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Operation of the 184" Cyclotron

L. R. Henrich, D. C. Sewell, J. Vale

May 17, 1949

Berkeley, California

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ABSTRACT

The operation of the 184" synchro-syclotron is reviewed in terms of the theory as developed by Bohm and Foldy. Certain relevant data on the properties of the magnet and rotating condenser are also presented.

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1. Introduction

The general theory of the operation of a frequency modulated cyclotron -- or synchro-cyclotron -- has been discussed in earlier papers by Bohm and Foldy[1,2]

[1] D. Bohm and L. Foldy, Phys.Rev. 70, 249, 1946. "The Theory of the Synchrotron," Herein after referred to as paper A. Equations and figures from this paper will be referred to as (A-X or A-Fig. X), respectively.

[2] D. Bohm and L. Foldy, Phys. Rev. 72, 649, 1947. "Theory of the Synchro-Cyclotron." Referred to as paper B.

among others. The purpose of the present paper is to present in more detail than previously reported[3] such operational data for 184" Berkeley synchro-cyclotron as might be of interest. It is also intended to interpret the performance of the machine in terms of the theory. We mention first general characteristics of the machine and then investigate the operation when certain conditions are varied.

[3] W. M. Brobeck, E. O. Lawrence, K. R. MacKenzie, E. M. McMillan, R. Serber, D. C. Sewell, K. M. Simpson, and R. L. Thornton, Phys.Rev. 71, 449, 1947.

2. Normal Acceleration of Ions.

The final design of the 184" magnet determines such things as the energy of the ion as a function of the radius, r, while the design of the rotating condenser affects the mean rate of energy gain per turn. Since these and other related quantities will be needed in subsequent discussions of the performance of the machine, the expressions for these quantities are given below. The actual values for the machine are later tabulated (Table 2) or graphed (Fig. 1).

a. $H(r)/H_c$ - where $H(r)$ is the value of the magnetic field in the median plane and perpendicular to it as a function of the radius (r); and H_c is the value of the field at the center of the machine.

b. $n(r) = -\frac{r}{H}\frac{dH}{dr}$ 1

c. $\beta^2(r) = \left[1 + \left(\frac{M_0c^2}{ZerH}\right)^2\right]^{-1} = \left(\frac{v}{c}\right)^2$ 2

where M_0 is the rest mass of the ion, c is the velocity of light, Ze is the charge on the ion, e is the unit electric charge, and v is the velocity of the ion.

d. $E(r) = \frac{E_0}{\sqrt{1-\beta^2}}$ - the total energy of the ion, including the rest energy (E_0). 3

e. $\omega = \frac{ZeH}{Mc}$ where ω is the angular frequency of rotation of the ion, and M the total mass of the ion. 4

f. $K = 1 + \frac{n}{1-n}\frac{c^2}{v^2}$ A necessary physical quantity (See A-16) 5

$K = 1 + h\frac{c^2}{\omega^2}$ an alternative expression of K valid in a parabolic 6

magnetic field (See B-5) where h expresses the rate of decrease of the magnetic field with radius, i.e., $H = H_c\,(1 - hr^2/2)$. 7

The above quantities depend on the shape of the magnetic field. In addition there are the quantities which depend on the physical characteristics of the rotating condenser that modulates the frequency of the dee voltage.[4] These may be expressed as a function of the phase τ of the rotating condenser, where we let $2\pi\tau$ represent the phase angle of the condenser and $\tau = 0$ when the oscillation is at maximum frequency.

[4] For the design of the frequency modulation equipment see

F. H. Schmidt, Rev. Sci. Instr. 17, 301, 1946.

K. R. MacKenzie, F. H. Schmidt, J. R. Woodyard, and L. F. Wouters, Rev. Sci. Instr. 20, 126, 1949.

g. $\omega_s(\tau)$ = the angular frequency of the oscillator. The subscript $_s$ denotes the angular frequency the ion must have if it is to be locked into the synchronous orbit at a fixed phase angle with no oscillations about this phase angle.

h. $\frac{V(\tau)}{\bar{V}}$ = the ratio of the actual maximum dee voltage across the gap to the mean value during a modulation cycle.

i. $-\frac{d \ln \omega_s}{d\tau}$: a quantity used in computing mean energy gain per revolution.

