# Onsager core of Abor-Miri and Mising languages 

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#### Abstract

We study an Abor-Miri to English dictionary and a Mising to English dictionary. We count words one by one. We draw the natural logarithm of the number of words, normalised, starting with a letter vs the natural logarithm of the rank of the letter, normalised( unnormalised). We observe that the graphs are closer to the curves of reduced magnetisation vs reduced temperature for various approximations of the Ising model. We find that behind the words of Abor-Miri language, the magnetisation curve is $\operatorname{BP}(4, \beta H=0.08)$, in the Bethe-Peierls approximation of Ising model with four nearest neighbours, in presence of liitle external magnetic field, $\beta H=0.08$; behind the words of Mising language the magnetisation curve is $\mathrm{BW}(\mathrm{c}=0)$, in the Bragg-Williams approximation of Ising model in absence of external magnetic field. Neverthless, once the Mising alphabet is reduced to that of Abor-Miri, the magnetisation curve behind the Mising language is $\operatorname{BP}(4, \beta H=0.08)$. Both seem to underlie the same type of magnetisation curve in the Spin-Glass phase in the presence of external magnetic field. Moreover, words of both Abor-Miri language and the Mising language in the reduced alphabet scheme, go over to Onsager solution, on few successive normalisations. $\beta$ is $\frac{1}{k_{B} T}$ where, T is temperature, H is external magnetic field and $k_{B}$ is Boltzmann constant.


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## I. INTRODUCTION

Saturation phenomenon is quite common in nature. We ourselves show symptoms of saturation. We restrict ourselves to friend circle, we stay within family, we move often within a linguistics group. We get bored so easily. In the physical world, in a solid say common salt sodium chloride, one sodium atom attaches to a chlorine atom on an average, in a gas of hydrogen one hydrogen atom attaches to one other hydrogen atom on an average, in a big nucleus one nucleon interacts with few other nucleons of the order of one. This saturation phenomenon is mimicked amply in Ising model, [1] with nearest neighbour interaction(n.n). Ising model of spins is a model of array of spins. In case of nearest neighbour interaction(n.n), one spin interacts with only the nearest neighbour spins, nothing else. The two dimensional Ising model, in absence of external magnetic field, is prototype of an Ising model. In case of square lattice of planar spins, one spin talks to four other nearest neighbour spins i.e. on an average to another one spin. Below a certain ambient temperature, denoted as $T_{c}$, the two dimensional array of spins reduces to a planar magnet with magnetic moment per site varying as a function of $\frac{T}{T_{c}}$. This function was inferred, [2]. by Lars Onsager way back in 1948, [3] and thoroughly deduced thereafter by C.N.Yang[4]. This function we refer to as Onsager solution. Moreover, systems, [5], showing behaviour like Onsager solution is rare to come across. What about languages as systems? Do some natural languages behave like Onsager solution? Is Ising model the inner truth behind the languages we speak in? In this paper, we find that in the Abor-Miri language [6], if we keep the Abor-Miri alphabet intact, but decide not to use the words starting with the first six letters(in that sense core), in terms of number of words, the graph of $\frac{\operatorname{lnf}}{\ln f_{n n n n n n-m a x}}$ vs $\frac{\operatorname{lnk}}{\ln k_{l i m}}$ is the Onsager solution. We find also that in the Mising language[7], if we ignore diacritical variation thereby reducing the Mising alphabet [7] to the Abor-Miri alphabet, [6], and decide not to use the words beginning with the first four letters in terms of ranking, [8], the graph of $\frac{\ln f}{\ln f_{n n n n-m a x}}$ vs $\frac{\operatorname{lnk}}{\ln k_{l i m}}$ is the Onsager solution.

Moreover, counting number of pages for a letter and multiplying by average number of words, number of words was deduced for each letter for Abor-Miri language in, [9]. But for nouns, verbs, adverbs, adjectives for Abor-Miri language in, [9], counting was done one by one from beginning to the end. Here we count words one by one from beginning to the end and redo the analysis. Abor-Miri broke into two languages. One Mising and another Adi.

We undertake study of Mising laguage in the later part of this paper.
The present author studied natural languages, [9] and have found, in the preliminary study, existence of a curve magnetisation related to two approximations of Ising model, under each language. We termed this phenomenon as graphical law. Then we looked into, [10], dictionaries of five discipline of knowledge and found existence of a curve magnetisation under each discipline. This was followed by finding of graphical law behind bengali, [11], Basque, [12], Romanian, [13] and five more disciplines of knowledge, [14].

We describe how a graphical law is hidden within the Abor-Miri and Mising languages, in this article. The planning of the paper is as follows. We introducte the standard curves of magnetisation of Ising model in the section II. In the section III, we describe reanalysis of Abor-Miri words, [6]. In the section IV, we carry out analysis of Mising words, [7]. We resort to reducing Mising alphabet, [7] to Abor-Miri alphabet, [6] and do the analysis of the resulting sets of words in the section V . This fecilitates comparison of sets of words collected under the title Abor-Miri and the title Mising after an interval of one hundred year. Section VI is discussion. The appendix section VII deals with the details of the magnetisation curves. The section describes how to obtain the formulas related to the comparator curves from the subject of statistical mechanics and how to draw the curves. Next section VIII is acknowledgement section. The last section is bibliography.

## II. CURVES OF MAGNETISATION

The Ising Hamiltonian, [1], [15], for a lattice of spins is $-\epsilon \Sigma_{n . n} \sigma_{i} \sigma_{j}-H \Sigma_{i} \sigma_{i}$, where n.n refers to nearest neighbour pairs, $\sigma_{i}$ is i-th spin, H is external magnetic field and $\epsilon$ is coupling between two nearest neighbour spins. $\sigma_{i}$ is binary i.e. can take values $\pm 1$. At a temperature T , below a certain temperature called phase transition temperature, $T_{c}$, for the two dimensional Ising model in absence of external magnetic field i.e. for H equal to zero, the exact, unapproximated, Onsager solution gives reduced magnetisation as a function of reduced temperature as, [4], 15],

$$
\frac{M}{M_{\max }}=\left[1-\left(\sinh \frac{.8813736}{\frac{T}{T_{c}}}\right)^{-4}\right]^{1 / 8} .
$$

Graphically, the Onsager solution appears as in fig.1. In the Bragg-Williams and Bethe-


FIG. 1. Reduced magnetisation vs reduced temperature curves for exact solution of two dimensional Ising model, due to Onsager, in absence of external magnetic field

Peierls approximations for an Ising model in any dimension, in absence and presence of external magnetic fields, reduced magnetisation as a function of reduced temperature, below the phase transition temperature, $T_{c}$, vary as in the figures 2.4. Related details are in the appendix. The graphs in the figures.14, are used in the sections to follow as refernce curves.


FIG. 2. Reduced magnetisation vs reduced temperature curve, $\mathrm{BW}(\mathrm{c}=0)$, in the Bragg-Williams approximation in absence of external magnetic field, curve $\mathrm{BW}(\mathrm{c}=0.01)$ in presence of magnetic field, $c=\frac{H}{\gamma \epsilon}=0.01$, and curve, $\operatorname{BP}(4, \beta \mathrm{H}=0)$, in the Bethe-Peierls approximation in absence of magnetic field, for four nearest neighbours


FIG. 3. Reduced magnetisation vs reduced temperature curves, $\mathrm{BP}(4, \beta \mathrm{H})$, for Bethe-Peierls approximation in presence of little external magnetic fields, for four nearest neighbours, with $\beta H=2 m$.


