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One-way elastic wave reverse-time migration

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SUMMARY

A one-way reverse-time migration method is developed for elastic wave imaging in this paper. It is based on the one-way method for acoustic waves and a newly derived separation formula for transverse and longitudinal waves. Numerical experiments confirm the efficiency of the method.

Key words: one-way elastic equation, reverse-time migration, wavefield decomposition.

1 INTRODUCTION

In order to suppress the multiple reflections of acoustic waves which appear in the reverse-time propagating process, the one-way equation method (Gazdag 1981; Baysal, Kosloff & Sherwood 1983) and the two-way non-reflecting wave equation method (Baysal, Kosloff & Sherwood 1984) have been developed for the reverse-time imaging of acoustic waves. For elastic wave reverse-time migration, there are also multiple reflections during the reverse-time propagation process. However, as yet, none of current methods developed for elastic reverse-time migration consider them (Chang & McMechan 1987; Teng & Dai 1989). Here we extend the one-way method for acoustic waves to elastic wave reverse-time imaging, and as the one-way equation is restricted to downward travelling waves (Gazdag 1981), multiple reflections are avoided during the back propagation of elastic waves. The basic idea is introduced in Section 2, and two numerical experiments are done in Section 3.

2 THEORY

Generally, either longitudinal or transverse waves may be reverse-time migrated by the one-way equation (Gazdag 1981):

$$\frac{\partial \varphi(\boldsymbol{x}, t)}{\partial t} = VFT^{-}[j \operatorname{sign}(k_2)\,|k|\,\tilde{\varphi}(\boldsymbol{k}, t)] \qquad (1)$$

where $\boldsymbol{k} = k_x \boldsymbol{e}_1 + k_y \boldsymbol{e}_2 + k_z \boldsymbol{e}_3$ is the wave vector, k_x, k_y, k_z the three components of $\boldsymbol{k}$ in Cartesian coordinates, $|k| = \sqrt{k_x^2 + k_y^2 + k_z^2}$, φ is the wave function, V is the wave velocity of the longitudinal or transverse wave, '$\sim$' represents the Fourier transform of the spatial coordinates, $j = \sqrt{-1}$, and FT^- is the inverse Fourier transform.

According to the principle of superposition, the one-way elastic equation is given by,

$$\frac{\partial \boldsymbol{u}(\boldsymbol{x}, t)}{\partial t} = V_p FT^{-}[j \operatorname{sign}(k_z)\,|k|\,\boldsymbol{U}_p(\boldsymbol{k}, t)] + V_s FT^{-}[j \operatorname{sign}(k_z)\,|k|\,\boldsymbol{U}_s(\boldsymbol{k}, t)] \qquad (2)$$

where $\boldsymbol{u}$ is the total displacement field, $\boldsymbol{u}_p, \boldsymbol{u}_s$ the displacement of the longitudinal and transverse waves, V_p, V_s the wave velocity for the two waves respectively, $\boldsymbol{U}_p = FT^{+}\boldsymbol{u}_p$, $\boldsymbol{U}_s = FT^{+}\boldsymbol{u}_s$.

Apparently, if $\boldsymbol{u}_p$ and $\boldsymbol{u}_s$ are given, equation (2) may be used to migrate the elastic wave, and if $\boldsymbol{u}_p$ is separated from $\boldsymbol{u}_s$ before migration, only equation (1), the acoustic method, is needed to complete the migration. However, it is not easy to separate the two waves in the domain (x, y, t), i.e. the time section, so a new method is developed in this paper to separate them in the domain (x, y, z), i.e. the depth section, during the process of reverse-time migration.

Based on the principle of superposition

$$\boldsymbol{u} = \boldsymbol{u}_p + \boldsymbol{u}_s \qquad (3)$$

and the polarization of the transverse wave

$$\operatorname{div} \boldsymbol{u}_s = 0 \qquad (4)$$

we have

$$\operatorname{div} \boldsymbol{u}_p = \operatorname{div} \boldsymbol{u} \qquad (5)$$

invoking the formula

$$\operatorname{rot}\operatorname{rot} \boldsymbol{u}_p = \operatorname{grad}\operatorname{div} \boldsymbol{u}_p - \Delta \boldsymbol{u}_p \qquad (6)$$

where $\Delta = \operatorname{div}\operatorname{grad}$, and using the polarization of the longitudinal wave

$$\operatorname{rot} \boldsymbol{u}_p = 0 \qquad (7)$$

we get

$$\Delta \boldsymbol{u}_p = \operatorname{grad}\operatorname{div} \boldsymbol{u}_p. \qquad (8)$$