The last three quantities have been expressed as a function of τ. However, the angular frequency of the ion is determined by the expression for ω given in (e) and depends on the magnetic field. For an ion to be in the synchronous orbit at any time its frequency must also be ω_s. This condition provides a unique correlation between the phase of the oscillator and the radius of an ion in the synchronous orbit. We accordingly denote the radius, energy, and angular frequency of such a synchronous particle by r_s, E_s and ω_s. This correlation also permits us to express $\frac{V}{\bar{V}}$ and $-\frac{d \ln \omega_s}{d\tau}$ as a function of radius.

An ion will stay at the synchronous frequency only if it gains energy at the proper rate. This may be expressed using A-15 by

$$\frac{dE}{dt} = -\frac{E_s}{K\omega_s}\frac{d\omega_s}{dt} = -\frac{CE_s}{K\omega_s}\frac{d\omega_s}{d\tau} \qquad 8$$

where t is the time. To express the energy gain in terms of the rate of frequency modulation we have let $\tau = Ct$ where C is the number of frequency modulation cycles per second. Multiply by the period of a revolution, $\frac{2\pi}{\omega_s}$, to get the energy gain per turn for the synchronous orbit, ΔE_s; so that

$$\frac{\Delta E_s}{C} = \frac{2\pi E_s}{K\omega_s}\left[-\frac{d \log \omega_s}{d\tau}\right] \qquad 9$$

The change in radius per turn will be given by A-14

$$\frac{\Delta r_s}{C} = \frac{r_s}{(1-n)\beta_s^2}\frac{\Delta E_s}{CE_s} \qquad 10$$

Now the phase angle, ϕ_s, for acceleration in the synchronous orbit is given by $e\bar{V}\sin\phi_s = \Delta E_s$. Due to the change in ΔE_s and in V during the acceleration period the phase angle will change during the acceleration. The magnitude of the variation in sin ϕ_s may be determined from the equation,

$$e\,\bar{V}\,\sin\phi_s = \frac{\bar{V}}{V}\,\Delta E_s. \qquad 11$$

3. Oscillations about the Synchronous Orbit.

The motion of a particle on the synchronous orbit is described by the expressions in the preceding section. Actually, few, if any, particles follow this orbit precisely. There are deviations from the orbit due to the fact that ions are not injected at the correct frequency corresponding to the central magnetic field, also due to the fact that ions are not injected at the synchronous phase angle ϕ_s but of necessity near a phase angle of 90° (See B-App. I). These causes will result in phase oscillations of the radius, energy and phase angle (See paper A).

The angular frequency of phase oscillation (A-21) is given approximately for small oscillations by

$$\omega_{ph} = \left(\frac{e\,V\,K\,\cos\phi_s}{2\pi E_s}\right)^{1/2} \omega_s \qquad 12$$

The period of this oscillation will be $T_{ph} = \dfrac{2\pi}{\omega_{ph}}$ 13

The amplitude of the energy oscillation will be (A-19),

$$\Delta E_{ph} = \pm \left(\frac{e\,V\,E_s}{\pi K}\right)^{1/2} \left[U(\phi_m) - U(\phi)\right]^{1/2} \qquad 14$$

where

$$U(\phi) = -\cos\phi - \phi\sin\phi_s \qquad 15$$

and ϕ_m is the maximum value of ϕ.

The amplitude of radial phase oscillation will be given using A-14.

$$\Delta r_{ph} = \frac{r_s}{(1-n)\beta_s^2}\,\frac{\Delta E_{ph}}{E_s} \qquad 16$$

There will also be additional radial oscillations -- due, for example, to inhomogeneities in the magnetic field -- which will be called "free radial oscillations." There will also be free vertical oscillations -- i.e. -- oscillations perpendicular to the median plane of the magnet gap, -- denoted by a z - coordinate -- due originally to the ions starting their orbits at positions displaced from the median plane.

The general equations of motion of the ion may be obtained from the Hamiltonian

$$H = c\left\{M_o^2c^2 + \left(p_r - \frac{e}{c}A_r\right)^2 + \left(\frac{p_\theta}{r} - \frac{e}{c}A_\theta\right)^2 + \left(p_z - \frac{e}{c}A_z\right)^2\right\}^{1/2} - e\Phi \qquad 16a$$

where $\vec{A} = (A_r, A_\theta, A_z)$ is the vector potential in cylindrical coordinates and Φ is the accelerating potential, and $\vec{p} = (p_r, p_\theta, p_z)$ are the canonical momenta. The investigation of the free vertical and radial oscillations near any specified radius, r_o, may be carried out while neglecting the accelerating dee voltage. The mass may be taken as constant. The magnetic field may be taken as a function of r and z only, where z is the distance from the median plane. In this case it is possible to set $A_r = A_z = 0$ and retain only $A_\theta = A\,(r,z)$. The equations of motion of the ion about a certain equilibrium orbit of radius, r_o, under these conditions can be written to the first approximation as