FIG. 4. Reduced magnetisation vs reduced temperature curves, $\mathrm{BP}(4, \beta \mathrm{H}=0.1)$ and $\mathrm{BP}(4, \beta \mathrm{H}=0.08)$ in the Bethe-Peierls approximation in presence of little external magnetic field, for four nearest neighbours.

| A | B | D | E | G | I | J | K | L | M | N | O | P | R | S | T | U | Y |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 766 | 352 | 356 | 254 | 269 | 223 | 109 | 557 | 426 | 491 | 320 | 136 | 437 | 228 | 461 | 557 | 66 | 274 |

TABLE I. Abor-Miri words: the first row represents letters of the Abor-Miri alphabet in the serial order

## III. REANALYSIS OF WORDS OF ABOR-MIRI LANGUAGE

"This work was compiled during a residence of two and a half years at Sadiya( from June 1900 to February 1903) where the Author and his colleague, Mr. Savidge, were studying the Abor and Miri dialects, with the hope of eventually settling among the Bor-Abors as Christian Missionaries.

This work was based ontwo dialect, (1) that which is spoken by the Bor-Abors or, Padam, who inhabit principally the southern slopes of the Himalayas lying between the gorges of the Dihong and Dibong Rivers and , (2) that which is used by the majority of Miris who live on the plains in the neighbourhood of Sadiya and also lower down the Brahmaputra Valley. These two dialects have very much in common and are also very closely allied to all the other Abor and Miri dialects. Collectively they form what may be termed the Abor-Miri language. It seems probable that the dialect spoken by the Bor-Abors is the stock from which all others have sprung." -preface to "A Dictionary of the Abor-Miri Language" --1st February, 1906.

It was compiled by Reverand Herbert Lorrain during his posting in Sadiya, before he got a sudden transfer to Mizoram. During his stay in Mizoram he compiled dictionary of Mizo to English. As one goes out of Lengpui airport of Mizoram and starts moving towards Izwal, the capital town, welcomes one a big portrait of Reverand Lorrain, no one else.

We take a tour through the Abor-Miri to English dictionary,[6]. Counting number of pages for a letter and multiplying by average number of words, number of words was deduced for each letter for Abor-Miri language in, [9]. Here we count words one by one from beginning to the end and redo the analysis. The result is the following tables, I and II
Highest number of words, seven hundred sixty six, start with the letter A followed by words numbering five hundred fifty seven beginning with K and T , four hundred ninety one with the letter M. To visualise we plot the number of words again respective letters in the dictionary sequence, [6] in the figure fig $[5$

For the purpose of exploring graphical law, we assort the letters according to the number of


FIG. 5. Vertical axis is number of words and horizontal axis is respective letters. Letters are represented by the number in the alphabet or, dictionary sequence, [6].
words, in the descending order, denoted by $f$ and the respective rank, denoted by $k . k$ is a positive integer starting from one. Moreover, we attach a limiting rank, $k_{\text {lim }}$, and a limiting number of words. The limiting rank is maximum rank plus one, here it is eighteen and the limiting number of words is one. As a result both $\frac{\ln f}{\ln f_{\max }}$ and $\frac{\operatorname{lnk}}{\ln k_{l i m}}$ varies from zero to one. Then we tabulate in the adjoining table, [I] and plot $\frac{\ln f}{\ln f_{\max }}$ against $\frac{\ln k}{\ln k_{l i m}}$ in the figure fig.6. We then ignore the letter with the highest of words, tabulate in the adjoining table, II and redo the plot, normalising the $\ln f \mathrm{~s}$ with next-to-maximum $\ln f_{\text {nextmax }}$, and starting from $k=2$ in the figure fig.7. Normalising the $\ln f$ s with next-to-next-to-maximum $\ln f_{\text {nextnextmax }}$, we tabulate in the adjoining table, II and starting from $k=3$ we draw in the figure fig.8. Normalising the $\ln f s$ with next-to-next-to-next-to-maximum $\ln f_{\text {nextnextnextmax }}$ we record in the adjoining table, [II] and plot starting from $k=4$ in the figure fig 9, Normalising the $\ln f_{\mathrm{S}}$ with next-to-next-to-next-to-next-to-maximum $\ln f_{\text {nnnnmax }}$ we record in the adjoining table, (II and plot starting from $k=5$ in the figure fig (10. Normalising the $\ln f \mathrm{~s}$ with nextnextnextnextnext-maximum $\ln f_{n n n n n m a x}$ we record in the adjoining table, (II and plot starting from $k=6$ in the figure fig.11. Normalising the $\ln f_{\mathrm{s}}$ with nextnextnextnextnextnext-maximum $\ln f_{\text {nnnпnпmax }}$ we record in the adjoining table, [II and plot starting from $k=7$ in the figure fig. 12 .

| $\mathrm{k} \operatorname{lnk}$ | $\operatorname{lnk} / \ln k_{l i m}$ |  | $\operatorname{lnf}$ | $\operatorname{lnf} / \ln f_{\text {max }}$ | $\operatorname{lnf} / \ln f_{\text {next }}$ max | $\ln / \ln f_{n m a x}$ | $\operatorname{lnf} / \ln f_{\text {nnmmax }}$ | $\operatorname{lnf} / \ln f_{n n n m a x}$ | $\operatorname{lnf} / \ln f_{n n n n m a x}$ | $\operatorname{lnf} / \ln f_{n n n n n m a x}$ | $\operatorname{lnf} /\left[n f_{n n n n n n n n n m a x}\right.$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 766 | 6.64 |  | Blank | Blank | Blank | Blank | Blank | Blank | Blank |
| 20.69 | 0.335 | 557 | 6.32 | 0.952 | 1 | Blank | Blank | Blank | Blank | Blank | Blank |
| 1.10 | 0.374 | 491 | 6.20 | 0.933 | 0.980 | 1 | Blank | Blank | Blank | Blank | Blank |
| 1.39 | 0.473 | 461 | 6.13 | 0.924 | 0.970 | 0.990 | 1 | Blank | Blank | Blank | Blank |
| 1.61 | 0.548 | 437 | 6.08 | 0.916 | 0.962 | 0.981 | 0.991 | 1 | Blank | Blank | Blank |
| 1.79 | 0.609 | 426 | 6.05 | 0.912 | 0.957 | 0.977 | 0.987 | 0.996 | 1 | Blank | Blank |
| 1.95 | 0.663 | 356 | 5.88 | 0.885 | 0.929 | 0.948 | 0.958 | 0.966 | 0.970 | 1 | Blank |
| 2.08 | 0.707 | 352 | 5.86 | 0.883 | 0.927 | 0.946 | 0.956 | 0.964 | 0.969 | 0.998 | Blank |
| 2.20 | 0.748 | 320 | 5.77 | 0.869 | 0.912 | 0.931 | 0.940 | 0.949 | 0.953 | 0.982 | Blank |
| 102.30 | 0.782 | 274 | 5.61 | 0.845 | 0.888 | 0.906 | 0.915 | 0.923 | 0.927 | 0.955 | Blank |
| 112.40 | 0.816 | 269 | 5.60 | 0.842 | 0.885 | 0.903 | 0.912 | 0.920 | 0.924 | 0.952 | 1 |
| 122.48 | 0.844 | 254 | 5.54 | 0.834 | 0.876 | 0.894 | 0.903 | 0.911 | 0.915 | 0.942 | 0.990 |
| 132.56 | 0.871 | 228 | 5.43 | 0.817 | 0.859 | 0.876 | 0.885 | 0.893 | 0.897 | 0.924 | 0.970 |
| 12.64 | 0.898 | 223 | 5.41 | 0.814 | 0.855 | 0.873 | 0.882 | 0.889 | 0.893 | 0.920 | 0.966 |
| 152.71 | 0.922 | 136 | 4.91 | 0.740 | 0.777 | 0.793 | 0.801 | 0.808 | 0.812 | 0.836 | 0.878 |
| 162.77 | 0.942 | 109 | 4.69 | 0.706 | 0.742 | 0.757 | 0.765 | 0.772 | 0.775 | 0.798 | 0.838 |
| 172.83 | 0.963 | 66 | 4.19 | 0.631 | 0.663 | 0.676 | 0.683 | 0.689 | 0.692 | 0.713 | 0.749 |
| 182.89 | 0.983 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

TABLE II. Abor-Miri words: ranking,natural logarithm, normalisations


FIG. 6. Vertical axis is $\frac{\ln f}{\ln f_{\text {max }}}$ and horizontal axis is $\frac{\operatorname{lnk}}{\ln k_{l i m}}$. The + points represent the words of the Abor-Miri language with the fit curve being Bethe-Peierls curve in presence of four nearest neighbours and little magnetic field, $m=0.005$ or, $\beta H=0.01$. The uppermost curve is the Onsager solution.


