Opaque Trading, Disclosure and Asset Prices: Implications for Hedge Fund Regulation

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Abstract

We investigate the effect of ambiguity about hedge fund strategies on market efficiency and aggregate welfare. We model some traders (mutual funds) as facing ambiguity about the equilibrium trading strategies of other traders (hedge funds). This ambiguity limits the ability of mutual funds to infer information from prices and has negative effects on market performance. We use this analysis to investigate the implications of regulations that affect the amount of ambiguity about hedge fund strategies or the cost of operating a hedge fund. Our analysis demonstrates how regulations affect asset prices and welfare through their influence on opaque trading.

Keywords: Opaque trading; asset prices; welfare; regulation.

JEL Classifications: G14, G12, G11, D82

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1 Introduction

Hedge funds control nearly two trillion dollars of investable funds. Although this amount is small relative to the total assets under management by mutual funds (Vanguard alone has about $1.6 trillion), it is large enough to move markets, and in the aftermath of the financial crises, big enough to concern regulators. Perhaps more important than their size, however, is the information that hedge funds have about their own trading strategies and about future asset values. Unlike mutual funds, which are required by the SEC to disclose publicly their holdings and positions on a regular basis, hedge funds face much more limited disclosure requirements. Current rules require hedge funds with $1 billion or more under management to file quarterly reports with regulators describing their trading and portfolio positions, but these reports are not disclosed to the public. There is now on-going debate as to whether hedge funds should face the same public disclosure requirements as mutual funds.[1]

Hedge funds offer a number of arguments for their need for opacity. Unlike mutual funds which are offered to the public, hedge funds are restricted to private investors. Disclosure rules for mutual funds facilitate monitoring by both current and future investors, but such monitoring for hedge funds can be accomplished by (less costly) internal disclosures. Moreover, forcing public disclosure by hedge funds could allow others to infer their trading strategies and information, inducing mimicking trade which could erode the profitability of strategies. In addition, the large scale of hedge fund positions means that any position adjustments impose liquidity demands on the market. Knowledge of fund positions could allow others to exploit this need for liquidity, thereby also reducing the profitability of hedge fund trading. For a variety of reasons, therefore, it is in the best interest of hedge funds to keep their trading secret and in the best interest of others in the market to discover as much as possible about their trading in order to profit from it.[2]

In this research, we investigate the implications of hedge fund regulation for market efficiency and welfare. Our particular focus is on rules requiring hedge funds to disclose

[1] As discussed in Aragon and Nanda (2012) Title IV of Dodd Frank directs the SEC to impose regulatory requirements on hedge funds as it deems necessary or appropriate in the public interest or the assessment of system risk. How, exactly, such requirements are to be implemented remains a subject of considerable controversy.

[2] Aragon, Hertzel and Shi (2011) find that confidential hedge fund positions earn significant, positive abnormal returns and that this profitability is an important determinant of the decision by hedge funds to maintain confidentiality.

their trading and regulations that change hedge funds' cost of doing business.[3] No useful conclusions can be reached about the impact of hedge fund regulation until we understand the implications of opacity of trading strategies for both individual investors and the market. To address this issue, we construct a rational expectations model in which traders all have private information, but elements of the trading strategies employed by some traders are not known to other traders. We model this uncertainty by assuming that some traders (mutual funds) view the trading strategies of other traders (hedge funds) as ambiguous. This ambiguity limits the ability of mutual funds to infer the information held by hedge funds and it affects both the profitability of hedge funds and market performance. After describing the rational expectations equilibrium (with ambiguity) in which the fraction of traders who are hedge funds is fixed, we find the equilibrium fraction of hedge funds by allowing institutions to choose between operating as a mutual fund or hedge fund.[4]

One important aspect of our analysis, in addition to the inclusion of ambiguity about hedge fund strategies, is that we give hedge funds access to private investment opportunities that are not available to mutual funds.[5] These extra investment opportunities could be direct investments in companies, options trading, and asset classes in which mutual funds are not permitted to invest. The correlation between returns on these additional investment opportunities and returns on the stock market are also the source of ambiguity about the hedge funds' trading strategies in equity markets. This correlation allows hedge funds to diversify in ways unavailable to mutual funds, and it results in hedge funds' effective risk aversion in the equity market being lower than that of mutual funds to an extent that is unknown by mutual funds. The ambiguity that mutual funds face about this effective risk aversion interferes with their ability to infer the private information held by hedge funds,

[3] Of course, regulations that impact hedge funds may have other implications for the economy that are outside of our analysis. One interesting possibility would be to consider regulations that address the systemic risk created by the perceived interdependence among hedge fund strategies. Analyzing this impact would require modelling systemic risk which goes beyond the scope of this paper.

[4] We do not consider competition between hedge funds and mutual funds arising from competition for managers or alternative compensations structures. For analysis of these issues see Deuskar, Pollet, Wang and Zhang (2011).

[5] Note that these extra investment opportunities make hedge funds socially valuable. Without these opportunities there would be no reason in our model for the existence of hedge funds. It would not be informative to study optimal hedge fund regulation in an environment in which they should not exist as the optimal regulation would simply prohibit them. The social value arising from the extra investment opportunities available to hedge funds is partially offset by the ambiguity that hedge funds introduce, and it is the balance of these two effects that we study in our efficiency and welfare analysis.

and it is thus one source of the attractiveness of becoming a hedge fund.

After describing the full equilibrium, we investigate the implications of changing the amount of ambiguity about hedge fund strategies and changing the cost of operating a hedge fund. We find that increasing the differential cost of operating a hedge fund will decrease the fraction of hedge funds, increase the equity premium, and decrease welfare. Thus, regulatory policies designed to limit the number of hedge funds simply be increasing their cost of doing business seem ill-advised. Regulations designed to reduce the ambiguity about hedge fund strategies have more complex effects. Not surprisingly, lower ambiguity reduces the attractiveness of becoming a hedge fund, and so results in fewer hedge funds in equilibrium. The effect on the equity premium is complex. Reducing ambiguity makes prices more informative and thus tends to reduce the equity premium, but as it reduces the number of hedge funds it reduces the fraction of aggressive traders in the market and this tends to increase the equity premium. Similarly, the effect on welfare is ambiguous. It thus remains an empirical question whether reducing ambiguity has the effects that regulators desire.

Our paper is most closely related to recent papers studying the process of information transmission through prices in the presence of ambiguity (Caskey, 2009; Condie and Ganguli, 2011; Mele and Sangiorgi, 2011; Ozsoylev and Werner, 2011).[6] Caskey (2009) uses Klibanoff, Marinacci, and Mukerji's (2005) smooth ambiguity aversion to demonstrate that ambiguity-averse investors may prefer aggregate information even when the aggregate signal is not a sufficient statistic for its components, thereby causing prices to underreact to public information. Condie and Ganguli (2011) demonstrate the existence and robustness of partially revealing rational expectations equilibria in economies with ambiguity-averse preferences. Mele and Sangiorgi (2011) and Ozsoylev and Werner (2011) analyze noisy rational expectations equilibria in economies with Gilboa and Schmeidler's (1989) max-min ambigu-

[6] Ambiguity aversion has been extensively used to explain many interesting phenomena in financial markets, including, inter alia, portfolio choice and non-market participation (Cao, Han, Hirshleifer, and Zhang, 2011; Cao, Wang, and Zhang, 2005; Dow and Werlang, 1992; Easley and O'Hara, 2009; Garlappi, Uppal, and Wang, 2007; Epstein and Schneider, 2010; Illeditsch, 2011; Uppal and Wang, 2003), the equity premium puzzle (Epstein and Wang, 1994; Ui, 2011), learning (Epstein and Schneider, 2007, 2008; Ju and Miao, 2012), liquidity (Caballero and Krishnamurthy, 2008; Routledge and Zin, 2009), market selection and survival (Condie, 2008), and microstructure (Easley and O'Hara, 2010a, 2010b). Several recent studies empirically investigate the impact of ambiguity with real market data (e.g., Anderson, Ghysels, and Juergens, 2009; Leippold, Trojani, and Vanini, 2008).

ity aversion preferences and study implications of ambiguity for volume, liquidity and price volatility.

Our paper is different from and thus complements these papers in two ways. First, the source of ambiguity is different. In these studies, investors are ambiguous about asset payoffs, or *exogenous* and public fundamental signals, such as earnings reports and analyst forecasts. In contrast, in our economy, investors are not ambiguous about fundamentals, but instead are ambiguous about the trading strategies of some sophisticated traders, whose trading in equilibrium causes prices to become an *endogenous* ambiguous signal regarding the asset payoff. Second, these studies, except Condie and Ganguli (2011), have to rely on noisy trading to prevent a fully revealing price system. The unmodeled noisy trading renders a complete welfare analysis impossible. Given that one of our main focuses is on the implications of regulation, welfare analysis is particularly important. In this sense, our model without exogenous noisy trading is more useful for the questions we ask.

In Section 2 we provide a brief overview of ambiguity aversion and motivate our use of ambiguity to model opaque trading of hedge funds. We then describe our model with ambiguity about hedge fund strategies in Section 3. In Section 4, we first compute the equilibrium in the asset market with a fixed fraction of hedge funds. We then use this analysis to determine the expected profits of hedge funds and mutual funds, and then we determine the equilibrium fraction of hedge funds. In Section 5 we determine the effect of various regulations on the equilibrium fraction of hedge funds, on the equity premium and on social welfare. Section 6 discusses the roles of our assumptions in driving our main results. Section 7 concludes.

2 Ambiguity and Hedge Fund Industry

Standard asset pricing models assume that investors behave according to expected utility preferences (EU). In the von Neumann-Morgenstern expected utility theory, individuals have preferences over, and make decisions between, objective distributions. In the Savage expected utility theory, individuals consider subjective distributions of payoffs which are derived from their preferences over stochastic consumption streams. In both cases, however, asset markets applications imply that each investor acts as if he or she has a single prior over the distributions of portfolio payoffs. In most applications of subjective expected utility, investors are additionally assumed to hold beliefs coinciding with the correct objective probabilities, which is usually justified with the rational expectations hypothesis.

There is much direct experimental evidence suggesting that individuals may not act as if they have a single prior. The most notable example is the Ellsberg Paradox (Ellsberg, 1961). In a simple version of the Ellsberg thought experiment, an individual is given an opportunity to bet on the draw of a ball from one of two urns. Urn one has fifty red and fifty black balls. Urn two has one hundred balls, which are some mix of red and black. First, subjects are offered a choice between two gambles: $1 if the ball drawn from urn one is red and nothing if it is black or $1 if the ball drawn from urn two is red and nothing if it is black. If a subject chooses the first gamble and has a prior on urn two, the predicted probability of red in urn two is less than 0.5. Next, subjects are offered a choice between two new gambles: $1 if the ball drawn from urn one is black and nothing if it is red or $1 if the ball drawn from urn two is black and nothing if it is red. If a subject again chooses the first gamble and has a prior on urn two the predicted probability of black in urn two is less than 0.5. This cannot be, so an individual making these choices does not act as if they have only one prior on urn two.

The Ellsberg Paradox, and other difficulties with the standard theory (see Becker and Brownson, 1964; Slovic and Tversky, 1974; Bossaerts et al., 2010), led decision theorists to revise the standard EU model in order to produce a decision theory consistent with the observed behavior. Most notably, Gilboa and Schmeidler (1989) weaken the independence axiom of the EU theory and produce a representation with a set of distributions over payoffs rather than a single distribution. The axioms also imply that the decision maker evaluates any act according to the minimum expected utility it yields.[7] In the Ellsberg framework, this model implies that the individual acts as if there is a set of distributions for urn two that includes a distribution in which the probability of red is less than 0.5 and a distribution in which the probability of black is less than 0.5. Since he acts as if evaluates each act according to its minimum expected utility, he will never chose urn two as in his pessimistic view it will be unlikely to pay off. In the recent finance literature, ambiguity aversion has

[7]The Gilboa and Schmeidler model has been generalized to allow for the possibility that the decision maker is not so pessimistic as to select an act that maximizes the minimum expected utility (e.g., Ghirardato, Maccheroni, and Marinacci, 2004; Klibanoff, Marinacci, and Mukerji, 2005).

also been successfully applied to various financial topics (see footnote 6).

Applying the ambiguity approach to investors beliefs about the opaque trading strategies of hedge funds seems particularly appropriate as ambiguity occurs when decision makers lack enough information or experience to assess the relevant distribution over payoffs (e.g., Heath and Tversky, 1991; Epstein and Schneider, 2008). This applies to investors beliefs about the hedge fund industry as historically, hedge funds have been exempted from many registration and reporting requirements that apply to other investment entities such as mutual funds. The justification for this exemption is that "qualified" hedge funds investors are believed to be able to make an informed decision about investment decisions without relying on regulatory oversight. For example, in the U.S., mutual funds are required to register with the SEC, while hedge funds historically were not required to register (prior to the 2010 Dodd-Frank Wall Street Reform Act). As a result of the light regulation, hedge funds have very limited transparency, and it's difficult for investors not in hedge funds to know what's going on in this industry or to assess the relevant distributions of payoffs generated by their sophisticated investment strategies.[8] So, in this sense, our approach based on ambiguity is well-suited to modelling trading opacity.[9]

3 The Model

3.1 Environment

In the economy we analyze two assets are traded: a risk-free asset, the bond, which has a constant value of 1; and, a risky asset, the stock, which has a price of $\tilde{p}$ per unit and an

[8] Stulz (2007) stated that it can easily cost \$50,000 for an investor to properly understand the risks of investing a particular hedge fund.

