# Online Mixture and Connection of Basic Motions for Humanoid Walking Control by Footprin t Specification 

Koic hi NISHIWAKI*, Tomomichi SUGIHARA, Satoshi KAGAMI, Masa yuki INABA, and Hirochik INOUE<br>Department of Mechano-Informatics, The University of Tokyo<br>7-3-1 Hongo, Bunkyo-ku, T okyo, 113-8656, Japan<br>${ }^{*}$ nishi@jsk.t.u-toky o.ac.jp


#### Abstract

This paper introduces and describes a novel method which enables the online generation of humanoid walking patterns that follow desired footprint locations. Online generation is realized by the dynamically stable mixture and connection of pre-designed motions. Characteristics of ZMP are utilized in order to maintain the overall dynamic stabilit $y$ of "mixed" motions. Experimen ts using an online pattern generation software system and online w alking control joystik interface for the humanoid robot H 6 are introduced to show the validity of the method.


## 1 Introduction

Humanoid robots ha ving a size, shape, and DOF arrangement similar to $h$ uman beings are considered to have the advantage of being capable of tasks such as handling tools and moving around in spaces designed for real humans.

The first humanoid robot in the world, WABOT-1 (WAseda roBOT 1), was developed in 1973 by Fiato [1]. Many other humanoid robots have subsequently been developed, and dynamic w alking was realized with some of them $[2,3,4]$.

Biped humanoids ha ve a more complicated dynamics model than biped walking robots that are dev eloped primarily to verify walking theories. Because of the high cost of calculating the dynamics, walking on biped humanoids has been realized by constructing a dynamically-stable trajectory in advance, and executing it $[2,3,4]$.

The goal of humanoid walking is ho wever to allow a variable control strategy that allows the robot to adapt its motion to a changing environmen $t$. Thus, the ability to generate a desired walking pattern according to online controls by a human or from sensory information such as visual feedback is indispensable.

In this paper, we introduce a real-time w alking pattern generation method that enables a humanoid robot to follow specified footprint locations online. ZMP is adopted as the criteria for dynamic stability. The characteristics of ZMP are utilized for both the construction of typical walking trajectories in advance and the real-time generation of dynamically-stable trajectories. Experimen ts controlling a walking humanoid using a


Figure 1: Mixture of Motions
joystick interface were conducted to demonstrate the validity and capability of this method.

## 2 Online Walking $P$ attern Generation

W e realized walking motions that can follow arbitrary footprints given online without turning. The time for a step is fixed. Setiaw an et al. realized online walking pattern generation for forward and backward walking with a constant step width[5]. They realized the walking by connecting prepared unit walking patterns.

W e realize online generation of desired walking patterns in 3 stages. The first stage is the construction of typical stepping patterns in advance. To realize walking that follows arbitrary footprints, 21 typical stepping patterns are constructed to follow the designed ZMP trajectory. The second stage is the mixture of pre-designed patterns. This mixture is carried out in two phases: 1) mixture along the same axis in the horizontal plane, and 2) mixture of motions perpendicular to each other (Figure 1). For the first mixture, approximate linearity is utilized to main tain dynamic stabilit $y$. The approximate independence of ZMP from perpendicular motion components is utilized for the second mixture. The third stage is the connection of stepping motions. Each stepping motion is constructed and mixed so that the torso speed at the boundaries is determined by the positional relationship between the two feet. Therefore, considering only the positional connection of the patterns, the boundary velocity of two stepping patterns also becomes the same.

W e also prepared turning walking pattern segmen ts that can be connected to the patterns described above


Figure 2: Coordinates for Calculating ZMP
in order to realize turning motions without stopping.

## 3 Dynamics Model of Humanoid for ZMP Criteria

Zero Momen t Point (ZMP) was proposed by Vukobratovic[6] and it is often used as the criterion for dynamic stabilit $y$ for biped walking robots. If the ZMP is inside the con vex hull of the contact points between a robot and the ground, the robot will not fall. Therefore motions in which the trajectory of the ZMP stays inside of the convex luull of contact points can be realized without the robot falling.