$$\frac{d^2(\Delta r)}{dt^2} = -\,(1-n)\,\omega_o^2\,\Delta r, \qquad 17$$

$$\frac{d^2 z}{dt^2} = -\,n\,\omega_o^2\,z, \qquad 18$$

where Δr ($= r - r_o$) is the deviation from the equilibrium orbit, ω_o is the angular frequency in the equilibrium orbit and n is the value of n for the equilibrium orbit. The free radial oscillations will have the angular frequency $\omega_r = \sqrt{1-n}\,\omega_o$ and the vertical oscillations will have the angular frequency $\omega_z = \sqrt{n}\,\omega_o$. The vertical motion will be stable only for $n > 0$ and the radial motion will be stable only for $n < 1$. At $n = 1$ there will be resonance between the periods of rotation and vertical oscillation, so in this region the vertical amplitude could build up due to resonance effects -- in the same manner as the radial amplitudes could build up at $n = 0$. Hence, for a stable orbit $0 < n < 1$. It follows that both ω_r and ω_z are smaller or slower than ω_o. Consequently, while the ion is performing one revolution, it will not complete a free radial or vertical oscillation. The result will be that the azimuthal position in the machine at which the ion has a maximum radial displacement will precess. The precessional period for radial oscillations will be

$$T_{pr} = \frac{2\pi}{\omega_o} \frac{1}{1-\sqrt{1-n}}, \qquad 18a$$

and precessional angular frequency will be

$$\omega_{pr} = \omega_o \left(1 - \sqrt{1-n}\right) \qquad 19$$

The change in mean radius during a precessional period will be given by

$$\Delta r_{pr} = \frac{1}{1-\sqrt{1-n}} \frac{r_s}{(1-n)\beta_s^2} \frac{\Delta E}{E_s}. \qquad 19a$$

4. Operation under Normal Conditions.

Typical operating conditions (e.g., magnetic field, tank pressure, etc.) are given in Appendix I. Unless otherwise mentioned the discussion and data refer to operation with deuterons. The deuteron beam current received on an internal target as a function of the radius is plotted in Fig. 2. The minor variations in the ion measurements between 25 and 70 inches presumably are not significant and probably arise due to the method of reading the current. The factors that cause decrease of beam intensity (neglecting the effects at the maximum radius reached) are effective primarily at the small radii. These are (1) gas scattering which is unimportant at higher energies; (2) return of ions to center which takes place primarily during the first phase oscillation so that most of these ions never get beyond a radius of about 10 inches; (3) loss of beam because of the increase of the synchronous phase angle so that ions can no longer be retained in stable orbits. (This occurs primarily before a radius of 20 inches is reached); (4) loss of beam at small radii due to inadequate vertical focusing. Since the probe cannot go closer to the center than about 20 inches the curve obtained (which starts at a radius of 25 inches) would not be expected to show any considerable change in beam intensity during acceleration.

5. Loss of Beam at Large Radius.

Originally it was expected that the beam could be accelerated to the radius at which n = 1, where the motion becomes radially unstable. Actually, however, most of the beam disappears at a radius of about 81.5 inches. Copper probes in the shape of a C (See Fig. 3a) -- with the open end of the C toward the center of the tank were placed in the path of the beam to find out what happened to the beam. Radioautographs were made of these probes (See Fig. 3b). They show that beam is received in the back part of the C at a radius of 75 inches. The radioautograph at the 82 inch radius shows most of the beam hitting on the top of the C. The lack of symmetry for top and bottom is probably due to not centering the probe in the surface of symmetry of the beam. This surface is determined as the surface about which the vertical oscillations of the ions take place. The loss of the beam could be considered as due to this surface being displaced. However, densitometer measurements of probe radioautographs at r = 75 inches indicate that the surface of symmetry is displaced upwards only one-half inch while at r = 80-1/2, 81-3/4, and 82-1/4 inches this surface is very close to the median plane of the magnet gap. The radioautograph at 83 inches shows all of the beam hitting the top or bottom of the C probe before the 83 inch radius is reached and none hitting at 83 inches. It was also observed that there was a "hot" spot on the dee at about this radius where presumably the amplitude of vertical oscillations increases sufficiently to cause the ions to hit the dee. The hot spot was on the top of the dee due to the fact that the surface of symmetry is about one-half inch above the median plane of the magnet gap while the dee is centered in the gap.

