + \text{h.c.}$	
$Q_{quqd}^{(1)}$	$(\bar q_p^j u_r)\epsilon_{jk}(\bar q_s^k d_t)$
$Q_{quqd}^{(8)}$	$(\bar q_p^j T^A u_r)\epsilon_{jk}(\bar q_s^k T^A d_t)$
$Q_{lequ}^{(1)}$	$(\bar l_p^j e_r)\epsilon_{jk}(\bar q_s^k u_t)$
$Q_{lequ}^{(3)}$	$(\bar l_p^j \sigma_{\mu\nu} e_r)\epsilon_{jk}(\bar q_s^k \sigma^{\mu\nu} u_t)$

Table 1. The 59 independent dimension-6 operators built from Standard Model fields which conserve baryon number, as given in ref. [31]. The operators are divided into eight classes: X^3, H^6, etc. Operators with +h.c. in the table heading also have hermitian conjugates, as does the $\psi^2 H^2 D$ operator Q_{Hud}. The subscripts p, r, s, t are flavour indices, The notation is described in [10].

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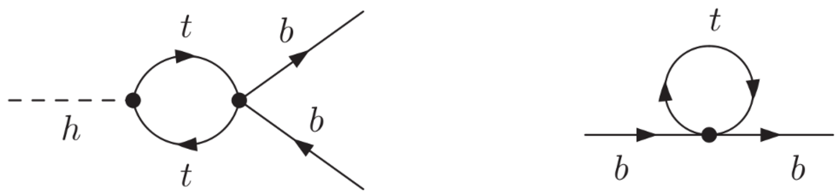

Figure 1. Examples of one-loop diagrams involving Class 8 operators to the $h \to b\bar{b}$ process (left), and to the b-quark two-point function (right). The corresponding diagrams for $h \to \tau\bar{\tau}$ are of similar form.

$$C_{8,b}^{L,(1),\mathrm{bare}} = \frac{1}{v_T}\Big[m_b(4-2\epsilon)I_8^b\left(C_{qb}^{(1)} + c_{F,3}C_{qb}^{(8)}\right) - 2m_\tau I_8^\tau C_{l\tau bq} \\ - m_t I_8^t\left((2N_c+1)C_{qtqb}^{(1)*} + c_{F,3}C_{qtqb}^{(8)*}\right)\Big], \tag{4.7}$$

and for $h \to \tau\bar{\tau}$ one has

$$C_{8,\tau}^{L,(1),\mathrm{bare}} = \frac{1}{v_T}\Big[m_\tau(4-2\epsilon)I_8^\tau C_{l\tau} + 2N_c m_t I_8^t C_{l\tau qt}^{(1)*} - 2N_c m_b I_8^b C_{l\tau bq}^*\Big]. \tag{4.8}$$

Results for the functions $C_{8,f}^{R,(1),\mathrm{bare}}$ are obtained through the relation

$$C_{8,f}^{R,(1),\mathrm{bare}} = C_{8,f}^{L,(1),\mathrm{bare}}\left(C_i^* \leftrightarrow C_i\right), \tag{4.9}$$

which clearly only effects the complex Wilson coefficients, i.e. those multiplying $(\bar{L}R)(\bar{R}L)$ or $(\bar{L}R)(\bar{L}R)$ operators which are labeled with four subscripts. In the above expressions for the bare matrix elements, the following notation has been introduced

$$I_8^j = A_0(m_j^2) - \frac{1}{2}\left(m_H^2 - 4m_j^2\right)B_0(m_H^2, m_j^2, m_j^2), \tag{4.10}$$

which appears for all diagrams. To make explicit the cancellation of UV divergences in the renormalisation procedure, the UV divergent contributions are extracted from the integrals according to

$$I_8^j = \frac{1}{\epsilon}\left(3m_j^2 - \frac{m_H^2}{2}\right) + \hat{A}_0(m_j^2) - \frac{1}{2}\left(m_H^2 - 4m_j^2\right)\hat{B}_0(m_H^2, m_j^2, m_j^2), \tag{4.11}$$

$$\equiv \frac{1}{\epsilon}\left(3m_j^2 - \frac{m_H^2}{2}\right) + \hat{I}_8^j. \tag{4.12}$$

Therefore, the bare one-loop $h \to b\bar{b}$ matrix element can be written as

$$C_{8,b}^{L,(1),\mathrm{bare}} = \frac{1}{v_T}\frac{1}{\epsilon}\Bigg[4m_b\left(3m_b^2 - \frac{m_H^2}{2}\right)\left(C_{qb}^{(1)} + c_{F,3}C_{qb}^{(8)}\right) + 2m_\tau\left(3m_\tau^2 - \frac{m_H^2}{2}\right)C_{l\tau bq}^* \\ - m_t\left(3m_t^2 - \frac{m_H^2}{2}\right)\left((1+2N_c)C_{qtqb}^{(1)*} + c_{F,3}C_{qtqb}^{(8)*}\right)\Bigg] + C_{8,b}^{L,(1),\mathrm{fin}}, \tag{4.13}$$

$$C_{8,b}^{L,(1),\mathrm{fin}} = \frac{1}{v_T}\Big[m_b\left(4\hat{I}_8^b - 6m_b^2 + m_H^2\right)\left(C_{qb}^{(1)} + c_{F,3}C_{qb}^{(8)}\right) + 2m_\tau \hat{I}_8^\tau C_{l\tau bq}^* \\ - m_t\hat{I}_8^t\left((2N_c+1)C_{qtqb}^{(1)*} + c_{F,3}C_{qtqb}^{(8)*}\right)\Big]. \tag{4.14}$$

The corresponding result for $h \to \tau\bar{\tau}$ is

$$C_{8,\tau}^{L,(1),\text{bare}} = \frac{1}{v_T}\frac{1}{\epsilon}\left[4m_\tau\left(3m_\tau^2 - \frac{m_H^2}{2}\right)C_{le} - 2N_c m_b\left(3m_b^2 - \frac{m_H^2}{2}\right)C_{l\tau bq}^* \right.$$
$$\left. + 2N_c m_t\left(3m_t^2 - \frac{m_H^2}{2}\right)C_{l\tau qt}^{(1)*}\right] + C_{8,\tau}^{L,(1),\text{fin}}, \tag{4.15}$$

$$C_{8,\tau}^{L,(1),\text{fin}} = \frac{1}{v_T}\left[m_\tau\left(4\hat{I}_8^\tau - 6m_\tau^2 + m_H^2\right)C_{le} - 2N_c m_b \hat{I}_8^b C_{l\tau bq}^* + 2N_c m_t \hat{I}_8^t C_{l\tau qt}^{(1)*}\right]. \tag{4.16}$$

4.2 Counterterms

As outlined in section 3, to cancel the poles in the bare matrix element we must construct the UV counterterms according to (3.18). The four-fermion operators contribute to operator renormalisation, as well as to fermion mass and wavefunction renormalisation.

The four-fermion contribution to δC_{fH}^* is calculated according to (3.8), where explicit results for $\dot{C}_{fH}^*$ can be taken from [10, 11]. To adapt those results to the broken phase of the theory, the Yukawa couplings and the parameter λ from the Higgs potential must be replaced with the physical parameters m_H and m_f, as in (2.6) and (2.7) respectively. Extracting the pieces involving only four-fermion contributions to δC_{bH} and $\delta C_{\tau H}$ gives

$$\delta C_{bH}^{(4f)} = \frac{\sqrt{2}}{v_T^3}\frac{1}{\epsilon}\left[\frac{1}{2}m_t(m_H^2 - 4m_t^2)\left((2N_c+1)C_{qtqb}^{(1)} + c_{F,3}C_{qtqb}^{(8)}\right)\right.$$
$$\left. - 2m_b(m_H^2 - 4m_b^2)\left(C_{qb}^{(1)} + c_{F,3}C_{qb}^{(8)}\right) + m_\tau(m_H^2 - 4m_\tau^2)C_{l\tau bq}^*\right], \tag{4.17}$$

$$\delta C_{\tau H}^{(4f)} = \frac{\sqrt{2}}{v_T^3}\frac{1}{\epsilon}\left[N_c m_b(m_H^2 - 4m_b^2)C_{l\tau bq}\right.$$
$$\left. - 2m_\tau(m_H^2 - 4m_\tau^2)C_{l\tau} - N_c m_t(m_H^2 - 4m_t^2)C_{l\tau qt}^{(1)}\right]. \tag{4.18}$$

The counterterms from mass and wavefunction renormalisation are calculated from two-point functions according to (3.4) and (3.5). The relevant one-loop diagrams are of the form of that shown on the right-hand side of figure 1. The results for the wavefunction and mass renormalisation for the b-quark are

$$\begin{aligned}
\delta m_b^{(6)} =& \frac{1}{\epsilon}\bigg[\frac{m_t^3}{2}\left((2N_c+1)\left(C_{qtqb}^{(1)}+C_{qtqb}^{(1)*}\right)+c_{F,3}\left(C_{qtqb}^{(8)}+C_{qtqb}^{(8)*}\right)\right)\\
&-4m_b^3\left(C_{qb}^{(1)}+c_{F,3}C_{qb}^{(8)}\right)+m_\tau^3\left(C_{l\tau bq}+C_{l\tau bq}^*\right)\bigg]+\delta m_b^{\text{fin}}(\mu)\,,\\
\delta m_b^{\text{fin}}(\mu) =& \frac{m_t}{2}\hat{A}_0(m_t^2)\left((2N_c+1)\left(C_{qtqb}^{(1)}+C_{qtqb}^{(1)*}\right)+c_{F,3}\left(C_{qtqb}^{(8)}+C_{qtqb}^{(8)*}\right)\right)\\
&+2m_b\left(m_b^2-2\hat{A}_0(m_b^2)\right)\left(C_{qb}^{(1)}+c_{F,3}C_{qb}^{(8)}\right)+m_\tau\hat{A}_0(m_\tau^2)\left(C_{l\tau bq}+C_{l\tau bq}^*\right)\,,\\
\delta Z_b^{(6),L} =& \frac{1}{\epsilon}\bigg[-\frac{m_t^3}{m_b}\left((2N_c+1)\left(C_{qtqb}^{(1)}-C_{qtqb}^{(1)*}\right)+c_{F,3}\left(C_{qtqb}^{(8)}-C_{qtqb}^{(8)*}\right)\right)\\
&+2\frac{m_\tau^3}{m_b}\left(C_{l\tau bq}-C_{l\tau bq}^*\right)\bigg]+\delta Z_b^{L,\text{fin}}(\mu)\,,\\
\delta Z_b^{L,\text{fin}}(\mu) =& -\frac{m_t}{m_b}\hat{A}_0(m_t^2)\left((2N_c+1)\left(C_{qtqb}^{(1)}-C_{qtqb}^{(1)*}\right)+c_{F,3}\left(C_{qtqb}^{(8)}-C_{qtqb}^{(8)*}\right)\right)\\
&+2\frac{m_\tau}{m_b}\hat{A}_0(m_\tau^2)\left(C_{l\tau bq}-C_{l\tau bq}^*\right)\,,\\
\delta Z_b^{(6),R} =& 0\,.
\end{aligned}\tag{4.19}$$