Using equation (5), equation (8) becomes

$$\Delta \boldsymbol{u}_p = \text{grad div } \boldsymbol{u}. \tag{9}$$

So we get

$$\boldsymbol{u}_p = \Delta^{-1} \text{grad div } \boldsymbol{u} \tag{10}$$

and

$$\boldsymbol{u}_s = \boldsymbol{u} - \Delta^{-1} \text{grad div } \boldsymbol{u}. \tag{11}$$

Using the transform method (Gazdag 1981), equation (10) and equation (11) become

$$\boldsymbol{U}_p = k^{-2}\boldsymbol{kk} \cdot \boldsymbol{U} \tag{12}$$

$$\boldsymbol{U}_s = \boldsymbol{U} - \boldsymbol{U}_p \tag{13}$$

where $\boldsymbol{U} = FT^{+}\boldsymbol{u}$, and equation (2) becomes

$$\frac{\partial \boldsymbol{u}(\boldsymbol{x}, t)}{\partial t} = V_p FT^{-}[j \text{ sign}(k_z)\, |k|\, k^{-2}\boldsymbol{kk} \cdot \boldsymbol{U}(\boldsymbol{k}, t)] + V_s FT^{-}\{j \text{ sign}(k_z)\, |k|\, [\boldsymbol{U}(\boldsymbol{k}, t) - k^{-2}\boldsymbol{kk} \cdot \boldsymbol{U}(\boldsymbol{k}, t)]\} \tag{14}$$

differentiating $\frac{\partial \boldsymbol{u}}{\partial t}$ by centre difference as

$$\boldsymbol{u}(\boldsymbol{x}, t - dt) = \boldsymbol{u}(\boldsymbol{x}, t + dt) - 2dt \frac{\partial \boldsymbol{u}(\boldsymbol{x}, t)}{\partial t} \tag{15}$$

where dt is the time step length.

Combining equation (14) and equation (15), five steps are needed to complete the one-way migration of the elastic wave:

(1) forward transformation to obtain $\boldsymbol{U}(\boldsymbol{k}, t)$.

(2) Wavefield separated by equation (12) and equation (13).

(3) Back transformation to obtain $\frac{\partial \boldsymbol{u}}{\partial t}$.

(4) Reverse-time propagation by equation (15).

(5) Imaging the displacement field $\boldsymbol{u}$ like the two-way method (Chang & McMechan 1987).

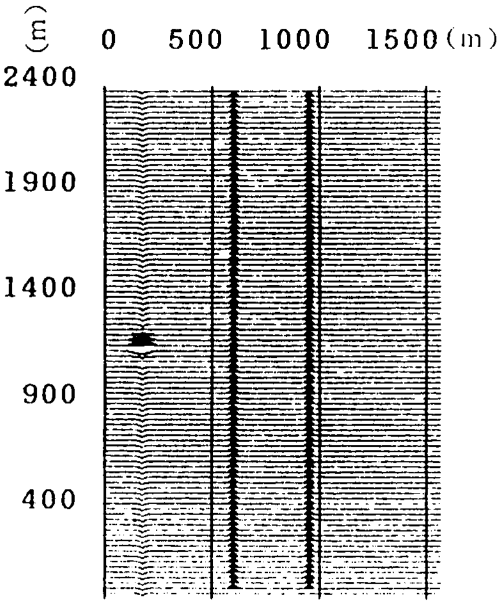
3 EXAMPLE

Forward modelling with the exact elastic equation (Chang *et al.* 1987) is performed using the transform method (Gazdag 1981). The seismic source is a line source of longitudinal waves in a 2-D medium. Fig. 1 shows a model with two horizontal interfaces, the flat event at the same level as the source is the geophone array, the other two flat events are the interfaces, and all the parameters are given under the figure. Fig. 2 shows the time sections, the longitudinal and converted transverse waves may be observed here. For example, in Fig. 2(a), the event located near 200 ms is the longitudinal wave, and the events near 300 ms are the transverse waves. The longitudinal wave is much stronger than the transverse wave in the vertical component (Fig. 2a), and the transverse wave is much stronger in the horizontal component (Fig. 2b). Fig. 3 shows the image sections of the two horizontal-layers model; the event locations are the same as the model (Fig. 1). As the longitudinal wave is symmetric about the source point, the transverse wave is anti-symmetric (inversed polarity) in the time sections (Fig. 2), and the vertical component mainly contains the longitudinal wave. The horizontal component mainly contains the transverse wave. The two-image depth sections appear the same—symmetric (Fig. 3a) and anti-symmetric (Fig. 3b)—as the longitudinal and transverse waves respectively.