5a. Coupling at n = 0.2.

It appears that the coupling between free radial and vertical oscillations at n = 0.2 (r = 81.2 inches) and the coupling between vertical oscillations and azimuthal inhomogeneities at n = 0.25 (r = 81.7 inches) is adequate to cause this loss of beam.

The amplitude, A_r, of the free radial oscillations is not known accurately but it is probably of the order on the average of one or two inches. The amplitude, A_z, of vertical oscillation is probably less than one inch. However, if all the energy of radial oscillation were fed into the vertical oscillation, the new amplitude of vertical oscillation, A'_z, would be

$$A'_z = \sqrt{A_z^2 + \left(\frac{\omega_r}{\omega_z}\right)^2 A_r^2} \qquad 20$$

At $n = 0.2$, $(\omega_r/\omega_z) = 2$. Hence, if $A_r \gtrsim 1$ inch and the energy in the radial oscillation were fed into the vertical oscillation, the amplitude A'_z would be at least 2 inches. Now the surface of symmetry at this radius is about one-half inch above the median plane of the magnet gap. The vertical clearance in the dee is 5 inches so that the distance from the surface of symmetry to the dee will be only 2 inches. Hence, all ions with $A_r >$ 1 inch would be intercepted by the dee if this process went on.

The extent to which this process goes on may be roughly calculated in two steps. We may first calculate the extent of coupling between the two modes of oscillation as follows. Let $r = r_o(1 + \rho)$, $z = r_o \zeta$ and $\tau = \omega_o t$ where ω_o is the angular frequency of rotation at the radius in question. (Note that τ as used in this section is not the same τ as used to denote period of frequency modulation.) Then the Hamiltonian, neglecting the accelerating voltage, may be written in the following form

$$H = \frac{M}{2} \left\{\frac{p^2}{M^2} + \frac{p_r^2}{M^2} \frac{p_z^2}{M^2} + \omega_r^2\, r_o^2\, \rho^2 + \omega_z^2\, r_o^2 \zeta^2 - \alpha\, \omega_o^2\, r_o^2 \rho\, \zeta^2 + - \right], \qquad 21$$

where p is the orbital angular momentum for the equilibrium orbit. The $\alpha =$ $\left(\frac{r^2}{H} \frac{\partial^2 H}{\partial r^2}\right)_{r_o}$ represents the coupling coefficient. Transforming to action and angle variables the Hamiltonian is averaged over the short term oscillations leaving only the long term oscillation due to the coupling term. It is found then that the duration of the coupling period, τ_J -- i.e., the period of the long term oscillation in which period the amplitude of the z oscillation will change from its minimum value to a maximum value and back to its minimum value again -- can be represented roughly by $\tau_J \simeq 2\pi \frac{0.8r_0}{\alpha A_r}$. For $\alpha = 6.4$ and $A_r = 1$ inch the coupling period is $\tau_J \simeq 2\pi \cdot 10$, i.e., the time it takes the ion to perform 10 revolutions, and during this coupling period at some time most of energy of radial oscillation will be in the vertical oscillation. Hence, if the duration of the coupling is as large as τ_J, we can expect a considerable variation in the amplitude of A_z. Actually, when there is a considerable change in A_r, or A_z this method of solution is not accurate. However, if the method indicates large changes, it is an indication of possible danger points.

Now actually as the ion increases its radius the value of n slowly increases. Whether much energy transfer takes place will depend on the length of time that the ion is close to resonance. An estimate of the effective period of resonance can be obtained as follows: let $n = n_0 + \sigma\tau$ where n_0 is the value of n at the resonance. Then substitute this expression for n in the uncoupled equations of motion, 17 and 18, and obtain the solutions for the uncoupled equation in ρ, ζ. Taking these as the solutions of the homogeneous equations we have to solve the non-homogeneous coupled equations

$$\frac{d^2\rho}{dt^2} = -(1-n)\rho + \frac{\alpha}{2}\zeta^2 , \qquad 22$$

and

$$\frac{d^2\zeta}{dt^2} = -n\zeta + \alpha\rho\zeta . \qquad 23$$

The method of variation of parameters is used with the above approximate solutions. The variable coefficient in the solution then contains a term of large amplitude which is a Fresnel integral. The time taken to go from the minimum to the maximum value of this integral gives an approximation to the effective resonance period, τ_R -- i.e., an approximation to the time during which the coupling may be considered effective as the particle passes through the resonance value $n = 0.2$. It is found that

$$\tau_R = 4 \sqrt{\frac{\pi}{\sigma} \cdot n_o \sqrt{1-n_o}} \; . \qquad 24$$

Now

$$\sigma = \frac{dn}{d\tau} = \frac{dn}{dr} \cdot \frac{dr}{d\tau} = \frac{n+n^2-\alpha}{(1-n)\beta^2} \frac{\Delta E_s}{2\pi E_s} \quad . \qquad 25$$