FIG. 7. Vertical axis is $\frac{\operatorname{lnf}}{\ln f_{n e x t-m a x}}$ and horizontal axis is $\frac{\ln k}{\ln k_{l i m}}$. The + points represent the words of the Abor-Miri language with the fit curve being Bethe-Peierls curve in presence of four nearest neighbours and little magnetic field, $m=0.04$ or, $\beta H=0.08$. The uppermost curve is the Onsager solution.


FIG. 8. Vertical axis is $\frac{\operatorname{lnf}}{\ln f_{n e x t n e x t-m a x ~}}$ and horizontal axis is $\frac{\ln k}{\ln k_{l i m}}$. The + points represent the words of the Abor-Miri language with the fit curve being Bethe-Peierls curve in presence of four nearest neighbours and little magnetic field, $m=0.05$ or, $\beta H=0.1$. The uppermost curve is the Onsager solution.


FIG. 9. Vertical axis is $\frac{\ln f}{\ln f_{n e x t n e x t n e x t-m a x ~}}$ and horizontal axis is $\frac{\ln k}{\ln k_{l i m}}$. The + points represent the words of the Abor-Miri language with the fit curve being Bethe-Peierls curve in presence of four nearest neighbours and little magnetic field, $m=0.05$ or, $\beta H=0.1$. The uppermost curve is the Onsager solution.


FIG. 10. Vertical axis is $\frac{\ln f}{\ln f_{n n n n-m a x}}$ and horizontal axis is $\frac{l n k}{\ln k_{l i m}}$. The + points represent the words of the Abor-Miri language with the fit curve being the Onsager solution.


FIG. 11. Vertical axis is $\frac{\operatorname{lnf}}{\ln f_{n n n n n-\max }}$ and horizontal axis is $\frac{\ln k}{\ln k_{l i m}}$. The + points represent the words of the Abor-Miri language with the fit curve being the Onsager solution.


FIG. 12. Vertical axis is $\frac{\operatorname{lnf}}{\operatorname{lnf} f_{n n n n n-m a x}}$ and horizontal axis is $\frac{l n k}{\operatorname{lnk} k_{l i m}}$. The + points represent the words of the Abor-Miri language with the fit curve being the Onsager solution.

## A. conclusion

From the figures (fig (6-fig.12), we observe that there is a curve of magnetisation, behind words of Abor-Miri. This is magnetisation curve, $\mathrm{BP}(4, \beta H=0.08)$, in the Bethe-Peierls approximation with four nearest neighbours, in presence of liitle magnetic field, $\beta H=0.08$. Moreover, the associated correspondance is,

$$
\begin{gathered}
\frac{\operatorname{lnf}}{\ln f_{\text {next-to-maximum }}} \longleftrightarrow \frac{M}{M_{\max }}, \\
\operatorname{lnk} \longleftrightarrow T .
\end{gathered}
$$

k corresponds to temperature in an exponential scale, [16]. On the top of it, on successive higher normalisations, words of Abor-Miri almost go over to Onsager solution in the $\frac{\operatorname{lnf}}{\operatorname{lnf} f_{\text {nnnnnn-max }}}$ vs $\frac{l n k}{\operatorname{lnk} k_{l i m}}$ graph.
Still to be sure, we draw $\frac{\operatorname{lnf}}{\ln f_{\text {max }}}$ and $\frac{\ln f}{\ln f_{\text {next }-\max }}$ against $\ln k$ in the figures fig. (13)(14) to explore for the possible existence of a magnetisation curve of a Spin-Glass in presence of an external magnetic field, underlying Abor-Miri words.


FIG. 13. Vertical axis is $\frac{\ln f}{\ln f_{\max }}$ and horizontal axis is $\ln k$. The + points represent the words of the Abor-Miri language.


FIG. 14. Vertical axis is $\frac{\operatorname{lnf}}{\ln f_{\text {next-max }}}$ and horizontal axis is $l n k$. The + points represent the words of the Abor-Miri language.

In the figures 13 and 14, the points has a clearcut transition, above transition the line is almost horizontal. Hence, the words of the Abor-Miri language, can be described to underlie a Spin-Glass magnetisation curve, [17], in the presence of magnetic field.

| O | O: | A |  | A | 1 | I: U | U U | U: E | E E |  | É | É: í |  | í: | K | G | NG | S | J | NY | T | D | N | P | B | M | R | L | Y |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 148 | 7 |  | 23 | 31 | 41 | 105 | 53 | 4 | 181 | 47 | 776 | 640 | 011 | 10 | 468 | 223 | 83 | 369 | 115 | 20 | 406 | 277 | 117 | 1702 | 266 | 302 | 186 | 275 |  |  |

TABLE III. Mising words: the first row represents letters of the Mising alphabet, [7] in the serial order


FIG. 15. Vertical axis is number of words and horizontal axis is respective letters. Letters are represented by the number in the alphabet or, dictionary sequence, [7].

## IV. ANALYSIS OF WORDS OF MISING LANGUAGE

Abor-Miri broke into two languages. One Mising and another Adi. We undertake study of Mising laguage in this and the next sections of the paper. Starting from the place of Sadiya, down the banks of Brahmaputra, are the places of the Indian state of Assam where Misings stay. people belonging to Adi tribe populates the state of Arunachal. An interesting dictionary, exclusively for the Mising, was compiled by Tabu Ram Taid, recently in 2010. We go through his Mising to English dictionary,[7]. We count the words, one by one from the beginning to the end, starting with different letters. The result is the table, III. Moreover, we have counted words like ad- but have not counted entries like ad-~kan-, ad-~nam, ad-~né, (pl. see page . 195 of) [7]. Highest number of words, four hundred sixty eight, start with the letter K followed by words numbering four hundred six beginning with T , three hundred sixty nine with the letter S . To visualise we plot the number of words again respective letters in the dictionary sequence, [7] in the figure fig (15,

For the purpose of exploring graphical law, we assort the letters according to the number of
words, in the descending order, denoted by $f$ and the respective rank, denoted by $k$. $k$ is a positive integer starting from one. Moreover, we attach a limiting rank, $k_{\text {lim }}$, and a limiting number of words. The limiting rank is maximum rank plus one, here it is twenty eight and the limiting number of words is one. As a result both $\frac{\operatorname{lnf}}{\ln f_{\text {max }}}$ and $\frac{\operatorname{lnk}}{\ln k_{l i m}}$ varies from zero to one. Then we tabulate in the adjoining table, IV and plot $\frac{\operatorname{lnf}}{\ln f_{\text {max }}}$ against $\frac{\operatorname{lnk}}{\ln k_{l i m}}$ in the figure fig. 16.
We then ignore the letter with the highest of words, tabulate in the adjoining table, IV and redo the plot, normalising the $\ln f \mathrm{~s}$ with next-to-maximum $\ln f_{\text {nextmax }}$, and starting from $k=2$ in the figure fig. 17 . Normalising the $\ln f \mathrm{~s}$ with next-to-next-to-maximum $\ln f_{\text {nextnextmax }}$, we tabulate in the adjoining table, IV, and starting from $k=3$ we draw in the figure fig. 18. Normalising the $\ln f \mathrm{~s}$ with next-to-next-to-next-to-maximum $\ln f_{\text {nextnextnextmax }}$ we record in the adjoining table, IV, and plot starting from $k=4$ in the figure fig 19 Normalising the $\ln f s$ with next-to-next-to-next-to-next-to-maximum $\ln f_{n n n n m a x}$ we record in the adjoining table, IV, and plot starting from $k=5$ in the figure fig 20, Normalising the $\ln f_{\mathrm{s}}$ with nextnextnextnextnext-maximum $\ln f_{\text {nnnnnmax }}$ we record in the adjoining table, IV, and plot starting from $k=6$ in the figure fig,21. Normalising the $\ln f \mathrm{~s}$ with nextnextnextnextnextnext-maximum $\ln f_{n n n n n n m a x ~}$ we record in the adjoining table, IV, and plot starting from $k=7$ in the figure fig 22 .