while those from τ-leptons are

$$\begin{aligned}
\delta m_\tau^{(6)} =& \frac{1}{\epsilon}\left[-4m_\tau^3C_{l\tau}+N_cm_b^3\left(C_{l\tau bq}+C_{l\tau bq}^*\right)-N_cm_t^3\left(C_{l\tau qt}^{(1)}+C_{l\tau qt}^{(1)*}\right)\right]+\delta m_\tau^{\text{fin}}(\mu)\,,\\
\delta m_\tau^{\text{fin}}(\mu) =& 2m_\tau\left(m_\tau^2-2\hat{A}_0(m_\tau^2)\right)C_{l\tau}+N_cm_b\hat{A}_0(m_b^2)\left(C_{l\tau bq}+C_{l\tau bq}^*\right)\\
&-N_cm_t^3\hat{A}_0(m_t^2)\left(C_{l\tau qt}^{(1)}+C_{l\tau qt}^{(1)*}\right)\,,\\
\delta Z_\tau^{(6),L} =& 2N_c\frac{1}{\epsilon}\left[\frac{m_t^3}{m_\tau}\left(C_{l\tau qt}^{(1)}-C_{l\tau qt}^{(1)*}\right)-\frac{m_b^3}{m_\tau}\left(C_{l\tau bq}-C_{l\tau bq}^*\right)\right]+\delta Z_\tau^{L,\text{fin}}(\mu)\,,\\
\delta Z_\tau^{L,\text{fin}}(\mu) =& 2N_c\left(\frac{m_t}{m_\tau}\hat{A}_0(m_t^2)\left(C_{l\tau qt}^{(1)}-C_{l\tau qt}^{(1)*}\right)-\frac{m_b}{m_\tau}\hat{A}_0(m_b^2)\left(C_{l\tau bq}-C_{l\tau bq}^*\right)\right),\\
\delta Z_\tau^{(6),R} =& 0\,.
\end{aligned}\tag{4.20}$$

Notably, only the real parts of the four-fermion Wilson coefficients contribute to mass renormalisation, while only the imaginary parts contribute to wavefunction renormalisation.

4.3 Renormalised matrix element

Adding together the bare matrix element and UV counterterms as in (3.19), we find that the UV divergences cancel. We write the remaining finite contribution as

$$i\mathcal{M}_{8,f}^{(1)}(h\to f\bar{f}) = -i\frac{1}{16\pi^2}\bar{u}(p_f)\left(C_{8,f}^{L,(1)}P_L+C_{8,f}^{R,(1)}P_R\right)v(p_{\bar{f}})\,.\tag{4.21}$$

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The renormalised one-loop matrix element for $h \to b\bar{b}$ decays is

$$\begin{aligned} v_T C_{8,b}^{L,(1)} =& m_b(m_H^2 - 4m_b^2)\left(1 - 2\hat{B}_0(m_H^2, m_b^2, m_b^2)\right)\left(C_{qb}^{(1)} + c_{F,3}C_{qb}^{(8)}\right) \\ &+ m_\tau(m_H^2 - 4m_\tau^2)\hat{B}_0(m_H^2, m_\tau^2, m_\tau^2)C_{l\tau bq} \\ &+ \frac{m_t}{2}(m_H^2 - 4m_t^2)\hat{B}_0(m_H^2, m_t^2, m_t^2)\left((2N_c+1)C_{qtqb}^{(1)*} + c_{F,3}C_{qtqb}^{(8)*}\right) . \end{aligned} \tag{4.22}$$

The μ-dependence of the one-loop matrix element is contained implicitly in the functions $\hat{B}_0$. We can make it explicit by writing

$$\hat{B}_0(m_H^2, m_t^2, m_t^2) = \hat{b}_0(m_H^2, m_t^2, m_t^2) - \ln\left(\frac{m_H^2}{\mu^2}\right) . \tag{4.23}$$

We then find

$$\begin{aligned} v_T C_{8,b}^{L,(1)} =& m_b(m_H^2 - 4m_b^2)\left(1 - 2\hat{b}_0(m_H^2, m_b^2, m_b^2)\right)\left(C_{qb}^{(1)} + c_{F,3}C_{qb}^{(8)}\right) \\ &+ m_\tau(m_H^2 - 4m_\tau^2)\hat{b}_0(m_H^2, m_\tau^2, m_\tau^2)C_{l\tau bq} \\ &+ \frac{m_t}{2}(m_H^2 - 4m_t^2)\hat{b}_0(m_H^2, m_t^2, m_t^2)\left((2N_c+1)C_{qtqb}^{(1)*} + c_{F,3}C_{qtqb}^{(8)*}\right) \\ &- \frac{1}{2}\frac{v_T^2}{\sqrt{2}}\dot{C}_{bH}^{(4f)*}\ln\left(\frac{m_H^2}{\mu^2}\right) . \end{aligned} \tag{4.24}$$

In obtaining this expression, we have used $\dot{C}_i(\mu) = 2\epsilon\,\delta C_i(\mu)$ to express (4.17) in a convenient form. The corresponding result for $h \to \tau\bar{\tau}$ decays reads

$$\begin{aligned} v_T C_{8,\tau}^{L,(1)} =& m_\tau(m_H^2 - 4m_\tau^2)\left(1 - 2\hat{b}_0(m_H^2, m_\tau^2, m_\tau^2)\right)C_{l\tau} \\ &+ N_c m_b(m_H^2 - 4m_b^2)\hat{b}_0(m_H^2, m_\tau^2, m_\tau^2)C_{l\tau bq}^{*} \\ &- N_c m_t(m_H^2 - 4m_t^2)\hat{b}_0(m_H^2, m_t^2, m_t^2)C_{l\tau qt}^{(1)*} \\ &- \frac{1}{2}\frac{v_T^2}{\sqrt{2}}\dot{C}_{\tau H}^{(4f)*}\ln\left(\frac{m_H^2}{\mu^2}\right) . \end{aligned} \tag{4.25}$$

Written in this way, it is clear that the μ-dependence in the one-loop results arises from the fact that the Wilson coefficients are renormalised in the $\overline{\text{MS}}$ scheme, and that this μ-dependence cancels that in the tree-level result (2.11), so that the renormalised matrix element is μ-independent up to one-loop order.

As the expressions for the scalar integrals appearing in (4.24) and (4.25) are particularly simple, we provide them explicitly for convenience. For the contributions from internal b-quark lines, the integral

$$\hat{b}_0(m_H^2, m_b^2, m_b^2) = 2 - \bar{z}\left[\ln\left(\frac{1+\bar{z}}{1-\bar{z}}\right) - i\pi\right] - \ln\left(\frac{m_b^2}{m_H^2}\right) , \tag{4.26}$$

where $\bar{z} = \sqrt{1 - 4m_b^2/m_H^2}$. The result for internal τ-lepton lines is then obtained after obvious replacements. In the case of top-quark contributions,

$$\hat{b}_0(m_H^2, m_t^2, m_t^2) = 2 - 2z\,\text{ArcCot}\,(z) - \ln\left(\frac{m_t^2}{m_H^2}\right) , \tag{4.27}$$

where $z = \sqrt{4m_t^2/m_H^2 - 1}$. The right-handed contributions $C_{8,f}^{R,(1)}$ are obtained as in (4.9). As discussed in section 2.2, the quantity v_T in the one-loop results should be replaced in favour of G_F using (2.18). In fact, neglecting terms of order $\mathcal{O}(1/\Lambda_{\rm NP}^4)$ and higher, the relation $\sqrt{2}v_T^2 = 1/G_F$ can be used.

These results are valid in the on-shell scheme for fermion masses, and we will use them to study the size of one-loop corrections from four-fermion operators in section 6. In a more detailed phenomenological analysis the $\overline{\rm MS}$ scheme for quark masses may be preferable. This is particularly true for the b-quark, for which accurate numerical extractions of the $\overline{m}_b(\overline{m}_b)$ exist [37]. At one-loop order, the $\overline{\rm MS}$ mass is related to the pole mass according to

$$\overline{m}_f(\mu) = m_f + \delta m_f^{\rm fin}(\mu)\,, \tag{4.28}$$

where the one-loop results $\delta m_{b,\tau}^{\rm fin}(\mu)$ were given in (4.19) and (4.20). It is straightforward to obtain the $\overline{\rm MS}$ results for the $h \to f\bar{f}$ matrix element. One eliminates the pole mass m_f in favour of its $\overline{\rm MS}$ counterpart using (4.28), and then re-expands the formula as appropriate at one-loop. One then finds

$$v_T \overline{C}_{8,f}^{L,(1)} = v_T C_{8,f}^{L,(1)} - \delta m_f^{\rm fin}(\mu)\,. \tag{4.29}$$

5 The one-loop contributions in the large-m_t limit

We have obtained the full set of corrections to both $h \to b\bar{b}$ and $h \to \tau\bar{\tau}$ decay rates in the limit of vanishing gauge couplings, and will present them in future work along with the results which include the full gauge coupling dependence [29]. In this section we focus instead on the leading corrections in the $m_t \to \infty$ limit, which are a well-defined subset of the full corrections and potentially dominant numerically. However, a number of the interesting features of the renormalisation procedure are subleading in this limit. In order to illustrate them, we keep exact mass dependence of contributions multiplying $1/\epsilon$ poles and μ-dependent logarithms. In fact, these μ-dependent terms can be deduced from results for the RG equations of the dimension-6 Wilson coefficients appearing in the tree-level result (2.11). These RG equations were calculated explicitly in the unbroken phase of the theory in [10, 11], and our calculation directly in the broken phase of the theory thus provides a very non-trivial consistency check on those results, as well as on our renormalisation procedure and explicit loop calculations.

As with the calculation of four-fermion contributions, we consider only the third generation contributions. We additionally make the assumption of real Wilson coefficients. To ease the calculation of the contributing diagrams, of which there are many even with the above mentioned simplifications, we have implemented the dimension-6 Lagrangian in `FeynRules` [41], and subsequently generated and computed the relevant Feynman diagrams with `FeynArts` [42] and `FormCalc` [43]. We give some details of our procedure for calculating the one-loop corrections mentioned above in appendix A, paying special attention to deriving results valid in the $m_t \to \infty$ limit. The renormalised one-loop results are obtained by evaluating (3.19). We first give results for the ingredients entering the counterterms, and then give the final results for the renormalised one-loop matrix elements at the end of the section.