Fig. 4 is another model, the flat event at the same level as the source is the geophone array, the other event is the interface (dipping at 45° then becoming horizontal); all the parameters are given under the figure. Fig. 5 shows the image depth sections, which also reconstruct the model correctly.

Here we only present a new method to image the vector elastic data, how these results should be interpreted remains a problem for the future. For example, the polarity is inversed on each side of the source in Fig. 3(b), and for real geological structures, the interface is not always a horizontal plane. The anti-symmetric point is also unclear, consequently the stack after single shot migration may result in a loss of transverse events. Perhaps, we need to define a new 'modified field' to represent the results, such as $u = \sqrt{u^2}$. This removes the reversed polarity, but the signal spectrum is changed too.

The amplitude characteristics of the one-way acoustic or elastic equation have not been very clear until now. Because the one-way equation is strictly derived for homogeneous media, and then assumed to be applicable to heterogeneous media (Gazdag 1981), how the amplitudes match the wave fields crossing an interface is still to be determined.



Model parameters: $NX = 96$, $NZ = 64$, $DX = DZ = 25$ m; source location: $X_s = 960$ m, $Z_s = 160$ m; recorder locations: $Z_r = 160$ m; layers 1 and 3: $V_p = 3.4\ \text{kms}^{-1}$, $V_s = 2.4\ \text{kms}^{-1}$, $\rho = 2.5 \times 10^3\ \text{kgm}^{-3}$; layer 2: $V_p = 3.1\ \text{kms}^{-1}$, $V_s = 2.1\ \text{kms}^{-1}$, $\rho = 2.0 \times 10^3\ \text{kgm}^{-3}$.

Figure 1. Two horizontal interface model.