[9] Although we view our ambiguity approach to be the most natural and realistic approach to modeling opacity about hedge funds strategies, we do not rule out other possible models of opaque trading. In particular, we also considered an asymmetric information model based on expected utility à la Wang (1994). In Wang's model, informed traders possess private information not only about the return of a risky asset, which both informed and uninformed traders can trade, but also about the expected return of a private investment. One can loosely interpret the informed traders in this setting as hedge funds and information asymmetry between informed and uninformed traders regarding informed traders' private investment as opacity about hedge funds' trading. However, this approach to modeling opacity is fundamentally different from our ambiguity approach. In particular, in this setting, the information contained in prices doesn't vary with the size of hedge fund population, which in turn can cause the benefit function of switching trader types to be upward sloping, generating unrealistic equilibrium predictions which differ from those generated by our ambiguity approach.

uncertain future value $\tilde{v}$. We assume that

$$\tilde{v} = \bar{v} + \tilde{\theta}_T + \tilde{\theta}_O + \tilde{\varepsilon}, \tag{1}$$

where $\bar{v} > 0$, $\tilde{\theta}_T \sim N(0, \sigma_{\theta T}^2)$ with $\sigma_{\theta T} > 0$, $\tilde{\theta}_O \sim N(0, \sigma_{\theta O}^2)$ with $\sigma_{\theta O} > 0$, $\tilde{\varepsilon} \sim N(0, \sigma_{\varepsilon}^2)$ with $\sigma_\varepsilon > 0$, and $\tilde{\theta}_T$, $\tilde{\theta}_O$ and $\tilde{\varepsilon}$ are mutually independent. Random variables $\tilde{\theta}_T$ and $\tilde{\theta}_O$ represent information that is observable to different traders. One can view this risky asset as a proxy for the stock market.

There is a $[0, 1]$ continuum of rational traders who are initially identical. Each trader is endowed with one share of the stock and zero units of the bond. These traders have constant-absolute-risk-aversion (CARA) utility functions with risk tolerance coefficient 1 defined over their final wealth. Prior to entering the asset market they choose to belong to one of two trader-types: "transparent-traders" and "opaque-traders." By default, they are transparent-traders, and if they want to switch to being opaque-traders, they have to pay a cost $c > 0$. We let μ denote the fraction of opaque-traders. We will interpret transparent-traders and opaque-traders as mutual funds and hedge funds, respectively.[10] The cost c can be interpreted as the cost that hedge funds incur in developing complex trading strategies, which will be introduced shortly, net of the difference between hedge funds and mutual funds cost of reporting and complying with regulations.

When the asset market opens, transparent-traders observe the equilibrium stock price $\tilde{p}$ and the information $\tilde{\theta}_T$, and trade the bond and the stock. Opaque-traders are different from transparent-traders in two ways. First, they have a different information set, i.e., they observe a different signal $\tilde{\theta}_O$ (in addition to the equilibrium price $\tilde{p}$).[11] Second, they have an enlarged investment opportunity set (which is developed from the cost c that they spent), i.e., in addition to the bond and the stock, opaque-traders can access additional investment opportunities that are not available to transparent-traders. One can interpret these extra investment opportunities as, for example, venture capital, foreign currencies, options, precious metals, and commodities, in which equity mutual funds typically do not

[10] Although our analysis is cast in terms of hedge funds and mutual funds, its implications can be more general. For example, opaque-traders and transparent-traders can be interpreted as institutional traders and individual traders, as individual traders may not understand how institutions trade.

[11] Opaque-traders do not observe $\tilde{\theta}_T$ and transparent-traders do not observe $\tilde{\theta}_O$.

invest.

3.2 Demand Function of Opaque-Traders

The presence of the additional investment opportunities will cause opaque-traders to behave *as if* they are more risk tolerant than transparent-traders to the extent that the returns on these additional investment opportunities are correlated with the returns on traded assets. That is, we can show that opaque-traders' demand for the stock is

$$D_O(\tilde{p}, \tilde{\theta}_O; k) = \frac{k(\bar{v} + \tilde{\theta}_T + \tilde{\theta}_O - \tilde{p})}{\sigma_\varepsilon^2}, \tag{2}$$

where $k > 1$ is their *effective* risk tolerance in the asset market. In equation (2), we explicitly incorporate k as an argument in the demand function.

To see where equation (2) comes from, let $1 + \tilde{\eta}$ be the (gross) returns on the extra investment opportunities, where $\tilde{\eta} \sim N\left(0, \sigma_\eta^2\right)$ with $\sigma_\eta > 0$. Assume that $\tilde{\varepsilon}$ and $\tilde{\eta}$ are correlated with a coefficient of $\rho \in (-1, 1)$, so that opaque-traders can use these investment opportunities to hedge stock investments. Let D_O and Z be opaque-traders' investment in the stock and the private investment opportunities. Then, given a starting wealth of $(\tilde{p} - c)$ before trade, which is the value of their stock endowment minus the cost of switching type, their future wealth at the end of date 1 is

$$\tilde{W}_O = (\tilde{p} - c) + (\tilde{v} - \tilde{p})\, D_O + Z\tilde{\eta}, \tag{3}$$

and the CARA-normal setup implies that their expected utility is

$$E\left[-\exp\left(-\tilde{W}_O\right)\Big|\, \tilde{p}, \tilde{\theta}_O\right] = -\exp\left[-\left(E(\tilde{W}_O|\tilde{p}, \tilde{\theta}_O) - \frac{1}{2}Var(\tilde{W}_O|\tilde{p}, \tilde{\theta}_O)\right)\right], \tag{4}$$

where $E(\cdot|\tilde{p}, \tilde{\theta}_O)$ and $Var(\cdot|\tilde{p}, \tilde{\theta}_O)$ are the conditional mean and variance operators.

In our economy, opaque-traders can infer the information $\tilde{\theta}_T$ of transparent-traders from prices (see equation (11)), while transparent-traders cannot infer opaque-traders' information $\tilde{\theta}_O$ from prices because of the opaque trading. This distinction between the amount of information that prices reveal to the two types of traders provides a trading advantage to opaque-traders and it is the reason that opaque-traders can have a substantial impact on the market even if they control only a small fraction of total wealth. Since opaque-traders can infer $\tilde{\theta}_T$ from prices we have,

$$E(\tilde{W}_O|\tilde{p},\tilde{\theta}_O) = (\tilde{p}-c) + (\bar{v}+\tilde{\theta}_T+\tilde{\theta}_O-\tilde{p})D_O, \tag{5}$$

$$Var(\tilde{W}_O|\tilde{p},\tilde{\theta}_O) = \sigma_\varepsilon^2 D_O^2 + \sigma_\eta^2 Z^2 + 2\rho\sigma_\varepsilon\sigma_\eta D_O Z. \tag{6}$$

Inserting these moment expressions into the objective function of opaque-traders (equation (4)) and solving their decision problem yields:

$$D_O^* = \frac{E(\tilde{v}|\tilde{p},\tilde{\theta}_O)-\tilde{p}}{(1-\rho^2)\,\sigma_\varepsilon^2} \text{ and } Z^* = -\frac{\rho\left[E(\tilde{v}|\tilde{p},\tilde{\theta}_O)-\tilde{p}\right]}{(1-\rho^2)\,\sigma_\varepsilon\sigma_\eta}. \tag{7}$$

Letting

$$k = \frac{1}{1-\rho^2}, \tag{8}$$

we see that the demand function in equation (7) is identical to equation (2).

Although we motivate the increased effective risk tolerance of opaque-traders using an enlarged investment opportunity set, it could equally well come from differences in the institutional environment. For example, because the ability of investors to withdraw their investments from hedge funds is limited, hedge funds tend to have more liquidity than mutual funds. As a consequence, hedge funds will appear to be less risk averse in the asset market than mutual funds. So, in subsequent text, we work with this more general notation k in determining the investment opportunity structure.

3.3 Trading Opacity

To capture the "opacity" of the trading strategy of opaque-traders, we assume that transparent-traders perceive ambiguity about the effective risk tolerance parameter k of opaque-traders; that is, transparent-traders perceive

$$k \in [\underline{k},\bar{k}] \text{ with } 1 \leq \underline{k} \leq \bar{k}, \tag{9}$$

and cannot assign a probability on this set. At a more micro-level, this trading strategy ambiguity reflects transparent-traders' uncertainty regarding the structure of investment opportunity set (for example, the parameter ρ in the previous subsection) or liquidity conditions of opaque-traders.

This ambiguity will cause stock prices, which incorporate opaque-traders' information $\tilde{\theta}_O$ through their trading, to become an ambiguous signal about the stock payoff $\tilde{v}$. We use a boldface $\mathbf{k} \geq 1$ to denote the true value of k, and assume that $\underline{k}$ and $\bar{k}$ are generated as follows:

$$\underline{k} = \mathbf{k} - \Delta k \text{ and } \bar{k} = \mathbf{k} + \Delta k, \tag{10}$$

where $\Delta k \geq 0$ is an exogenous parameter which determines the amount of ambiguity faced by transparent-traders.

The timeline of the model is given by Figure 1. Initially, each trader is endowed with one share of the stock and decides whether to pay c to switch from being a transparent-trader to an opaque-trader in the asset market. Before the asset market opens, transparent-traders observe information $\tilde{\theta}_T$, and opaque-traders observe information $\tilde{\theta}_O$ and learn the structure of the additional investment opportunities (and hence, they realize their new effective risk tolerance parameter k). Then the asset market opens and all traders trade the bond and the stock at prices 1 and $\tilde{p}$, respectively; opaque-traders also invest in the additional investment opportunities. After trading ends, traders receive the payoffs on their portfolios and consume.

[INSERT FIGURE 1 HERE]

To summarize, the tuple

$$\mathcal{E} = (\sigma_{\theta T}, \sigma_{\theta O}, \sigma_{\varepsilon}, \mathbf{k}, \Delta k, c)$$

defines an economy. We are interested in the implications for asset prices, trader distributions, and welfare of changes in the parameters Δk and c, which are in turn influenced by regulatory polices.

4 Equilibrium

We consider rational expectations equilibriua (REE). We start by analyzing trading behavior and prices in the financial market, given some fraction μ of opaque-traders. We then analyze traders' decisions about whether to become opaque or stay transparent, and determine the equilibrium fraction μ^* of opaque-traders. In the text, we focus on economies with interior $\mu^* \in (0,1)$; i.e. ones in which both transparent- and opaque-traders are active in the asset market, as this is the empirically relevant case. Analysis of less interesting economies in which $\mu^* = 0$ or $\mu^* = 1$ is provided in Appendix B.

4.1 Financial Market Equilibrium

4.1.1 Characterization

For any given k, we assume that transparent-traders rationally conjecture that the price function is:

$$\tilde{p} = \bar{v} + \tilde{\theta}_T + \tilde{\theta}_O - f(k), \tag{11}$$

where the function $f(k)$ will be endogenously determined in equilibrium. We will show that $f(k)$ is the transparent-traders' *perceived* equity premium for a given belief k: $E_k(\tilde{v} - \tilde{p})$, where $E_k(\cdot)$ denotes an expectation using the belief k.

Since transparent-traders are ambiguous about k, they view the stock price $\tilde{p}$ as an ambiguous signal about $\tilde{\theta}_O$. Thus, the decision problem for transparent-traders is

$$\max_{D_T} \min_{k \in [\underline{k}, \bar{k}]} E_k \left(-e^{-\tilde{W}_T} \middle| \tilde{p}, \tilde{\theta}_T \right), \tag{12}$$

subject to the standard budget constraint:

$$\tilde{W}_T = \tilde{p} + D_T (\tilde{v} - \tilde{p}), \tag{13}$$

where in equation (12), $E_k(\cdot|\tilde{p}, \tilde{\theta}_T)$ denotes a conditional expectation given the belief k, and in equation (13), the first term, $\tilde{p}$, is the value of transparent-traders' stock endowment, and D_T is their demand for the stock. It follows immediately from our normal distribution structure, that the above decision problem is equivalent to:

$$\max_{D_T} \min_{k\in[\underline{k},\bar{k}]} \left(\left[E_k(\tilde{v}|\tilde{p},\tilde{\theta}_T) - \tilde{p} \right] D_T - \frac{1}{2} Var_k(\tilde{v}|\tilde{p},\tilde{\theta}_T) D_T^2 \right). \tag{14}$$

By equation (11), $E_k(\tilde{v}|\tilde{p},\tilde{\theta}_T)$ and $Var_k(\tilde{v}|\tilde{p},\tilde{\theta}_T)$, the conditional moments of $\tilde{v}$ taken under a particular belief k, are given by:

$$E_k(\tilde{v}|\tilde{p},\tilde{\theta}_T) = \tilde{p} + f(k) \text{ and } Var_k(\tilde{v}|\tilde{p},\tilde{\theta}_T) = \sigma_\varepsilon^2.$$

and as a result, for a fixed investment D_T, we have:

$$\left[E_k(\tilde{v}|\tilde{p},\tilde{\theta}_T) - \tilde{p} \right] D_T - \frac{1}{2} Var_k(\tilde{v}|\tilde{p},\tilde{\theta}_T) D_T^2 = -\frac{1}{2}\sigma_\varepsilon^2 D_T^2 + f(k) D_T.$$

Define the minimum and maximum values that $f(\cdot)$ takes on as

$$f_{\min} \triangleq \min_{k\in[\underline{k},\bar{k}]} f(k) \text{ and } f_{\max} \triangleq \max_{k\in[\underline{k},\bar{k}]} f(k). \tag{15}$$

Then the objective function of a transparent-trader can be written as

$$\min_{k\in[\underline{k},\bar{k}]} \left(\left[E_k(\tilde{v}|\tilde{p},\tilde{\theta}_T) - \tilde{p} \right] D_T - \frac{1}{2} Var_k(\tilde{v}|\tilde{p},\tilde{\theta}_T) D_T^2 \right) = \begin{cases} -\frac{1}{2}\sigma_\varepsilon^2 D_T^2 + f_{\min} D_T, & \text{if } D_T \geq 0, \\ -\frac{1}{2}\sigma_\varepsilon^2 D_T^2 + f_{\max} D_T, & \text{if } D_T < 0, \end{cases} \tag{16}$$

Thus a transparent-trader's demand function is:

$$D_T(\tilde{p},\tilde{\theta}_T) = \begin{cases} \frac{f_{\min}}{\sigma_\varepsilon^2}, & \text{if } f_{\min} > 0, \\ 0, & \text{if } f_{\min} \leq 0 \leq f_{\max}, \\ \frac{f_{\max}}{\sigma_\varepsilon^2}, & \text{if } f_{\max} < 0. \end{cases} \tag{17}$$

Using the two demand functions we can determine the functional form of $f(k)$ from the market clearing condition

$$\mu D_O(\tilde{p},\tilde{\theta}_O;k) + (1-\mu) D_T(\tilde{p},\tilde{\theta}_T) = 1. \tag{18}$$

The transparent-trader's demand $D_T(\tilde{p}, \tilde{\theta}_T)$ can take the three possible values in equation (17), so we will discuss market clearing case by case. First, suppose $D_T(\tilde{p}, \tilde{\theta}_T) = \frac{f_{\min}}{\sigma_\varepsilon^2}$. Substituting this expression into the market clearing condition (equation (18)), we can solve for the equilibrium price $\tilde{p}$:

$$\tilde{p} = \bar{v} + \tilde{\theta}_T + \tilde{\theta}_O - \frac{-(1-\mu) f_{\min} + \sigma_\varepsilon^2}{\mu k}.$$

Comparing this price function with the conjectured price function in equation (11), we see that if the transparent-traders' conjecture is rational, then:

$$f(k) = \frac{-(1-\mu) f_{\min} + \sigma_\varepsilon^2}{\mu k}. \tag{19}$$

In the current case in which $D_T(\tilde{p}, \tilde{\theta}_T) = \frac{f_{\min}}{\sigma_\varepsilon^2}$ in equation (17), we must have $f_{\min} > 0$, and hence $f(k) > 0$ for all possible k. So $f(k)$ achieves its minimum at $\bar{k}$ in equation (19); that is,

$$f_{\min} = \frac{-(1-\mu) f_{\min} + \sigma_\varepsilon^2}{\mu \bar{k}} \Rightarrow f_{\min} = \frac{\sigma_\varepsilon^2}{1-\mu+\mu\bar{k}}. \tag{20}$$

Substituting this value back into equation (19) and solving for the function f we have

$$f(k) = \frac{\bar{k}\sigma_\varepsilon^2}{k\left(1-\mu+\mu\bar{k}\right)}. \tag{21}$$

Second, suppose that $D_T(\tilde{p}, \tilde{\theta}_T) = 0$. Then, the market clearing condition implies $f(k) = \frac{\sigma_\varepsilon^2}{\mu k}$. However, this means that $f_{\min} > 0$, which is inconsistent with the case $D_T(\tilde{p}, \tilde{\theta}_T) = 0$, as from equation (17) we see that $D_T(\tilde{p}, \tilde{\theta}_T) = 0$ implies that $f_{\min} \leq 0$. Similarly, we can eliminate the case in which the transparent traders' demand is $\frac{f_{\max}}{\sigma_\varepsilon^2}$.