Let $\mathbf{r}_{i}, m_{i}$, and $\mathrm{I}_{i}$ respectively represent the position, mass, and inertia tensor of the $i$-th link in the coordinates shown in Figure 2. The equation of motion concerning the rotation around the point $\mathrm{p}=\left(x_{p}, y_{p}, 0\right)^{T}$ is:
$\sum_{i=0}^{n}\left\{m_{i}\left(\mathbf{r}_{i}-\mathbf{p}\right) \times \ddot{\mathbf{r}}_{i}+\mathbf{I}_{i} \cdot \dot{\omega}_{i}\right\}=\sum_{i=0}^{n} m_{i}\left(\mathbf{r}_{i}-\mathbf{p}\right) \times \mathbf{G}+\mathbf{T}$,
where $\mathbf{G}$ and $\mathbf{T}$ represent the acceleration of gravity and the torque caused by the ground respectively.

The ZMP is the poin $t$ where the $x$-componen and $y$-component of $\mathbf{T}$ are 0 . Therefore the equation to derive ZMP from the position and the acceleration of the links can be obtained as:

$$
\left\{\begin{array}{l}
x_{p}=\frac{\sum m_{i} z_{i} \ddot{x}_{i}-\sum\left\{m_{i}\left(\ddot{z}_{i}+g\right) x_{i}+(0,1,0)^{\mathrm{T}} \mathbf{I}_{i} \dot{\omega}_{i}\right\}}{-\sum m_{i}\left(\ddot{z}_{i}+g\right)}  \tag{2}\\
y_{p}=\frac{\sum m_{i}\left(\ddot{z}_{i}+g\right) y_{i}-\sum\left\{m_{i} z_{i} \ddot{y}_{i}+(1,0,0)^{\mathrm{T}} \mathbf{I}_{i} \dot{\omega}_{i}\right\}}{\sum m_{i}\left(\ddot{z}_{i}+g\right)}
\end{array},\right.
$$

where $\mathbf{G}=(0,0,-g)^{\mathbf{T}}, \mathbf{r}_{i}=\left(x_{i}, y_{i}, z_{i}\right)^{\mathrm{T}}$.
According to Eq. (2), the x -component of the ZMP $\left(x_{p}\right)$ is independent of the motion along the y -axis. Similarly the $y$-componen $t$ of the ZMP $y_{p}$ ) is independent of the motion along the $x$-axis. This $c$ haracteristic is utilized for the mixture of motions at a right angle.

The x -component of the ZMP $\left(x_{p}\right)$ can be regarded as a linear com bination of $x$ and $\ddot{x}$. (The same applies to the $y$-component of the ZMP.) This characteristic is utilized for the mixture of the motions along the same axis.

4 Offline Construction of Typical $P$ atterns

### 4.1 Fast Generation Method of Motion Trajectories

W e previously proposed a fast generation method of humanoid motion trajectories that follo w desired ZMP trajectories [7]. For the construction of typical patterns of walking segments, we extended this method to arbitrary velocity boundary conditions.

### 4.2 Construction of $T$ ypical $P$ atterns for Mixture

Typical patterns are constructed satisfying the following conditions:

- Patterns are designed using the trajectories of the position of the upper body and both feet.
- Three groups of patterns are constructed: 1) initial steps (starting from rest), 2) normal steps (continuous walking), and 3) final steps (coming to a stop).
- Motions in the $x-z$ plane and motions in the $y-z$ plane are separately constructed for ead group.
- Vertical and rotational components of each motion is constructed to be identical among the same group of trajectories.
- Motion time for a step is the same in each group.

The mixture is carried out using the pre-designed motions in each group.