| k | lnk | $\operatorname{lnk} / \ln k_{\text {lim }}$ | f | $\operatorname{lnf}$ | $\operatorname{lnf} / \ln f_{\text {max }}$ | $\operatorname{lnf} / \ln f_{\text {nextmax }}$ | $\operatorname{lnf} / \ln f_{\text {nnmax }}$ | $\operatorname{lnf} / \ln f_{\text {nnnmax }}$ | $\operatorname{lnf} / \ln f_{\text {nnnnmax }}$ | $\operatorname{lnf} / l n f_{\text {nnnnnmax }}$ | $\operatorname{lnf} / l n f_{\text {nnnnnnnnnmax }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 0 | 468 | 6.15 | 1 | Blank | Blank | Blank | Blank | Blank | Blank |
| 2 | 0.69 | 0.207 | 406 | 6.01 | 0.977 | 1 | Blank | Blank | Blank | Blank | Blank |
| 3 | 1.10 | 0.330 | 369 | 5.91 | 0.961 | 0.984 | 1 | Blank | Blank | Blank | Blank |
| 4 | 1.39 | 0.417 | 323 | 5.78 | 0.940 | 0.962 | 0.977 | 1 | Blank | Blank | Blank |
| 5 | 1.61 | 0.483 | 302 | 5.71 | 0.929 | 0.951 | 0.966 | 0.988 | 1 | Blank | Blank |
| 6 | 1.79 | 0.538 | 277 | 5.624 | 0.915 | 0.936 | 0.951 | 0.973 | 0.985 | 1 | Blank |
| 7 | 1.95 | 0.586 | 275 | 5.617 | 0.914 | 0.935 | 0.950 | 0.972 | 0.984 | 0.999 | Blank |
| 8 | 2.08 | 0.625 | 266 | 5.58 | 0.908 | 0.930 | 0.945 | 0.966 | 0.978 | 0.993 | Blank |
| 9 | 2.20 | 0.661 | 223 | 5.41 | 0.879 | 0.900 | 0.915 | 0.936 | 0.947 | 0.961 | Blank |
| 10 | 2.30 | 0.691 | 208 | 5.34 | 0.868 | 0.889 | 0.903 | 0.924 | 0.935 | 0.949 | Blank |
| 11 | 2.40 | 0.721 | 186 | 5.23 | 0.850 | 0.870 | 0.884 | 0.904 | 0.915 | 0.929 | 1 |
| 12 | 2.48 | 0.745 | 148 | 5.00 | 0.813 | 0.832 | 0.845 | 0.865 | 0.875 | 0.889 | 0.956 |
| 13 | 2.56 | 0.769 | 117 | 4.762 | 0.775 | 0.793 | 0.806 | 0.824 | 0.834 | 0.847 | 0.911 |
| 14 | 2.64 | 0.793 | 115 | 4.745 | 0.772 | 0.790 | 0.803 | 0.821 | 0.831 | 0.844 | 0.908 |
| 15 | 2.71 | 0.814 | 83 | 4.42 | 0.719 | 0.736 | 0.748 | 0.765 | 0.774 | 0.786 | 0.846 |
| 16 | 2.77 | 0.832 | 77 | 4.34 | 0.707 | 0.723 | 0.735 | 0.752 | 0.761 | 0.772 | 0.831 |
| 17 | 2.83 | 0.850 | 53 | 3.97 | 0.646 | 0.661 | 0.672 | 0.687 | 0.695 | 0.706 | 0.760 |
| 18 | 2.89 | 0.868 | 41 | 3.71 | 0.604 | 0.618 | 0.628 | 0.643 | 0.650 | 0.660 | 0.711 |
| 19 | 2.94 | 0.883 | 40 | 3.69 | 0.600 | 0.614 | 0.624 | 0.638 | 0.646 | 0.656 | 0.706 |
| 20 | 3.00 | 0.901 | 31 | 3.43 | 0.559 | 0.572 | 0.581 | 0.594 | 0.601 | 0.611 | 0.657 |
| 21 | 3.04 | 0.913 | 20 | 3.00 | 0.487 | 0.499 | 0.507 | 0.519 | 0.525 | 0.533 | 0.573 |
| 22 | 3.09 | 0.928 | 18 | 2.89 | 0.470 | 0.481 | 0.489 | 0.500 | 0.506 | 0.514 | 0.553 |
| 23 | 3.14 | 0.943 | 14 | 2.64 | 0.429 | 0.439 | 0.446 | 0.457 | 0.462 | 0.469 | 0.505 |
| 24 | 3.18 | 0.955 | 10 | 2.30 | 0.375 | 0.383 | 0.390 | 0.399 | 0.403 | 0.409 | 0.441 |
| 25 | 3.22 | 0.967 | 7 | 1.95 | 0.317 | 0.324 | 0.329 | 0.337 | 0.341 | 0.346 | 0.372 |
| 26 | 3.26 | 0.979 | 6 | 1.79 | 0.291 | 0.298 | 0.303 | 0.310 | 0.314 | 0.319 | 0.343 |
| 27 | 3.30 | 0.991 | 4 | 1.39 | 0.225 | 0.231 | 0.234 | 0.240 | 0.243 | 0.246 | 0.265 |
| 28 | 3.33 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

TABLE IV. Mising words: ranking, natural logarithm, normalisations


FIG. 16. Vertical axis is $\frac{l n f}{\ln f_{\text {max }}}$ and horizontal axis is $\frac{l n k}{\ln k_{l i m}}$. The + points represent the words of the Mising language with the fit curve being Bragg-Williams approximation curve in absence of magnetic field. The uppermost curve is the Onsager solution.


FIG. 17. Vertical axis is $\frac{\operatorname{lnf}}{\ln f_{n e x t-m a x}}$ and horizontal axis is $\frac{l n k}{\ln k_{l i m}}$. The + points represent the words of the Mising language with the fit curve being Bragg-Williams approximation curve in presence of magnetic field, $c=\frac{H}{\gamma \epsilon}=0.01$. The uppermost curve is the Onsager solution.

## A. conclusion

From the figures (fig.16-fig.22), we observe that there is a curve of magnetisation, behind words of Mising language, [7]. This is magnetisation curve, BW ( $\mathrm{c}=0$ ), in the Bragg-Williams


FIG. 18. Vertical axis is $\frac{\operatorname{lnf}}{\operatorname{lnf} f_{\text {nextnext-max }}}$ and horizontal axis is $\frac{\operatorname{lnk}}{\ln k_{l i m}}$. The + points represent the words of the Mising language with the fit curve being Bragg-Williams approximation curve in presence of magnetic field, $c=\frac{H}{\gamma \epsilon}=0.01$. The uppermost curve is the Onsager solution.