The argument above is summarized in the following proposition which describes the rational expectations equilibrium.[12]

Proposition 1 *Suppose* $0 < \mu < 1$. *There exists a REE in which the price function is*

$$\tilde{p} = \bar{v} + \tilde{\theta}_T + \tilde{\theta}_O - f(k),$$

[12] Note that this REE price function reveals $\tilde{\theta}_T$ to opaque traders confirming the conjecture made in equation (2).

where

$$f(k) = \frac{\bar{k}\sigma_\varepsilon^2}{k\left(1 - \mu + \mu\bar{k}\right)}.$$

4.1.2 Implications for the Equity Premium

The equity premium of the stock is defined as:

$$EP \triangleq \mathbf{E}(\tilde{v} - \tilde{p}) = \frac{(\mathbf{k} + \Delta k)\,\sigma_\varepsilon^2}{\mathbf{k}\left[(1-\mu) + \mu(\mathbf{k} + \Delta k)\right]}, \tag{22}$$

where the boldfaced $\mathbf{E}(\cdot)$ indicates that the expectation is taken under the true value $\mathbf{k}$ of the parameter k, and where the second equality follows from equation (21) and the definition of $\bar{k}$ in equation (10).

Clearly, for any fixed $\mu \in (0,1)$, we have:

$$\frac{\partial EP}{\partial \Delta k} > 0;$$

that is, a decrease in the amount of ambiguity will decrease the equity premium. This occurs because in equilibrium, transparent-traders must hold some of the stock, and if they are less uncertain about what opaque-traders are doing, for any given price, they are more optimistic about the payoff of the stock and hence require a lower equity premium to be compensated for holding it. Formally we have the following corollary.

Corollary 1 *When $\mu \in (0,1)$ is fixed, reducing the amount of ambiguity will lower the equity premium.*

This result has important policy implications. Requiring hedge funds to disclose their positions more frequently can be interpreted in our model as reducing Δk, as more frequent disclosure helps the market to understand what is going on in the black box of the hedge fund industry. Note that disclosure does not have to be interpreted as eliminating ambiguity, instead disclosure may reduce ambiguity, that is reduce Δk, without necessarily driving it to zero.[13] Thus, implementing this practice would tend to lower the equity premium in the

[13] Specifically, imagine the following dynamic setting where the effective risk tolerance k_t of opaque-traders evolves according to the following process: $k_t = ak_{t-1} + b_t$. Here, $a \in (0,1)$ is a known constant, and b_t is an ambiguous innovation term to transparent-traders: they perceive $b_t \in \left[\underline{b}, \bar{b}\right]$ and cannot assign a probability

short run when the number of hedge funds active in the market has not been affected (i.e., when μ is fixed).

4.1.3 The Ex Post Performance of Opaque-Traders

Empirical studies find that actively managed stock mutual funds as a group do not perform better than the stock market, while hedge funds beat mutual funds after adjusting for beta risk (i.e., the risk arising from exposure to common market movements).[14] In this section we show that this difference in returns occurs in equilibrium in our model. This is no surprising as hedge funds managers in our economy have an information advantage and are actively developing additional investment opportunities which produce alpha returns.

When we interpret the tradable risky asset as the whole stock market, the return on the market portfolio in excess of the interest rate (which is normalized to 1 in our model) is

$$\tilde{R}_M = \frac{\tilde{v}}{\tilde{p}} - 1. \tag{23}$$

Substituting the relevant branch of the transparent-traders demand function, that is, $D_T(\tilde{p}, \tilde{\theta}_T) = \frac{f_{\min}}{\sigma_\varepsilon^2}$, into their budget constraint, and dividing by their initial wealth $\tilde{p}$ shows that the excess return on the equilibrium portfolio of transparent-traders is:

$$\tilde{R}_T = \frac{\tilde{W}_T}{\tilde{p}} - 1 = \frac{f_{\min}}{\sigma_\varepsilon^2} \tilde{R}_M. \tag{24}$$

An econometrician with access to historical data would compute the beta of transparent-traders' portfolio as:

$$\boldsymbol{\beta}_T \triangleq \frac{\mathbf{Cov}\left(\tilde{R}_T, \tilde{R}_M \middle| \tilde{p}\right)}{\mathbf{Var}\left(\tilde{R}_M \middle| \tilde{p}\right)} = \frac{f_{\min}}{\sigma_\varepsilon^2}, \tag{25}$$

where the boldfaced $\mathbf{Cov}(\cdot|\tilde{p})$ and $\mathbf{Var}(\cdot|\tilde{p})$ indicate that the conditional moments are taken

on this set. If there is no disclosure about opaque-traders' previous positions, then transparent-traders have no idea of the realizations of k_{t-j}, and as a result, at date t, transparent-traders would view k_t to be possible on the whole set of $\left[\frac{\underline{b}}{1-a}, \frac{\bar{b}}{1-a}\right]$. If, in contrast, opaque-traders disclose their positions with one period lag, then transparent-traders can use equations (2) and (11) to back out k_{t-1}, so that at date t, their ambiguity set about k_t shrinks to a smaller set $\left[k_{t-1} + \underline{b}, k_{t-1} + \bar{b}\right] \subset \left[\frac{\underline{b}}{1-a}, \frac{\bar{b}}{1-a}\right]$.

[14] See Stulz (2007) for a survey on the performance of hedge funds and mutual funds.

under the true value $\mathbf{k}$ of the parameter k. We assume that the beta is computed conditional on price $\tilde{p}$ because in practice, price information is indeed available to econometricians.

Thus, by equations (23)-(25), the market-adjusted returns on transparent-traders' equilibrium portfolio is

$$\boldsymbol{\alpha}_T \triangleq \mathbf{E}(\tilde{R}_T|\tilde{p}) - \boldsymbol{\beta}_T \mathbf{E}(\tilde{R}_M|\tilde{p}) = 0.$$

This result is intuitive: since transparent-traders hold only the risk-free bond with a zero rate of return and the market portfolio, their portfolio returns do not beat the market.

Inserting the optimal investments in the tradable assets and the additional investment opportunities (equation (7)) into the opaque-traders' budget constraint (equation (3)), and dividing by their total invested amount $(\tilde{p}-c)$ show that the excess return on the equilibrium portfolio of opaque-traders is:

$$\tilde{R}_O = \frac{\tilde{W}_O}{\tilde{p}-c} - 1 = \frac{1}{\tilde{p}-c}\left[\frac{(\tilde{\varepsilon}+f(k))\,f(k)}{(1-\rho^2)\,\sigma_\varepsilon^2} - \frac{\rho f(k)}{(1-\rho^2)\,\sigma_\varepsilon\sigma_\eta}\tilde{\eta}\right]. \tag{26}$$

Equations (23) and (26) and the definition of k in equation (8) together imply that an outside econometrician would compute the beta and alpha of opaque-traders' portfolio as:

$$\boldsymbol{\beta}_O \triangleq \frac{\mathbf{Cov}\left(\tilde{R}_O, \tilde{R}_M \middle| \tilde{p}\right)}{\mathbf{Var}\left(\tilde{R}_M \middle| \tilde{p}\right)} = \frac{\tilde{p}}{\tilde{p}-c}\frac{EP}{\sigma_\varepsilon^2}, \tag{27}$$

$$\boldsymbol{\alpha}_O \triangleq \mathbf{E}(\tilde{R}_O|\tilde{p}) - \boldsymbol{\beta}_O \mathbf{E}(\tilde{R}_M|\tilde{p}) = \frac{\mathbf{k}-1}{\tilde{p}-c}\frac{EP^2}{\sigma_\varepsilon^2}, \tag{28}$$

where EP is the equity premium of the stock given by equation (22). Since $\mathbf{k} > 1$, opaque-traders appear to generate positive alphas in our economy, as long as the net wealth $(\tilde{p}-c)$ of opaque traders is positive.

4.2 Trader Distribution Equilibrium

We now analyze trader's choices of whether to remain transparent or pay the cost c and become opaque. Whether a transparent-trader would like to switch depends on the comparison between the expected utility of staying transparent and that of becoming opaque.

Substituting the relevant branch of demand function of transparent-traders, that is,

$D_T(\tilde{p},\tilde{\theta}_T) = \frac{f_{\min}}{\sigma_\varepsilon^2}$, into their budget constraint and objective function shows that the indirect utility of being a transparent-trader in a market with price $\tilde{p}$ is:

$$V_{T1}(\tilde{p},\tilde{\theta}_T) = \min_{k\in[\underline{k},\bar{k}]} E_k\left[-e^{-\left(\tilde{p}+D_T\left(\tilde{p},\tilde{\theta}_T\right)(\tilde{v}-\tilde{p})\right)}\middle|\,\tilde{p},\tilde{\theta}_T\right] = -\exp\left[-\left(\tilde{p}+\frac{f_{\min}^2}{2\sigma_\varepsilon^2}\right)\right], \tag{29}$$

where the second equality follows from equation (16). Given the recursive multiple-priors utility representation, the utility of staying transparent is

$$V_{T0} = \min_{k\in[\underline{k},\bar{k}]} E_k\left[V_{T1}(\tilde{p},\tilde{\theta}_T)\right] = -\exp\left[-\left(\bar{v}-f_{\max}+\frac{f_{\min}^2}{2\sigma_\varepsilon^2}-\frac{\sigma_{\theta T}^2+\sigma_{\theta O}^2}{2}\right)\right], \tag{30}$$

where the second equality follows from equation (29) and the equilibrium price function in Proposition 1.

Similarly, inserting the optimal investments of an opaque-trader (equation (7)) into equation (4), and using the moment expressions of equations (5)-(6) and the definition of k in equation (8), we have the indirect utility of an opaque-trader in a market with price $\tilde{p}$ as

$$V_{O1}(\tilde{p},\tilde{\theta}_O;k) = -\exp\left[-\left(\tilde{p}-c+\frac{k}{2\sigma_\varepsilon^2}\left(\bar{v}+\tilde{\theta}_T+\tilde{\theta}_O-\tilde{p}\right)^2\right)\right], \tag{31}$$

and the ex ante expected utility of becoming opaque is

$$V_{O0} = \min_{k\in[\underline{k},\bar{k}]} E_k\left[V_{O1}(\tilde{p},\tilde{\theta}_O;k)\right].$$

Using the equilibrium price function in Proposition 1, we have:

$$V_{O0} = -\exp\left(-\left[\bar{v}-c+\min_k\left[-f(k)+k\frac{[f(k)]^2}{2\sigma_\varepsilon^2}\right]-\frac{\sigma_{\theta T}^2+\sigma_{\theta O}^2}{2}\right]\right). \tag{32}$$

Thus, by equations (30) and (32), the benefit of switching from being a transparent-trader to an opaque-trader as a function of the fraction μ of opaque-traders is:

$$B(\mu) \triangleq \min_k\left[-f(k)+k\frac{[f(k)]^2}{2\sigma_\varepsilon^2}\right]-\left(-f_{\max}+\frac{f_{\min}^2}{2\sigma_\varepsilon^2}\right). \tag{33}$$

In order for an interior fraction $\mu^* \in (0,1)$ of opaque-traders to be an equilibrium, it must be that every transparent-trader is indifferent between becoming opaque versus remaining transparent, i.e., $B(\mu^*) = c$. Note that, in equilibrium, the cost c of becoming an opaque-trader just balances the excess returns achieved by opaque-traders. In Appendix A1 we show that such an equilibrium exists when c takes values in a range whose size is positively associated with the degree of ambiguity of transparent-traders about the trading behavior of opaque-traders. In addition, we show that the benefit function B is decreasing in μ, that is, increasing the fraction of opaque-traders in the market will decrease the benefit to a transparent-trader of becoming opaque. Thus, any interior fraction of opaque-traders $(0 < \mu^* < 1)$ must be unique.

Formally, we have the following proposition regarding equilibrium fraction μ^* of opaque-traders (The proof is provided in Appendix A1).

Proposition 2 *Let* $\underline{c} \triangleq \left(\frac{1}{\underline{k}} - \frac{1}{\bar{k}^2}\right)\frac{\sigma_\varepsilon^2}{2}$, $\bar{c} \triangleq \left(\min\left\{\frac{\bar{k}^2}{\underline{k}} - 1, \frac{2\bar{k}}{\underline{k}} + \bar{k} - 3\right\}\right)\frac{\sigma_\varepsilon^2}{2}$. *When* $\underline{c} < c < \bar{c}$, *there exists a unique interior trader distribution equilibrium* $0 < \mu^* < 1$. *In addition, the size of the range* $(\underline{c}, \bar{c})$ *increases with the degree of ambiguity; that is,* $\frac{\partial(\bar{c} - \underline{c})}{\partial \Delta k} > 0$.