Following the procedure taken for the four-fermion contributions, we construct the UV counterterms from operator renormalisation by adapting the results of [10, 11] to the broken phase of theory. The results are:

$$\begin{aligned}\frac{v_T^2}{\sqrt{2}}\delta C_{bH} &= \frac{1}{\epsilon}\frac{1}{v_T}\Bigg[-m_b\left(6m_b^2+m_H^2\right)\frac{C_{H,\rm kin}}{v_T^2}+\frac{1}{4}m_b\left(2m_b^2-m_H^2\right)C_{HD}\\ &\quad+\frac{v_T}{2\sqrt{2}}\big((10N_c+21)m_b^2+12m_H^2+(6N_c-3)m_t^2+6m_\tau^2\big)C_{bH}\\ &\quad-\frac{(3-2N_c)v_T m_b m_t C_{tH}}{\sqrt{2}}+\sqrt{2}v_T m_b m_\tau C_{\tau H}-4m_b m_\tau^2 C_{H\tau}^{(3)}\\ &\quad-m_b\left(4N_c m_b^2-3m_H^2+(4N_c-6)m_t^2\right)C_{Hq}^{(3)}+m_b\left(2m_b^2+m_H^2\right)\left(C_{Hq}^{(1)}-C_{Hb}\right)\\ &\quad-m_t\left(-4N_c m_b^2+m_H^2+2m_t^2\right)C_{Htb}\Bigg]+\frac{v_T^2}{\sqrt{2}}\delta C_{bH}^{(4f)}\,, \qquad (5.1)\\ \frac{v_T^2}{\sqrt{2}}\delta C_{\tau H} &= \frac{1}{\epsilon}\frac{1}{v_T}\Bigg[-m_\tau\left(6m_\tau^2+m_H^2\right)\frac{C_{H,\rm kin}}{v_T^2}+\frac{1}{4}m_\tau\left(m_H^2-2m_\tau^2\right)C_{HD}\\ &\quad+\frac{v_T}{2\sqrt{2}}\left(12m_H^2+31m_\tau^2+6N_c\left(m_b^2+m_t^2\right)\right)C_{\tau H}+\sqrt{2}N_c v_T m_\tau(m_b C_{bH}+m_t C_{tH})\\ &\quad+m_\tau\Bigg((3m_H^2-4m_\tau^2)C_{H\tau}^{(3)}+(m_H^2+2m_\tau^2)\left(C_{H\tau}^{(1)}-C_{H\tau}\right)\\ &\quad+4N_c\left(m_b m_t C_{Htb}-(m_b^2+m_t^2)C_{Hq}^{(3)}\right)\Bigg)\Bigg]+\frac{v_T^2}{\sqrt{2}}\delta C_{\tau H}^{(4f)}\,, \qquad (5.2)\\ v_T^2\delta C_{HD} &= \frac{1}{\epsilon}\Big[(3m_H^2+4N_c(m_b^2+m_t^2)+4m_\tau^2)C_{HD}+8N_c m_b^2 C_{Hb}-8N_c m_t^2 C_{Ht}\\ &\quad-8N_c(m_b^2-m_t^2)C_{Hq}^{(1)}-8N_c m_b m_t C_{Htb}+8m_\tau^2(C_{H\tau}-C_{H\tau}^{(1)})\Big]\,, \qquad (5.3)\\ \delta C_{H,\rm kin} &= \left(\delta C_{H\Box}-\frac{\delta C_{HD}}{4}\right)v_T^2\,,\\ [4pt] &= \frac{1}{\epsilon}\Bigg[2\left(3m_H^2+2\left(m_\tau^2+N_c\left(m_b^2+m_t^2\right)\right)\right)\frac{C_{H,\rm kin}}{v_T^2}+\frac{3}{4}m_H^2 C_{HD}\\ &\quad-6\left(m_\tau^2 C_{H\tau}^{(3)}+N_c\left((m_b^2+m_t^2)C_{Hq}^{(3)}-m_b m_t C_{Htb}\right)\right)\Bigg]\,, \qquad (5.4)\\ v_T^2\delta C_{HWB} &= \frac{1}{\epsilon}\left(m_H^2+2\left(m_\tau^2+N_c(m_b^2+m_t^2)\right)\right)C_{HWB}\,. \qquad (5.5)\end{aligned}$$

We calculate counterterms from wavefunction, mass, and electric charge renormalisation from two-point functions as described in section 3. The results for the counterterms (but not for the renormalised amplitude) are in general gauge-dependent. We quote here the results in in 't Hooft-Feynman gauge.[4] They read

$$\begin{aligned}
\frac{\delta m_b^{(4)}}{m_b} &= \frac{C_\epsilon}{v_T^2}\left[\frac{3}{2\epsilon}\left(m_b^2-m_t^2\right)-\frac{5}{4}m_t^2\right], \\
\delta m_b^{(6)} &= C_\epsilon\left[\frac{1}{\epsilon}\left(3m_b^3\frac{C_{H,\mathrm{kin}}}{v_T^2}+\frac{1}{4}m_b^3C_{HD}-\frac{3v_T}{2\sqrt{2}}\left(2m_b^2+m_H^2\right)C_{bH}+m_b^3\left(C_{Hb}-C_{Hq}^{(1)}\right)\right.\right. \\
&\quad -3m_bm_t^2C_{Hq}^{(3)}+m_t^3C_{Htb}-4m_b^3\left(C_{qb}^{(1)}+c_{F,3}C_{qb}^{(8)}\right)+m_t^3\left((2N_c+1)C_{qtqb}^{(1)}+c_{F,3}C_{qtqb}^{(8)}\right) \\
&\quad \left.\left.+2m_\tau^3C_{l\tau bq}\right)-\frac{5}{2}m_bm_t^2C_{Hq}^{(3)}+m_t^3\left(C_{Htb}+(2N_c+1)C_{qtqb}^{(1)}+c_{F,3}C_{qtqb}^{(8)}\right)\right], \\
\frac{\delta m_\tau^{(4)}}{m_\tau} &= \frac{C_\epsilon}{v_T^2}\frac{3m_\tau^2}{2\epsilon}, \\
\delta m_\tau^{(6)} &= C_\epsilon\left[\frac{1}{\epsilon}\left(3m_\tau^3\frac{C_{H,\mathrm{kin}}}{v_T^2}+\frac{1}{4}m_\tau^3C_{HD}-\frac{3v_T}{2\sqrt{2}}(2m_\tau^2+m_H^2)C_{\tau H}+m_\tau^3\left(C_{H\tau}-C_{H\tau}^{(1)}\right)\right.\right. \\
&\quad \left.\left.-4m_\tau^3C_{l\tau}+2N_c\left(m_b^3C_{l\tau bq}-m_t^3C_{l\tau qt}^{(1)}\right)\right)-2N_cm_t^3C_{l\tau qt}^{(1)}\right], \\
\delta Z_b^{(4),L} &= \frac{C_\epsilon}{v_T^2}\left[\frac{1}{\epsilon}\left(-m_b^2-m_t^2\right)-\frac{3}{2}m_t^2\right], \\
\delta Z_b^{(6),L} &= C_\epsilon\left[\frac{1}{\epsilon}\left(-m_b^2\frac{C_{H,\mathrm{kin}}}{v_T^2}+\frac{1}{4}m_b^2C_{HD}+\frac{v_T}{\sqrt{2}}m_bC_{bH}+m_b^2C_{Hb}+2\left(m_t^2-m_b^2\right)C_{Hq}^{(3)}\right.\right. \\
&\quad \left.\left.+m_bm_tC_{Htb}\right)+m_t^2C_{Hq}^{(3)}\right], \\
\delta Z_\tau^{(4),L} &= -\frac{C_\epsilon}{v_T^2}\frac{m_\tau^2}{\epsilon}, \\
\delta Z_\tau^{(6),L} &= \frac{C_\epsilon}{\epsilon}\left[-m_\tau^2\frac{C_{H,\mathrm{kin}}}{v_T^2}+\frac{1}{4}m_\tau^2C_{HD}+\frac{v_T}{\sqrt{2}}m_\tau C_{\tau H}+m_\tau^2\left(C_{H\tau}-2C_{H\tau}^{(3)}\right)\right], \\
\delta Z_b^{(4),R} &= -\frac{C_\epsilon}{v_T^2}\frac{2m_b^2}{\epsilon}, \\
\delta Z_b^{(6),R} &= \frac{C_\epsilon}{\epsilon}\left[-m_b^2\frac{C_{H,\mathrm{kin}}}{v_T^2}+\frac{1}{4}m_b^2C_{HD}+\frac{v_T}{\sqrt{2}}m_bC_{bH}-m_b^2\left(C_{Hq}^{(1)}+3C_{Hq}^{(3)}\right)\right], \\
\delta Z_\tau^{(4),R} &= -\frac{C_\epsilon}{v_T^2}\frac{2m_\tau^2}{\epsilon}, \\
\delta Z_\tau^{(6),R} &= \frac{C_\epsilon}{\epsilon}\left[-m_\tau^2\frac{C_{H,\mathrm{kin}}}{v_T^2}+\frac{1}{4}m_\tau^2C_{HD}+\frac{v_T}{\sqrt{2}}m_\tau C_{\tau H}-m_\tau^2\left(C_{H\tau}^{(1)}+3C_{H\tau}^{(3)}\right)\right], \\
\delta Z_h^{(4)} &= \frac{C_\epsilon}{v_T^2}\left[-\frac{2}{\epsilon}\left(N_c(m_b^2+m_t^2)+m_\tau^2\right)+\frac{4}{3}N_cm_t^2\right],
\end{aligned}$$

[4] We have also performed the calculation of the renormalised one-loop amplitude in unitary gauge and found full agreement with the Feynman gauge results.

