Online Model Predictive Torque Control for Permanent Magnet Synchronous Motors

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Outline

- **ODYS** is a private SME founded in 2011, originally as a university spin-off
- **Core business**: consultancy and software for the development of advanced control systems, with special focus on **model predictive control (MPC)**
- **Main expertise:** advanced multivariable control design, efficient real-time optimization algorithms, and tools for their deployment in embedded systems

Application domains

- automotive
- aerospace
- energy
- process control

Background

The standard control method for PMSMs is the **Field Oriented Control (FOC)** (cascade structure, with three **linear regulators**)

$\rightarrow$The work deals with the application of **Model Predictive Control (MPC)**, to PMSM **torque control**.

MPC in electrical drives control is, mainly, motivated by two facts:

- mathematical models of these systems are relatively well known
- several constraints are needed for safety reasons

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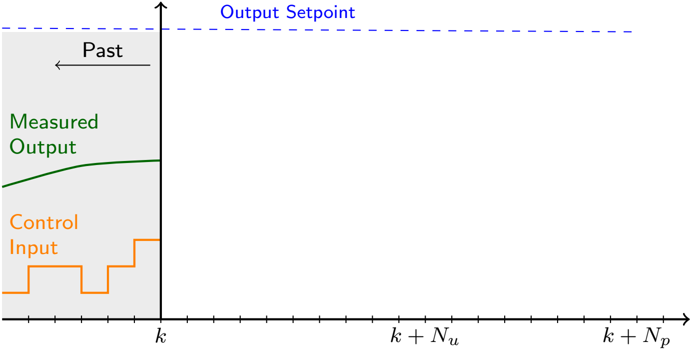
The main idea of MPC

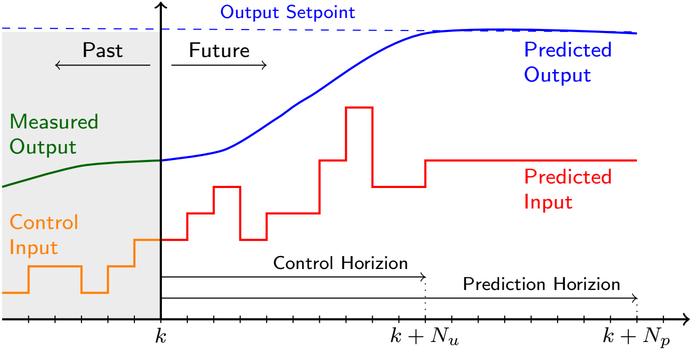
Obtain the control actions by solving, at each sampling time, a **finite-horizon optimal control problem**

Model Predictive Control



- Linear prediction model
- Control and Prediction Horizon
- Hard input and soft output constraints

Model Predictive Control



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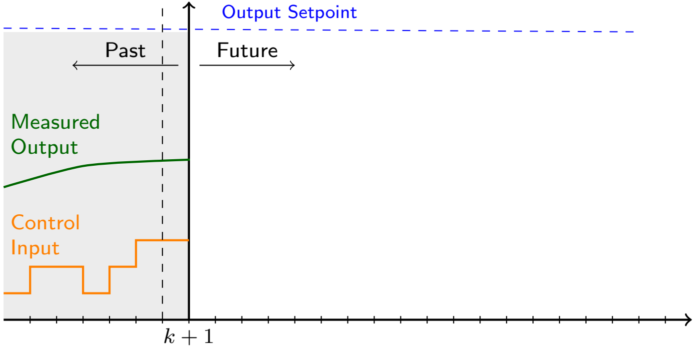

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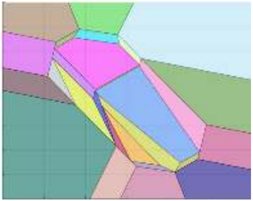
Existing solutions

Explicit MPC[a]: the optimization problem is pre-solved **offline** via multiparametric Quadratic Programming (mpQP)



- ✓ it is an easy-to-implement PieceWise Affine (PWA) function of the parameters
- ✓ fast binary search tree algorithm can be used
- ✗ it is applicable only to small problems
- ✗ high memory requirements

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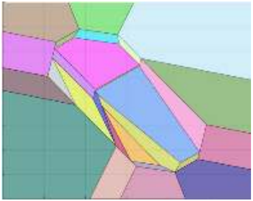