5 Regulation, Prices and Welfare

In this section, we discuss the implications of changes in the cost, c, of becoming opaque and changes in the extent of ambiguity, Δk, for the equilibrium trader distribution μ^*, the equity premium EP^*, and welfare. We focus on changes in c and Δk because these parameters relate to current policy debates. The recent SEC registration requirement under the Dodd-Frank Act for hedge funds can be understood to imply an increase in the cost c of operating a hedge fund.[15] The suggestion that hedge funds disclose their positions to the public (with some delay) implies a decrease in Δk. Note that for our focused economies with interior $\mu^* \in (0, 1)$, welfare can be measured by the expected utility of transparent-traders, since all traders are ex ante identical and in equilibrium they all have the same welfare.

[15] See Sjostrom (2011) for a discussion on the history of hedge funds registration with SEC.

5.1 Implication of Registration: c

In this subsection we examine the impact of increasing c while keeping Δk fixed. We begin by analysing the effect on the equilibrium fraction μ^* of opaque-traders, which is determined by the condition $B(\mu^*) = c$. Note that, the benefit, $B(\cdot)$, of switching types is affected only by ambiguity parameters $\underline{k}$ and $\bar{k}$, and not by the cost c. As a result, an increase in cost c will reduce the incentive of transparent-traders to become opaque, leading to fewer opaque traders, i.e. a lower μ^*. That is, if c increases μ^* must decrease to restore the equality $B(\mu^*) = c$.

In Figure 2 we use a numerical example to illustrate this result. We take one period as one year and suppose the cost increases from 0.05 to 0.1, while other relevant parameter values are fixed as follows: $\sigma_\varepsilon^2 = 0.04$, $\mathbf{k} = 2$ and $\Delta k = 1$. The upper panel plots the benefit function B and various horizontal cost functions. For any cost, the equilibrium fraction μ^* of opaque-traders is determined by the intersection of function B and the horizontal cost function. We see that increasing c from 0.05 to 0.1 decreases μ^* from 0.39 to 0.07.

[INSERT FIGURE 2 HERE]

The equilibrium equity premium is given by equation (22) evaluated at the equilibrium μ^*:

$$EP^* = \frac{\bar{k}\sigma_\varepsilon^2}{\mathbf{k}\left[(1-\mu^*) + \mu^*\bar{k}\right]} = \frac{(\mathbf{k} + \Delta k)\,\sigma_\varepsilon^2}{\mathbf{k}\left[1 + \mu^*(\mathbf{k}-1) + \mu^*\Delta k\right]}. \tag{34}$$

Because opaque-traders have a greater effective risk tolerance coefficient (the term $\mu^*(\mathbf{k}-1)$ in the denominator of the above equation) and do not face ambiguity (the term $\mu^*\Delta k$ in the denominator of the above equation), they trade more aggressively than transparent-traders. Therefore, a reduction in μ^* due to an increase in c will lower asset prices and increase the equity premium. We illustrate this effect by continuing the example illustrated by Figure 2. The middle panel of Figure 2 plots equation (34), and it shows that the drop in μ^* increases the equilibrium equity premium from 3.35% to 5.23%.

We measure welfare by the certainty equivalent of transparent-traders' ex ante equilibrium utility (adjusted by subtracting a constant $\bar{v} - \frac{\sigma_{\theta T}^2 + \sigma_{\theta O}^2}{2}$), which, by equation (30), is

$$WEL^* = -f_{\max}^* + \frac{f_{\min}^{*2}}{2\sigma_\varepsilon^2}, \tag{35}$$

where $f_{\max}^*$ and $f_{\min}^*$ are transparent-traders' *perceived* maximum and minimum equilibrium equity premium (given by equation (15) evaluated at μ^*). Note that these two variables affect welfare differently: an increase in $f_{\max}^*$ reduces welfare, because it reflects transparent-traders' perceived discount of their wealth (term $\tilde{p}$ in equation (29)), while an increase in $f_{\min}^*$ increases welfare, because it captures the perceived benefit resulting from future trading in asset market.[16]

Given the definitions of $f_{\max}^*$, $f_{\min}^*$ and the true equity premium EP^*, we know that $f_{\max}^* = \frac{\mathbf{k}}{\underline{k}} EP^*$ and $f_{\min}^* = \frac{\mathbf{k}}{\bar{k}} EP^*$. Substituting these expressions into equation (35) provides the following relationship between equilibrium welfare WEL^* and the equity premium EP^*:

$$WEL^* = -\frac{\mathbf{k}}{\underline{k}} EP^* + \frac{\left(\mathbf{k}/\bar{k}\right)^2}{2\sigma_\varepsilon^2} EP^{*2}. \tag{36}$$

The first term $-\frac{\mathbf{k}}{\underline{k}} EP^*$ shows that a high EP^* tends to reduce welfare through its impact on wealth, while the second term $\frac{\left(\mathbf{k}/\bar{k}\right)^2}{2\sigma_\varepsilon^2} EP^{*2}$ shows that a high EP^* tends to increase welfare through trading. The net effect of an increase in EP^* on welfare depends on the relative strength of these two terms. In our economy, the first term dominates, so that when the equity premium becomes larger, welfare will decrease.[17] Thus, an increase in the cost of

[16] Note that transparent-traders perceive the equity premium differently at the trading stage and at the trader type decision stage. This difference does not mean that they are time inconsistent, since the belief construction in our economy satisfies Epstein and Schneider (2003)'s "rectangularity condition" and the preference has a recursive structure. Instead, this difference comes from the ambiguity-averse investor's acting conservatively at both stages. At the trading stage, transparent-traders are buyers and a high equity premium will benefit them, since it means that they have bought the stock at a cheaper price. So, to be conservative in their buying position, they act as if the equity premium is at its lowest, $f_{\min}^*$. In contrast, at the stage of deciding whether or not to become opaque, the future uncertain price also affects traders' expected wealth level, and now a high equity premium means a high discount on their endowment, which is consistent with being conservative in evaluating ex ante welfare. Thus, at this stage of their decision problem they act as if the equity premium is at its highest level $f_{\max}^*$. The price function in equation (11) makes the different treatment of equity premiums most striking, because the trading benefit is a constant, so that at the stage of deciding which type of trader to be, transparent-traders only need to consider the impact of prices on their future wealth.

[17] To see this, notice that when Δk is fixed, $\frac{\partial WEL^*}{\partial EP^*} < 0$ if and only if $EP^* < \frac{\bar{k}^2 \sigma_\varepsilon^2}{\mathbf{k}\underline{k}}$, which is true for any $0 < \mu^* < 1$ by equation (34).

becoming an opaque-trader, c, increases the equity premium, EP^*, and reduces welfare, WEL^*. Continuing the example in Figure 2, we see in its lower panel which plots equation (36), that the increase in WEL^* decreases welfare from -0.06 to -0.09.

The results in this subsection are summarized in the following proposition.

Proposition 3 *Suppose* $0 < \mu^* < 1$. *An increase in* c *will decrease the fraction* μ^* *of opaque-traders, increase the equity premium* EP^**, and decrease welfare* WEL^**.*

Figure 3 uses the same numerical example as in Figure 2 to illustrate Proposition 3. Indeed, as we gradually increase c, μ^* decreases, EP^* increases, and WEL^* decreases.

[INSERT FIGURE 3 HERE]

5.2 Implication of Disclosure: Δk

In this subsection we examine the impact of decreasing the extent of ambiguity, Δk, while keeping c fixed. A decrease in Δk will reduce the benefit of switching from being a transparent-trader to being an opaque-trader as the reduced Δk causes price to be a more informative signal about the stock payoff. As a consequence, the equilibrium fraction μ^* of opaque-traders decreases as Δk decreases.

Figure 4 uses an example similar to the economy in Figure 2 to illustrate this result. We decrease Δk from 1 to 0.5, while other relevant parameters are fixed as follows: $\sigma_\varepsilon^2 = 0.04$, $\mathbf{k} = 2$ and $c = 0.05$. The upper panel plots benefit functions B when $\Delta k = 1$ (blue, solid curve) and when $\Delta k = 0.5$ (red, dashed curve). The equilibrium fraction μ^* of opaque-traders is determined by the intersection of the benefit functions and the horizontal cost function. We find that decreasing Δk from 1 to 0.5 shifts the benefit function downward, leading to a decrease in μ^* from 0.39 to 0.06.

[INSERT FIGURE 4 HERE]

The impact of a change in Δk on the equity premium EP^*, is complex because, according to equation (34), Δk affects the equity premium in two opposing ways. First, as captured by the numerator of equation (34) and formalized by Corollary 1, when μ^* is fixed, a decrease in

Δk will directly decrease the equity premium, because the price becomes a more informative signal for transparent-traders. Second, there is an indirect effect which works through the endogenous μ^*: as captured by the denominator of equation (34), a decease in Δk causes μ^* to decrease, which, in turn, tends to increase the equity premium, since opaque-traders trade more aggressively than transparent-traders. The total effect of decreasing Δk on EP^* depends on the relative strength of these two effects.

In Appendix A2 we show that a sufficient condition for the second effect to dominate is:

$$\mu^* > \frac{(\bar{k}/2) - 1}{\bar{k} - 1}. \tag{37}$$

That is, if μ^* is sufficiently large, then decreasing Δk will increase the equity premium. To see why this is true consider the extreme case in which μ^* is close to 1. In this case, since almost all traders are opaque-traders and face no ambiguity, varying Δk has almost no direct effect on the price. However, as μ^* begins to decline due to a decrease in Δk, the indirect effect is still effective, which lowers the price and increases the equity premium.

The range in equation (37) is large. In fact, when $\bar{k} \leq 2$, the lower bound $\frac{(\bar{k}/2)-1}{\bar{k}-1}$ is negative, and thus for all interior values of $\mu^* \in (0, 1)$, a decrease in Δk will raise the equity premium, EP^*. Also, note that the range includes all values of $\mu^* > \frac{1}{2}$, since the lower bound $\frac{(\bar{k}/2)-1}{\bar{k}-1}$ is smaller than $\frac{1}{2}$. So a more frequent disclosure requirement on hedge fund positions may initially lower the equity premium (as predicted by Corollary 1 for a fixed μ), but the equity premium will eventually increase as the fraction of hedge funds adjusts to its new equilibrium value.

We illustrate these effects by continuing our example in Figure 4. The middle panel plots equation (34) when $\Delta k = 1$ (blue, solid curve) and when $\Delta k = 0.5$ (red, dashed curve). Consistent with Corollary 1, a decrease in Δk shifts the curve downward, showing that when μ^* is fixed at its initial value of 0.39, decreasing Δk will reduce the equity premium EP^*. However, decreasing Δk also reduces μ^* from 0.39 to 0.06, and this decreased μ^* causes the equity premium, EP^*, to increase from 3.35% to 4.61%.

Unlike the previous subsection where we saw that changing c always affects the equity premium and welfare in opposite ways, it is possible for a decrease in Δk to improve welfare while increasing the equity premium. This can occur because when ambiguity-averse traders evaluate their welfare, they tend to overestimate the effect of equity premium in discounting wealth and to underestimate the effect of equity premium in increasing the trading benefit. This occurs because in equation (36), the coefficient $\frac{\mathbf{k}}{\underline{k}}$ is greater than one and the coefficient $\frac{(\mathbf{k}/\bar{k})^2}{2\sigma_\varepsilon^2}$ is less than $\frac{1}{2\sigma_\varepsilon^2}$. As a result, when disclosure reduces ambiguity, so that Δk decreases, their estimates of k are closer to the true value, which improves traders' welfare ex ante. When this positive effect is strong enough to dominate the potential negative effect of the increased equity premium, traders actually are made better off by more disclosure. Appendix A2 provides a sufficient condition for a decrease in Δk to improve equilibrium welfare: $\underline{k} < \frac{\bar{k}}{2}\left(\sqrt{1+2\bar{k}}-1\right)$.

The lower panel of Figure 4 illustrates this point. Here, we plot welfare WEL^* as a function of the equity premium EP^* (i.e., equation (36)) when $\Delta k = 1$ (blue, solid curve) and when $\Delta k = 0.5$ (red, dashed curve). We find that decreasing Δk from 1 to 0.5 moves the whole curve upward, meaning that if the equity premium is fixed at its initial value 3.35%, then disclosure will improve welfare from -0.061 to -0.037. However, decreasing Δk also raises the equity premium from 3.35% to 4.61%, and as a result, welfare does not improve as much as it would for a fixed equity premium; that is, WEL^* increases from -0.061 to -0.044. Still, in total, in this example disclosure is welfare-improving.

The following proposition summarizes the results in this subsection (The proof is given in Appendix A2):

Proposition 4 *Suppose* $0 < \mu^* < 1$. *A decrease in* Δk *will reduce the fraction* μ^* *of opaque-traders. If* $\mu^* > \frac{(\bar{k}/2)-1}{\bar{k}-1}$, *then a decrease in* Δk *will increase the equity premium* EP^* *and if, in addition,* $\underline{k} < \frac{\bar{k}}{2}\left(\sqrt{1+2\bar{k}}-1\right)$, *then a decrease in* Δk *will increase welfare* WEL^*.

Although Proposition 4 only provides sufficient conditions for the increase in welfare, this result holds in various numerical examples. Figure 5 provides a typical example, where we follow Figure 2 in taking the period as one year and setting the other relevant parameter values as follows: $\sigma_\varepsilon^2 = 0.04$, $\mathbf{k} = 2$ and $c = 0.019$. Figure 5 shows that a decrease in Δk consistently decreases μ^* and increases EP^* and WEL^*.

[INSERT FIGURE 5 HERE]

6 The Role of Information Transmission

To this point, we have analyzed the impact of policy parameters, in particular, of disclosure policies reducing trading opacity Δk, on market outcomes in both the short and long run. As we noted in Section 3, opaque-traders have two advantages over transparent-traders. First, opaque-traders have an additional investment opportunity, whose structure creates opacity to transparent-traders. Second, opaque-traders observe a different signal from transparent-traders, which, combined with the price, endogenously creates an information advantage to opaque-traders. Although we believe that both features are realistic, one might wonder what role each assumption plays in driving our results. To address this question, in this section we analyze an economy, "Economy$^{\text{sym}}$", which shuts down the information asymmetry feature. Specifically, we set $\sigma_{\theta T} = \sigma_{\theta O} = 0$ in Economy$^{\text{sym}}$, so that the two signals $\tilde{\theta}_T$ and $\tilde{\theta}_O$ are reduced to their unconditional mean 0, and there is no information transmission.[18] Comparing Economy$^{\text{sym}}$ with the economy in our main model isolates the role of the endogenous information asymmetry in delivering our results.