$$
\begin{aligned}
\delta Z_h^{(6)} &= C_\epsilon\Bigg[\frac{1}{\epsilon}\Bigg(-\left(4m_\tau^2+4N_c\left(m_b^2+m_t^2\right)+14m_H^2\right)\frac{C_{H,\mathrm{kin}}}{v_T^2}-m_H^2C_{HD}\\
&\quad+2\sqrt{2}v_T\left(N_c\left(m_bC_{bH}+m_tC_{tH}\right)+m_\tau C_{\tau H}\right)\Bigg)+\frac{8}{3}N_cm_t^2\frac{C_{H,\mathrm{kin}}}{v_T^2}\Bigg],\\
\frac{\delta M_W^{(4)}}{M_W} &= \frac{C_\epsilon}{v_T^2}\left[-\frac{1}{\epsilon}\left(N_c(m_b^2+m_t^2)+m_\tau^2\right)-\frac{1}{2}N_cm_t^2\right],\\
\frac{\delta M_W^{(6)}}{M_W} &= C_\epsilon\left[\frac{2}{\epsilon}\left(N_cm_bm_tC_{Htb}-N_c\left(m_b^2+m_t^2\right)C_{Hq}^{(3)}-m_\tau^2C_{H\tau}^{(3)}\right)-N_cm_t^2C_{Hq}^{(3)}\right],\\
\frac{\delta M_Z^{(4)}}{M_Z} &= -\frac{C_\epsilon}{v_T^2}\frac{1}{\epsilon}\left(N_c\left(m_b^2+m_t^2\right)+m_\tau^2\right),\\
\frac{\delta M_Z^{(6)}}{M_Z} &= \frac{C_\epsilon}{\epsilon}\Bigg[\frac{1}{4}\left(2N_c\left(m_b^2+m_t^2\right)+2m_\tau^2+3m_H^2\right)C_{HD}\\
&\quad+m_H^2\hat{c}_w\hat{s}_wC_{HWB}+2m_\tau^2\left(C_{H\tau}-C_{H\tau}^{(1)}-C_{H\tau}^{(3)}\right)\\
&\quad+2N_c\left(m_b^2C_{Hb}-m_t^2C_{Ht}-(m_b^2-m_t^2)C_{Hq}^{(1)}-(m_b^2+m_t^2)C_{Hq}^{(3)}\right)\Bigg],\\
\frac{\delta\bar{e}^{(4)}}{\bar{e}} &= 0,\\
\frac{\delta\bar{e}^{(6)}}{\bar{e}} &= -\frac{C_\epsilon}{\epsilon}m_H^2\hat{c}_w\hat{s}_wC_{HWB}\,.
\end{aligned}
\tag{5.6}
$$

We note that the SM results agree with those quoted in [44]. Using the results above along with the definition (3.13), we find

$$
\begin{aligned}
\frac{\delta\bar{s}_w^{(4)}}{\bar{s}_w} &= \frac{\hat{c}_w^2}{2\hat{s}_w^2}\frac{N_cm_t^2}{v_T^2},\\
\frac{\delta\bar{s}_w^{(6)}}{\bar{s}_w} &= \frac{C_\epsilon}{\epsilon}\frac{m_H^2\hat{c}_w}{2\hat{s}_w}\left(\hat{c}_w^2-\hat{s}_w^2\right)C_{HWB}+\frac{\hat{c}_w^2}{2\hat{s}_w^2}N_cm_t^2\left[C_{HD}+2C_{Hq}^{(3)}\right]\\
&\quad+\frac{N_cm_t^2\hat{c}_w}{\hat{s}_w}\left(1-\frac{1}{4\hat{s}_w^2}\right)C_{HWB}-\frac{\hat{c}_w\hat{v}_T^2}{4\hat{s}_w}\left(\dot{C}_{HWB}+\frac{\hat{c}_w}{2\hat{s}_w}\dot{C}_{HD}\right)\ln\left(\frac{m_t^2}{\mu^2}\right).
\end{aligned}
\tag{5.7}
$$

In the above equations we have defined the quantity

$$
C_\epsilon=\left(\frac{\mu^2}{m_t^2}\right)^\epsilon=1-\epsilon\ln\left(\frac{m_t^2}{\mu^2}\right)+\mathcal{O}(\epsilon^2)\,.
\tag{5.8}
$$

This is a natural definition for the large-m_t contributions, where only logarithms of the form $\ln(m_t^2/\mu^2)$ can appear. When expanding the counterterms in ϵ this definition generates additional, finite terms which are subleading in the large-m_t limit, i.e. terms of the form $m_b^2\ln m_t^2/\mu^2$. We choose to keep these subleading terms in our final analytic results for the renormalised amplitudes, since then the μ-independence up to one-loop order is manifest, also away from the large-m_t limit. While these subleading terms may appear more naturally as logarithms of the form, e.g. $m_b^2\ln m_b^2/\mu^2$, the argument of the μ-dependent logarithms is not fixed to leading order in the large-m_t limit considered here — such ambiguities will be resolved by the full calculation [29].

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The full UV counterterm contribution to the amplitude (3.18) can be constructed from these results, and when added to the bare one-loop matrix element (computed from the sum of diagrams depicted in figure 2, for example), one finds that all UV poles and gauge dependence cancels. The final step in the calculation is to eliminate v_T dependence using (3.20). At one-loop order, we can immediately use the results above to calculate the expansion coefficients (3.22) of Δr. We find that

$$\begin{aligned}
\Delta r^{(4,1)} &= -\frac{\hat{c}_w^2}{\hat{s}_w^2}\frac{N_c m_t^2}{v_T^2}\,, \\
\Delta r^{(6,1)} &= -\frac{\hat{c}_w^2}{\hat{s}_w^2}N_c m_t^2\left(C_{HD}+2C_{Hq}^{(3)}\right)-\frac{N_c m_t^2\hat{c}_w}{\hat{s}_w}\left(4-\frac{1}{\hat{s}_w^2}\right)C_{HWB} \\
&\quad+\frac{\hat{c}_w\hat{v}_T^2}{\hat{s}_w}\left(\dot{C}_{HWB}+\frac{\hat{c}_w}{4\hat{s}_w}\dot{C}_{HD}\right)\ln\left(\frac{m_t^2}{\mu^2}\right)\,.
\end{aligned} \tag{5.9}$$

Obtaining the one-loop contribution $\Delta R^{(6,1)}$ requires a separate but straightforward calculation. In the large-m_t limit, the only non-logarithmic finite contributions arise from the insertions of four-fermion operators onto the usual tree-level W boson exchange graph, resulting in the diagrams in figure 3. Evaluating those diagrams in the large-m_t limit (and including the μ-dependent terms implied by the RG equations) we find that

$$\Delta R^{(6,1)} = \frac{N_c m_t^2}{\sqrt{2}v_T^2}\left(C^{(3)}_{\substack{lq\\ \mu\mu 33}}+C^{(3)}_{\substack{lq\\ ee33}}\right)-\frac{1}{2\sqrt{2}}\left(\dot{C}^{(3)}_{\substack{Hl\\ ee}}+\dot{C}^{(3)}_{\substack{Hl\\ \mu\mu}}-\frac{1}{2}\left(\dot{C}_{\substack{ll\\ \mu ee\mu}}+\dot{C}_{\substack{ll\\ e\mu\mu e}}\right)\right)\ln\left(\frac{m_t^2}{\mu^2}\right)\,. \tag{5.10}$$

We are now in position to give the final results for the renormalised one-loop corrections in the large-m_t limit. We write the results in terms of expansion coefficients as

$$i\mathcal{M}(h\to f\bar{f}) = -i\bar{u}(p_f)v(p_{\bar{f}})\left[A_f^{(4,0)}+A_f^{(6,0)}+\frac{1}{16\pi^2}\left(A_f^{(4,1)}+A_f^{(6,1)}\right)\right]\,. \tag{5.11}$$

The results for $h\to b\bar{b}$ are, at tree-level,

$$A_b^{(4,0)} = \left(\sqrt{2}G_F\right)^{\frac{1}{2}}m_b\,, \tag{5.12}$$

$$A_b^{(6,0)} = A_b^{(4,0)}C_{H,\mathrm{kin}}-\frac{C_{bH}}{2G_F}+A_b^{(4,0)}\frac{\Delta R^{(6,0)}}{2G_F}\,. \tag{5.13}$$

The one-loop results are

$$A_b^{(4,1)} = A_b^{(4,0)}G_F m_t^2\left(\frac{-18+7N_c}{3\sqrt{2}}\right)\,, \tag{5.14}$$

$$\begin{aligned}
A_b^{(6,1)} &= A_b^{(4,0)}m_t^2\left(\frac{3G_F}{\sqrt{2}}(-2+N_c)C_{H,\mathrm{kin}}+(-1+N_c)\,C_{Hq}^{(3)}\right)+\frac{(-15+4N_c)m_t^2}{12}\frac{C_{bH}}{\sqrt{2}} \\
&\quad+\frac{1}{2}\left[A_b^{(4,0)}\dot{C}_{H,\mathrm{kin}}-\frac{1}{2G_F}\dot{C}_{bH}\right]\ln\left(\frac{m_t^2}{\mu^2}\right) \\
&\quad+\frac{1}{2G_F}\left(A_b^{(4,0)}\Delta R^{(6,1)}+3A_b^{(4,1)}\Delta R^{(6,0)}\right)\,.
\end{aligned} \tag{5.15}$$

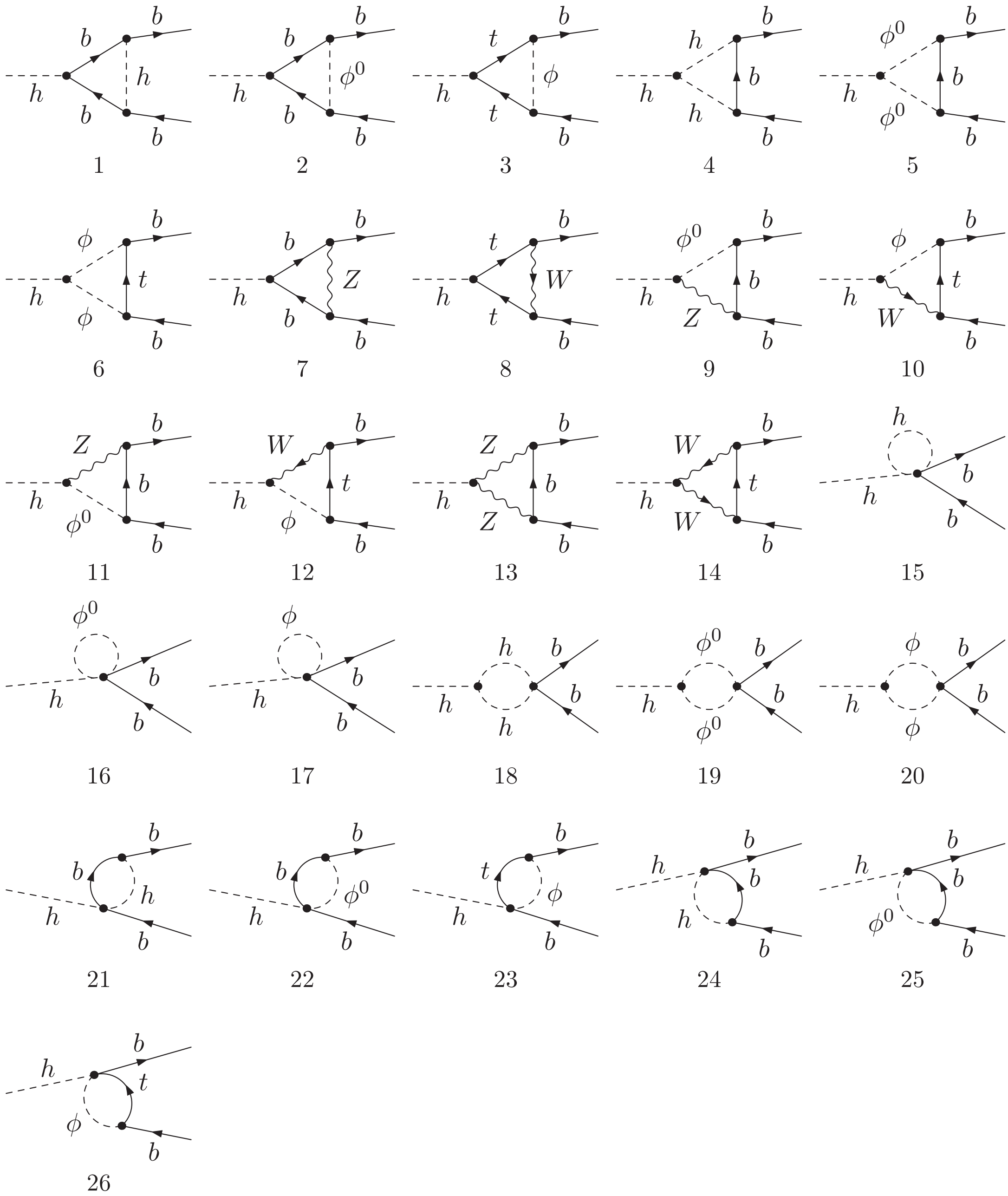

Figure 2. Representative diagrams contributing to the process $h \to b\bar{b}$ in 't Hooft-Feynman gauge. The dimension-6 contributions are inserted onto the relevant vertices, and contributions to $\mathcal{O}(1/\Lambda_{NP}^2)$ are kept. The fields ϕ, ϕ^0 refer to the charged and neutral Goldstone bosons. In addition to the usual SM diagrams, note the presence of Diagrams 15–17 which are generated solely by Class 5 operators. The contributions from Class 8 operators are depicted on the left side of figure 1.