The position and velocity of the upper body at the boundary time is designed to follow the equations below in order to guarantee that the position and velocity become the the same at the boundary between two connected mixed patterns.

$$
\begin{gather*}
x_{t}\left(t_{b}\right)=\frac{x_{l}\left(t_{b}\right)+x_{r}\left(t_{b}\right)}{2}+x_{T}  \tag{3}\\
y_{l}\left(t_{b}\right)=\frac{y_{l}\left(t_{b}\right)+y_{r}\left(t_{b}\right)}{2}  \tag{4}\\
\left|\dot{x}_{t}\left(t_{b}\right)\right|=K_{x}\left|x_{l}\left(t_{b}\right)-x_{r}\left(t_{b}\right)\right|  \tag{5}\\
\left|\dot{y}_{t}\left(t_{b}\right)\right|=K_{y}\left|y_{l}\left(t_{b}\right)-y_{r}\left(t_{b}\right)\right|+V_{y} \tag{6}
\end{gather*}
$$

where $x_{t}(t), y_{t}(t), x_{l}(t), y_{l}(t), x_{r}(t), y_{r}(t)$ represents the trajectory of the upper body and both feet, and $t_{b}$ represents the boundary time.

## 5 Real-time Generation Method of Desired W alking Patterns

The real-time mixture of motions that maintains overall dynamic stability is realized in tw o phases. First, the motions along the same axis are mixed, and then the motions at a right angle.


Figure 3: The Same Axis Mixture (Sideward)

### 5.1 Mixture of Motions along the Same Axis

As mentioned in section 3, the $x$-componen $t$ of the ZMP is expressed as a linear combination of $x$ and $\ddot{x}$. Let $\mathbf{A}(t)$ and $\mathbf{B}(t)$ each represent the trajectories of the torso and feet for stepping motions. Then a new stepping motion $\mathbf{C}(t)$ is constructed as:

$$
\begin{equation*}
\mathbf{C}(t)=k_{1} \mathbf{A}(t)+k_{2} \mathbf{B}(t), \tag{7}
\end{equation*}
$$

where $k_{1}+k_{2}=1$.
Taking some conditions iuto consideration, sudı as:

- linearity of the differential,
- no redundancy between each foot and torso,
(There are 6 DOF for each leg.)
- the motion component along the z -axis and the rotational component of the motion are the same in $\mathbf{A}(t)$ and $\mathbf{B}(t)$,
the trajectory of the ZMP for the motion $\mathbf{C}(t)$ is approximately represen ted as:

$$
\begin{equation*}
\mathrm{ZMP}_{\mathrm{C}}=k_{1} \mathrm{ZMP}_{\mathrm{A}}+k_{2} \mathrm{ZMP}_{\mathrm{B}} \tag{8}
\end{equation*}
$$

where $Z M P_{A}$ and $Z M P_{B}$ are the trajectories of the ZMP for motions $\mathbf{A}(t)$ and $\mathbf{B}(t)$ respectively.

Consequently, the new motion can be generated by the linear com bination of trajectories of pre-designed motions. This motion will be dyuamically stable if the mixture is carried out such that the same linear com bination of ZMP becomes the desired trajectory of the mixed motion as described below for some cases.

Since this mixture is a linear com bination of predesigned typical motions, the mixed motion also satisfies the equation (3), (4), (5), (6) that describes the upper body position and velocity at the boundary time. When connecting tw o mixed motions, by just considering the continuity of the feet position, the continuity of the upper body position and velocity is achieved at the same time.

Generation of Sideward Step A stepping motion of $y[\mathrm{~mm}]$ to a side (Figure 3) can be generated from 2 typical motions. $\mathbf{D}_{1}(t)$ is the trajectory of a predesigned stepping motion to a point $Y[\mathrm{~mm}]$ to one side. $\mathbf{D}_{2}(t)$ is the trajectory of a pre-designed stepping motion "in place" (no sideward motion). Let

$$
\begin{equation*}
\mathrm{M}_{1}(t)=\frac{y}{Y} \mathbf{D}_{1}(t)+\frac{Y-y}{Y} \mathbf{D}_{2}(t) . \tag{9}
\end{equation*}
$$



Figure 4: the Same Axis Mixture (Forward)
$\mathbf{M}_{\mathbf{1}}(t)$ is the trajectory of a stepping motion to a point $y[\mathrm{~mm}]$ to one side. The trajectory of the ZMP for $\mathbf{M}_{1}(t)$ is expressed by:

$$
\begin{equation*}
\mathrm{ZMP}_{\mathrm{M}_{1}}=\frac{y}{Y} \mathrm{ZMP}_{\mathrm{D}_{1}}+\frac{Y-y}{Y} \mathrm{ZMP}_{\mathrm{D}_{2}} . \tag{10}
\end{equation*}
$$

Here the linear combination of the desired ZMP trajectories of the pre-designed motions with the same ratio as the mixture of the motions becomes the desired ZMP trajectory of the mixed motion. Therefore the motion generated with this method of mixture becomes dynamically stable.