Note that the price function in Proposition 1 does not depend on the parameters $\sigma_{\theta T}$ and $\sigma_{\theta O}$, suggesting that we cannot simply take the limit of $\sigma_{\theta T}$ and $\sigma_{\theta O}$ in our main model to get the results in Economy$^{\text{sym}}$. This occurs because transparent-traders in Economy$^{\text{sym}}$ do not need to infer information from prices to make decisions and so the form of their demand function is different. Thus we re-derive the equilibrium in Economy$^{\text{sym}}$.

Specifically, we can show that the opaque- and transparent-traders' demand functions for the stock are, respectively,

$$D_O\left(p^{sym}\right) = \frac{k\left(\bar{v} - p^{sym}\right)}{\sigma_\varepsilon^2} \text{ and } D_T\left(p^{sym}\right) = \frac{\bar{v} - p^{sym}}{\sigma_\varepsilon^2}. \tag{38}$$

Substituting these demand functions into the market clearing condition, $\mu D_O\left(p^{sym}\right) + (1-\mu)\, D_T\left(p^{sym}\right) = 1$, we see that the functional form of the price p^{sym} is

$$p^{sym} = \bar{v} - f^{sym}\left(k\right), \tag{39}$$

[18]For simplicity, we shut down the uncertainty of both signals $\tilde{\theta}_T$ and $\tilde{\theta}_O$. Actually, in order to eliminate the information asymmetry between opaque- and transparent-traders, it would be sufficient to set $\sigma_{\theta O} = 0$ (and keep $\sigma_{\theta T} > 0$), because opaque-traders can infer the information $\tilde{\theta}_T$ from prices.

where

$$f^{sym}\left(k\right)=\frac{\sigma_{\varepsilon}^{2}}{1-\mu+\mu k} \tag{40}$$

is the perceived equity premium for a given belief k in Economy$^{\text{sym}}$.

The equity premium of the stock in this economy is

$$EP^{sym}\triangleq\mathbf{E}\left(\tilde{v}-p^{sym}\right)=\frac{\sigma_{\varepsilon}^{2}}{1-\mu+\mu\mathbf{k}}. \tag{41}$$

Comparing equations (22) and (41), we immediately notice two features that differentiate Economy$^{\text{sym}}$ from our main model. First, in Economy$^{\text{sym}}$, the opacity measure Δk does not directly affect the equity premium EP^{sym}. Consequently, our earlier finding in Corollary 1 that disclosure policies designed to reduce opacity do affect the equity premium is driven by the endogenous information asymmetry. Second, for any given μ, the equity premium in Economy$^{\text{sym}}$ is smaller than it is in our main model; that is, $EP^{sym}<EP$. This is because the endogenous information asymmetry creates its own risk for the transparent-traders and lowers their demand in the economy studied in our main model.

We now derive the benefit of switching from being a transparent-trader to an opaque-trader. In Appendix A3, we show that the ex ante expected utility of staying transparent is

$$V_{T0}^{sym}=-\exp\left[-\left(\bar{v}-f_{\max}^{sym}+\frac{1}{2}\frac{\left(f_{\max}^{sym}\right)^{2}}{\sigma_{\varepsilon}^{2}}\right)\right], \tag{42}$$

where

$$f_{\max}^{sym}\triangleq\max_{k\in\left[\underline{k},\bar{k}\right]}f^{sym}\left(k\right)=\frac{\sigma_{\varepsilon}^{2}}{1-\mu+\mu\underline{k}}. \tag{43}$$

In equation (42), the linear and quadratic terms of $f_{\max}^{sym}$, respectively, capture the two roles of the perceived equity premium on traders' ex ante utility—discounting the expected wealth level associated with the unit stock endowment and determining the trading benefit of traders as buyers in the financial market. Unlike equation (30), now transparent-traders use the same perceived equity premium, $f_{\max}^{sym}$, in both terms of equation (42). This is because at the trading stage, observing the price p^{sym} enables transparent-traders to infer the value of k.

The ex ante expected utility of becoming opaque is more complicated. But, we can show that when $\mu > \frac{1}{2}$, it takes the following simple form:

$$V_{O0}^{sym} = \exp\left[-\left(\bar{v} - c - f_{\max}^{sym} + \frac{k}{2\sigma_\varepsilon^2}\left(f_{\max}^{sym}\right)^2\right)\right]. \tag{44}$$

Again, the term $-f_{\max}^{sym}$ is the discount applied to the expected wealth level, and the term $\frac{k}{2\sigma_\varepsilon^2}\left(f_{\max}^{sym}\right)^2$ captures the trading benefit. Therefore, in the region of $[1/2, 1]$, the benefit $B^{sym}(\mu)$ of switching trader types is:

$$B^{sym}(\mu) = \frac{k-1}{2\sigma_\varepsilon^2}\left(f_{\max}^{sym}\right)^2, \tag{45}$$

which is the difference in the trading benefit of the two types of traders. Function $B^{sym}(\mu)$ is still downward sloping, so that the equilibrium fraction μ^* of opaque-traders is unique.

Unlike in our main model where decreasing Δk always lowers the benefit function B, decreasing Δk has two opposing effects on B^{sym} in Economy$^{\text{sym}}$. First, by equation (43), it will decrease the maximum perceived equity premium $f_{\max}^{sym}$, and thus decrease the trading benefit for each unit of stock purchased. Second, it will increase the conservative estimation of the extra trading benefit associated with the more aggressive trading of opaque-traders, and this effect is captured by the term $\frac{k-1}{2\sigma_\varepsilon^2}$ in equation (45). As a result of these two opposing effects, decreasing Δk does not necessarily lower the benefit B^{sym}.[19]

These results are very different from Proposition 4 which shows that decreasing Δk always decreases μ^* and increase the equity premium (for $\mu^* > \frac{1}{2}$) in our main model with endogenous information asymmetry.[20] Intuitively, in our main model, because opaque-traders have an extra information advantage whose strength is positively related to Δk, lowering Δk will lower the benefit generated by this information advantage, and as a result, decreasing Δk will always shift the curve $B(\mu)$ downward. We believe that keeping this information asymmetry feature active is more in line with reality, as hedge funds often claim that disclosure harms their interests through revealing their trading secrets and reducing their information advantage (c.f., Aragon, Hertzel, and Shi, 2011).

[19] Specifically, we can show that this is true if and only if $\mu^* > \frac{1}{\underline{k}-1}$. That is, if $\mu^* > \frac{1}{\underline{k}-1}$, decreasing Δk will decrease B^{sym}, and hence decrease μ^* and increase EP^{sym} (by equation (41)); if $\frac{1}{2} < \mu^* < \frac{1}{\underline{k}-1}$, decreasing Δk will increase B^{sym}, and hence increase μ^* and decrease EP^{sym}.

[20] Note that the condition of $\mu^* > \frac{(\bar{k}/2)-1}{\underline{k}-1}$ in Proposition 4 includes the range $[1/2, 1]$.

In sum, the analysis in this section suggests that the feature of endogenous information asymmetry is central to our results on the equity premium in both the short run and the long run.

7 Conclusion

This paper demonstrates the effect of ambiguity about hedge fund strategies on the equity premium and aggregate welfare. We use this analysis to investigate the implications of regulations that affect the cost of operating a hedge fund or disclosure requirements that influence the amount of ambiguity about hedge fund strategies. We find that increasing the differential cost of operating a hedge fund decreases the equilibrium fraction of hedge funds, increases the equity premium, and decreases welfare. Thus regulatory policies designed simply to limit the number of hedge funds by increasing their costs seem ill-advised. Increased disclosure requirements, however, have different effects because they can reduce the ambiguity about hedge fund strategies. Not surprisingly, the reduction in ambiguity reduces the attractiveness of becoming a hedge fund, and so results in fewer hedge funds in equilibrium. But the effect on the equity premium is now more complicated. Reducing ambiguity makes prices more informative and thus tends to reduce the equity premium, but as it reduces the number of hedge funds it reduces the fraction of aggressive traders in the market and this tends to increase the equity premium. Similarly, the effect on welfare is ambiguous. It thus remains an empirical question whether reducing ambiguity has the effects that regulators desire.

Appendix

A. Proofs

A1. Proof of Proposition 2

We first give a more explicit expression of function B, defined by equation (33). By equation (21), we can express k as a function of $f(k)$:

$$k = \frac{\bar{k}\sigma_{\varepsilon}^2}{f(k)\left(1-\mu+\mu\bar{k}\right)}.$$

Using the above expression, the minimization problem in the definition of B is simply:

$$\min_{k}\left[-f(k)+k\frac{[f(k)]^2}{2\sigma_\varepsilon^2}\right]=\min_{f(k)}\left[\left(-1+\frac{\bar{k}}{2\left(1-\mu+\mu\bar{k}\right)}\right)f(k)\right].$$

Depending on the sign of the coefficient $\left(-1+\frac{\bar{k}}{2\left(1-\mu+\mu\bar{k}\right)}\right)$, the minimized value is achieved at either $f_{\min}$ or $f_{\max}$. It can be easily shown that

$$\left(-1+\frac{\bar{k}}{2\left(1-\mu+\mu\bar{k}\right)}\right)>0\Leftrightarrow\mu<\frac{\left(\bar{k}/2\right)-1}{\bar{k}-1}.$$

Thus, if $\bar{k}<2$, then

$$B(\mu)=B_1(\mu)\triangleq\left(-1+\frac{\bar{k}}{2\left(1-\mu+\mu\bar{k}\right)}\right)f_{\max}-\left(-f_{\max}+\frac{f_{\min}^2}{2\sigma_\varepsilon^2}\right),$$

and otherwise, the benefit function B can be further expressed as a combination of two branches:

$$B(\mu)=\begin{cases}B_0(\mu)\triangleq\left(-1+\frac{\bar{k}}{2\left(1-\mu+\mu\bar{k}\right)}\right)f_{\min}-\left(-f_{\max}+\frac{f_{\min}^2}{2\sigma_\varepsilon^2}\right), & \text{if } \mu<\frac{\bar{k}/2-1}{\bar{k}-1},\\ B_1(\mu)\triangleq\left(-1+\frac{\bar{k}}{2\left(1-\mu+\mu\bar{k}\right)}\right)f_{\max}-\left(-f_{\max}+\frac{f_{\min}^2}{2\sigma_\varepsilon^2}\right), & \text{otherwise,}\end{cases}\tag{46}$$

where the subscripts "0" and "1" in functions B_0 and B_1 indicate that those are values of B when μ is close to 0 and 1 respectively.

By equation (21), we know:

$$f_{\min}=\frac{\sigma_\varepsilon^2}{1-\mu+\mu\bar{k}}\text{ and }f_{\max}=\frac{\bar{k}\sigma_\varepsilon^2}{\underline{k}\left(1-\mu+\mu\bar{k}\right)}.\tag{47}$$

Inserting these expressions into functions B_0 and B_1 delivers:

$$B_0(\mu)=\left[\frac{\bar{k}-\underline{k}}{\underline{k}}+\frac{\bar{k}-1}{2\left(1-\mu+\mu\bar{k}\right)}\right]\frac{\sigma_\varepsilon^2}{1-\mu+\mu\bar{k}},\tag{48}$$

$$B_1(\mu)=\frac{1}{2\left(1-\mu+\mu\bar{k}\right)}\left(\frac{\bar{k}^2}{\underline{k}}-1\right)\frac{\sigma_\varepsilon^2}{1-\mu+\mu\bar{k}}.\tag{49}$$

Since $B_0(\mu)=B_1(\mu)$ at $\mu=\frac{\left(\bar{k}/2\right)-1}{\bar{k}-1}$ and since both B_0 and B_1 are decreasing in μ, function $B(\mu)$ is continuous and decreasing in μ.

The values of $\underline{c}$ and $\bar{c}$ in Proposition 2 are defined by $\underline{c}=B(1)$ and $\bar{c}=B(0)$. Clearly, $\underline{c}=B(1)=B_1(\mu)=\left(\frac{\bar{k}^2}{\underline{k}}-1\right)\frac{\sigma_\varepsilon^2}{2\bar{k}^2}$. For the value of $\bar{c}$, there are two possible cases. First, if $\bar{k}<2$, then function B_0 is irrelevant, and $B(\mu)=B_1(\mu)$ for all values of $\mu\in[0,1)$. So,

$\bar{c} = B_1(0) = \frac{\sigma_\varepsilon^2}{2}\left(\frac{\bar{k}^2}{\underline{k}} - 1\right)$. Second, if $\bar{k} \geq 2$, then for μ close to 0, $B(\mu) = B_0(\mu)$, and thus, $\bar{c} = B_0(0) = \left(\frac{2\bar{k}}{\underline{k}} + \bar{k} - 3\right)\frac{\sigma_\varepsilon^2}{2}$. In sum, $\bar{c} = \min\{B_1(0), B_0(0)\}$. Thus, if $\underline{c} < c < \bar{c}$, then there is a unique *interior* equilibrium fraction $\mu^* \in (0,1)$, which is determined by $B(\mu^*) = c$.

Finally, we show that $(\bar{c} - \underline{c})$ increases with Δk. By the definitions of $\bar{c}$ and $\underline{c}$, we have

$$(\bar{c} - \underline{c}) = \min\left\{\frac{\bar{k}^2}{\underline{k}} - 1 - \left(\frac{1}{\underline{k}} - \frac{1}{\bar{k}^2}\right), \frac{2\bar{k}}{\underline{k}} + \bar{k} - 3 - \left(\frac{1}{\underline{k}} - \frac{1}{\bar{k}^2}\right)\right\}\frac{\sigma_\varepsilon^2}{2},$$

which is a continuous function of Δk. Because

$$\frac{\partial\left[\frac{\bar{k}^2}{\underline{k}} - 1 - \left(\frac{1}{\underline{k}} - \frac{1}{\bar{k}^2}\right)\right]}{\partial\Delta k} = \frac{\bar{k}^2 - 1}{\underline{k}^2} + 2\left(\frac{\bar{k}}{\underline{k}} - \frac{1}{\bar{k}^3}\right) > 0,$$

$$\text{and}\quad \frac{\partial\left[\frac{2\bar{k}}{\underline{k}} + \bar{k} - 3 - \left(\frac{1}{\underline{k}} - \frac{1}{\bar{k}^2}\right)\right]}{\partial\Delta k} = \frac{2\bar{k} - 1}{\underline{k}^2} + 1 + 2\left(\frac{1}{\underline{k}} - \frac{1}{\bar{k}^3}\right) > 0,$$

we have $\frac{\partial(\bar{c} - \underline{c})}{\partial\Delta k} > 0$.