FIG. 19. Vertical axis is $\frac{\operatorname{lnf}}{\operatorname{lnf} f_{n e x t n e x t n e x t-m a x ~}}$ and horizontal axis is $\frac{\operatorname{lnk}}{\operatorname{lnk} k_{l i m}}$. The + points represent the words of the Mising language with the fit curve being Bethe-Peierls curve in presence of four neighbours. The uppermost curve is the Onsager solution.
approximation in absence of external magnetic field.
Moreover, the associated correspondance is,

$$
\frac{\ln f}{\ln f_{\text {maximum }}} \longleftrightarrow \frac{M}{M_{\max }}
$$



FIG. 20. Vertical axis is $\frac{\ln f}{\ln f_{n e x t n e x t n e x t n e x t-m a x ~}}$ and horizontal axis is $\frac{\operatorname{lnk}}{\ln k_{l i m}}$. The + points represent the words of the Mising language with the fit curve being Bethe-Peierls curve in presence of four neighbours. The uppermost curve is the Onsager solution.


FIG. 21. Vertical axis is $\frac{\ln f}{\ln f_{n n n n n-\max }}$ and horizontal axis is $\frac{\ln k}{\ln k_{l i m}}$. The + points represent the words of the Mising language with the fit curve being Bethe-Peierls curve in presence of four neighbours.The uppermost curve is the Onsager solution.

$$
\ln k \longleftrightarrow T
$$

k corresponds to temperature in an exponential scale, [16].
To explore for possible existence of a magnetisation curve of a spin-glass transition, under-


FIG. 22. Vertical axis is $\frac{\operatorname{lnf}}{\operatorname{lnf} 10 n-\max }$ and horizontal axis is $\frac{l n k}{\ln k_{l i m}}$. The + points represent the words of the Abor-Miri language with the fit curve being Bethe-Peierls curve in presence of four neighbours. The uppermost curve is the Onsager solution.


FIG. 23. Vertical axis is $\frac{\operatorname{lnf}}{\ln f_{\text {max }}}$ and horizontal axis is $\ln k$. The + points represent the words of the Abor-Miri language.
lying Mising words, $\frac{\ln f}{\ln f_{\text {max }}}$ and $\frac{\operatorname{lnf}}{\ln f_{\text {next-max }}}$ are drawn against $l n k$ in the figures fig.(23|24).
In the figure 23-24, the pointslines do not have clearcut transitions Hence, the words of the Mising language, are not suited to be described, to underlie a Spin-Glass magnetisation curve, [17], in the presence of magnetic field.
Moreover, on successive higher normalisations, words of Mising language do not go over to, the Onsager solution.


FIG. 24. Vertical axis is $\frac{\ln f}{\ln f_{n e x t-m a x}}$ and horizontal axis is $\ln k$. The + points represent the words of the Abor-Miri language with.

| A | B | D | E | G | I | J | K | L | M | N | O | P | R | S | T | U | Y |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 354 | 266 | 277 | 115 | 223 | 101 | 115 | 468 | 275 | 302 | 220 | 155 | 302 | 186 | 369 | 406 | 57 | 208 |

TABLE V. Mising words(reduced alphabet scheme): the first row represents letters of the Mising alphabet, in the reduced alphabet scheme, in the serial order

## V. ANALYSIS OF WORDS OF MISING LANGUAGE, IN THE REDUCED ALPHABET SCHEME

In this section, we reduce the alphabet of Mising-English dictionary, [7] to that of the AborMiri to English dictionary, [6]. We combine $O$ and $O$ : into one letter O. Total number of words become one hundred forty eight plus seven i.e. one hundred fifty five. Similarly, we combine $A$ and $A$ : into one letter $A$ with the resulting number of words starting with $A$ being three hundred fifty four. We combine I, I:, Í and I: into I with words starting with I being one hundred one. We combine $U$ and $U$ : into $U$. Total number of words starting with U then is fifty seven. Moreover, E, E: É and É: are put into E with words having it as initial numbering one hundred fifteen. NG, NY, N are brought under N with resultant number of words beginning with being two hundred twenty. We arrange the letters as in the AborMiri to English dictionary, [6]. This method we refer to as reduced alphabet scheme. This results in the table, $\overline{\mathrm{V}}$. This is in contarst to the alphabet of Mising-English dictionary, [7], which we refer to as extended alphabet scheme and denoted in short as misingex whereas words of alphabet of Mising Language, in the reduced alphabet scheme is denoted as misingr. Highest number of words, four hundred sixty eight, start with the letter K followed by words numbering four hundred six beginning with T , three hundred sixty nine with the letter S . To visualise we plot the number of words against respective letters in the reduced alphabet scheme, in the figure fig.25. For the purpose of exploring graphical law, we assort the letters according to the number of words, in the descending order, denoted by $f$ and the respective rank, denoted by $k . k$ is a positive integer starting from one. Moreover, we attach a limiting rank, $k_{l i m}$, and a limiting number of words. The limiting rank is maximum rank plus one, here it is seventeen and the limiting number of words is one. As a result both $\frac{\ln f}{\ln f_{\max }}$ and $\frac{\ln k}{\ln k_{l i m}}$ varies from zero to one. Then we tabulate in the adjoining table, VI and plot $\frac{\ln f}{\ln f_{\max }}$ against $\frac{l n k}{\ln k_{l i m}}$ in the figure fig. 26 ,
We then ignore the letter with the highest of words, tabulate in the adjoining table, VI and redo the plot, normalising the $\ln f \mathrm{~s}$ with next-to-maximum $\ln f_{\text {nextmax }}$, and starting from $k=$


FIG. 25. Vertical axis is number of words and horizontal axis is respective letters. Letters are represented by the number in the reduced alphabet scheme.

2 in the figure fig.27. Normalising the $\ln f \mathrm{~s}$ with next-to-next-to-maximum $\ln f_{\text {nextnextmax }}$, we tabulate in the adjoining table, VI and starting from $k=3$ we draw in the figure fig. 28 , Normalising the $\ln f_{\mathrm{s}}$ with next-to-next-to-next-to-maximum $\ln f_{\text {nextnextnextmax }}$ we record in the adjoining table, VI and plot starting from $k=4$ in the figure fig 29, Normalising the $\ln f_{\mathrm{S}}$ with next-to-next-to-next-to-next-to-maximum $\ln f_{n n n n m a x}$ we record in the adjoining table, VI and plot starting from $k=5$ in the figure fig 30 . Normalising the $\ln f \mathrm{~s}$ with nextnextnextnextnext-maximum $\ln f_{\text {nnnnпmax }}$ we record in the adjoining table, VI and plot starting from $k=6$ in the figure fig 31 .

| k | $\operatorname{lnk}$ | $\operatorname{lnk} / \operatorname{lnk} k_{\text {lim }}$ | f | $\operatorname{lnf}$ | $\operatorname{lnf} / \ln f_{\text {max }}$ | $\operatorname{lnf} / \ln f_{n e x t-m a x}$ | $\operatorname{lnf} / \ln f_{n n m a x}$ | $\operatorname{lnf} / \ln f_{n n n m a x}$ | $\operatorname{lnf} / \ln f_{n n n n m a x}$ | $\operatorname{lnf} / \ln f_{n n n n n a x}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 0 | 0 | 468 | 6.15 | 1 | Blank | Blank | Blank | Blank | Blank |
| 2 | 0.69 | 0.244 | 406 | 6.01 | 0.977 | 1 | Blank | Blank | Blank | Blank |
| 3 | 1.10 | 0.389 | 369 | 5.91 | 0.961 | 0.984 | 1 | Blank | Blank | Blank |
| 4 | 1.39 | 0.491 | 354 | 5.87 | 0.955 | 0.977 | 0.993 | 1 | Blank | Blank |
| 5 | 1.61 | 0.569 | 302 | 5.71 | 0.929 | 0.951 | 0.966 | 0.973 | 1 | Blank |
| 6 | 1.79 | 0.633 | 277 | 5.624 | 0.915 | 0.936 | 0.951 | 0.958 | 0.985 | 1 |
| 7 | 1.95 | 0.689 | 275 | 5.617 | 0.914 | 0.935 | 0.950 | 0.957 | 0.984 | 0.999 |
| 8 | 2.08 | 0.735 | 266 | 5.58 | 0.908 | 0.930 | 0.945 | 0.951 | 0.978 | 0.993 |
| 9 | 2.20 | 0.777 | 223 | 5.41 | 0.879 | 0.900 | 0.915 | 0.921 | 0.947 | 0.961 |
| 10 | 2.30 | 0.813 | 220 | 5.39 | 0.877 | 0.898 | 0.913 | 0.919 | 0.945 | 0.959 |
| 11 | 2.40 | 0.848 | 208 | 5.34 | 0.868 | 0.889 | 0.903 | 0.910 | 0.935 | 0.949 |
| 12 | 2.48 | 0.876 | 186 | 5.23 | 0.850 | 0.870 | 0.884 | 0.890 | 0.915 | 0.929 |
| 13 | 2.56 | 0.905 | 155 | 5.04 | 0.820 | 0.840 | 0.853 | 0.859 | 0.883 | 0.897 |
| 14 | 2.64 | 0.933 | 115 | 4.75 | 0.772 | 0.790 | 0.803 | 0.808 | 0.831 | 0.844 |
| 15 | 2.71 | 0.958 | 101 | 4.62 | 0.751 | 0.768 | 0.781 | 0.786 | 0.808 | 0.821 |
| 16 | 2.77 | 0.979 | 57 | 4.04 | 0.658 | 0.673 | 0.684 | 0.689 | 0.708 | 0.719 |
| 17 | 2.83 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

