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Open-Ended Rectangular Waveguide for Nondestructive Thickness Measurement and Variation Detection of Lossy Dielectric Slabs Backed by a Conducting Plate

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***Abstract*—Solutions of fields inside a slab of a generally lossy dielectric medium backed by a conducting plate, placed outside a waveguide-fed rectangular aperture, are evoked in the application to the microwave nondestructive thickness measurement of such dielectric slabs. Upon construction of the waveguide terminating admittance expression from its variational form, an inverse problem is then solved to extract the slab thickness from the conductance and susceptance in a recursive manner. A comparison between the experimental and theoretical results showed that the significance of higher order modes is minimal; hence, the dominant mode assumption is, in general, valid to describe the aperture field distribution. The validity of this assumption has led to the construction of a simple integral solution which is fast converging for generally lossy dielectric slabs, and may easily be implemented for real-time applications. Experiments were conducted to verify the theoretical findings. Good agreement was obtained between the theoretical and experimental results. Multiple thicknesses of two different dielectric samples were estimated in this way.**

Introduction

Nondestructive measurement of the thickness (and monitoring its variation) of generally lossy dielectric slab products is of great interest in many facets of industry. Examples of this would be thickness variation monitoring of ceramic and synthetic rubber coatings on metallic substrates. Composite ceramic materials are used in high-temperature and pressure situations like those encountered in the aerospace industry. The control of the geometry and properties of these materials is of utmost importance. Usually, only one side of such materials is accessible, and the other side is backed by a conducting plate. Thus, a nondestructive, fast, and accurate technique is desirable for monitoring the thickness or properties of these coatings. The ability of microwaves to penetrate inside a dielectric medium and their sensitivity to the presence of a boundary between two dissimilar layers make them quite suitable for this type of measurement [1]–[3].

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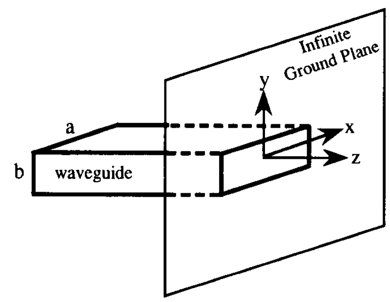
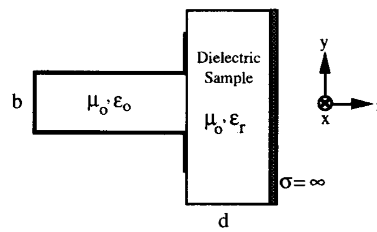


(a)



(b)

Fig. 1. (a) Rectangular waveguide radiating into an infinite half-space. (b) Cross-section of the geometry.

This paper discusses a fast-converging solution to an inverse problem in which the dielectric slab thickness is determined from the expression for admittance of an open-ended rectangular waveguide radiating into a conductor-backed dielectric slab, as shown in Fig. 1. The theoretical formulation for the admittance expression is parallel to those used by Galejs and Compton [4], [5]. The admittance of an open-ended rectangular waveguide radiating into a dielectric slab backed by a conducting plate for a given microwave frequency is a function of the relative dielectric properties of the slab ($\epsilon_r = \epsilon_r' - j\epsilon_r''$, $\tan\delta = \epsilon_r''/\epsilon_r'$) and its thickness ($d$). Thus, the admittance measurement will render thickness information if the dielectric properties of the slab are known, and vice versa. The aperture conductance and susceptance G and B can be measured via the measurement of the VSWR and stand-

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ing-wave null displacement (relative to the null locations for a shorted waveguide). With some further manipulation of the theoretical model, a noncontact case, where an air gap exists between the aperture and the sample, can also be considered.

The effect of higher order modes was ignored in the theoretical formulation, and only the dominant mode was considered. The validity of this consideration is based not only on the standard practice [4], [5], but was also confirmed by the experimental results that were found to be in good agreement with the theory.