A2. Proof of Proposition 4

We first show that decreasing Δk will reduce the equilibrium fraction μ^* of opaque-traders, by demonstrating that a decrease in Δk will shift downward both functions B_0 and B_1 defining the benefit function B in equation (46).

For a given μ, direct computation shows

$$\frac{\partial B_0}{\partial\Delta k} = \left[\frac{\bar{k} + \underline{k}}{\underline{k}^2} + \frac{1 - \mu\left(\bar{k} - 1\right)}{2\left(1 - \mu + \mu\bar{k}\right)^2} - \frac{\mu\left(\bar{k} - \underline{k}\right)}{\underline{k}}\frac{1}{\left(1 - \mu + \mu\bar{k}\right)}\right]\frac{\sigma_\varepsilon^2}{\left(1 - \mu + \mu\bar{k}\right)}.$$

By $\mu < 1$:

$$\begin{aligned}
&\frac{\bar{k} + \underline{k}}{\underline{k}^2} + \frac{1 - \mu\left(\bar{k} - 1\right)}{2\left(1 - \mu + \mu\bar{k}\right)^2} - \frac{\mu\left(\bar{k} - \underline{k}\right)}{\underline{k}}\frac{1}{\left(1 - \mu + \mu\bar{k}\right)} \\
>\;& \frac{\bar{k} + \underline{k}}{\underline{k}^2} + \frac{1 - \mu\left(\bar{k} - 1\right)}{2\left(1 - \mu + \mu\bar{k}\right)^2} - \frac{\left(\bar{k} - \underline{k}\right)}{\underline{k}}\frac{1}{\bar{k}} \\
=\;& \frac{\bar{k}^2 + \underline{k}^2}{\underline{k}^2\bar{k}} + \frac{1 - \mu\left(\bar{k} - 1\right)}{2\left(1 - \mu + \mu\bar{k}\right)^2}.
\end{aligned}$$

As long as $\frac{\bar{k}^2 + \underline{k}^2}{\underline{k}^2\bar{k}} + \frac{1 - \mu(\bar{k} - 1)}{2(1 - \mu + \mu\bar{k})^2} > 0$, then $\frac{\partial B_0}{\partial\Delta k} > 0$. We can show that this is indeed true for values of $\mu < \frac{\bar{k}/2 - 1}{\bar{k} - 1}$, when B_0 is relevant.

Specifically, if $\frac{1 - \mu(\bar{k} - 1)}{2(1 - \mu + \mu\bar{k})^2} > 0$, then clearly, $\frac{\bar{k}^2 + \underline{k}^2}{\underline{k}^2\bar{k}} + \frac{1 - \mu(\bar{k} - 1)}{2(1 - \mu + \mu\bar{k})^2} > 0$. If $\frac{1 - \mu(\bar{k} - 1)}{2(1 - \mu + \mu\bar{k})^2} < 0$, then:

$$
\begin{aligned}
\mu &< \frac{\bar{k}/2-1}{\bar{k}-1} \Rightarrow \frac{1}{\left(1-\mu+\mu\bar{k}\right)} < \frac{2}{\bar{k}} \text{ and } \mu\left(\bar{k}-1\right) < \bar{k}/2-1 \\
&\Rightarrow \frac{1-\mu\left(\bar{k}-1\right)}{2\left(1-\mu+\mu\bar{k}\right)^2} > \frac{1-\left(\bar{k}/2-1\right)}{2}\left(\frac{2}{\bar{k}}\right)^2,
\end{aligned}
$$

and hence

$$
\frac{\bar{k}^2+\underline{k}^2}{\underline{k}^2\bar{k}} + \frac{1-\mu\left(\bar{k}-1\right)}{2\left(1-\mu+\mu\bar{k}\right)^2} > \frac{\bar{k}^2+\underline{k}^2}{\underline{k}^2\bar{k}} + \frac{1-\bar{k}/2+1}{2}\left(\frac{2}{\bar{k}}\right)^2 = \frac{\bar{k}^2\bar{k}+4\underline{k}^2}{\underline{k}^2\bar{k}^2} > 0.
$$

For a given value of μ, direct computation shows

$$
\frac{\partial B_1}{\partial \Delta k} = \left[\left(2\bar{k}\underline{k}+\bar{k}^2\right) - 2\frac{\mu\left(\bar{k}^2\underline{k}-\underline{k}^2\right)}{\left(1-\mu+\mu\bar{k}\right)}\right]\frac{1}{\underline{k}^2\left(1-\mu+\mu\bar{k}\right)^2}.
$$

Note that $\frac{\mu\left(\bar{k}^2\underline{k}-\underline{k}^2\right)}{\left(1-\mu+\mu\bar{k}\right)}$ is increasing in μ, and thus

$$
\left(2\bar{k}\underline{k}+\bar{k}^2\right) - 2\frac{\mu\left(\bar{k}^2\underline{k}-\underline{k}^2\right)}{\left(1-\mu+\mu\bar{k}\right)} > \left(2\bar{k}\underline{k}+\bar{k}^2\right) - 2\frac{\left(\bar{k}^2\underline{k}-\underline{k}^2\right)}{\bar{k}} = \bar{k}^2 + 2\frac{\underline{k}^2}{\bar{k}} > 0.
$$

So, $\frac{\partial B_1}{\partial \Delta k} > 0$.

We next show that decreasing Δk will reduce the equity premium when $\mu^* > \frac{\bar{k}/2-1}{\bar{k}-1}$. When $\mu^* > \frac{\bar{k}/2-1}{\bar{k}-1}$, function B_1 is relevant. Thus, the equilibrium condition for the trader type decision is

$$
B_1\left(\mu^*\right) = \frac{1}{2\left(1-\mu^*+\mu^*\bar{k}\right)}\left(\frac{\bar{k}^2}{\underline{k}}-1\right)\frac{\sigma_\varepsilon^2}{1-\mu^*+\mu^*\bar{k}} = c, \tag{50}
$$

which implies

$$
EP^* = \frac{\bar{k}\sigma_\varepsilon^2}{\mathbf{k}\left(1-\mu^*+\mu^*\bar{k}\right)} = \sqrt{\frac{1}{\underline{k}^{-1}-\bar{k}^{-2}}}\frac{\sqrt{2c\sigma_\varepsilon^2}}{\mathbf{k}}. \tag{51}
$$

Clearly, a decrease in Δk will increase EP^*.

In addition, equation (50) also implies an analytical expression of μ^*:

$$
\mu^* = \frac{\sqrt{\frac{\left(\bar{k}^2\underline{k}^{-1}-1\right)\sigma_\varepsilon^2}{2c}}-1}{\bar{k}-1}. \tag{52}
$$

Putting this expression into the condition of $\mu^* > \frac{\bar{k}/2-1}{\bar{k}-1}$ can characterize this condition in terms of exogenous parameters.

Plugging equation (51) into equation (36), we can express the welfare as follows:

$$WEL^{*}=-\sqrt{\frac{2c\sigma_{\varepsilon}^{2}}{\underline{k}-\left(\underline{k}/\bar{k}\right)^{2}}}+\frac{c}{\bar{k}^{2}\underline{k}^{-1}-1}, \tag{53}$$

where the first term captures the expected wealth and the second term captures the trading benefit. Clearly, decreasing Δk will increase the second term. Direct computation shows that

$$\frac{\partial}{\partial\Delta k}\left(-\sqrt{\frac{2c\sigma_{\varepsilon}^{2}}{\underline{k}-\left(\underline{k}/\bar{k}\right)^{2}}}\right)>0\Leftrightarrow 2\left(\underline{k}\bar{k}+\underline{k}^{2}\right)<\bar{k}^{3}\Leftrightarrow\underline{k}<\frac{\bar{k}}{2}\left(\sqrt{1+2\bar{k}}-1\right).$$

Thus, if $\underline{k}<\frac{\bar{k}}{2}\left(\sqrt{1+2\bar{k}}-1\right)$, then $\frac{\partial WEL^{*}}{\partial\Delta k}>0$.

A3. Compute the Ex Ante Expected Utilities of Traders in Section 6

Transparent-traders. The indirect utility of being a transparent-trader in a market with price p^{sym} is:

$$V_{T1}^{sym}\left(p^{sym}\right)=-e^{-\left\{p^{sym}+\frac{\left[f^{sym}(k)\right]^{2}}{2\sigma_{\varepsilon}^{2}}\right\}}=-e^{-\left\{\bar{v}-f^{sym}(k)+\frac{\left[f^{sym}(k)\right]^{2}}{2\sigma_{\varepsilon}^{2}}\right\}}.$$

Thus, by the recursive multiple-priors structure, we have

$$V_{T0}^{sym}=\min_{k}V_{T1}^{sym}\left(p^{sym}\right)=-\exp\left[\bar{v}+\min_{k}\left\{-f^{sym}\left(k\right)+\frac{\left[f^{sym}\left(k\right)\right]^{2}}{2\sigma_{\varepsilon}^{2}}\right\}\right]. \tag{54}$$

Note that the quadratic function $-x+\frac{x^{2}}{2\sigma_{\varepsilon}^{2}}$ is downward sloping at the range of $(0,\sigma_{\varepsilon}^{2})$ and that $f^{sym}\left(k\right)\in(0,\sigma_{\varepsilon}^{2})$ by equation (40). Thus,

$$\min_{k}\left\{-f^{sym}\left(k\right)+\frac{1}{2}\frac{\left[f^{sym}\left(k\right)\right]^{2}}{\sigma_{\varepsilon}^{2}}\right\}=-f_{\max}^{sym}+\frac{1}{2}\frac{\left(f_{\max}^{sym}\right)^{2}}{\sigma_{\varepsilon}^{2}}.$$

Plugging the above equation into equation (54) delivers equation (42).

Opaque-traders. The indirect utility of being an opaque-trader in a market with price p^{sym} is:

$$\begin{aligned}V_{O1}^{sym}\left(p^{sym}\right) &= \exp\left[-\left(p^{sym}-c+\frac{k}{2\sigma_{\varepsilon}^{2}}\left[f^{sym}\left(k\right)\right]^{2}\right)\right]\\ &= \exp\left[-\left(\bar{v}-c-f^{sym}\left(k\right)+\frac{k}{2\sigma_{\varepsilon}^{2}}\left[f^{sym}\left(k\right)\right]^{2}\right)\right].\end{aligned}$$

So, the ex ante expected utility of becoming opaque is:

$$V_{O0}^{sym} = \min_k V_{O1}\left(p^{sym}\right) = \exp\left[-\left(\bar{v} - c + \min_k\left[-f^{sym}(k) + \frac{k}{2\sigma_\varepsilon^2}\left[f^{sym}(k)\right]^2\right]\right)\right]. \quad (55)$$

By equation (40), we have

$$k = \frac{\sigma_\varepsilon^2}{\mu f^{sym}(k)} - \frac{1-\mu}{\mu}.$$

Inserting the above equation into $-f^{sym}(k) + \frac{k}{2\sigma_\varepsilon^2}\left[f^{sym}(k)\right]^2$ yields:

$$\begin{aligned} & -f^{sym}(k) + \frac{k}{2\sigma_\varepsilon^2}\left[f^{sym}(k)\right]^2 \\ = & -\frac{2\mu-1}{2\mu} f^{sym}(k) - \frac{1-\mu}{2\sigma_\varepsilon^2\mu}\left[f^{sym}(k)\right]^2. \end{aligned}$$

So, if $\frac{2\mu-1}{2\mu} > 0$, i.e. if $\mu > \frac{1}{2}$, the quadratic function $-\frac{2\mu-1}{2\mu}x - \frac{1-\mu}{2\sigma_\varepsilon^2\mu}x^2$ is decreasing in the range of $(0,\infty)$, so that the minimum of $\left[-f^{sym}(k) + \frac{k}{2\sigma_\varepsilon^2}\left[f^{sym}(k)\right]^2\right]$ is achieved at $f_{\max}^{sym}$ (when k is equal to $\underline{k}$). That is, if $\mu > \frac{1}{2}$, we have:

$$\begin{aligned} & \min_k\left[-f^{sym}(k) + \frac{k}{2\sigma_\varepsilon^2}\left[f^{sym}(k)\right]^2\right] \\ = & -\frac{2\mu-1}{2\mu} f_{\max}^{sym} - \frac{1-\mu}{2\sigma_\varepsilon^2\mu}\left(f_{\max}^{sym}\right)^2 \\ = & -f_{\max}^{sym} + \frac{\underline{k}}{2\sigma_\varepsilon^2}\left(f_{\max}^{sym}\right)^2. \end{aligned}$$

Plugging the above equation into equation (55) delivers equation (44).

B. Polar Economies

In the text, we have focused on economies with interior $\mu^* \in (0,1)$. In this appendix, we will derive the results for economies in which $\mu^* = 0$ or $\mu^* = 1$.

B1. Case 1: $\mu^* = 0$

Let us first examine the financial market equilibrium. Now all traders are transparent and observe the same signal $\tilde{\theta}_T$. Price does not contain extra information for predicting the future stock payoff $\tilde{v}$, and as a result, the demand function of all traders is $\frac{E\left(\tilde{v}|\tilde{\theta}_T\right)-\tilde{p}}{Var\left(\tilde{v}|\tilde{\theta}_T\right)} = \frac{\bar{v}+\tilde{\theta}_T-\tilde{p}}{\sigma_{\theta O}^2+\sigma_\varepsilon^2}$. Equating this total demand with the total unit supply of the stock determines the equilibrium market price: $\tilde{p} = \bar{v} + \tilde{\theta}_T - (\sigma_{\theta O}^2 + \sigma_\varepsilon^2)$.