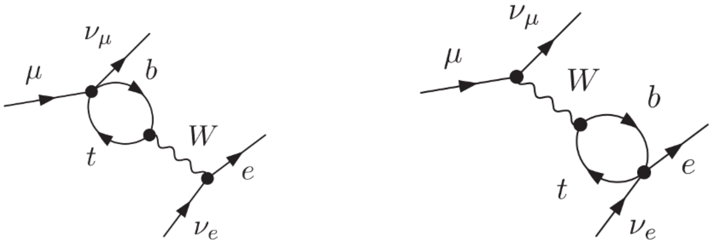

Figure 3. Feynman diagrams which contribute to the finite corrections to $\Delta R^{(6,1)}$ in the large-m_t limit.

Similarly, for $h \to \tau\bar{\tau}$, at tree-level,

$$A_\tau^{(4,0)} = \left(\sqrt{2}G_F\right)^{\frac{1}{2}} m_\tau \,, \tag{5.16}$$

$$A_\tau^{(6,0)} = A_\tau^{(4,0)} C_{H,\mathrm{kin}} - \frac{C_{\tau H}}{2G_F} + A_\tau^{(4,0)} \frac{\Delta R^{(6,0)}}{2G_F} \,. \tag{5.17}$$

The one-loop results are

$$A_\tau^{(4,1)} = A_\tau^{(4,0)} G_F m_t^2 \frac{7N_c}{3\sqrt{2}} \,, \tag{5.18}$$

$$\begin{aligned} A_\tau^{(6,1)} = {} & A_\tau^{(4,0)} N_c m_t^2 \left(\frac{3}{\sqrt{2}} G_F C_{H,\mathrm{kin}} + C_{Hq}^{(3)}\right) + \frac{N_c m_t^2}{3} \frac{C_{\tau H}}{\sqrt{2}} \\ & + \frac{1}{2}\left[A_\tau^{(4,0)} \dot{C}_{H,\mathrm{kin}} - \frac{1}{2G_F} \dot{C}_{\tau H}\right] \ln\left(\frac{m_t^2}{\mu^2}\right) \\ & + \frac{1}{2G_F}\left(A_\tau^{(4,0)} \Delta R^{(6,1)} + 3A_\tau^{(4,1)} \Delta R^{(6,0)}\right) \,. \end{aligned} \tag{5.19}$$

The main results of this section are the renormalised one-loop contributions to the decay amplitudes in (5.15) and (5.19). We have written them in a form which makes clear that the renormalised decay amplitudes are μ-independent up to one-loop. We have calculated the coefficients of the μ-dependent logarithms directly in the broken phase of the theory, and emphasise that it is a non-trivial cross-check that are results are consistent with those by the RG equations of [10, 11]. The non-logarithmic one-loop corrections, on the other hand, cannot be deduced from RG equations and are a new result. Interestingly, the potentially dominant non-logarithmic contributions proportional to m_t^3 occurring in the bare matrix elements are cancelled by those in the mass renormalisation counterterms $\delta m_{b,\tau}^{(6)}$ in (5.6). Obviously, this is a scheme-dependent result that would not hold if these masses were instead renormalised in the $\overline{\mathrm{MS}}$ scheme.

6 Impact on phenomenology

In this section we explore the implications of the one-loop corrections provided in (5.15) and (5.19) on the interpretation of Higgs decay data. We begin by commenting on the application of RG-improved perturbation theory to the interpretation of data from experiment, before discussing the sensitivity of Higgs decay measurements to various Wilson coefficients.

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In the context of the current calculation, the relevant physical scale for the process is $\mu = \mu_t \sim m_t$. By setting the scale which appears in both one-loop and tree-level dimension-six amplitudes to this value, the large logarithms which appear in the one-loop matrix elements in (5.15) and (5.19) are absorbed into the Wilson coefficients $C_i(\mu_t)$. The decay rates which are then computed from squaring the sum of these amplitudes are then a function of purely finite terms and Wilson coefficients defined at the scale $C_i(\mu_t)$. Constraints on the possible values of these Wilson coefficients (defined at the scale μ_t) can then be obtained by performing a fit to the available data.[5]

In the (hopeful) scenario where such a fit prefers non-zero values for some of these Wilson coefficients $C_i(\mu_t)$, it will be possible to interpret such a scenario in terms of new physics. This can be done without making reference to a specific model, by evolving these Wilson coefficients from the scale μ_t to the scale $\Lambda_{\rm NP}$ by solving the RG equations. In doing so, potentially large logarithms are resummed into RG evolution factors which relate Wilson coefficients at different scales. In fact, provided the scale $\Lambda_{\rm NP}$ does not exceed the scale μ_t by several orders of magnitude, the relation between Wilson coefficients at different scales can be approximated through the one-loop solution to the RG equations:

$$C_i(\mu_t) = C_i(\Lambda_{\rm NP}) + \frac{1}{2}\frac{1}{16\pi^2}\dot{C}_i(\Lambda_{\rm NP})\ln\left(\frac{\mu_t^2}{\Lambda_{\rm NP}^2}\right). \tag{6.1}$$

The constraints on the values of the Wilson coefficients $C_i(\mu_t)$ obtained in this way are therefore translated into constraints on the Wilson coefficients defined at the scale $\Lambda_{\rm NP}$. The benefit of such an approach is that solution to (6.1) is fully known at one-loop [8, 10, 11]. Therefore, it is possible to directly test specific new physics models by matching them to the SMEFT at a scale $\mu \sim \Lambda_{\rm NP}$, and comparing the consistency of the set of non-vanishing Wilson coefficients $C_i(\Lambda_{\rm NP})$ generated in this matching procedure with those obtained from data (having been evolved to the scale $\Lambda_{\rm NP}$). In general, the main goal of NLO calculations within the SMEFT is to evaluate the purely finite contributions to the one-loop amplitudes, as we have provided in (5.15) and (5.19). The importance of evaluating these contributions is to determine whether or not they have any impact on the extraction of the values of the $C_i(\mu_t)$. Of course the NLO calculations also provide a cross check of the previous anomalous dimension calculations.

In the following, we will assess the impact our results on the decay rates at the scale μ_t, and then briefly discuss the potential interpretation of a non-zero extraction of Wilson coefficients at this scale in terms of new physics in a model independent way. We calculate decay rates as a double expansion in loop factors and $1/\Lambda_{\rm NP}$, neglecting self-interference of dimension-6 operators as well as that of one-loop contributions. We thus decompose the decay rate as

$$\Gamma(h \to f\bar{f}) = B_f\left[\Gamma_f^{(4,0)} + \Gamma_f^{(6,0)} + \Gamma_f^{(4,1)} + \Gamma_f^{(6,1)}\right] + \dots, \tag{6.2}$$

[5] In the situation where a global fit is performed to a large data set, which may involve differential measurements or processes with different scales, a relevant scale should be chosen for each data point and the constraints obtained on the Wilson coefficients in this way should be presented at a common scale.

where

$$B_\tau = \frac{m_H}{8\pi}\left(1-\frac{4m_\tau^2}{m_H^2}\right)^{\frac{3}{2}}, \qquad B_b = N_c\frac{m_H}{8\pi}\left(1-\frac{4m_b^2}{m_H^2}\right)^{\frac{3}{2}}. \tag{6.3}$$

Taking advantage of the fact that the matrix elements are real, we can define the SM contributions by

$$\Gamma_f^{(4,0)} = \left[A_f^{(4,0)}\cdot A_f^{(4,0)}\right], \qquad \Gamma_f^{(4,1)} = \frac{1}{16\pi^2}\left[2A_f^{(4,0)}\cdot A_f^{(4,1)}\right], \tag{6.4}$$

while the dimension-6 contributions are

$$\Gamma_f^{(6,0)} = \left[2A_f^{(4,0)}\cdot A_f^{(6,0)}\right], \qquad \Gamma_f^{(6,1)} = \frac{1}{16\pi^2}\left[2\left(A_f^{(6,0)}\cdot A_f^{(4,1)} + A_f^{(4,0)}\cdot A_f^{(6,1)}\right)\right]. \tag{6.5}$$

To evaluate these expression numerically, we use the following set of input parameters: $m_t = 173.3\,\text{GeV}$, $m_b = 4.75\,\text{GeV}$, $m_\tau = 1.777\,\text{GeV}$, $m_H = 125.0\,\text{GeV}$, $G_F = 1.16638\cdot 10^{-5}\,\text{GeV}^{-2}$. To make the suppression of dimension-6 contributions explicit, we define the dimensionless quantities

$$C_i(\mu_t) \equiv \frac{\tilde{C}_i(\mu_t)}{\Lambda_{\text{NP}}^2}, \qquad C_i(\Lambda_{\text{NP}}) \equiv \frac{\hat{C}_i(\Lambda_{\text{NP}})}{\Lambda_{\text{NP}}^2}. \tag{6.6}$$

6.1 $h \to b\bar{b}$ decays

We now consider the relative size of the different types of corrections for the process $h \to b\bar{b}$. First of all, the ratio of tree-level dimension-6 and SM contributions is given by

$$\frac{\Gamma_b^{(6,0)}}{\Gamma_b^{(4,0)}} = -\frac{1}{G_F(\sqrt{2}G_F)^{\frac{1}{2}}m_b}\frac{\tilde{C}_{bH}}{\Lambda_{\text{NP}}^2} + \frac{1}{G_F}\left(\sqrt{2}\left(\frac{\tilde{C}_{H\Box}}{\Lambda_{\text{NP}}^2} - \frac{1}{4}\frac{\tilde{C}_{HD}}{\Lambda_{\text{NP}}^2}\right) + \frac{\Delta\tilde{R}^{(6,0)}}{\Lambda_{\text{NP}}^2}\right), \tag{6.7}$$

where the explicit definition of $C_{H,\text{kin}}$ in (2.5) has been used. Numerically, at a scale of $\Lambda_{\text{NP}} = 1\,\text{TeV}$, this amounts to

$$\frac{\Gamma_b^{(6,0)}}{\Gamma_b^{(4,0)}} = -4.44\tilde{C}_{bH} + 0.03\left(4\tilde{C}_{H\Box} - \tilde{C}_{HD}\right) + 0.09\Delta\tilde{R}^{(6,0)}. \tag{6.8}$$

The dimension-6 contributions are large if $\tilde{C}_{bH} \sim \mathcal{O}(1)$. On the other hand, if one assumes that $\tilde{C}_{bH} \sim y_b$, as would be the case in MFV, then the result is

$$\frac{\Gamma_b^{(6,0)}}{\Gamma_b^{(4,0)}} = -0.12\frac{\tilde{C}_{bH}}{y_b} + \ldots, \tag{6.9}$$

where the ellipses denote the remaining terms of (6.8), whose sensitivity is numerically comparable in an MFV-like scenario.