Theoretical Formulation

A rectangular aperture in an infinite ground plane, fed by a waveguide, is shown in Fig. 1(a). Considering the dominant TE_{10} mode incident on the aperture, the terminating admittance of the waveguide can be written as [5], [6].

$$Y = G + jB = \frac{\iint_{\text{aperture}} [\overline{E}(x, y, 0) \times \overline{W}(x, y)] \cdot \hat{a}_z \, dx \, dy}{\left[\iint_{\text{aperture}} \overline{E}(x, y, 0) \cdot \overline{e}_0(x, y) \, dx \, dy \right]^2} \tag{1}$$

where

$$\overline{W}(x, y) = \overline{H}(x, y, 0) + \sum_{n=1}^{\infty} Y_n \overline{h}_n(x, y) \iint_{\text{aperture}} \overline{E}(\eta, \xi, 0) \cdot \overline{e}_n(\eta, \xi) \, d\eta \, d\xi \tag{2}$$

where $\boldsymbol{E}(x, y, 0)$ and $\boldsymbol{H}(x, y, 0)$ are the aperture field distributions. This admittance expression is constructed using transverse vector mode functions and their orthogonal properties [7]. The nth vector mode functions are $\boldsymbol{e}_n$ and $\boldsymbol{h}_n$, and Y_n is the characteristic admittance of the waveguide for the nth mode. The large and small transverse dimensions of the waveguide are represented by a and b; hence, the TE_{10} mode aperture field distribution is given by

$$E_y(x, y, 0) = \overline{e}_0(x, y) = \begin{cases} \sqrt{\dfrac{2}{ab}} \cos\left(\dfrac{\pi x}{a}\right), & (x, y) \in \text{aperture} \\ 0, & (x, y) \notin \text{aperture}. \end{cases} \tag{3}$$

As shown in Fig. 1(b), the waveguide is in contact with a dielectric slab which is terminated by a conducting layer. The slab is assumed to be homogenous and with free-space permeability. It has been shown that (1) is stationary with respect to the variations of the E-field about its exact value [4], [6]. Thus, an approximation for the E-field would result in a good estimate of the admittance.

By evoking the vector potential formulation, the fields in region 1 (dielectric slab) can be constructed in the following manner. Fields in this region satisfy the wave equation

$$\nabla^2 \overline{A} + k_1^2 \overline{A} = 0 \tag{4}$$

where the vector potential has two components:

$$\overline{A} = \Phi \hat{a}_x + \Psi \hat{a}_y \tag{5}$$

with

$$\overline{E}(x, y, z) = -\nabla \times \overline{A} \tag{6a}$$

$$\overline{H}(x, y, z) = \frac{1}{j\omega\mu_0} [k_1^2 \overline{A} + \nabla\nabla \cdot \overline{A}]. \tag{6b}$$

The solution for the field components can be written as

$$E_{x1}(x, y, z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [-jk_{z1} C_\Psi^+ e^{-jk_{z1}z} + jk_{z1} C_\Psi^- e^{jk_{z1}z}] e^{-jk_x x} e^{-jk_y y} \, dk_x \, dk_y \tag{7}$$

$$E_{y1}(x, y, z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [jk_{z1} C_\Phi^+ e^{-jk_{z1}z} - jk_{z1} C_\Phi^- e^{jk_{z1}z}] e^{-jk_x x} e^{-jk_y y} \, dk_x \, dk_y \tag{8}$$

$$H_{x1}(x, y, z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ \frac{(k_1^2 - k_x^2)}{j\omega\mu_0} [C_\Phi^+ e^{-jk_{z1}z} + C_\Phi^- e^{jk_{z1}z}] - \frac{k_x k_y}{j\omega\mu_0} \cdot [C_\Psi^+ e^{-jk_{z1}z} + C_\Psi^- e^{jk_{z1}z}] \right\} \times e^{-jk_x x} e^{-jk_y y} \, dk_x \, dk_y \tag{9}$$

$$H_{y1}(x, y, z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ \frac{(k_1^2 - k_y^2)}{j\omega\mu_0} [C_\Psi^+ e^{-jk_{z1}z} + C_\Psi^- e^{jk_{z1}z}] - \frac{k_x k_y}{j\omega\mu_0} \cdot [C_\Phi^+ e^{-jk_{z1}z} + C_\Phi^- e^{jk_{z1}z}] \right\} \times e^{-jk_x x} e^{-jk_y y} \, dk_x \, dk_y \tag{10}$$

$$k_{z1} = \sqrt{k_1^2 - k_x^2 - k_y^2} \tag{11}$$

where k_1 is the wavenumber inside the material, and k_{z1} is chosen such that

$$\text{Re}\{k_{z1}\} \geq 0; \ \text{Im}\{k_{z1}\} \leq 0. \tag{12}$$

Using the Fourier properties of the above expressions and applying the appropriate boundary conditions at $z =$