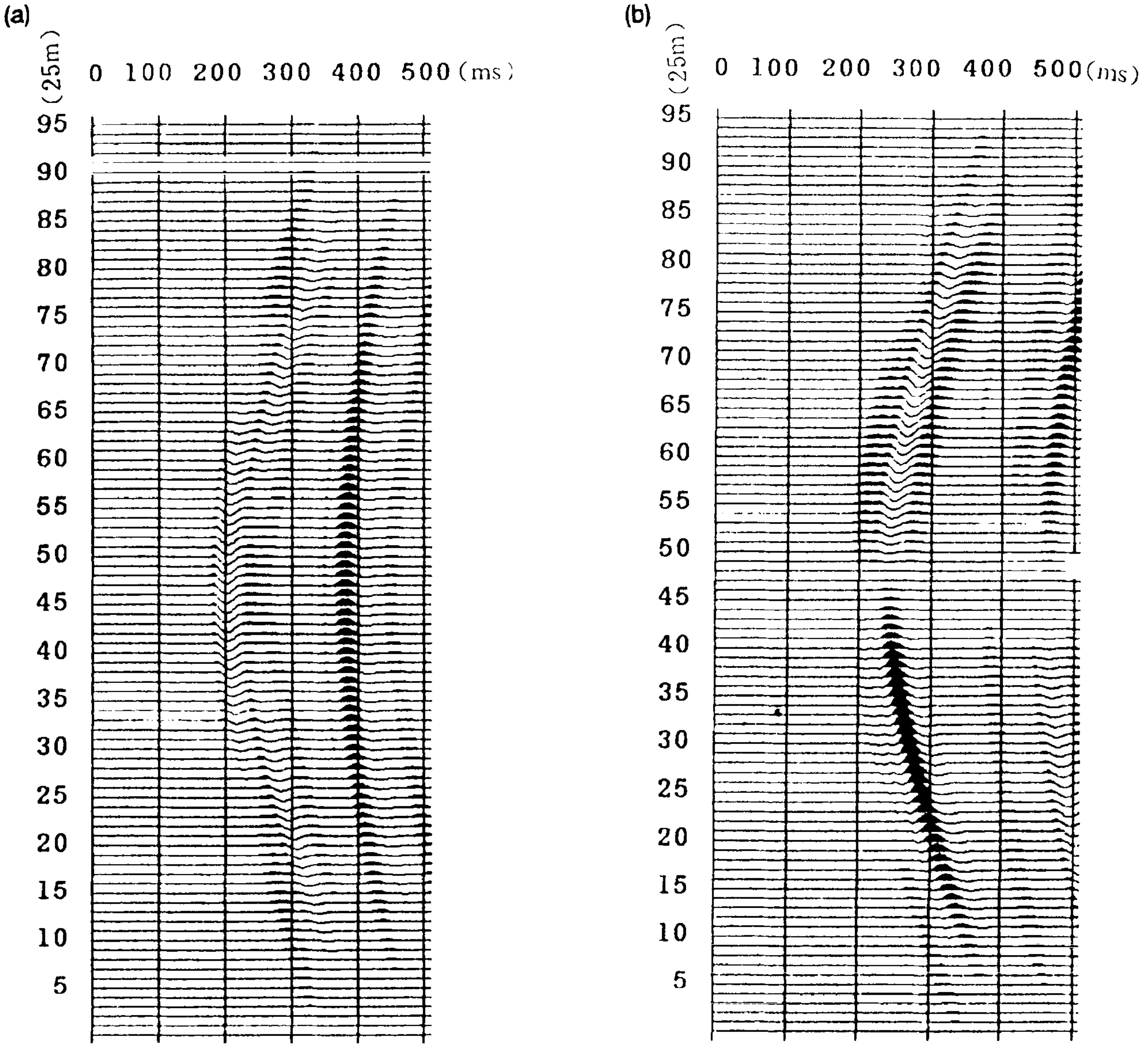


Figure 2. Displacement-field time section. (a) Vertical displacement. (b) Horizontal displacement.

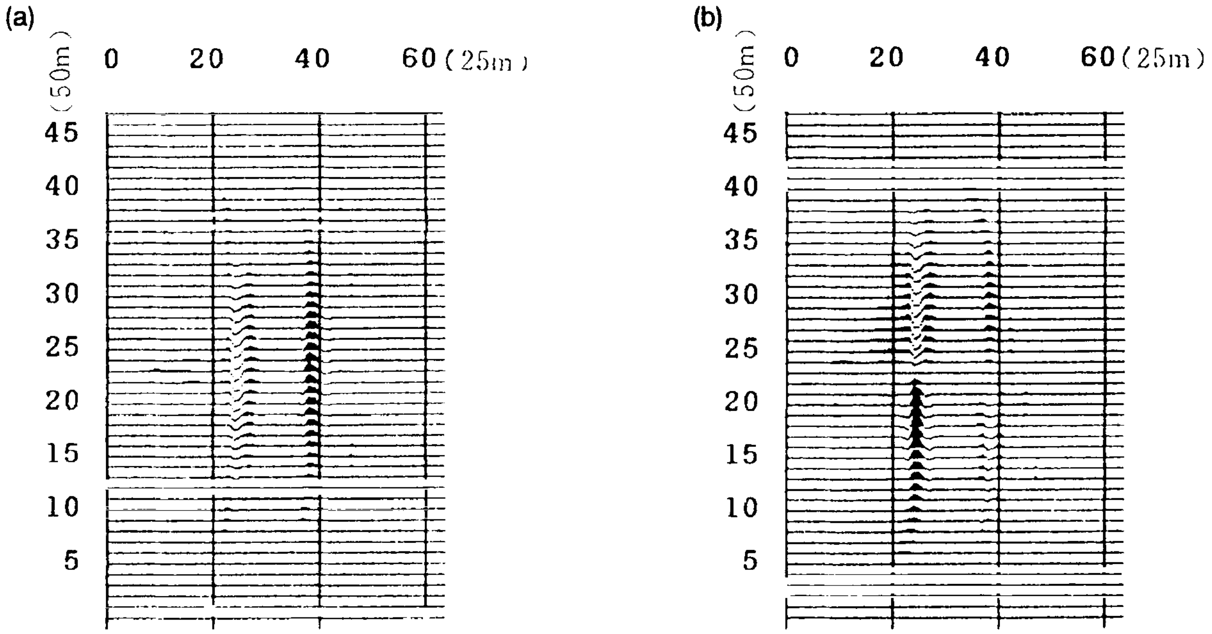
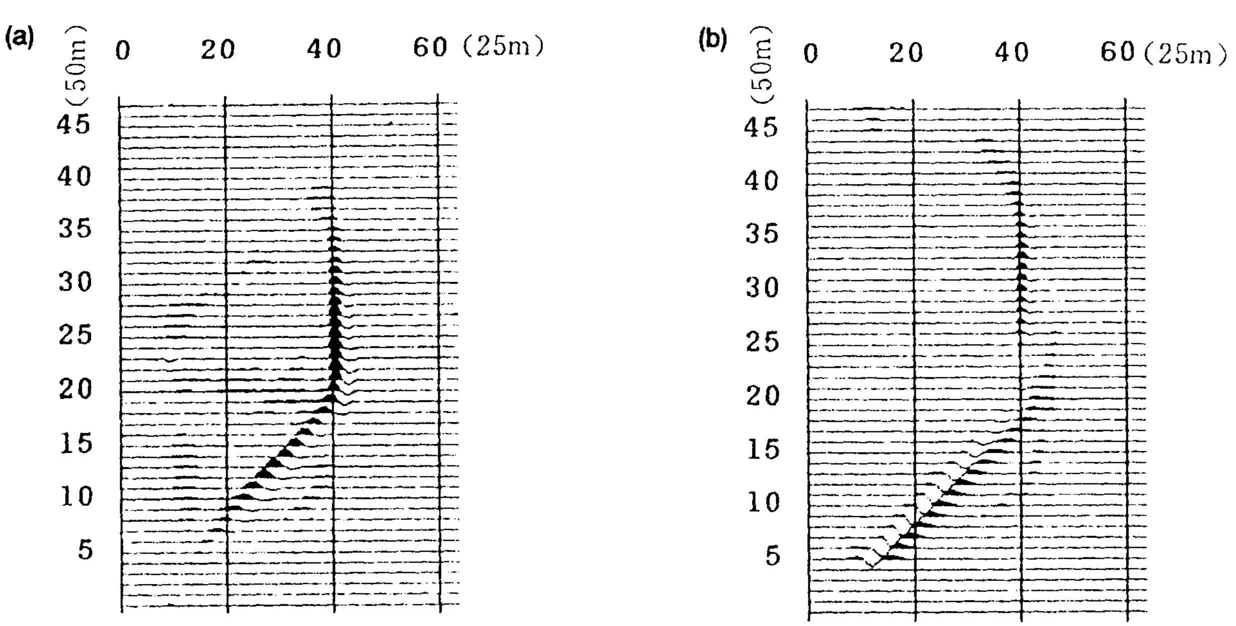