We next study the size of one-loop corrections. The ratio of the one-loop to tree-level corrections in the SM is

$$\frac{\Gamma_b^{(4,1)}}{\Gamma_b^{(4,0)}} = \frac{G_F m_t^2}{8\pi^2}\left(\frac{-18+7N_c}{3\sqrt{2}}\right) = 0.003, \tag{6.10}$$

in agreement with previous results [30]. The one-loop corrections are quite small due to a large cancellation between the N_c-dependent and N_c-independent terms in the numerator.

The ratio of the one-loop SMEFT and tree-level SM predictions can also be obtained in a similar fashion, written in terms of Wilson coefficients defined at the scale μ_t, we find

$$\begin{aligned}\frac{\Gamma_b^{(6,1)}}{\Gamma_b^{(4,0)}} &= -\frac{m_t^2}{(\sqrt{2}G_F)^{\frac{1}{2}}m_b}\frac{(-21+10N_c)}{96\pi^2\sqrt{2}}\frac{\tilde{C}_{bH}}{\Lambda_{\rm NP}^2} - m_t^2\frac{(9-4N_c)}{48\pi^2}\left(4\frac{\tilde{C}_{H\Box}}{\Lambda_{\rm NP}^2} - \frac{\tilde{C}_{HD}}{\Lambda_{\rm NP}^2}\right) \\ &\quad + m_t^2\frac{(-1+N_c)}{8\pi^2}\frac{\tilde{C}_{Hq}^{(3)}}{\Lambda_{\rm NP}^2} + \frac{1}{16\pi^2 G_F}\frac{\Delta\tilde{R}^{(6,1)}}{\Lambda_{\rm NP}^2} + m_t^2\frac{(-18+7N_c)}{12\pi^2\sqrt{2}}\frac{\Delta\tilde{R}^{(6,0)}}{\Lambda_{\rm NP}^2}\,, \\ &\simeq -0.01\tilde{C}_{bH} + 10^{-3}\left(0.19\left(4\tilde{C}_{H\Box} - \tilde{C}_{HD} + 4\tilde{C}_{Hq}^{(3)}\right) + 0.54\left(\Delta\tilde{R}^{(6,0)} + \Delta\tilde{R}^{(6,1)}\right)\right)\,, \\ &\simeq -0.0003\frac{\tilde{C}_{bH}}{y_b} + \dots\,. \end{aligned} \tag{6.11}$$

By comparing the numerical pre-factor of $\tilde{C}_{bH}$ in the expression above with that in the corresponding LO expression (6.8), we see that these finite one-loop corrections are numerically unimportant. As noted in the previous section, the potentially dominant non-logarithmic terms proportional to m_t^3 in the bare matrix element and multiplying the C_{Htb}, $C_{qtqb}^{(1)}$ and $C_{qtqb}^{(8)}$ coefficients are cancelled exactly by the mass counterterm for the b-quark in the on-shell scheme. Our calculation therefore justifies using a LO SMEFT analysis, at least in the on-shell scheme, to constrain the Wilson coefficients $\tilde{C}_i/\Lambda_{\rm NP}^2$ appearing in (6.8), and then in turn using an RG analysis to interpret such constraints at the scale $\Lambda_{\rm NP}$. In fact, the anomalous dimension calculation which is required to do such an analysis has already been presented [13], where the authors also studied the phenomenological implications of their results.

We perform a similar analysis below by expressing the Higgs decay rate in terms of Wilson coefficients at the scale $\Lambda_{\rm NP} \sim 1\,\text{TeV}$, where we use the hatted notation for the Wilson coefficients $\hat{C}_i(\Lambda_{\rm NP})$ introduced in (6.6) to differentiate them from $\tilde{C}_i(\mu_t)$. We therefore compute a compact expression for the ratio of the SMEFT decay rate with respect to the tree-level SM prediction. Retaining only the numerically important terms, which correspond to those which appear at tree-level and in addition a subset of the mixing contributions generated by the running of C_{bH}, we find

$$\begin{aligned}\frac{\Gamma_b^{(6,0)}+\Gamma_b^{(6,1)}}{\Gamma_b^{(4,0)}} &\simeq -\frac{1}{G_F(\sqrt{2}G_F)^{\frac{1}{2}}m_b}\frac{\hat{C}_{bH}}{\Lambda_{\rm NP}^2} + \frac{1}{G_F}\left(\sqrt{2}\left(\frac{\hat{C}_{H\Box}}{\Lambda_{\rm NP}^2} - \frac{1}{4}\frac{\hat{C}_{HD}}{\Lambda_{\rm NP}^2}\right) + \frac{\Delta\hat{R}^{(6,0)}}{\Lambda_{\rm NP}^2}\right) \\ &\quad + \frac{1}{16\pi^2}\frac{1}{\Lambda_{\rm NP}^2}\Bigg[\frac{3}{\sqrt{2}(\sqrt{2}G_F)^{\frac{1}{2}}m_b}\left(4m_H^2 + m_t^2(-1+2N_c)\right)\hat{C}_{bH} \\ &\quad - 2\frac{m_t}{m_b}(m_H^2+2m_t^2)\hat{C}_{Htb} - \frac{m_t}{m_b}(4m_t^2-m_H^2) \\ &\quad \left((2N_c+1)\hat{C}_{qtqb}^{(1)} + c_{F,3}\hat{C}_{qtqb}^{(8)}\right)\Bigg]\ln\left(\frac{\Lambda_{\rm NP}^2}{m_t^2}\right)\,. \end{aligned} \tag{6.12}$$

Evaluating (6.12) at the scale $\Lambda_{\rm NP} = 1\,{\rm TeV}$, we find

$$\frac{\Gamma_b^{(6,0)} + \Gamma_b^{(6,1)}}{\Gamma_b^{(4,0)}} \simeq -3.93\hat{C}_{bH} - 0.59\hat{C}^{(1)}_{qtqb} - 0.12\hat{C}_{Htb} + 0.12\hat{C}_{H\Box} - 0.11\hat{C}^{(8)}_{qtqb} + 0.09\Delta\hat{R}^{(6,0)} - 0.03\hat{C}_{HD}\,. \tag{6.13}$$

The above analyses demonstrates that this decay rate is numerically most sensitive to the coefficients $\hat{C}_{bH}$ and $\hat{C}^{(1)}_{qtqb}$. Interestingly, these particular Wilson coefficients are also not well experimentally constrained. For instance, the four-fermion operator multiplying $C^{(1)}_{qtqb}$ does not contribute to Zbb couplings at one-loop level due to its Dirac structure. This can be observed by direct calculation, or by examining the anomalous dimension matrix of the Wilson coefficients $C^{(1)}_{Hb}$ and $C^{(3)}_{Hq}$ which alter the Z boson couplings to fermions [11]. Consequently, $C^{(1)}_{qtqb}$ is not subject to strong constraints from LEP data. Nor does the operator $Q^{(1)}_{qtqb}$ give large contributions to top-quark pair production at hadron colliders, since the tree-level partonic process $b\bar{b} \to t\bar{t}$ is highly suppressed as result of the exceedingly small $b\bar{b}$-quark PDF luminosity. This leads us to consider a simplified analysis, where all Wilson coefficients except for C_{bH} and $C^{(1)}_{qtqb}$, which are currently unconstrained phenomenologically, vanish.

Under such conditions, it also straightforward to place experimental constraints on these Wilson coefficients by including the available Higgs decay data [45]. To do so, we can identify the extracted signal strength μ^{bb} with the SMEFT and SM decay rates in the following way $\mu^{bb} = 1 + \Gamma_b^{(6)}/\Gamma_b^{(4)}$. Under the assumption that new physics does not alter Higgs boson production, we can use the experimental extraction of μ^{bb} from the combined CMS and ATLAS analysis of $\mu^{bb} = 0.69^{+0.29}_{-0.27}$. Using the tree-level formula (6.7), leads to the following constraint

$$\frac{\tilde{C}_{bH}(\mu_t)}{G_F\Lambda^2_{\rm NP}} = \left(5.98^{+5.20}_{-5.59}\right) \times 10^{-3}\,. \tag{6.14}$$

To interpret the impact of the measurement of μ^{bb} in terms of Wilson coefficients defined at the scale $\Lambda_{\rm NP}$, we can simply use the compact formula (6.12) assuming non-vanishing C_{bH} and $C^{(1)}_{qtqb}$ Wilson coefficients. The solution, which depends both linearly and logarithmically on the choice of $\Lambda_{\rm NP}$, is presented for the choices $\Lambda_{\rm NP} = 1, 2\,{\rm TeV}$ in figure 4 (along side the results for the $h \to \tau\bar{\tau}$ which will be discussed below). Interestingly, the available data already constrains the values of these Wilson coefficients to be $\mathcal{O}(1)$, and prefers positive values of both $\hat{C}_{bH}$ and $\hat{C}^{(1)}_{qtqb}$ to accommodate the slightly low value of μ^{bb} observed in data. It should be noted that zero values of these Wilson coefficients are consistent with the data at 1σ CL.