d, the unknown field coefficients C_{Φ}^{-}, C_{Φ}^{-}, C_{Ψ}^{+}, C_{Ψ}^{-} can be determined. By substituting these into (7)–(10), the complete set of solutions for the field components is constructed. The E-field in region 1 is transverse to the x-axis. Upon normalization with respect to the free-space wavenumber k_O and applying a polar coordinate transformation, the admittance expression (1) and its related parameters can be presented as

$$y_s = \frac{j}{(2\pi)^2 \sqrt{1 - \left(\frac{\lambda_0}{2a}\right)^2}} \int_{R=0}^{\infty} \int_{\theta=0}^{2\pi} \cdot \mathfrak{J}\left\{ (K^2 - R^2 \cos^2 \theta)\left(2C_{\Phi} + \frac{j\mathfrak{J}}{\chi_z}\right)\right\} R \, d\theta \, dR \tag{13}$$

$$\mathfrak{J} = \sqrt{\frac{2A}{B}} \frac{(4\pi) \sin\left(\frac{\chi_y B}{2}\right) \cos\left(\frac{\chi_x A}{2}\right)}{\chi_y\left[\pi^2 - (\chi_x A)^2\right]} \tag{14}$$

$$C_{\Phi} = -\frac{\mathfrak{J} e^{j\chi_z D}}{2\chi_z \sin(\chi_z D)} \tag{15}$$

$$A = k_0 a, \quad B = k_0 b, \quad D = k_0 d, \quad K = \frac{k_1}{k_0} \tag{16}$$

$$\chi_x = R\cos\theta, \; \chi_y = R \sin\theta, \; \chi_y = \sqrt{K^2 - R^2} \tag{17}$$

where R and θ are the new variables of integration in polar coordinates. It should be mentioned that for a lossless dielectric slab, singularities occur when integrating (13). One can overcome this problem by integrating over a contour around singular points [9]. This has not been dealt with in this work since only generally *lossy* dielectric materials are of interest here.

Verification of the Theoretical Results

To gain a better insight into the nature of the problem and understand the limitations of the admittance formulation presented earlier, several tests were conducted. For the special case of an infinite half-space lossy dielectric (e.g., very long electrical length), the results of the theory were compared with those obtained using Lewin's infinite half-space approach [6]. Table I shows the computed admittance results for three dielectrics with different characteristics at 10 GHz. The dielectric values for this comparison were chosen to represent materials such as Plexiglas and two types of synthetic rubbers. Clearly, there is good agreement between the two approaches, considering that the values calculated from (13) are based on a finite number of integration points (a Gauss quadrature method was used).

To investigate the effect of the dielectric slab thickness variation on the admittance, variations of G and B as a function of slab thickness for two types of dielectric materials at 10 GHz are shown in Figs. 2 and 3. The thin line represents a low-loss dielectric (Plexiglas with $\epsilon_r' = 2.59$, $\tan\delta = 0.007$), and the other a dielectric of higher loss (synthetic rubber with $\epsilon_r' = 7.25$, $\tan\delta = 0.103$). The dielectric values used here are consistent with samples that will be used later to experimentally verify the theory. Dielectric properties of these samples were measured using a short-circuited waveguide method [10]. The changing of the sign of B can be interpreted as the aperture being terminated by a capacitive or inductive load for different sample thicknesses.

G and B can be measured experimentally using measurable quantities such as VSWR [or Attenuation (dB) = 20 log (VSWR)] and phase shift in the waveguide with

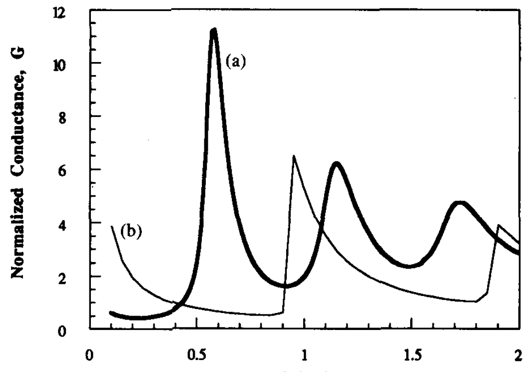

Fig. 2. Theoretical variations of conductance versus thickness for two types of dielectric slabs at 10 GHz: (a) $\epsilon_r' = 7.25$, $\tan\delta = 0.103$, and (b) $\epsilon_r' = 2.59$, $\tan\delta = 0.007$.