Generation of Forw ard Step A forward stepping motion of the left foot from $x_{1}[\mathrm{~mm}]$ behind to $x_{2}[\mathrm{~mm}]$ forward relative to the right foot can be generated by the mixture of 4 t ypical motions (Figure 4). $\mathbf{D}_{3}(t), \mathbf{D}_{4}(t), \mathbf{D}_{5}(t)$, and $\mathbf{D}_{6}(t)$ are trajectories of predesigned motions sho wn as Figure 4. ( $\mathbf{D}_{2}$ and $\mathbf{D}_{4}$ are not the same: $\mathbf{D}_{2}$ is not concerned with the motion along x -axis, and $\mathbf{D}_{4}$ is not concerned with the motion along $y$-axis.) Let

$$
\begin{align*}
\mathbf{M}_{2}(t)= & \frac{x_{1}\left(X_{2}-x_{2}\right)}{X_{1} X_{2}} \mathbf{D}_{3}(t) \\
& +\frac{\left(X_{1}-x_{1}\right)\left(X_{2}-x_{2}\right)}{X_{1} X_{2}} \mathbf{D}_{4}(t) \\
& +\frac{\left(X_{1}-x_{1}\right) x_{2}}{X_{1} X_{2}} \mathbf{D}_{5}(t)+\frac{x_{1} x_{2}}{X_{1} X_{2}} \mathbf{D}_{6} \tag{t}
\end{align*}
$$

$\mathbf{M}_{2}(t)$ is a trajectory of the stepping motion from $x_{1}[\mathrm{~mm}]$ behind to $x_{2}[\mathrm{~mm}]$ forward. The motion generated with this method becomes dynamically stable as in the first example.

In order to confirm the validity of this mixture method, stepping motions from $60[\mathrm{~mm}]$ behind to $90[\mathrm{~mm}]$ forward were generated and the ZMP trajectories of mixed motions w ere calculated in a simlation environment. (Here $X_{1}$ is $-180[\mathrm{~mm}]$, and $X_{2}$ is $180[\mathrm{~mm}]$.) The average error of the ZMP trajectory


Figure 5: Mixture of P erpendicular Motions
from the desired one is $0.6[\mathrm{~mm}]$. Considering that the typical motions are constructed so that the average ZMP error is less than $1.0[\mathrm{~mm}]$, the error after the mixture is small enough.

### 5.2 Mixture of Motions at a Right Angle

A motion in the $\mathrm{x}-\mathrm{z}$ plane and a motion in the $\mathrm{y}-\mathrm{z}$ plane can be mixed if the z -axis and rotational components of two motions are the same. As mentioned in section 3, the ZMP along x -axis is independent of the motion component along the $y$-axis and vice versa. Consequently, the x-component of the ZMP trajectory for the mixed motion is appro ximately the same as the ZMP trajectory for the motion in the $\mathrm{x}-\mathrm{z}$ plane used for the mixture. The same applies to the $y$-component of the ZMP trajectory. Figure 5 shows an example of this mixture. Let motion $M_{1}$ be dynamically stable stepping motion in the $\mathrm{x}-\mathrm{z}$ plane from point A to point B , and let $M_{2}$ be a dynamically stable stepping motion in the $y$-z plane from $A$ to $C$. The stepping motion from A to D can be generated with a mixture of $M_{1}$ and $M_{2}$.

To confirm the v alidity of this technique, a mixture of forward motion and sideward motion was carried out, and the ZMP trajectory of the mixed motion was calculated in a sim ulation environment. In this case, the forward motion ranged from $60[\mathrm{~mm}]$ backward to $90[\mathrm{~mm}]$ forward, and the sideward motion ranged from the nentral position to $100[\mathrm{~mm}]$ to one side. The a erage ZMP error of the mixed motion was $0.4[\mathrm{~mm}]$. It is small enough considering that typical motions are constructed so that the ZMP errors are less than $1.0[\mathrm{~mm}]$.

