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OPACITY-LIMITED HIERARCHICAL FRAGMENTATION AND THE MASSES OF PROTOSTARS

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SUMMARY

Hierarchical fragmentation of a collapsing cloud terminates when opacity prevents individual sub-clouds from radiating their gravitational binding energy on a free-fall time scale. The approximate mass of the final fragments is derived in a form which clarifies why it is insensitive to physical conditions in the cloud. This mass is $\sim 10^{-3}\, T^{1/4} f^{-1/2}$ times the Chandrasekhar mass, where T (K) is the gas temperature when the fragments first significantly reabsorb their own radiation, and f is the radiative efficiency of a fragment at that stage relative to a black body.

FRAGMENTATION AND PROTOSTAR MASS

A collapsing gas cloud is gravitationally unstable to fragmentation on mass-scales exceeding the Jeans mass

$$M_{\mathrm{J}} \simeq \left(\frac{\pi k}{G m_{\mathrm{p}} \mu}\right)^{3/2} T^{3/2} \rho^{-1/2}, \tag{1}$$

where m_{p} is the proton mass and μ the mean molecular weight. As Hoyle (1953) emphasized, fragments may themselves be liable to further break-up if the gas (which is being heated adiabatically during the collapse) can radiate efficiently enough for $T^{3/2}\rho^{-1/2}$ to *decrease* as ρ rises. Hoyle argued that, if cooling were efficient, collapsing clouds would undergo 'hierarchical fragmentation' into progressively small masses, and that this process would terminate only when individual fragments became opaque enough to trap their radiation. He showed that a protogalaxy in which the main coolants were H I and H II could collapse isothermally at $\sim 10^4$ K, and that the final 'non-fragmenting fragments' (protostars?) had masses $\sim 1\, M_{\odot}$.

Subsequently, several other authors interested in star formation (Gaustad 1963; Yoneyama 1972; Suchkov & Shchekinov 1976; Low & Lynden-Bell 1976; Silk 1976) have calculated the influence of extra cooling agents, e.g. H_2, heavy elements, molecules and dust, and introduced further refinements. When a variety of coolants are present, a collapsing cloud can trace out a more complicated track in the T–ρ diagram. When cooling is 'efficient', in the sense that the radiative cooling time is $\lesssim (G\rho)^{-1/2}$, and the optical depth is small, $T^{3/2}\rho^{-1/2}$ decreases and hierarchical fragmentation can proceed; but when cooling is *in*efficient (e.g. if the density is low, or grains evaporate) a cloud with $M \simeq M_{\mathrm{J}}$ deflates quasi-statically on the cooling time scale, remaining close to virial equilibrium with $T \propto \rho^{1/3}$, until a more efficient coolant comes into play and the fragmentation can proceed.

When the fragments finally become opaque enough to reabsorb their radiation, $d(\log T)/d(\log \rho)$ becomes $> \frac{1}{3}$ and fragmentation stops. The most significant (and perhaps gratifying) outcome of such calculations is that—even though the adopted cooling rates and opacities may differ by many orders of magnitude—the inferred mass-scale M_F at which opacity intervenes and terminates the fragmentation process lies in the general range 10^{-2}–$1\ M_\odot$.

This note outlines a general argument which aims to clarify *why* Hoyle-style hierarchical fragmentation predicts an M_F that seems surprisingly insensitive to the cooling parameters *and* is comparable with lower-main-sequence stellar masses.

Clouds whose masses exceed M_J collapse at almost the free-fall rate, a fraction $\sim (M/M_J)^{-2/3}$ of the binding energy released being expended in '$P\,dV$ work'. M_J will decrease during the collapse only if $\gtrsim \frac{1}{2}$ of this can be radiated away. At any stage in the hierarchical fragmentation process, the typical fragment mass is $\sim M_J$. A necessary condition for further fragmentation is therefore that *opacity effects should not prevent a fragment from radiating its binding energy on a free-fall time scale.*

Since a cloud automatically becomes opaque when its radiation rate per unit area approaches that of a black body, this condition implies a *lower limit* on T. In expressing this thermodynamic constraint quantitatively, it proves rather illuminating to express the fragment's radius r in terms of the Schwarzschild radius $r_s = 2GM/c^2$, and the temperature in terms of $m_p c^2/k$ (despite the fact that r/r_s and $m_p c^2/kT$ are both of course $\gg 1$ when fragmentation occurs in astrophysically realistic environments).