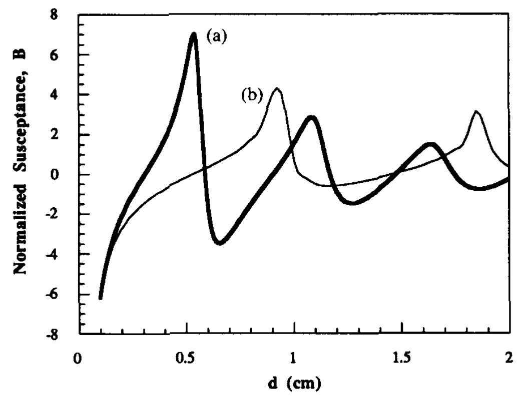

Fig. 3. Theoretical variations of susceptance versus thickness for two types of dielectric slabs at 10 GHz: (a) $\epsilon_r' = 7.25$, $\tan\delta = 0.103$, and (b) $\epsilon_r' = 2.59$, $\tan\delta = 0.007$.

TABLE I
Comparison of the Results of (13) with Those Using Lewin's Infinite Half-Space [6]

ϵ_r'	$\tan\delta$	Y (13)	Y (Lewin)
2.6	0.01	2.074 + j0.555	2.068 + j0.556
7.0	0.1	3.441 + j0.062	3.432 + j0.062
10.5	0.238	4.166 − j0.169	4.166 − j0.166

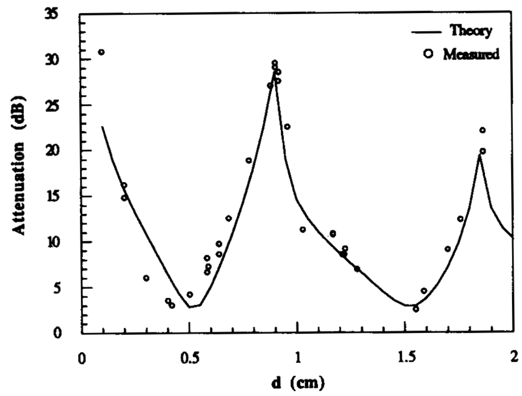

Fig. 4. Comparison of theory and measurement of attenuation versus thickness at 10 GHz for slab of $\epsilon_r' = 2.59$, $\tan\delta = 0.007$.

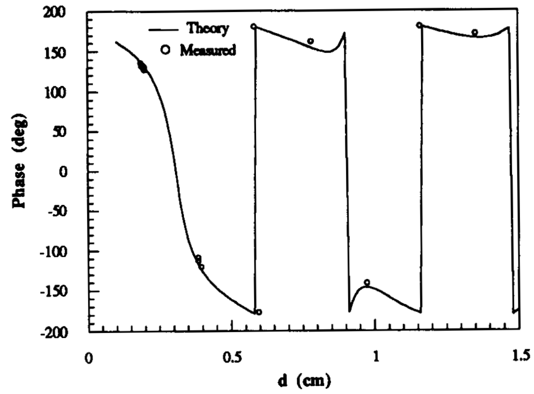

Fig. 7. Comparison of theory and measurement of phase versus thickness at 10 GHz for slab of $\epsilon_r' = 7.25$, $\tan\delta = 0.103$.

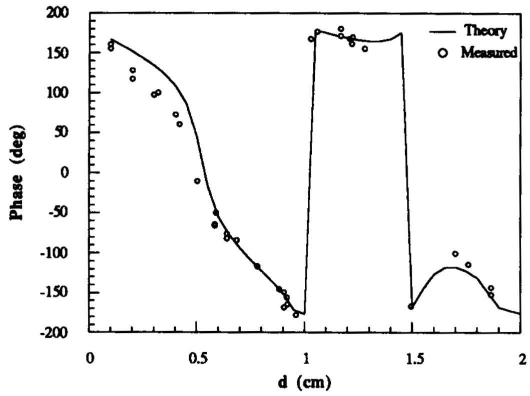

Fig. 5. Comparison of theory and measurement of phase versus thickness at 10 GHz for slab of $\epsilon_r' = 2.59$, $\tan\delta = 0.007$.