## 6 Experiments

### 6.1 Humanoid Robot H6

The Humanoid H6 [8] is used for the experiment. It is $1361[\mathrm{~mm}]$ in height, and $w$ eighs $55[\mathrm{~kg}]$. The distance betw cen the hip joint and the ankle joint is $500[\mathrm{~mm}]$. It has a total of 35 DOFs. Each leg has 7 DOFs including a 1-DOF toe joint, each arm has 7 DOFs with a 1-DOF gripper, and the head has 4 DOFs. It has an onboard PC and batteries inside the torso.


Figure 6: Left: Geometric Model in Euslisp, Center: DOF arrangement, Right: Photo of H6

|  | start | normal | stop |
| :--- | ---: | ---: | ---: |
| fwd \& bwd | 3 | 7 | 3 |
| sideward | 2 | 4 | 2 |

Table 1: Number of Typical Motions

### 6.2 Construction of T ypical Motions

Considering the symmetry of the robot, only the stepping motions for the left foot w ere prepared. 21 patterns in total (Table 1) were prepared in order to realize arbitrary footstep locations (except for turning while w alking).

In order to realize turning walking continnously with a normal walk, we also prepared turning walk segmen ts. It is difficult to realize a turning step of an arbitrary angle rotation by the mixture method. Thus, turning walks for discrete angles are realized. Four types of turning walks were prepared, 10 degrees per step turn in forward walking, 20 degrees turm in forward walking, and 10 and 20 degrees turn in backward walking.

Three groups of patterns were prepared for each type of turning walk: 1) start of turning motion, 2) regular turning (continuous motion), and 3) end of turning.

For start of turuing, eight patterus were prepared, four for the inner foot step, and four for the outer foot. Four patterns for the imner foot step are shown in Figure 7. Start of turning step from the position in hatched area in Figure 7 can be generated by the mixture of these four patterns. Then the generated pattern can be connected to the corresponding normal patterns. For regular turning two patterns were prepared: inner foot step, and outer foot step. Finally, eight patterns were prepared for the end of turning in the same was as the start of turning.

### 6.3 Control Softw are

W e implemented a softw are system for realizing online control of the walking motion and additional modules for interfacing a joystick controller as shown in Figure 8. RT-Lin ux[9] V2.2 was used as the real-time


Figure 8: Con trol Flow for Online Generation and Joystic k Experiment


Figure 7: Typical Patterns for Start of Turning
operating system. The function of each module is described below.

Motor Serv ois a software PD servo module. It runs at 1 [ms.] cycle.

Online Balancer is a sensor feedback module that stabilizes the walking. The ZMP position is measured using force sensors distributed at the feet. In order to reduce the error from the desired ZMP, the horizontal position of the torso is adjusted. Then the desired leg angles are calculated using inverse kinematics. It runs at 1 [ms.] cycle.
Sequence Manager stores motion patterns and outputs poses by interpolating them. It can execute patterns continuously when successive patterns are designated. It also has the ability to notify other modules when the execution time reac hes specified values. For online generation of walking, this function is used as the signal to request the next step pattern. The notification time is set to 100 [ms.] before execution of the current walking pattern segmen $t$ ends.
W alking $\mathbf{P}$ attern Generators the implementation of the method described in this paper. It generates one step of walking that follows a commanded footprint location. In order to a void making the robot fall down when the control stops, it outputs successive motion patterns that forces
the robot to stop at the same time. It took about $3.8[\mu \mathrm{~s}]$ to generate $1[\mathrm{~s}]$ stepping pattern.
Step Planner is the module that decides the desired footprint from the input vector. The input is the combination of a two dimensional vector that represents the direction and magnitude of motion in the horizontal plane, and a value that represents the turning rotation.
Main Planner is the controller for the joystick interface experiment. It queries the joystick server which runs on another pc for the current status of the joystick, and calculates the direction and magnitude of the horizontal motion and the rotation angle.