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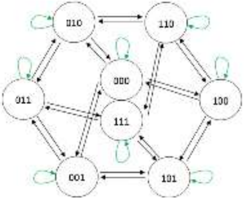
Finite Control Set[b]: exploits the **discrete nature** of the system by manipulating the inverter switch positions directly



- ✓ MPC is solved on line, by enumeration
- ✓ very fast with short prediction horizon
- ✗ computational load scales exponentially with the horizon
- ✗ high sampling frequency, not decoupled from control frequency
- ✗ variable frequency

[a] Bolognani, S.; Bolognani, S.; Peretti, L.; Zigliotto, M., *"Design and Implementation of Model Predictive Control for Electrical Motor Drives"*

[b] Rodriguez, J. and Kazmierkowski, M.P. and Espinoza, J.R. and Zanchetta, P. and Abu-Rub, H. and Young, H.A and Rojas, *"C.A, State of the Art of Finite Control Set Model Predictive Control in Power Electronics"*

MPC with online optimization

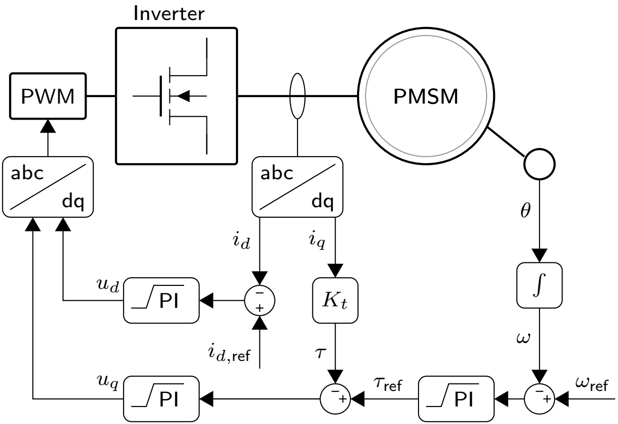
Embedded QP solver:
Model Predictive Control → Quadratic Programming with affine constraints

- ✓ MPC is solved online
- ✓ Low memory Requirements
- ✓ Switching frequency decoupled from sampling frequency
- ✓ Fixed switching frequency
- ✗ High computational burden

Main Objective

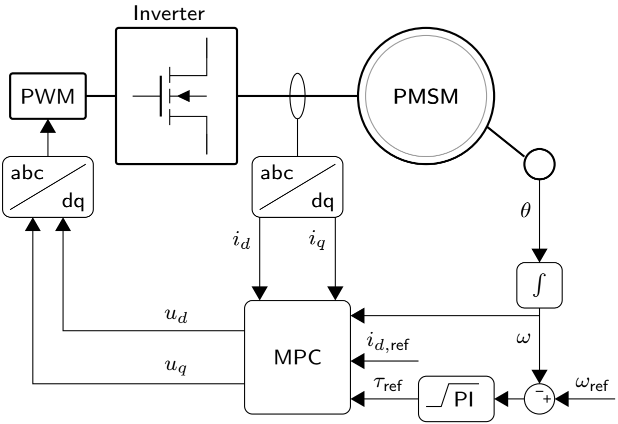
- Verify the feasibility of **embedded, online** MPC
- Application in **low-power DSP**, commonly used for motion control

Standard PI Field Oriented Control



The aim is replacing the standard PI-based torque controller, with ...

Proposed MPC Torque Control



...an MPC torque control, **solved with online optimization**.

Outline

Prediction Model

Consider an **isotropic PMSM** , with one pole pair. The nonlinear mathematical model is:

$$
\begin{aligned}
\dot{i}_d(t) &= -\frac{R}{L} i_d(t) + \omega(t) i_q(t) + \frac{1}{L} u_d(t) \\
\dot{i}_q(t) &= -\frac{R}{L} i_q(t) - \omega(t) i_d(t) + \frac{1}{L} u_q(t) - \frac{\lambda}{L}\omega(t) \\
\dot{\omega}(t) &= \frac{B}{J}\omega(t) + \frac{K_t}{J} i_q(t) - \frac{1}{J}\tau_l(t),
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\end{aligned}$$