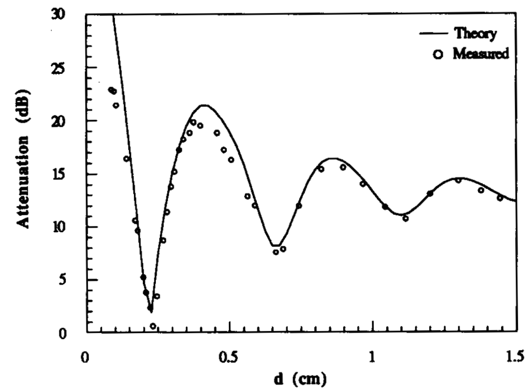

Fig. 8. Comparison of theory and measurement of attenuation versus thickness at 10 GHz for slab of $\epsilon_r' = 12.6$, $\tan\delta = 0.19$.

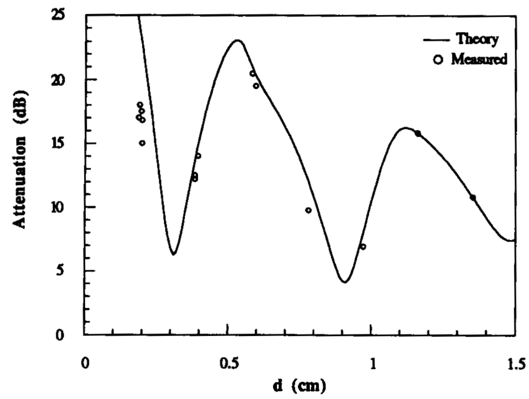

Fig. 6. Comparison of theory and measurement of attenuation versus thickness at 10 GHz for slab of $\epsilon_r' = 7.25$, $\tan\delta = 0.103$.

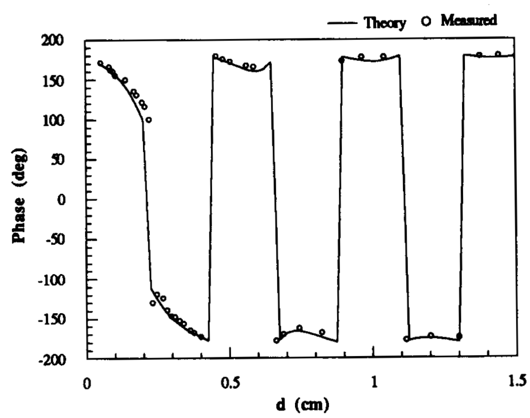

Fig. 9. Comparison of theory and measurement of phase versus thickness at 10 GHz for slab of $\epsilon_r' = 12.6$, $\tan\delta = 0.19$.

respect to a short circuit [10]. To verify the theoretical results for the admittance expression, the attenuation and phase of the reflection coefficient as a function of slab thickness were compared with a series of experimental results for three different dielectric samples at 10 GHz. These samples consisted of a low-loss and two higher loss dielectric slabs. Figs. 4 and 5 show these results for a slab with $\epsilon_r' = 2.59$, $\tan\delta = 0.007$; Figs. 6 and 7 for a slab with $\epsilon_r' = 7.25$, $\tan\delta = 0.103$; and Figs. 8 and 9 for a slab with $\epsilon_r' = 12.6$, $\tan\delta = 0.19$, respectively. For all cases, the theoretical and experimental results follow each other closely.

Calculation of Thickness

To calculate the thickness of an unknown sample, the conductance and susceptance expressions were treated separately in a root-finding code similar to the one described in [11]. Starting with initial lower and upper bounds for thickness, G and B were iterated alternately to find the root which would be a close estimation of the thickness of the dielectric sample. For both cases, the root is found within some prescribed range which can initially be estimated from plots of the variations of the parameters over some larger interval. This would also decrease the calculation time since a finer range can be estimated in advance for the position of the root. In general, for monitoring thickness variations, an *a priori* knowledge of the thickness is usually available.