TABLE VI. Mising words(reduced alphabet scheme): ranking,natural logarithm, normalisations


FIG. 26. Vertical axis is $\frac{\ln f}{\ln f_{\text {max }}}$ and horizontal axis is $\frac{\operatorname{lnk}}{\ln k_{l i m}}$. The + points represent the words of the Mising language, in the reduced alphabet scheme, with the fit curve being Bethe-Peierls curve in presence of four nearest neighbours and little magnetic field, $m=0.02$ or, $\beta H=0.04$. The uppermost curve is the Onsager solution.


FIG. 27. Vertical axis is $\frac{\operatorname{lnf}}{\operatorname{lnf} f_{n e x t-m a x}}$ and horizontal axis is $\frac{l n k}{\operatorname{lnk} k \text { lim }}$. The + points represent the words of the Mising language, in the reduced alphabet scheme, with the fit curve being Bethe-Peierls curve in presence of four nearest neighbours and little magnetic field, $m=0.04$ or, $\beta H=0.08$. The uppermost curve is the Onsager solution.


FIG. 28. Vertical axis is $\frac{\operatorname{lnf}}{\operatorname{lnf} f_{\text {nextnext-max }}}$ and horizontal axis is $\frac{l n k}{\ln k_{l i m}}$. The + points represent the words of the Mising language, in the reduced alphabet scheme, with the fit curve being Bethe-Peierls curve in presence of four nearest neighbours and little magnetic field, $m=0.05$ or, $\beta H=0.1$. The uppermost curve is the Onsager solution.


FIG. 29. Vertical axis is $\frac{\operatorname{lnf}}{\operatorname{lnf} f_{n e x t n e x t n e x t-m a x ~}}$ and horizontal axis is $\frac{l n k}{\operatorname{lnk} k_{l i m}}$. The + points represent the words of the Mising language, in the reduced alphabet scheme, with the fit curve being Bethe-Peierls curve in presence of four nearest neighbours and little magnetic field, $m=0.05$ or, $\beta H=0.1$. The uppermost curve is the Onsager solution.


FIG. 30. Vertical axis is $\frac{\ln f}{\ln f_{n e x t n e x t n e x t n e x t-m a x ~}}$ and horizontal axis is $\frac{\ln k}{\ln k_{l i m}}$. The + points represent the words of the Mising language, in the reduced alphabet scheme, with the fit curve being the Onsager solution.


FIG. 31. Vertical axis is $\frac{\ln f}{\ln f_{n n n n n-\max }}$ and horizontal axis is $\frac{\ln k}{\ln k_{l i m}}$. The + points represent the words of the Mising language, in the reduced alphabet scheme, with the fit curve being the the Onsager solution.


FIG. 32. Vertical axis is $\frac{\ln f}{\ln f_{\text {max }}}$ and horizontal axis is $\operatorname{lnk}$. The + points represent the words of the Mising language, in the reduced alphabet scheme.

## A. conclusion

From the figures (fig.26-fig.31), we observe that there is a curve of magnetisation, behind words of Mising language in the reduced alphabet scheme. This is magnetisation curve, $\mathrm{BP}(4, \beta H=0.08)$, in the Bethe-Peierls approximation with four nearest neighbours, in presence of liitle magnetic field, $\beta H=0.08$.

Moreover, the associated correspondance is,

$$
\begin{gathered}
\frac{\ln f}{\ln f_{\text {next-to-maximum }}} \longleftrightarrow \frac{M}{M_{\max }}, \\
\ln k \longleftrightarrow T
\end{gathered}
$$

k corresponds to temperature in an exponential scale, [16].
On the top of it, on successive higher normalisations, words of Mising language, in the reduced alphabet scheme, almost go over to Onsager solution in the $\frac{\operatorname{lnf}}{\ln f_{n n n n-m a x}}$ vs $\frac{\operatorname{lnk}}{\ln k_{\text {lim }}}$ graph.
Still to be sure, we draw $\frac{\operatorname{lnf}}{\ln f_{\text {max }}}$ and $\frac{\ln f}{\ln f_{\text {next }-\max }}$ against $\ln k$ in the figures fig.(32,33) to explore for the possible existence of a magnetisation curve of a Spin-Glass in presence of an external magnetic field, underlying Mising language, in the reduced alphabet scheme. In the figure $32[33$ the points has a clearcut transition, above transition the line is almost horizontal Hence, the words of the Mising language, in the reduced alphabet scheme,can be described, to underlie a Spin-Glass magnetisation curve, [17], in the presence of external magnetic field.


FIG. 33. Vertical axis is $\frac{\operatorname{lnf}}{\operatorname{lnf} f_{\text {next-max }}}$ and horizontal axis is $l n k$. The + points represent the words of of the Mising language, in the reduced alphabet scheme.

## VI. DISCUSSION

We have observed that there is a curve of magnetisation, behind words of Abor-Miri language, [6]. This is magnetisation curve, $\mathrm{BP}(4, \beta H=0.08)$, in the Bethe-Peierls approximation with four nearest neighbours, in presence of liitle magnetic field, $\beta H=0.08$. We have found also that words of Mising language, [7], once the alphabet is reduced to that of Abor-Miri language, [6], underlies the same magnetisation curve, $\mathrm{BP}(4, \beta \mathrm{H}=0.08)$. Moreover, words of the both go over under successive normalisations to the Onsager solution. i.e both have Onsager core. Both seems to be suited, [9], to be described to underlie a SpinGlass magnetisation curve, [17], in the presence of magnetic field. Maxima and minima of both fall on the same letters in the figure 34. The sameness is interesting in the light of these pertaining to two different dictionaries compiled by two different persons in the span of one hundred year and Mising being an offshoot of Abor-Miri. Dictionary of Adi, another offshoot of Abor-Miri, not available to us right now, will be eagerly awaited from the standpoint of uniqueness, as pointed out in [14]. Moreover, we have concluded that there is a curve of magnetisation, behind words of Mising language, 7], in the extended alphabet scheme. This is magnetisation curve, BW $(\mathrm{c}=0)$, in the Bragg-Williams approximation of Ising model, in the absence of external magnetic field.


FIG. 34. Vertical axis is number of words and horizontal axis is respective letters. Letters are represented by the number in the alphabet or, dictionary sequence, [6]. Upper dashed curve represents Abor-Miri words and lower solid line is for Misingr words.