The linearized model of the electrical subsystem is obtained imposing nominal speed $\omega(t) = \omega_0$.

$$\begin{aligned}
x(k+1) &= Ax(k) + Bu(k) + Gv(k) \\
y(k) &= Cx(k)
\end{aligned}$$

$$A = e^{A_c T_s}, \quad B = \int_0^{T_s} e^{A_c \tau} d\tau B_c, \quad G = \int_0^{T_s} e^{A_c \tau} d\tau G_c, \quad C = C_c$$

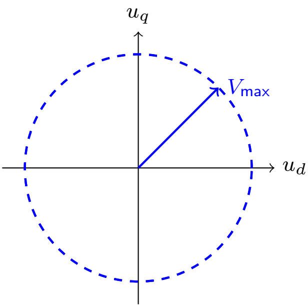
$$A_c = \begin{bmatrix} -\frac{R}{L} & \omega_0 \\ -\omega_0 & -\frac{R}{L} \end{bmatrix} \quad B_c = \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{1}{L} \end{bmatrix} \quad G_c = \begin{bmatrix} 0 \\ -\frac{\lambda}{L} \end{bmatrix} \quad C_c = \begin{bmatrix} 1 & 0 \\ 0 & K_t \end{bmatrix}$$

System Constraints

The most critical constraints concern the electrical components

$$u \in \tilde{\mathbb{U}} = \{u \in \mathbb{R}^2 : \|u\|_2 \leq V_{\text{max}}\} \tag{1}$$

$$x \in \tilde{\mathbb{X}} = \{x \in \mathbb{R}^2 : \|x\|_2 \leq I_{\text{max}}\} \tag{2}$$



(a) Voltage constraints (b) Current constraints

$V_{\text{max}} \rightarrow$ Depending from the *maximum DC-bus* and from the *modulation scheme* ($V_{\text{max}} = V_{\text{DC}}/\sqrt{3}$ for PWM and SVM)

$I_{\text{max}} \rightarrow$ Chosen to prevent overheating. Peaks larger than this limit are allowed for short time intervals .

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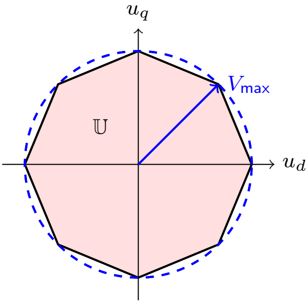
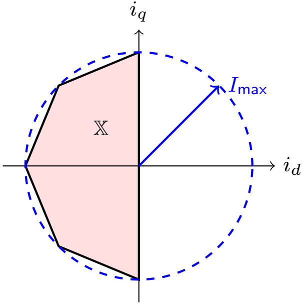


(c) Voltage constraints



(d) Current constraints

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How about embedded online optimization?

MPC problem formulation

$$
\begin{aligned}
\min_{\Delta u} \quad & \sum_{i=0}^{N-1} \|Q(y_{k+i|k} - r(k))\|_2^2 + \sum_{j=0}^{N_u-1} \|R\Delta u_{k+j|k}\|_2^2 + \\
& + \|P(y_{k+N|k} - r(k))\|_2^2 \\
\text{s.t.} \quad & x_{k|k} = x(k) \\
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Quadratic problem (QP)

$$\begin{aligned}\min_z \quad & q(z) \triangleq \frac{1}{2} z' H z + p(t)' F' z \\ \text{s.t.} \quad & Gz \leq W + Sp(t)\end{aligned}$$

- convex quadratic objective function
- polyhedral constraints
- QPs are easier to solve than standard formulation

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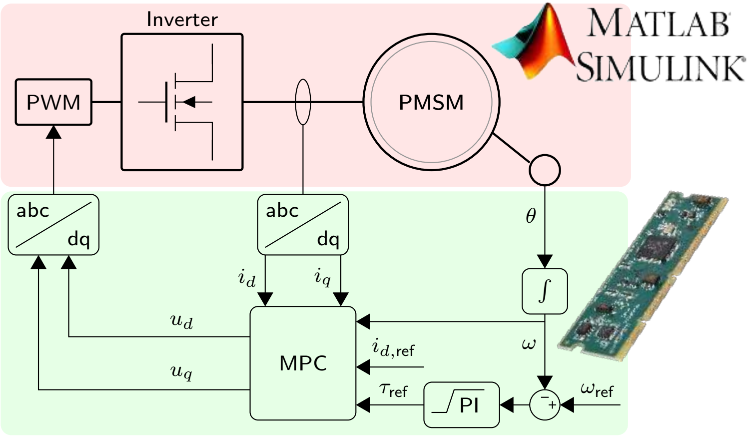
We developed an embedded QP solver