As the loss tangent decreases, the integration in (13) requires that finer increments be evaluated due to slower convergence. This simply means that the integrand approaches the singular behavior which would eventually appear for a lossless dielectric slab. However, as the loss tangent increases, the integrand becomes more smooth, and the integration becomes more manageable (faster convergence). Due to the fact that one of the main concerns in this type of measurements is calculation speed, open-interval integration methods which commonly use convergence tests are rendered useless. Here, a Gauss quadrature method with a fixed number of intervals was used to perform the integration. For such a case, it is vital to find, *a priori*, the necessary number of intervals for appropriate convergence in the range of dielectric characteristics of the medium under test. Once an upper bound on the necessary number of intervals is chosen, the integrations can be performed quite efficiently with the desired accuracy.

Experimental Results

Thicknesses of several lossy dielectric slabs from two types of synthetic rubber at X-band were measured using the procedure outlined above. Different samples in sheet form with known permittivity characteristics and different thicknesses were chosen. Using the measured permittivity values, the thicknesses of the samples were calculated using the above procedure. The results of these experiments are shown in Tables II and III. For both samples, the calculated thickness values were within 3% of their measured values.

The first column shows the physically measured values of slab thickness. The variations of the calculated results in column three are an indication of how close the root could be estimated in the domain of *both* B and G. Better resolution can be achieved by choosing an optimum frequency based on material thickness and dielectric property.

These experiments were performed to emphasize some important points that must be taken into account in this type of measurement. To increase the sensitivity for thickness measurements, the electrical length of the dielectric slab should not be such that it would constitute an infinite half-space. This fact is clear from Figs. 8 and 9, which show more abrupt variations at smaller thicknesses. Further, the optimum operating frequency must be chosen such that any possible dielectric constant variation would have a minimal effect on the thickness measurement due to the sensitivity of the microwaves to dielectric property variations.

TABLE II
Thickness Measurement Result at 10 GHz for $\epsilon_r' = 7.25$ and $\tan \delta = 0.103$

Measured d (mm)	Y_{measured}	Calculated d(mm)	% Mean Error
3.91 ± 0.04	$0.698 + j1.433$	3.9 ± 0.05	0.26
5.92 ± 0.04	$10.030 + j1.610$	5.83 ± 0.05	1.52
7.77 ± 0.05	$2.374 - j1.212$	7.8 ± 0.1	0.40
11.56 ± 0.05	$5.570 + j1.850$	11.38 ± 0.19	1.56

TABLE III
Thickness Measurement Result at 10 GHz for $\epsilon_r' = 12.6$ and $\tan \delta = 0.19$

Measured d (mm)	Y_{measured}	Calculated d (mm)	% Mean Error
3.63 ± 0.07	$3.67 + j4.25$	3.59 ± 0.02	0.1
3.76 ± 0.05	$4.85 + j4.83$	3.76 ± 0.02	0.1
5.90 ± 0.07	$2.80 + j1.69$	5.89 ± 0.06	0.2
7.43 ± 0.06	$3.08 + j1.64$	7.45 ± 0.05	0.3
8.20 ± 0.05	$3.88 + j2.55$	8.03 ± 0.05	2.2
12.05 ± 0.06	$4.49 + j0.98$	12.20 ± 0.10	1.2
13.0 ± 0.05	$5.18 + j1.17$	12.64 ± 0.06	2.8

Conclusions

A fast and accurate method has been discussed for the microwave thickness measurement of generally lossy dielectric slabs backed by a conducting plate. A simple integral solution was developed for the admittance of an open-ended rectangular waveguide radiating into a lossy dielectric slab backed by a conducting plate. The limited thickness measurement results presented here were for the worst case within 3% of the physically measured values. Some important issues were pointed out in the application of this technique for thickness measurements. The results show this method to be a fast and reliable technique for microwave nondestructive measurement of dielectric slabs or composite coatings backed by conducting plates. For slab thickness variation detection, it is best to operate in the region in which attenuation and phase experience more rapid fluctuations which can generally be achieved by operating at an optimum frequency. This technique renders measurement accuracies of better than 0.5% for dielectric slab thicknesses of less than 5 mm and no worse than 3% for thicker slabs. For this simple technique, these accuracies are better than desired for most industrial applications (in Table III and for thicknesses less than 5 mm, the accuracies are better than 40 μm). It must also be pointed out that even better accuracies may be obtained if the measurement parameters (specifically, the operating frequency) are optimally chosen for a given slab thickness and dielectric properties. In general, techniques such as the ultrasonic method render accuracies of about 1–3%. However, it is questionable whether an ultrasonic signal can penetrate into relatively thick dielectric materials with high loss factors.

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