Figure 3. Images for the two horizontal interfaces model. (a) Vertical displacement. (b) Horizontal displacement.



Model parameters: $NX = 96$, $NZ = 64$, $DX = DZ = 25$ m; source location: $X_s = 960$ m, $Z_s = 160$ m; recorders: $Z_r = 160$ m; layer 1: $V_p = 4.0\,\mathrm{kms^{-1}}$, $V_s = 2.7\,\mathrm{kms^{-1}}$, $\rho = 3.0 \times 10^3\,\mathrm{kgm^{-3}}$; layer 2: $V_p = 3.2\,\mathrm{kms^{-1}}$, $V_s = 2.2\,\mathrm{kms^{-1}}$, $\rho = 2.0 \times 10^3\,\mathrm{kms^{-1}}$.

Figure 4. 45° dipping-interface model.

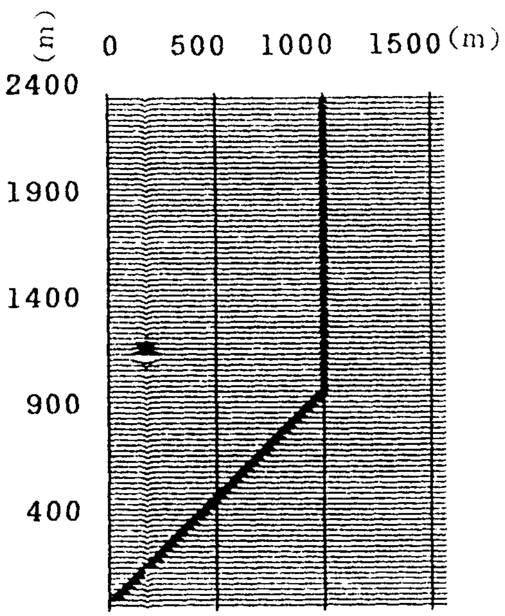


Figure 5. Images for the 45° dipping-interface model. (a) Vertical displacement. (b) Horizontal displacement.

4 CONCLUSION

The one-way elastic wave migration method developed in this article, based on the one-way method for acoustic waves and wavefield decomposition of transverse and longitudinal waves, provides a new choice for processing vector seismic data. Preliminary numerical tests given in this paper show its potential for the reconstruction of complex geological structures.

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