## VII. APPENDIX:MAGNETISATION

## A. Bragg-Williams approximation

Let us consider a coin. Let us toss it many times. Probability of getting head or, tale is half i.e. we will get head and tale equal number of times. If we attach value one to head, minus one to tale, the average value we obtain, after many tossing is zero. Instead let us consider a one-sided loaded coin, say on the head side. The probability of getting head is more than one half, getting tale is less than one-half. Average value, in this case, after many tossing we obtain is non-zero, the precise number depends on the loading. The loaded coin is like ferromagnet, the unloaded coin is like paramagnet, at zero external magnetic field. Average value we obtain is like magnetisation, loading is like coupling among the spins of the ferromagnetic units. Outcome of single coin toss is random, but average value we get after long sequence of tossing is fixed. This is long-range order. But if we take a small sequence of tossing, say, three consecutive tossing, the average value we obtain is not fixed, can be anything. There is no short-range order.

Let us consider a row of spins, one can imagine them as spears which can be vertically up or, down. Assume there is a long-range order with probability to get a spin up is two third.

That would mean when we consider a long sequence of spins, two third of those are with spin up. Moreover, assign with each up spin a value one and a down spin a value minus one. Then total spin we obtain is one third. This value is referred to as the value of longrange order parameter. Now consider a short-range order existing which is identical with the long-range order. That would mean if we pick up any three consecutive spins, two will be up, one down. Bragg-Williams approximation means short-range order is identical with long-range order, applied to a lattice of spins, in general. Row of spins is a lattice of one dimension.

Now let us imagine an arbitrary lattice, with each up spin assigned a value one and a down spin a value minus one, with an unspecified long-range order parameter defined as above by $L=\frac{1}{N} \Sigma_{i} \sigma_{i}$, where $\sigma_{i}$ is i-th spin, N being total number of spins. L can vary from minus one to one. $N=N_{+}+N_{-}$, where $N_{+}$is the number of up spins, $N_{-}$is the number of down spins. $L=\frac{1}{N}\left(N_{+}-N_{-}\right)$. As a result, $N_{+}=\frac{N}{2}(1+L)$ and $N_{-}=\frac{N}{2}(1-L)$. Magnetisation or, net magnetic moment, $M$ is $\mu \Sigma_{i} \sigma_{i}$ or, $\mu\left(N_{+}-N_{-}\right)$or, $\mu N L, M_{\max }=\mu N . \frac{M}{M_{\max }}=L \cdot \frac{M}{M_{\max }}$ is referred to as reduced magnetisation. Moreover, the Ising Hamiltonian, [1], for the lattice of spins, setting $\mu$ to one, is $-\epsilon \Sigma_{n . n} \sigma_{i} \sigma_{j}-H \Sigma_{i} \sigma_{i}$, where n.n refers to nearest neighbour pairs. The difference $\triangle E$ of energy if we flip an up spin to down spin is, [18], $2 \epsilon \gamma \bar{\sigma}+2 H$, where $\gamma$ is the number of nearest neighbours of a spin. According to Boltzmann principle, $\frac{N_{-}}{N_{+}}$ equals $\exp \left(-\frac{\Delta E}{k_{B} T}\right),[19]$. In the Bragg-Williams approximation, [20], $\bar{\sigma}=L$, considered in the thermal average sense. Consequently,

$$
\begin{equation*}
\ln \frac{1+L}{1-L}=2 \frac{\gamma \epsilon L+H}{k_{B} T}=2 \frac{L+\frac{H}{\gamma \epsilon}}{\frac{T}{\gamma \epsilon / k_{B}}}=2 \frac{L+c}{\frac{T}{T_{c}}} \tag{1}
\end{equation*}
$$

where, $c=\frac{H}{\gamma \epsilon}, T_{c}=\gamma \epsilon / k_{B},[21] . \frac{T}{T_{c}}$ is referred to as reduced temperature.
Plot of $L$ vs $\frac{T}{T_{c}}$ or, reduced magentisation vs. reduced temperature is used as reference curve. In the presence of magnetic field, $c \neq 0$, the curve bulges outward. Bragg-Williams is a Mean Field approximation. This approximation holds when number of neighbours interacting with a site is very large, reducing the importance of local fluctuation or, local order, making the long-range order or, average degree of freedom as the only degree of freedom of the lattice. To have a feeling how this approximation leads to matching between experimental and Ising model prediction one can refer to FIG. 12.12 of [18]. W. L. Bragg was a professor of Hans Bethe. Rudlof Peierls was a friend of Hans Bethe. At the suggestion of W. L. Bragg, Rudlof Peierls following Hans Bethe improved the approximation scheme, applying quasi-chemical
method.

## B. Bethe-peierls approximation in presence of four nearest neighbours, in absence

 of external magnetic fieldIn the approximation scheme which is improvement over the Bragg-Williams, [1], [18], [19], [20], [21], due to Bethe-Peierls, [15], reduced magnetisation varies with reduced temperature, for $\gamma$ neighbours, in absence of external magnetic field, as

$$
\begin{equation*}
\frac{\ln \frac{\gamma}{\gamma-2}}{\ln \frac{\text { factor }-1}{\text { factor } \frac{\gamma-1}{\gamma}-\text { factor }^{\frac{1}{\gamma}}}}=\frac{T}{T_{c}} ; \text { factor }=\frac{\frac{M}{M_{\max }}+1}{1-\frac{M}{M_{\max }}} . \tag{2}
\end{equation*}
$$

$\ln \frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma=4$ is 0.693 . For a snapshot of different kind of magnetisation curves for magnetic materials the reader is urged to give a google search "reduced magnetisation vs reduced temperature curve". In the following, we describe datas generated from the equation(11) and the equation(2) in the table, VII and curves of magnetisation plotted on the basis of those datas. BW stands for reduced temperature in Bragg-Williams approximation, calculated from the equation(1). $\mathrm{BP}(4)$ represents reduced temperature in the Bethe-Peierls approximation, for four nearest neighbours, computed from the equation(2). The data set is used to plot fig.2. Empty spaces in the table, VII, mean corresponding point pairs were not used for plotting a line.

## C. Bethe-peierls approximation in presence of four nearest neighbours, in pres-

 ence of external magnetic fieldIn the Bethe-Peierls approximation scheme, [15], reduced magnetisation varies with reduced temperature, for $\gamma$ neighbours, in presence of external magnetic field, as

$$
\begin{equation*}
\frac{\ln \frac{\gamma}{\gamma-2}}{\ln \frac{\text { factor }-1}{e^{\frac{2 B H}{\gamma}} \text { factor } \frac{\gamma-1}{\gamma}-e^{-\frac{2 \beta H}{\gamma}} \text { factor } \frac{1}{\gamma}}}=\frac{T}{T_{c}} ; \text { factor }=\frac{\frac{M}{M_{\max }}+1}{1-\frac{M}{M_{\max }}} . \tag{3}
\end{equation*}
$$