The *required* rate of radiation of binding energy is then

$$\sim \frac{GM^2}{r}(G\rho)^{1/2} \simeq \frac{c^5}{G}\left(\frac{r_s}{r}\right)^{5/2}. \tag{2}$$

The cloud cannot emit more than f ($\lesssim 1$) times as much as a 'black sphere' of radius r and temperature T, the value of f depending on the detailed physics of the cooling and opacity. Writing the radiation constant a as $\pi^2k^4/15c^3\hbar^3$, this *maximum* radiation rate is

$$f.\frac{\pi^3k^4T^4}{15c^2\hbar^3}r^2. \tag{3}$$

For continuing fragmentation, (3) must exceed (2), i.e.

$$\frac{\pi^3}{15}f\left(\frac{kT}{m_pc^2}\right)^4\left(\frac{M}{m_p}\right)^2\left(\frac{Gm_p^2}{\hbar c}\right)^3 \gtrsim \left(\frac{r_s}{r}\right)^{9/2}. \tag{4}$$

Now $(\hbar c/Gm_p^2)^{3/2}$ is approximately the number of particles in the Chandrasekhar mass. Writing this 'large number' as M_c/m_p (and dropping the factor $\pi^3/15$, since our whole argument is just an 'order-of-magnitude' one), we can rewrite (4) as

$$\frac{M}{M_c} \gtrsim \left(\frac{kT}{m_pc^2}\right)^{1/4} f^{1/2}\left[\left(\frac{r_s}{r}\right)\left(\frac{kT}{m_pc^2}\right)^{-1}\right]^{9/4}. \tag{5}$$

The quantity in square brackets is $\sim \mu^{-1}$ if $M \simeq M_J$. Thus the minimum Jeans mass is given by

$$M_F \simeq M_c \mu^{-9/4} f^{-1/2}\left(\frac{kT}{m_pc^2}\right)^{1/4}, \tag{6}$$

the appropriate value of T being that prevailing when opacity just becomes important. If the relevant temperature lies in the range 10–10^4 K, this argument predicts M_F to be $(1–6) \times 10^{-3} f^{-1/2} \mu^{-9/4} M_c$.*

If scattering can be ignored, the minimum Jeans mass is obtained when the optical depth $\kappa_{abs} r$ is only ~ 1 (Low & Lynden-Bell 1976). This means that, at the crucial stage when fragmentation terminates, the temperature contrast between the centre and surface of each fragment is small, justifying our use of a single temperature T.

An extra complication arises when scattering†, rather than absorption, makes the main contribution to the total opacity (i.e. when $\kappa_{scat} \gg \kappa_{abs}$): for example, electron scattering may provide the dominant opacity if a cloud of pure H and He collapses at $\sim 10^4$ K. Even if κ_{abs} were negligible, scattering can trap radiation within the cloud for a whole free-fall time scale if

$$\kappa_{scat} r > \left(\frac{r}{r_S}\right)^{1/2}. \tag{7}$$

When (7) is fulfilled, further fragmentation is impossible, even if T is so high that $\kappa_{abs} r < 1$; but fragmentation would never actually be terminated by this effect unless $\kappa_{scat} \gtrsim 10^5\, \kappa_{abs}$, which is plausible only for a cloud of H and He whose opacity is dominated by electron scattering.

Note also that if $\kappa_{scat} \gg \kappa_{abs}$, implying a high albedo, then the efficiency factor f must be $\lesssim (\kappa_{abs}/\kappa_{scat})$. Other situations which could reduce f well below unity, and thus raise M_F, include: (i) when the cooling mechanism involves emission in a few narrow lines, (ii) when the cooling is due to dust which remains colder than the gas (*cf.* Gaustad 1963; Yoneyama 1972; Suchkov & Shchekinov 1976; Low & Lynden-Bell 1976; Silk 1976); or (iii) if the fragment is bathed in external radiation (e.g. from neighbouring fragments, stars, or primordial background) with an effective temperature comparable with T (Silk 1976 and Smith & Wright 1975).

The value of f, and the relevant temperature T at which (6) becomes applicable, depend on the track traced out in the T–ρ plane by the collapsing clouds before they become opaque and therefore on their composition and environment. The crude general argument presented here is obviously no substitute for detailed calculation incorporating realistic cooling rates, but it does perhaps help us to understand various general features of hierarchical fragmentation, some of which have already emerged as consequences of detailed specific calculations:

(i) The fact that M_c appears explicitly in (6) shows why M_F is automatically of stellar order (rather than corresponding to a planetary or galactic mass).

(ii) The final fragment mass M_F is insensitive to the temperature, composition and environment of the collapsing cloud: μ cannot itself vary much, but the

* Note that even though M_F is insensitive to the temperature T at which opacity intervenes, the density at which the fragmentation stops depends steeply on T ($\propto T^{5/2}$). At higher temperatures, the collapse and fragmentation can continue to a higher ρ before opacity intervenes, and this leads to a net dependence of the minimum Jeans mass—equation (1)—on merely the fourth root of T.

† It is important to distinguish κ_{abs} from κ_{scat}: a high value of κ_{abs} implies efficient cooling, and thus tends to lower M_F; but a high κ_{scat} leads to trapping of radiation and (from (7)) *inhibits* fragmentation. Low & Lynden-Bell (1976) show that M_F depends on a low power of the opacity. Their argument is closely analogous to that given here *if* the opacity is purely absorptive, but would need modification if $\kappa_{scat} \gtrsim \kappa_{abs}$.

differences between the masses calculated by different authors result mainly from the dependence of the cooling (and therefore the relevant T and f) on the assumed composition.

(iii) This suggests that the stellar mass spectrum may be controlled primarily by the dynamical details of the fragmentation, and that further work on star formation should perhaps focus on this aspect of the problem.

(iv) The first generation of stars, forming (perhaps at large redshifts) in protogalaxies composed only of H and He, need not necessarily be massive.

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