Outline

Processor In the Loop Experiments



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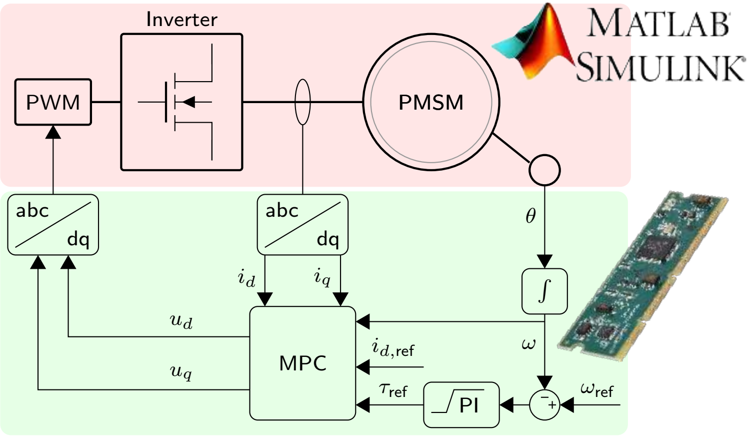

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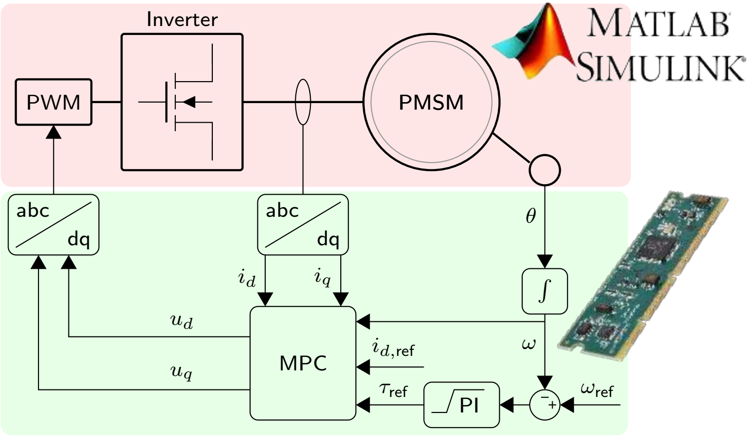

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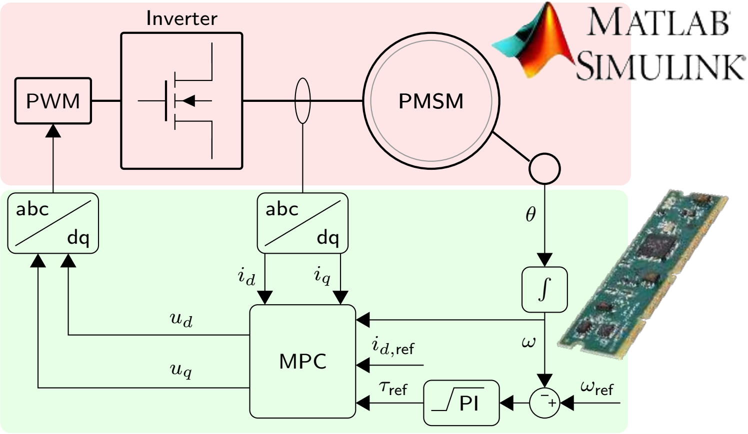

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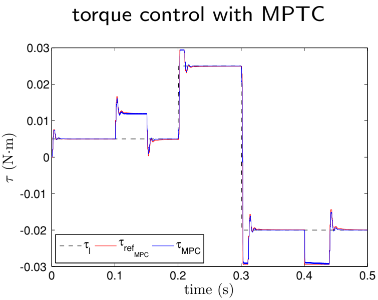
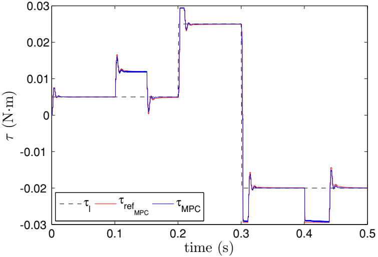
This board is commonly used for motion control!