We next go back to date 0 to examine the condition for $\mu^* = 0$ to be a trader distribution equilibrium. We need to figure out the benefit of becoming opaque when traders hold the belief that price function takes the form of $\tilde{p} = \bar{v} + \tilde{\theta}_T - (\sigma^2_{\theta O} + \sigma^2_\varepsilon)$.

The date 1 indirect utility of transparent-traders is:

$$V_{T1}\left(\tilde{p}, \tilde{\theta}_T\right) = -\exp\left[-\left(\tilde{p} + \frac{\sigma^2_{\theta O} + \sigma^2_\varepsilon}{2}\right)\right].$$

So, their date 0 utility is:

$$V_{T0} = E\left[V_{T1}\left(\tilde{p}, \tilde{\theta}_T\right)\right] = -\exp\left[-\left(\bar{v} - \frac{\sigma^2_{\theta O} + \sigma^2_{\theta T} + \sigma^2_\varepsilon}{2}\right)\right]. \tag{56}$$

By equation (31), the date 1 indirect utility of opaque-traders would be

$$\begin{aligned} V_{O1}\left(\tilde{p}, \tilde{\theta}_O; k\right) &= -\exp\left[-\left(\tilde{p} - c + \frac{k}{2\sigma^2_\varepsilon}\left(\bar{v} + \tilde{\theta}_T + \tilde{\theta}_O - \tilde{p}\right)^2\right)\right] \\ &= -\exp\left[-\left(\bar{v} + \tilde{\theta}_T - (\sigma^2_{\theta O} + \sigma^2_\varepsilon) - c + \frac{k}{2\sigma^2_\varepsilon}\left[\tilde{\theta}_O + (\sigma^2_{\theta O} + \sigma^2_\varepsilon)\right]^2\right)\right] \end{aligned} \tag{57}$$

Thus, the date 0 utility of becoming opaque is

$$V_{O0} = \min_k E_k\left[U_{O1}\left(p, \tilde{\theta}\right)\right].$$

Under a given belief k, applying the moment generation functions of the normally distributed random variable $\tilde{\theta}_T$ and the non-central Chi-distributed random variable $\left[\tilde{\theta}_O + (\sigma^2_{\theta O} + \sigma^2_\varepsilon)\right]^2$ in equation (57), we obtain:

$$E_k\left[V_{O1}\left(p, \tilde{\theta}\right)\right] = -\frac{1}{\sqrt{1 + k\sigma^2_{\theta O}\sigma^{-2}_\varepsilon}} e^{-\left[\bar{v} - \left(\sigma^2_{\theta O} + \sigma^2_\varepsilon + \frac{1}{2}\sigma^2_{\theta T}\right) - c + \frac{k\left(\sigma^2_{\theta O} + \sigma^2_\varepsilon\right)^2}{2\sigma^2_\varepsilon\left(1 + k\sigma^2_{\theta O}\sigma^{-2}_\varepsilon\right)}\right]},$$

which achieves its minimum at $k = \underline{k}$. As a consequence, the ex ante utility of switching from transparent to opaque is:

$$V_{O0} = -\frac{1}{\sqrt{1 + \underline{k}\sigma^2_{\theta O}\sigma^{-2}_\varepsilon}} e^{-\left[\bar{v} - \left(\sigma^2_{\theta O} + \sigma^2_\varepsilon + \frac{1}{2}\sigma^2_{\theta T}\right) - c + \frac{\underline{k}\left(\sigma^2_{\theta O} + \sigma^2_\varepsilon\right)^2}{2\sigma^2_\varepsilon\left(1 + \underline{k}\sigma^2_{\theta O}\sigma^{-2}_\varepsilon\right)}\right]}. \tag{58}$$

$\mu^* = 0$ is an equilibrium if and only if $V_{O0} \leq V_{T0}$. Using expressions of V_{O0} and V_{T0} in equations (58) and (56), we can show that

$$V_{O0} \leq V_{T0} \Leftrightarrow c > C_0 \triangleq \log\left(\sqrt{1 + \underline{k}\sigma^2_{\theta O}\sigma^{-2}_\varepsilon}\right) + \frac{(\underline{k} - 1)\left(\sigma^2_{\theta O} + \sigma^2_\varepsilon\right)}{2\left(1 + \underline{k}\sigma^2_{\theta O}\sigma^{-2}_\varepsilon\right)}. \tag{59}$$

B2. Case 2: $\mu^* = 1$

When $\mu^* = 1$, all traders are opaque and observe the signal $\tilde{\theta}_O$. Following a similar derivation as in the main text, we can show that their demand function and indirect utility as:

$$D_O\left(\tilde{p}, \tilde{\theta}_O; k\right) = \frac{\bar{v} + \tilde{\theta}_O - \tilde{p}}{\sigma_{\theta T}^2 + k^{-1}\sigma_\varepsilon^2}, \tag{60}$$

$$V_{O1}\left(\tilde{p}, \tilde{\theta}_O; k\right) = -\exp\left[-\left(\tilde{p} - c + \frac{\left(\bar{v} + \tilde{\theta}_O - \tilde{p}\right)^2}{2\left(\sigma_{\theta T}^2 + k^{-1}\sigma_\varepsilon^2\right)}\right)\right]. \tag{61}$$

Equating the total demand with the total unit supply of the stock determines the equilibrium market price at date 1: $\tilde{p} = \bar{v} + \tilde{\theta}_T - (\sigma_{\theta T}^2 + k^{-1}\sigma_\varepsilon^2)$. Thus, the equity premium is

$$f(k) = \sigma_{\theta T}^2 + k^{-1}\sigma_\varepsilon^2 \tag{62}$$

and we can similarly define

$$f_{\min} \triangleq \sigma_{\theta T}^2 + \bar{k}^{-1}\sigma_\varepsilon^2 \text{ and } f_{\max} \triangleq \sigma_{\theta T}^2 + \underline{k}^{-1}\sigma_\varepsilon^2. \tag{63}$$

Using the indirect utility of opaque-traders and the equilibrium pricing function, we can compute the date 0 utility of becoming opaque is:

$$\begin{aligned} V_{O0} &= \min_k E_k\left[V_{O1}\left(\tilde{p}, \tilde{\theta}_O; k\right)\right] \\ &= \min_k E_k\left[-\exp\left[-\left(\bar{v} + \tilde{\theta}_O - c - \frac{f(k)}{2}\right)\right]\right] \\ &= -\exp\left[-\left(\bar{v} - c - \frac{f_{\max}}{2} - \frac{\sigma_{\theta O}^2}{2}\right)\right]. \end{aligned} \tag{64}$$

Now let us examine what a transparent-trader would behave in this economy. Standard computations show that at date 1, equipped with the information $\{\tilde{p}, \tilde{\theta}_T\}$ and facing the pricing function $\tilde{p} = \bar{v} + \tilde{\theta}_T - (\sigma_{\theta T}^2 + k^{-1}\sigma_\varepsilon^2)$, a transparent-trader's demand function and indirect utility would be:

$$D_T\left(\tilde{p}, \tilde{\theta}_T\right) = \begin{cases} \frac{f_{\min} + \tilde{\theta}_T}{\sigma_\varepsilon^2}, & \text{if } \tilde{\theta}_T > -f_{\min}, \\ 0, & \text{if } -f_{\max} \le \tilde{\theta}_T \le -f_{\min}, \\ \frac{f_{\max} + \tilde{\theta}_T}{\sigma_\varepsilon^2}, & \text{if } \tilde{\theta}_T < -f_{\max}. \end{cases} \tag{65}$$

and

$$V_{T1}\left(\tilde{p}, \tilde{\theta}_T\right) = \begin{cases} -\exp\left[-\left(\tilde{p} + \frac{\left(f_{\min} + \tilde{\theta}_T\right)^2}{2\sigma_\varepsilon^2}\right)\right], & \text{if } \tilde{\theta}_T > -f_{\min}, \\ -\exp(-\tilde{p}), & \text{if } -f_{\max} \le \tilde{\theta}_T \le -f_{\min}, \\ -\exp\left[-\left(\tilde{p} + \frac{\left(f_{\max} + \tilde{\theta}_T\right)^2}{2\sigma_\varepsilon^2}\right)\right], & \text{if } \tilde{\theta}_T < -f_{\max}. \end{cases} \tag{66}$$

So, using the equilibrium pricing function $\tilde{p} = \bar{v} + \tilde{\theta}_T - (\sigma_{\theta T}^2 + k^{-1}\sigma_\varepsilon^2)$, we can show that the date 0 utility of staying transparent is:

$$\begin{aligned} V_{T0} &= \min_k \left[V_{T1}\left(\tilde{p}, \tilde{\theta}_T\right)\right] \\ &= -H \exp\left[-\left(\bar{v} - f_{\min} - \frac{\sigma_{\theta O}^2}{2}\right)\right] \end{aligned} \tag{67}$$

where

$$H = E\left[e^{-\frac{(f_{\min}+\tilde{\theta}_T)^2}{2\sigma_\varepsilon^2} 1_{\tilde{\theta}_T > -f_{\min}} - \frac{(f_{\max}+\tilde{\theta}_T)^2}{2\sigma_\varepsilon^2} 1_{\tilde{\theta}_T < -f_{\max}}}\right]. \tag{68}$$

$\mu^* = 1$ is an equilibrium if and only if $V_{O0} \geq V_{T0}$. Using expressions of V_{O0} and V_{T0} in equations (64) and (67), we can show that

$$V_{O0} \geq V_{T0} \Leftrightarrow c < C_1 \triangleq \log(H) + f_{\min} - \frac{f_{\max}}{2}. \tag{69}$$

Note that in general, C_0 and C_1 in equations (59) and (69) are different from $\bar{c}$ and $\underline{c}$ in Proposition 2, which implies multiple equilibria or non-existence of equilibrium ("market breakdown") for some parameter configurations. For example, if $C_0 < \bar{c}$, then for c in the range of $[C_0, \bar{c})$, there will be two equilibrium fractions of opaque-traders: $\mu_1^* = 0$ and $\mu_2^* > 0$. In contrast, if $C_0 > \bar{c}$, then for c in the range of $[\bar{c}, C_0)$, there will be no equilibrium fraction μ^* of opaque-traders in the economy.

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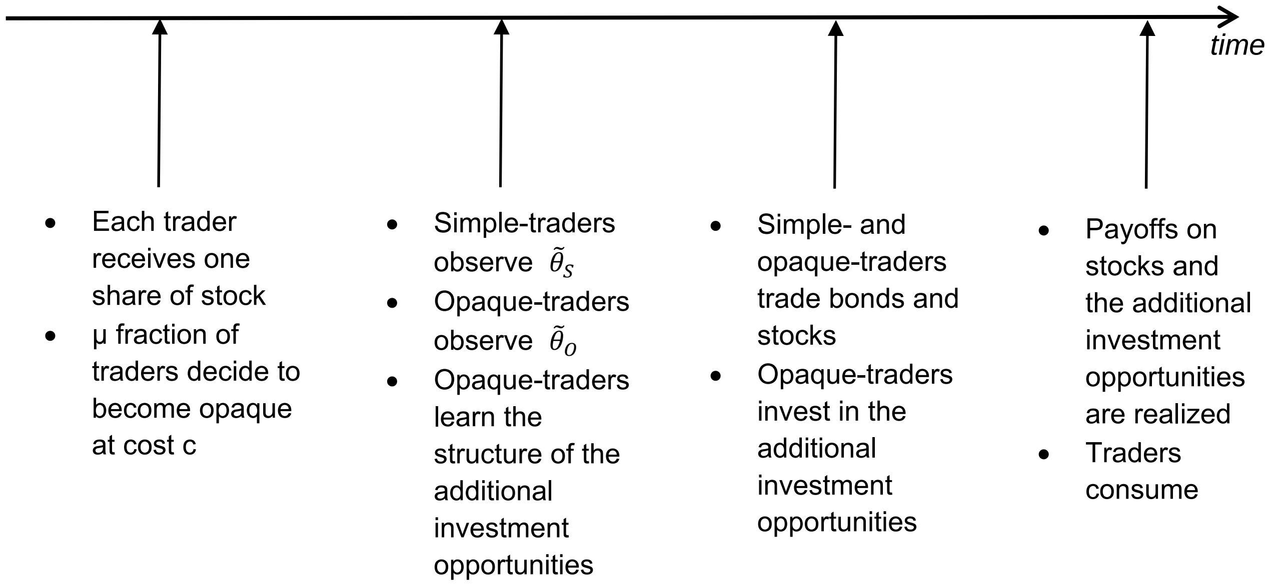
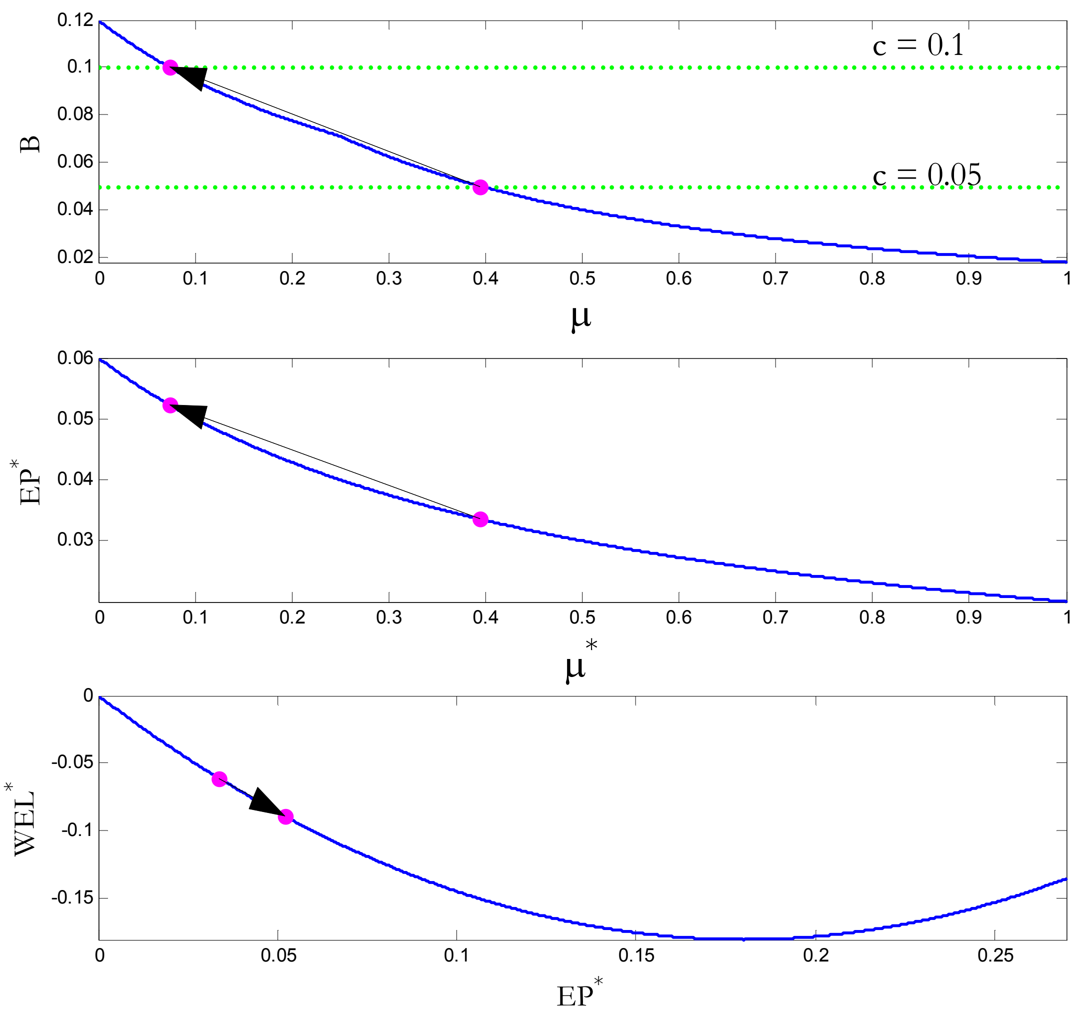
Figure 1: Timeline