### 6.4 Joystick Con trol Interface Experimerts

Experiments controlling the robot walking by a joystick were performed to show the capabilit $y$ of the online pattern generation method. The direction, step width and rotation angle of walking were commanded by using a 3 DOF joystick. Then the soft ware system described in the previous section plans the desired footprints and executes them. The maximum distance between the two feet was limited to $160[\mathrm{~mm}]$ for the forward direction, and $316[\mathrm{~mm}$ ] ( $140[\mathrm{~mm}$ ] away from the neutral position) for the sideward direction. The time for a normal step was $1.0[\mathrm{sec}$.$] , and for the first$ and last step was $1.2[\mathrm{sec}$.$] . Several snapshots taken$ during the experimen ts are shown in Figure 9.

## 7 Conclusion

A real-time walking pattern generation method was introduced. In order to generate the desired walking pattern without complicated dynamics calculations or lengthy iterative calculations, the linearity and independence characteristics of the ZMP from the robot motion is utilized. Consequently, dynamically stable walking patterns that follow desired footprint locations can be generated online using just a mixture of a predesigned motions. ( 21 pre-designed motions w ere prepared for the experiment described above to realize the stepping motion betw een arbitrary positions without rotation.)


Figure 9: Experiment of Walking Controlled by Joystick. (Av oiding Obstacles)

An online walking con trol system $w$ as constructed, and an online walking control experimen ts using a jgstick demonstrated the capability of the method.

Adapting the $w$ alk in reaction to changes in the environment by utilizing sensory information, especially vision, is the next research topic which is currently under investigation.

This work is supported by the Japan Society for the Promotion of Science Gran $t$ for Research For The Future JSPS-RFTF96P00801.

## REFERENCES

[1] Ichiro Kato. Dev elopment of wabot-1. Biome chanism 2, The University of Tokiyo $P_{r e s s}$, pages 173-214, 1973.
[2] Kazuo Hirai. Current and future perspective of Honda humanoid robot. In In Proceeding of 1997 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS'97), pages 500-508, 1997.
[3] Jin'ichi Yamaguchi, Sadatoshi Inone, Daisuke Nishino, and Atsuo T akanishi. Development of a bipedal humanoid robot having an tagonistic driven joints and three dof trunk. In Proc. of the 1998 IEEE/RSJ Int. Conf. on Intelligent Robots and Systems, pages 96-101, 1998.
[4] Ken'ichirou Nagasaka, Masa yuki Inaba, and Hirochika Inoue. W alking pattern generation for a humanoid
robot based on optimal gradient method. In Proc. of 1999 IEEE Int. Conf. on Systems, Man, and Cyb ernetics, 1999.
[5] Samuel Agus SETIA WNN, Sang Ho HYON, Jin'ichi Yamaguchi, and A. Takanishi. Physical interaction between human and a bipedal humanoid robot-realization of human-follow walking-. In Proceedings of the IEEE International Conference on Robotics 8 A utomation, pages 361-367, 1999.
[6] D.Juricic M.Vnkobratovic, A.A.F rank. On the stabilit y of biped locomotion. In IEEE T. on Biome d. Engg. BME-17, number 1, pages $25-36,1970$.
[7] Satoshi Kagami, Koichi Nishiwaki, Tomonobu Kitagawa, Tomomichi Sugihara, Masayuki Inaba, and Hirochika Inoue. A fast generation method of a dynamically stable humanoid robot trajectory with enhanced zmp constraint. In Proceedings of IEEE International Conference on Humanoid Robotics, 2000.
[8] K. Nishiwaki, T. Sugihara, S. Kagami, F. Kanehiro, M. Inaba, and H. Inone. Design and development of rescarch platform for perception-action integration in humanoid robot : H6. In Proceedings of the 2000 IEEE/RSJ International Conference on Intelligent Robotics and Systems (IROS 2000), pages 15591564, November 2000.
[9] Michael Barabanov. A linux-based real-time operating system. Master's thesis, New Mexico Institute of Mining and Technology, 1997.