In the above scenario, we have placed constraints on the Wilson coefficients without reference to any particular UV completion. However, in a broad range of UV completions, such as those studied in [13], these Wilson coefficients are expected to scale as $\hat{C}^{\rm MFV}_{bH} \sim y_b\hat{C}_{bH}$ and $\hat{C}^{{\rm MFV},(1)}_{qtqb} \sim y_b y_t\hat{C}^{(1)}_{qtqb}$, where the $\hat{C}_i$ are order one quantities (we have so far referred to such a scenario as MFV-like). In this case, useful bounds on the rescaled coefficients $\hat{C}_i$ can be expected only once experimental measurements improve in precision

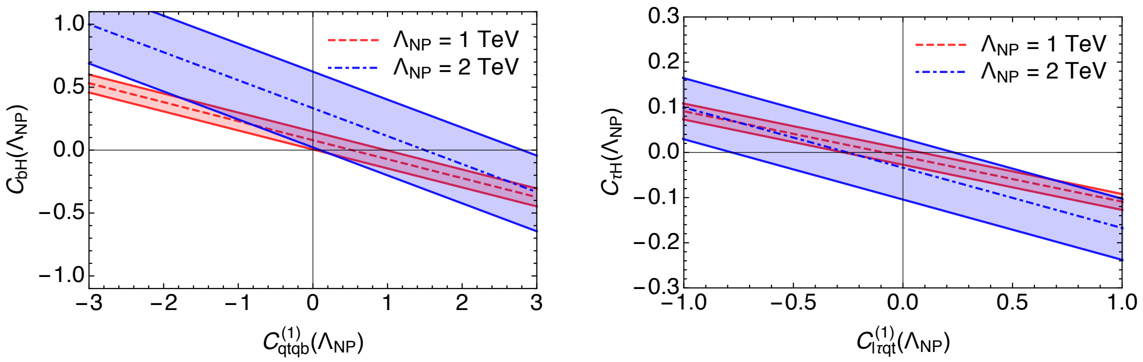

Figure 4. Constraints in the plane of the Wilson coefficients $C_{bH}(\Lambda_{\rm NP}) - C^{(1)}_{qtqb}(\Lambda_{\rm NP})$ (left) and $C_{\tau H}(\Lambda_{\rm NP}) - C^{(1)}_{l\tau qt}(\Lambda_{\rm NP})$ (right), based on a simplified analysis of the combined CMS and ATLAS Run-I data [45].

by at least an order of magnitude.[6] Such precision may be experimentally challenging at the LHC, even with the large amount of data expected during Run II [46]. However, such precision can be achieved at an e^+e^- machine [47–50], where sub-percent level precision is estimated for both b-quark and τ-lepton final states for particular e^+e^- programs.

6.2 $h \to \tau\bar{\tau}$ decays

The analysis for the case of $h \to \tau\bar{\tau}$ decays proceeds along similar lines. In fact, the ratio of the tree-level dimension-6 contributions to the tree-level SM contributions is given by (6.9) after replacing $b \to \tau$. The one-loop corrections in the SM are instead

$$\frac{\Gamma_\tau^{(4,1)}}{\Gamma_\tau^{(4,0)}} = \frac{G_F m_t^2}{8\pi^2}\left(\frac{7N_c}{3\sqrt{2}}\right) = 0.022\,, \tag{6.15}$$

Compared to the decay into b-quarks, this contribution is generated solely by the finite terms present in the counterterm for δM_W and δZ_h, and so there is no large cancellation.

To assess the impact of the purely finite one-loop SMEFT corrections, we take the ratio of this decay rate (at the scale μ_t) with that of the tree-level SM decay rate, finding

$$\begin{aligned}\frac{\Gamma_\tau^{(6,1)}}{\Gamma_\tau^{(4,0)}} &= -\frac{5N_c m_t^2}{48\pi^2(2\sqrt{2}G_F)^{\frac{1}{2}}m_\tau}\frac{\tilde{C}_{\tau H}}{\Lambda_{\rm NP}^2} + \frac{N_c m_t^2}{12\pi^2}\left(4\frac{\tilde{C}_{H\Box}}{\Lambda_{\rm NP}^2} - \frac{\tilde{C}_{HD}}{\Lambda_{\rm NP}^2}\right)\\ &\quad + \frac{N_c m_t^2}{8\pi^2}\frac{\tilde{C}^{(3)}_{Hq}}{\Lambda_{\rm NP}^2} + \frac{1}{16\pi^2 G_F}\frac{\Delta\tilde{R}^{(6,1)}}{\Lambda_{\rm NP}^2} + \frac{7N_c m_t^2}{12\pi^2\sqrt{2}}\frac{\Delta\tilde{R}^{(6,0)}}{\Lambda_{\rm NP}^2}\,,\\ &\simeq -0.09\tilde{C}_{\tau H} + 10^{-2}\Big(0.08\Big(4\tilde{C}_{H\Box} - \tilde{C}_{HD}\Big) + 0.11\tilde{C}^{(3)}_{Hq} + 0.38\Delta\tilde{R}^{(6,0)} + 0.054\Delta\tilde{R}^{(6,1)}\Big)\,,\\ &\simeq -0.0009\frac{\tilde{C}_{\tau H}}{y_\tau} + \ldots\,.\end{aligned} \tag{6.16}$$

[6]For further, model-dependent phenomenological studies in new physics scenarios such as supersymmetry, we refer the reader to [13].

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In this case, the finite corrections are more important than in the $h \to b\bar{b}$ decay, particularly for $\tilde{C}_{\tau H}$. However, in analogy to the $h \to b\bar{b}$ decay, the potentially numerically large term proportional to m_t^3 in the bare matrix element (which multiplies $\tilde{C}_{l\tau qt}^{(1)}$) is exactly cancelled by that in the τ-lepton mass counter-term. We can also perform a simplified analysis for τ-leptons, under the assumption that only non-zero values for the Wilson coefficients $\hat{C}_{\tau H}$ and $\hat{C}_{l\tau qt}^{(1)}$ are allowed. In this case, we use the experimentally extracted value of $\mu^{\tau\tau}$ from the combined CMS and ATLAS analysis, given by $\mu^{\tau\tau} = 1.12^{+0.25}_{-0.23}$. We find

$$\frac{\tilde{C}_{\tau H}(\mu_t)}{G_F \Lambda_{\rm NP}^2} = \left(-0.87^{+1.66}_{-1.80}\right) \times 10^{-3} \qquad \text{LO}\,, \tag{6.17}$$

$$\frac{\tilde{C}_{\tau H}(\mu_t)}{G_F \Lambda_{\rm NP}^2} = \left(-0.86^{+1.65}_{-1.79}\right) \times 10^{-3} \qquad \text{LMT}\,. \tag{6.18}$$

In the first case (labelled LO) the Wilson coefficient is extracted using the LO formula $\Gamma_\tau^{(i)} = \Gamma_\tau^{(i,0)}$. In the latter (labelled LMT), the large m_t-limit one-loop corrections are also included in the extraction as $\Gamma_\tau^{(i)} = \Gamma_\tau^{(i,0)} + \Gamma_\tau^{(i,1)}$. This demonstrates that the purely finite corrections are indeed not important for the interpretation of the experimental data in this case either. To extract constraints on the Wilson coefficients at the scale $\Lambda_{\rm NP}$ directly from $\mu^{\tau\bar{\tau}}$, we also provide a general compact analytic expression in terms of Wilson-coefficients defined at the scale $\Lambda_{\rm NP}$.

$$\begin{aligned}\frac{\Gamma_\tau^{(6,0)} + \Gamma_\tau^{(6,1)}}{\Gamma_\tau^{(4,0)}} \simeq & -\frac{1}{G_F(\sqrt{2}G_F)^{\frac{1}{2}} m_\tau}\frac{\hat{C}_{\tau H}}{\Lambda_{\rm NP}^2} + \frac{1}{G_F}\left(\sqrt{2}\left(\frac{\hat{C}_{H\Box}}{\Lambda_{\rm NP}^2} - \frac{1}{4}\frac{\hat{C}_{HD}}{\Lambda_{\rm NP}^2}\right) + \frac{\Delta\hat{R}^{(6,0)}}{\Lambda_{\rm NP}^2}\right) \\ & + \frac{1}{16\pi^2}\frac{1}{\Lambda_{\rm NP}^2}\left[-\frac{5N_c m_t^2}{3\sqrt{2}(\sqrt{2}G_F)^{\frac{1}{2}} m_\tau}\hat{C}_{\tau H} - \left(\frac{6(2m_H^2 + N_c m_t^2)}{\sqrt{2}(\sqrt{2}G_F)^{\frac{1}{2}} m_\tau}\hat{C}_{\tau H}\right.\right. \\ & \left.\left. + \frac{m_t}{m_\tau} 2N_c(4m_t^2 - m_H^2)\hat{C}_{l\tau qt}^{(1)}\right)\ln\left(\frac{\Lambda_{\rm NP}^2}{m_t^2}\right)\right]. \end{aligned} \tag{6.19}$$

The extracted values of $\hat{C}_{\tau H}(\Lambda_{\rm NP})$ of $\hat{C}_{l\tau qt}^{(1)}(\Lambda_{\rm NP})$, assuming otherwise vanishing Wilson coefficients, are presented in the right plot of figure 4. Once again, we have chosen provide the solutions for the scale choices $\Lambda_{\rm NP} = 1, 2\,\text{TeV}$.

We therefore arrive at similar conclusions for τ-leptons as in the case for b-quarks. That is, measurements of the Higgs decay rate already provide (better than) $\mathcal{O}(1)$ constraints on the combination of the Wilson coefficients $\hat{C}_{\tau H}(\Lambda_{\rm NP})$ and $\hat{C}_{l\tau qt}^{(1)}(\Lambda_{\rm NP})$ for values of $\Lambda_{\rm NP}$ in the few TeV range. In a scenario where these Wilson coefficients scale as $\hat{C}_{\tau H}^{\rm MFV} \sim y_\tau \hat{C}_{\tau H}$ and $\hat{C}_{l\tau qt}^{{\rm MFV},(1)} \sim y_\tau y_t \hat{C}_{l\tau qt}^{(1)}$, measurements in a clean e^+e^- environment will be necessary to constrain $\mathcal{O}(1)$ values of $\hat{C}_{\tau H}$ and $\hat{C}_{l\tau qt}^{(1)}$.

7 Conclusions

We have calculated a set of one-loop corrections to $h \to b\bar{b}$ and $h \to \tau\bar{\tau}$ decay rates within the SMEFT. In particular, we gave exact one-loop results from four-fermion operators, and in addition the leading electroweak corrections in the large-m_t limit. We also calculated the one-loop corrections to muon decay in the same limit, which is necessary to implement the G_F input-parameter scheme.

Our SMEFT calculations were carried out within an extension of the on-shell renormalisation scheme for electroweak corrections, which was described in section 3. In this procedure counterterms related to wavefunction, mass, and electric charge renormalisation are determined from one-loop two-point functions directly in the broken phase of the theory as in the on-shell scheme used in SM calculations. These counterterms receive contributions from both SM and dimension-6 operators, which we calculated explicitly within the approximations described above. The counterterms related to operator renormalisation, on the other hand, are defined in the $\overline{\rm MS}$ scheme and constructed using results from RG equations for Wilson coefficients determined in the unbroken phase of the theory. While the idea behind this procedure is simple, the specifics are rather involved, and we have shown explicitly how it correctly cancels UV-divergent contributions from a total of 21 different operators in the limit of vanishing gauge couplings. As a non-trivial check of our results, we also computed the coefficients of all μ-dependent logarithms in this same limit, and showed explicitly that they have the form dictated by the RG equations.