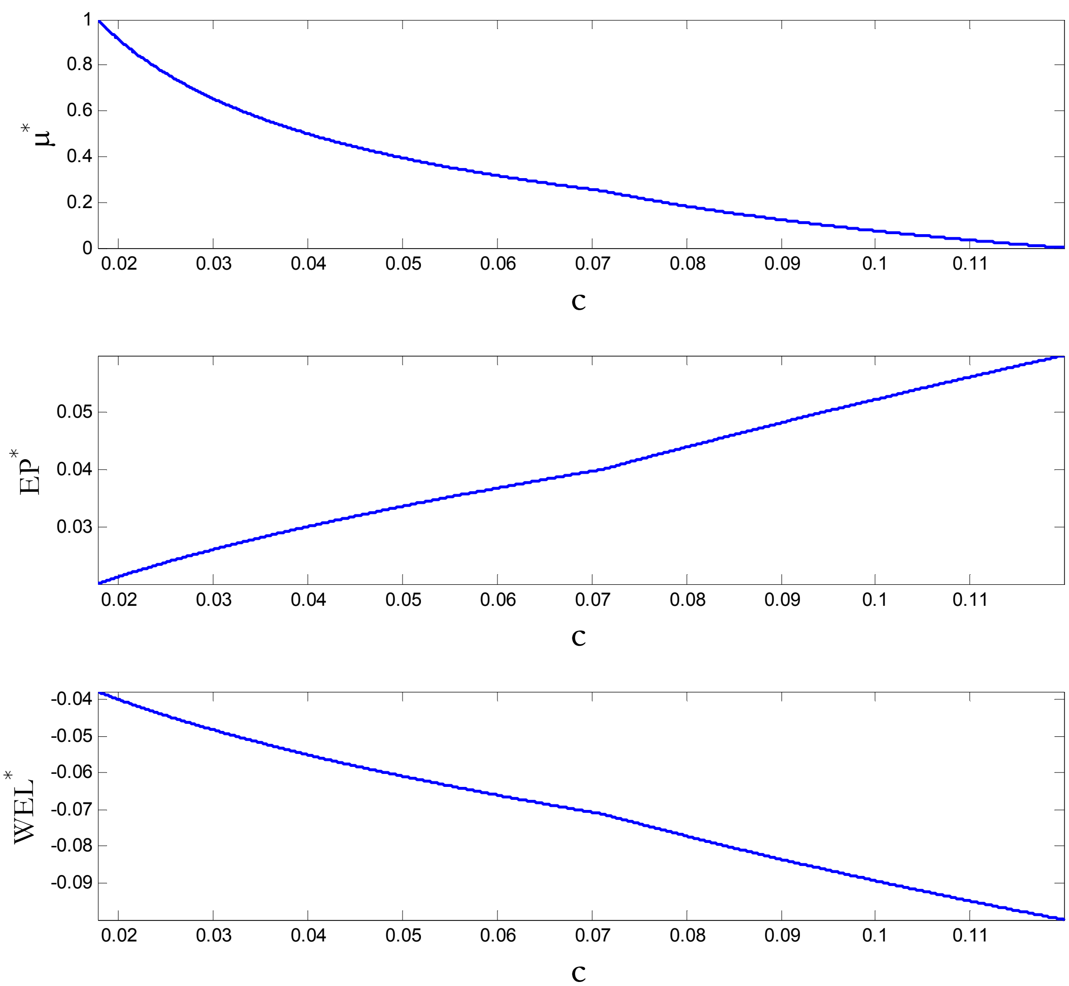
This figure plots the order of events.

Figure 2: Implications for Equilibrium Market Outcomes of Increasing c



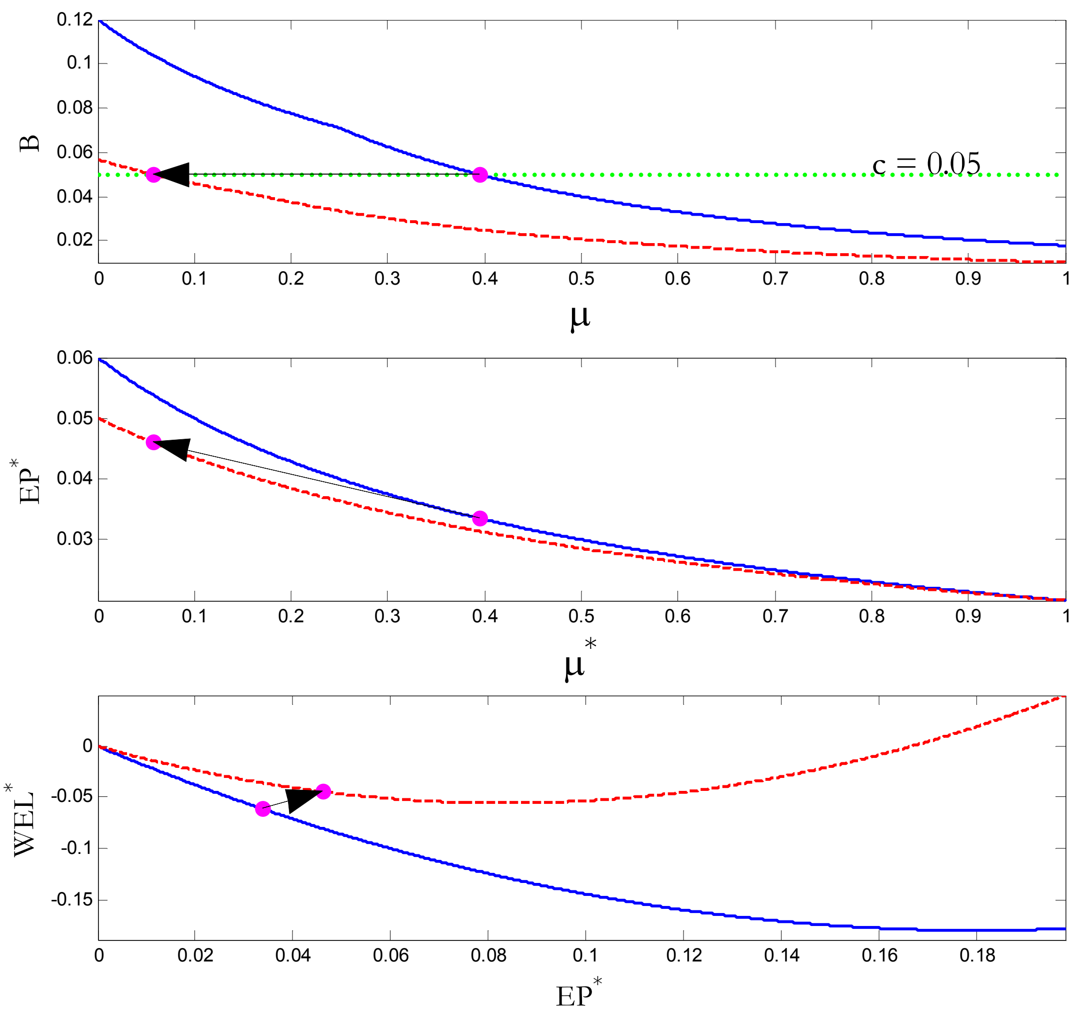
This figure shows the impact of increasing c from 0.05 to 0.1 on the equilibrium fraction μ* of opaque-traders, the equity premium EP*, and welfare WEL*. The other parameter values are: σ_ε^2=0.04, **k** = 2 and Δk = 1. The upper panel plots benefit function B (i.e., equation (33)) and the equilibrium fraction μ* of opaque-traders is determined by the intersection of function B and the horizontal cost functions. This panel shows that increasing c from 0.05 to 0.1 decreases μ* from 0.39 to 0.07. The middle panel plots equilibrium equity premium EP* as a function of μ* (i.e., equation (34)), and it shows that the drop in μ* increases EP* from 3.35% to 5.23%. The lower panel plots equilibrium welfare WEL* as a function of the equilibrium equity premium EP* (i.e., equation (36)), and it shows that the increase in EP* decreases welfare from −0.06 to −0.09.

Figure 3: Equilibrium Market Outcomes as Functions of c



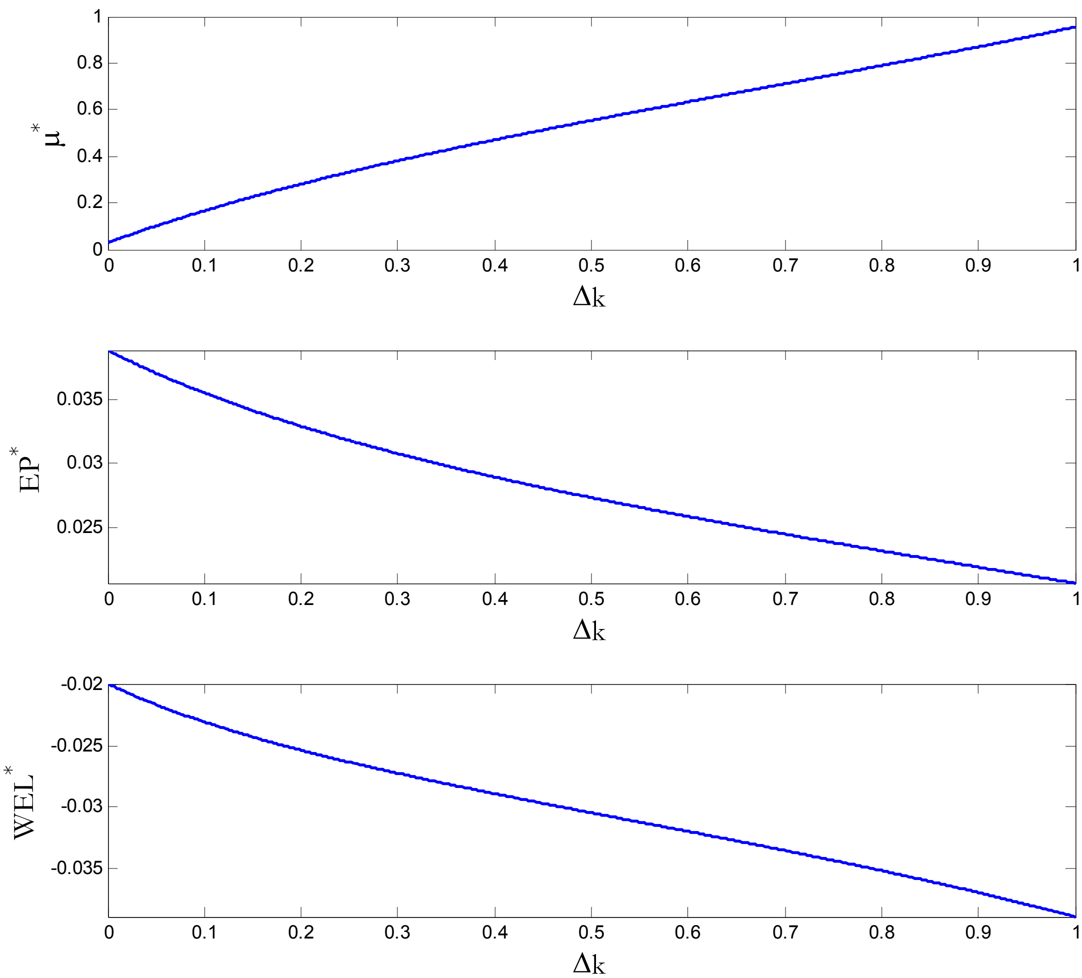
This figure plots the equilibrium fraction μ* of opaque-traders, the equity premium EP*, and welfare WEL* as functions of c. The parameter values are: σ_ε^2=0.04, **k** = 2 and Δk = 1.

Figure 4: Implications for Equilibrium Market Outcomes of Decreasing Δk



This figure shows the impact of decreasing Δk from 1 to 0.5 on the equilibrium fraction μ* of opaque-traders, the equity premium EP*, and welfare WEL*. The other parameter values are: σ_ε^2=0.04, **k** = 2 and c = 0.05. The upper panel plots benefit functions B (i.e., equation (33)) and the equilibrium fraction μ* of opaque-traders is determined by the intersection of functions B and the horizontal cost function. This panel shows that decreasing Δk from 1 to 0.5 decreases μ* from 0.39 to 0.06. The middle panel plots equilibrium equity premium EP* as functions of μ* (i.e., equation (34)), and it shows that the drop in μ* increases EP* from 3.35% to 4.61%. The lower panel plots equilibrium welfare WEL* as functions of the equilibrium equity premium EP* (i.e., equation (36)), and it shows that the decrease in Δk increases welfare from −0.06 to −0.04.

Figure 5: Equilibrium Market Outcomes as Functions of Δk



This figure plots the equilibrium fraction μ* of opaque-traders, the equity premium EP*, and welfare WEL* as functions of Δk. The parameter values are: $\sigma_\varepsilon^2 = 0.04$, $\mathbf{k} = 2$ and c = 0.019.