Derivation of this formula ala [15] is given in the appendix of [14].
$\ln \frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma=4$ is 0.693 . For four neighbours,

$$
\begin{equation*}
\frac{0.693}{\ln \frac{\text { actor }-1}{e^{\frac{2 B H}{\gamma}} \text { factor } \frac{\gamma-1}{\gamma}-e^{-\frac{2 \beta H}{\gamma}} \text { factor } \frac{1}{\gamma}}}=\frac{T}{T_{c}} ; \text { factor }=\frac{\frac{M}{M_{\max }}+1}{1-\frac{M}{M_{\max }}} . \tag{4}
\end{equation*}
$$

| BVV | $\mathrm{BVW}(\mathrm{c}=0.01)$ | BP(4, SII $=0)$ | reduced magnetisation |
| :---: | :---: | :---: | :---: |
| 0 | O | 0 | 1 |
| 0.435 | 0.439 | 0.563 | 0.978 |
| 0.439 | 0.443 | 0.568 | 0.977 |
| 0.491 | 0.495 | 0.624 | 0.961 |
| 0.501 | 0.507 | 0.630 | 0.957 |
| 0.514 | 0.519 | 0.648 | 0.952 |
| 0.559 | 0.566 | 0.654 | 0.931 |
| 0.566 | 0.573 | 0.7 | 0.927 |
| 0.584 | 0.590 | 0.7 | 0.917 |
| 0.601 | 0.607 | 0.722 | 0.907 |
| 0.607 | 0.613 | 0.729 | 0.903 |
| 0.653 | 0.661 | 0.770 | 0.869 |
| 0.659 | 0.668 | 0.773 | 0.865 |
| 0.669 | 0.676 | 0.784 | 0.856 |
| 0.679 | 0.688 | 0.792 | 0.847 |
| 0.701 | 0.710 | 0.807 | 0.828 |
| 0.723 | 0.731 | 0.828 | 0.805 |
| 0.732 | 0.743 | 0.832 | 0.796 |
| 0.756 | 0.766 | 0.845 | 0.772 |
| 0.779 | 0.788 | 0.864 | 0.740 |
| 0.838 | 0.853 | 0.911 | 0.651 |
| 0.850 | 0.861 | 0.911 | 0.628 |
| 0.870 | 0.885 | 0.923 | 0.592 |
| 0.883 | 0.895 | 0.928 | 0.564 |
| 0.899 | 0.918 |  | 0.527 |
| 0.904 | 0.926 | 0.941 | 0.513 |
| 0.946 | 0.968 | 0.965 | 0.400 |
| 0.967 | 0.998 | 0.965 | 0.300 |
| 0.987 |  | 1 | 0.200 |
| 0.997 |  | 1 | O. 100 |
| 1 | 1 | 1 | 0 |

TABLE VII. Reduced magnetisation vs reduced temperature datas for Bragg-Williams approximation, in absence of and in presence of magnetic field, $c=\frac{H}{\gamma \epsilon}=0.01$, and Bethe-Peierls approximation in absence of magnetic field, for four nearest neighbours .

In the following, we describe datas in the table, VIII, generated from the equation(4) and curves of magnetisation plotted on the basis of those datas. $\mathrm{BP}(4, \beta H=0.06)$ stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H , such that $\beta H=0.06$. calculated from the equation(4). $\mathrm{BP}(4, \beta H=0.05)$ stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H , such that $\beta H=0.05$. calculated from the equation(4). $\mathrm{BP}(4, \beta H=0.04)$ stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H , such that $\beta H=0.04$. calculated from the equation(4). $\mathrm{BP}(4, \beta H=0.02)$ stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H , such that

| $\mathrm{BP}(4, \beta H=0.1)$ | $\mathrm{BP}(4, \beta H=0.08)$ | $\mathrm{BP}(4, \beta H=0.06)$ | $\mathrm{BP}(4, \beta H=0.05)$ | $\mathrm{BP}(4, \beta H=0.04)$ | $\mathrm{BP}(4, \beta H=0.02)$ | $\mathrm{BP}(4, \beta H=0.01)$ | reduced magnetisation |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0.597 | 0.589 | 0.583 | 0.580 | 0.577 | 0.572 | 0.569 | 0.978 |
| 0.603 | 0.593 | 0.587 | 0.584 | 0.581 | 0.575 | 0.572 | 0.977 |
| 0.660 | 0.655 | 0.647 | 0.643 | 0.639 | 0.632 | 0.628 | 0.961 |
| 0.673 | 0.665 | 0.657 | 0.653 | 0.649 | 0.641 | 0.637 | 0.957 |
| 0.688 | 0.679 | 0.671 | 0.667 |  | 0.654 | 0.650 | 0.952 |
|  |  |  | 0.716 |  |  | 0.696 | 0.931 |
| 0.745 | 0.734 | 0.723 | 0.718 | 0.713 | 0.702 | 0.697 | 0.927 |
| 0.766 | 0.754 | 0.743 | 0.737 | 0.731 | 0.720 | 0.714 | 0.917 |
| 0.787 | 0.775 | 0.762 | 0.756 | 0.749 | 0.737 | 0.731 | 0.907 |
| 0.796 | 0.783 | 0.770 | 0.764 | 0.757 | 0.745 | 0.738 | 0.903 |
| 0.848 | 0.832 | 0.816 | 0.808 | 0.800 | 0.785 | 0.778 | 0.869 |
| 0.854 | 0.837 | 0.821 | 0.813 | 0.805 | 0.789 | 0.782 | 0.865 |
| 0.866 | 0.849 | 0.832 | 0.823 | 0.815 | 0.799 | 0.791 | 0.856 |
| 0.878 | 0.859 | 0.841 | 0.833 | 0.824 | 0.807 | 0.799 | 0.847 |
| 0.902 | 0.882 | 0.863 | 0.853 | 0.844 | 0.826 | 0.817 | 0.828 |
| 0.931 | 0.908 | 0.887 | 0.876 | 0.866 | 0.846 | 0.836 | 0.805 |
| 0.940 | 0.917 | 0.895 | 0.884 | 0.873 | 0.852 | 0.842 | 0.796 |
| 0.966 | 0.941 | 0.916 | 0.904 | 0.892 | 0.869 | 0.858 | 0.772 |
| 0.996 | 0.968 | 0.940 | 0.926 | 0.914 | 0.888 | 0.876 | 0.740 |
| 1 |  |  | 0.929 |  |  | 0.877 | 0.735 |
|  | 0.977 |  | 0.936 |  |  | 0.883 | 0.730 |
|  | 0.989 |  | 0.944 |  |  | 0.889 | 0.720 |
|  | 0.990 |  | 0.945 |  |  |  | 0.710 |
|  | 1.00 |  | 0.955 |  |  | 0.897 | 0.700 |
|  |  |  | 0.963 |  |  | 0.903 | 0.690 |
|  |  |  | 0.973 |  |  | 0.910 | 0.680 |
|  |  |  |  |  |  | 0.909 | 0.670 |
|  |  |  | 0.993 |  |  | 0.925 | 0.650 |
|  |  |  |  | 0.976 | 0.942 |  | 0.651 |
|  |  |  | 1.00 |  |  |  | 0.640 |
|  |  |  |  | 0.983 | 0.946 | 0.928 | 0.628 |
|  |  |  |  | 1.00 | 0.963 | 0.943 | 0.592 |
|  |  |  |  |  | 0.972 | 0.951 | 0.564 |
|  |  |  |  |  | 0.990 | 0.967 | 0.527 |
|  |  |  |  |  |  | 0.964 | 0.513 |
|  |  |  |  |  | 1.00 |  | 0.500 |
|  |  |  |  |  |  | 1.00 | 0.400 |
|  |  |  |  |  |  |  | 0.300 |
|  |  |  |  |  |  |  | 0.200 |
|  |  |  |  |  |  |  | 0.100 |
|  |  |  |  |  |  |  | o |

TABLE VIII. Bethe-Peierls approx. in presence of little external magnetic fields
$\beta H=0.02$. calculated from the equation(4). $\mathrm{BP}(4, \beta H=0.01)$ stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H , such that $\beta H=0.01$. calculated from the equation(4). The data set is used to plot fig. 3 and fig.4. Empty spaces in the table, VIII, mean corresponding point pairs were not used for plotting a line.

## D. Spin-Glass

In the case coupling between( among) the spins, not necessarily n.n, for the Ising model is( are) random, we get Spin-Glass, [17, 22-27]. When a lattice of spins randomly coupled and in
an external magnetic field, goes over to the Spin-Glass phase, magnetisation increases steeply like $\frac{1}{T-T_{c}}$ upto the the phase transition temperature, followed by very little increase, [17, 26], in magnetisation, as the ambient temperature continues to drop. This happens at least in the replica approach of the Spin-Glass theory, [23, 24].

## VIII. ACKNOWLEDGEMENT

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