In section 6 we assessed the impact of the SM and SMEFT contributions to the $h \to b\bar{b}$ and $h \to \tau\bar{\tau}$ decay rates. To do so, we compute the ratios of the decay rates in SMEFT, for Wilson coefficients defined at the scale $\mu = m_t$, with respect to the SM. We find that potentially large non-logarithmic contributions of $\mathcal{O}(m_t^2)$ are numerically unimportant. In particular, m_t^3 contributions in the bare matrix element multiplying poorly constrained four-fermion scalar operators are cancelled exactly by those those appearing in the on-shell mass counterterms. The current analysis suggests, at least in the large-m_t limit, that a simplified leading-logarithmic analysis is sufficient. That is, one can calculate the decay rate matrix elements at LO in the SMEFT, use the results to constrain Wilson coefficients at the scale $\mu \sim m_t$, and then use one-loop RG equations to translate the results into constraints at the scale $\mu \sim \Lambda_{\rm NP}$. We emphasise that it is still important to obtain the full NLO expression for the SMEFT decay rate, and such a computation is currently in progress [29]. In the meantime, we have performed a such simplified analysis focussing on the Wilson coefficients $C_{fH}(\mu_t)$, which are not subject to strong experimental constraints. Finally, making use of the RG equations, compact analytic formulas for the b-quark and τ-lepton decay rates in terms of Wilson coefficients at the generic scale $\Lambda_{\rm NP}$ are provided. For values of $\Lambda_{\rm NP}$ in the several TeV range, these decay rates are mostly sensitive to the pairs of Wilson coefficients $C_{bH}(\Lambda_{\rm NP}) - C^{(1)}_{qtqb}(\Lambda_{\rm NP})$ (for b-quarks) and $C_{\tau H}(\Lambda_{\rm NP}) - C^{(1)}_{l\tau qt}(\Lambda_{\rm NP})$ (for τ-leptons). Within the current experimental precision for these decay rates [45], $\mathcal{O}(1)$ constraints can be placed on these pairs of Wilson coefficients as shown in figure 4. We note that if instead these Wilson coefficients scale according to $\hat{C}^{\rm MFV}_{fH} \sim y_f \hat{C}_{fH}$, $\hat{C}^{\rm MFV,(1)}_{qtqb} \sim y_b y_t \hat{C}^{(1)}_{qtqb}$ and $\hat{C}^{\rm MFV,(1)}_{l\tau qt} \sim y_\tau y_t \hat{C}^{(1)}_{l\tau qt}$, as may be expected in a scenario with MFV, then the simplified analysis applied in section 6 is clearly not adequate. In the potential future scenario where sub-percent precision is achievable for measurements of Higgs decay rates [47–50], it will become increasingly important to include information from multiple processes, and in addition to improve the precision of the calculations which enter a global fit to dimension-6 Wilson coefficients. Extending SMEFT calculations to NLO will improve the accuracy of the theoretical predictions allowing for a more precise comparison of data in terms of non-vanishing Wilson coefficients.

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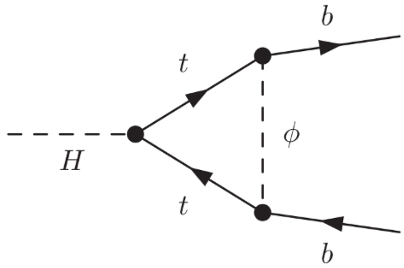
A The large-m_t limit

To clarify the procedure of taking the large-m_t limit, we consider the calculation of the following contribution to the process $h \to b\bar{b}$ which appears in Feynman gauge



We consider both the SM, as well as the contribution due to the Class 7 operator $Q_{Hq}^{(3)}$, $(H^\dagger i\overleftrightarrow{D}_\mu^I H)(\bar{q}_p \tau^I \gamma^\mu q_r)$. For the diagram considered here this operator alters the coupling of the quarks to the Goldstone bosons. The amplitude for this diagram is (setting V_{tb} to unity)

$$\begin{aligned}\mathcal{A} = i\frac{2m_t}{v_T^3}\int \frac{d^d l}{(2\pi)^d}\bar{u}(p_b)\Bigg[&\left[m_b P_L - m_t P_R - v_T^2(\not{l} - \not{p}_b)C_{Hq}^{(3)}P_L\right]\frac{1}{(l-p_b)^2 - m_W^2}\\ &\frac{\not{l} + m_t}{l^2 - m_t^2}\frac{\left(\not{l} - \not{p}_b - \not{p}_{\bar{b}}\right) + m_t}{(l - p_b - p_{\bar{b}})^2 - m_t^2}\left[m_t P_L - m_b P_R + v_T^2(\not{l} - \not{p}_b)C_{Hq}^{(3)}P_L\right]\Bigg]v(p_{\bar{b}})\end{aligned} \tag{A.1}$$

After performing the reduction to scalar integrals this can be written as

$$\mathcal{A} = -i\bar{u}(p_b)v(p_{\bar{b}})\frac{1}{16\pi^2}\frac{m_b}{v_T^3}\left(\mathcal{A}^{\text{div.}} + \mathcal{A}^{\text{fin.}}\right) \tag{A.2}$$

The divergent contribution is

$$\mathcal{A}^{\text{div.}} = \frac{m_t^2}{\epsilon}\left(2 + 5v_T^2 C_{Hq}^{(3)}\right). \tag{A.3}$$

Separating the finite SM and dimension-6 contributions, we find

$$\begin{aligned}\mathcal{A}^{\text{fin.}(4)} = \frac{2m_t^2}{4m_b^2 - m_H^2}\Bigg[&2(m_b^2 - m_t^2)\hat{B}_0(m_b^2, m_t^2, m_W^2)\\ &+ \left(2m_b^2 - m_H^2 + 2m_t^2\right)\hat{B}_0(m_H^2, m_t^2, m_t^2)\\ &- \left(2\left(m_b^2 - m_t^2\right)^2 + m_W^2\left(m_H^2 - 2m_b^2 - 2m_t^2\right)\right)C_0(m_H^2, m_b^2, m_b^2, m_t^2, m_t^2, m_W^2)\Bigg],\end{aligned} \tag{A.4}$$

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$$
\begin{aligned}
\mathcal{A}^{\text{fin.}(6)} = v_T^2 C_{Hq}^{(3)} \Bigg[& \frac{m_t^2}{m_b^2} \Big(\hat{A}_0(m_t^2) - \hat{A}_0(m_W^2) \Big) + \frac{4m_t^2}{4m_b^2 - m_H^2} (2m_b^2 - m_H^2 + 2m_t^2) \hat{B}_0(m_H^2, m_t^2, m_t^2) \\
& + \frac{1}{4m_b^2 - m_H^2} \Bigg(\frac{m_t^2}{m_b^2} \Big(m_b^2 \left(12(m_b^2 - m_t^2) - m_H^2 + 4m_W^2\right) \\
& + m_H^2 \left(m_t^2 - m_W^2\right) \Big) \hat{B}_0(m_b^2, m_t^2, m_W^2) \\
& - 4m_t^2 \left(2\left(m_b^2 - m_t^2\right)^2 + m_W^2 \left(m_H^2 - 2m_b^2 - 2m_t^2\right) \right) C_0(m_H^2, m_b^2, m_b^2, m_t^2, m_t^2, m_W^2) \Bigg) \Bigg],
\end{aligned}
$$

where $\hat{A}_0(s)$ is the finite part of the integral $A_0(s)$ defined in (4.1). We define the large-m_t limit of the finite parts of the integrals appearing in the results as a series in $1/m_t$. The large-m_t limit of the $\hat{A}_0(m_t^2)$ integral is trivial, and the corresponding limit of the $\hat{B}_0$ scalar integrals appearing in these expressions can be obtained by expanding (4.4) as

$$
\lim_{m_t \to \infty} \hat{B}_0(m_1^2, m_t^2, m_2^2) \to 1 + \frac{1}{m_t^2} \left(\frac{m_1^2}{2} + m_2^2 \ln\left[\frac{m_2^2}{m_t^2}\right] \right) - \ln\left[\frac{m_t^2}{\mu^2}\right], \tag{A.5}
$$

$$
\lim_{m_t \to \infty} \hat{B}_0(m_1^2, m_t^2, m_t^2) \to \frac{m_1^2}{6m_t^2} - \ln\left[\frac{m_t^2}{\mu^2}\right]. \tag{A.6}
$$

For the triangle integrals, it is possible to further simplify the integrals appearing in the amplitude by ignoring all fermion masses, except that of the top quark. This is a suitable simplification to make in the context of the current phenomenological study. In the limit of vanishing gauge couplings, where contributions proportional to positive powers of M_W^2 can be neglected, the evaluation of the C_0 functions is further simplified. Note that neither of these limits introduces any extra singularities in the triangle integrals used. Explicitly we have,

$$
\lim_{m_t \to \infty} C_0(m_H^2, m_b^2, m_b^2, m_t^2, m_t^2, m_W^2) \to \lim_{m_t \to \infty} C_0(m_H^2, 0, 0, m_t^2, m_t^2, 0), \tag{A.7}
$$

and similarly for other C_0 functions. The integral appearing in (A.4) becomes

$$
\lim_{m_t \to \infty} C_0(m_H^2, 0, 0, m_t^2, m_t^2, 0) \to -\frac{1}{m_t^2} - \frac{m_H^2}{12m_t^4}, \tag{A.8}
$$

while in the full calculation we also make use of

$$
\lim_{m_t \to \infty} C_0(0, m_H^2, 0, m_t^2, 0, 0) \to -\frac{1}{m_t^2} \left(1 + i\pi + \ln\left[\frac{m_t^2}{m_H^2}\right] \right), \tag{A.9}
$$

both of which are obtained from [30]. Thus, we find the expressions for the finite contributions appearing in (A.4) simplify to

$$
\begin{aligned}
\lim_{m_t \to \infty} \mathcal{A}^{\text{fin.}(4)} &\to -m_t^2 \left(1 + 2\ln\left[\frac{m_t^2}{\mu^2}\right] \right). \\
\lim_{m_t \to \infty} \mathcal{A}^{\text{fin.}(6)} &\to -m_t^2 v_T^2 C_{Hq}^{(3)} \left(\frac{3}{2} + 5\ln\left[\frac{m_t^2}{\mu^2}\right] \right).
\end{aligned} \tag{A.10}
$$

The procedure outlined above is applied to all the finite corrections provided in section 5.

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