Control Performance

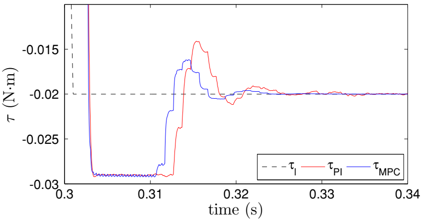


Control Performance

torque control with MPTC



MPTC vs PI-FOC torque transient



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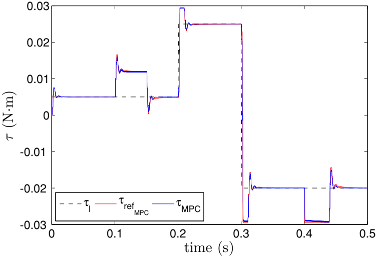

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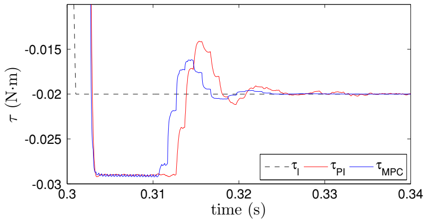
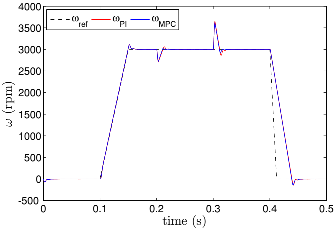


MPTC vs PI-FOC speed tracking



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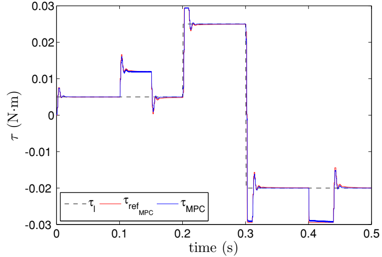

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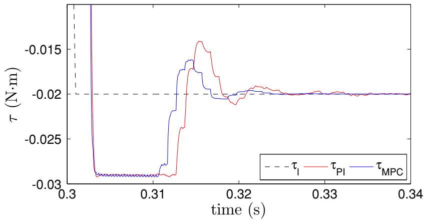

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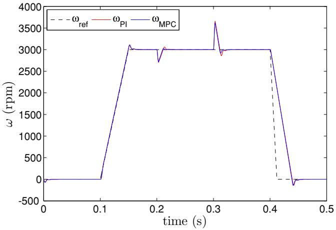

MPC provides 2.3% and 4.2% improvement in speed and torque tracking, respectively (ISE index)

Online Optimization

- The sampling interval is 0.3ms
- Dimension of the QP problem: 5 decision variables and 41 constraints
- DSP with 150MHz processor
- One hardware multiplier 32×32 bit
- Single precision

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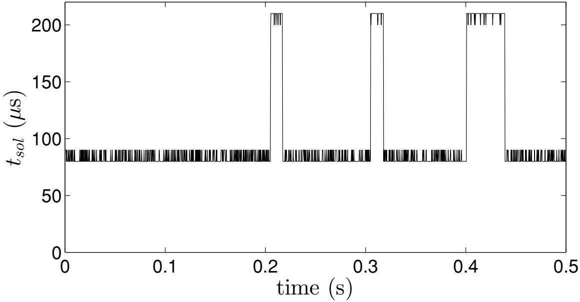

Memory occupancy: **2.5kB** out of the **34kB** of memory provided by the DSP

Outline

Conclusions

Summary

The feasibility of MPC with online optimization in terms of

- required **computational power**
- **memory** occupancy

has been demonstrated through processor-in-the-loop tests on a **low power DSP**

Future Activities

- Extension of the proposed strategy to consider the speed loop
- Flux weakening handling
- Evaluation of the effects of winding resistance increase due to temperature rise
- Application to automotive problems, such as Electronic Throttle Control and Hybrid Electric Motor Control

Thank you for your attention!!

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