The criterion may now be adopted that if $\tau_R > \tau_J$ most of the energy of radial oscillation will be transferred to vertical oscillation at least once as the ion goes through the resonance. Similarly, if a machine is being constructed and we wish to decide what physical condition must be met to have incomplete energy transfer for n = 0.2, the condition would be approximately that $\tau_R < \tau_J$; i.e., we obtain the condition that

$$\frac{\alpha^2}{0.24-\alpha} < \frac{0.55\, r_0^2}{\beta^2 A_r^2} \frac{\Delta E_s}{E_s} . \qquad 26$$

Calculation of τ_R for the 184" synchro-cyclotron with $E_s = 2070$ Mev and $\Delta E_s = 6$ Kev gives $\tau_R = 2\pi \cdot 100$, i.e., the passage through the resonance takes about 100 revolutions. Hence, we can expect almost complete energy transfer and so can expect that ions with amplitude of radial oscillation, A_r, greater than 1 inch will be lost near the n = 0.2 radius due to striking the dee.

Now we have discussed above an experiment in which C shaped probes were used to detect the beam. In these experiments it was found that some of the beam struck the central part of the probe placed at a radius of 82 inches, but that none struck the central part of a probe placed at a radius of 83 inches. [In view of the data discussed in §15 as to the true center of rotation of the ions, we should probably correct these to 82.2 inches and 83.2 inches from the actual center of rotation of the ions.] Ions which strike in the central region of the probe may belong in one of two groups. If $A_r < 1$ inch we may expect the ions to be accelerated past the radius at which n = 0.2 without striking the dee. (The possible loss of these ions will be discussed below.) Ions with $A_r > 1$ can be expected to get to a radial distance greater than where n = 0.2 during part of their orbit, since the resonant coupling is effective where the average value of n during a revolution is 0.2. Calculation of the average value of n indicates that if $A_r > 2\text{-}1/2$ inches, the ion should get to a radius of 83 inches; if $A_r >$ 5 inches the ion should get to a radius of 83-1/2 inches. Hence, we may conclude that in the 184" ions with 1 inch $< A_r <$ 5 inches may indeed be lost by coupling near the radius where $n = 0.2$ and not show up on the central part of a probe placed at a radius of 83-1/2 inches.

It is possible by using (26) to determine a limit for α to insure that coupling be unimportant at $n = 0.2$. For the 184" synchro-cyclotron letting $\Delta E =$ 6 Kev and $A_r = 1$ inch it is found that the coupling coefficient $(-\alpha)$ should be less than 0.14. Actually, it is about 6.4. Hence, if this is the reason for losing the beam at $n = 0.2$, it would be rather difficult to correct the field in this case as this means determining $\left(\frac{1}{H}\frac{\partial^2 H}{\partial r^2}\right)_{r=r_0}$ so that it is less than .00002/in.^{2} which means very high accuracy in machine construction and field measurements. The physical process can be seen as follows. When $-\alpha$ is large the strength of the vertical forces increases rapidly with radius since the field lines are more curved at the larger radii. Hence, when an ion is at large radii due to a radial oscillation, it will experience large vertical forces. At smaller radii the vertical forces will be weaker. With a suitable phase relation then it can be seen that the ion can have its vertical amplitude built up by the strong forces at large radii which are not counterbalanced by forces of similar strength at small radii.

In the case of the 37" synchro-cyclotron, with the same value of A_r and $\Delta E = 5$ Kev, at the radius where $n = 0.2$, $(-\alpha)$ again should be less than 0.14. The actual value seems to be smaller than this. Radioautographs of C shaped probes show that the beam does not blow up in the 37" synchro-cyclotron at the radius where $n = 0.2$ ($r_0 = 16$ inches) but that actually a considerable amount of the beam does reach the radius where $n = 1$. The difference between the two machines may be due to the fact that the rate of decrease of the field is changing rapidly for the 184" synchro-cyclotron near this radius. This is not true in the case of the 37" synchro-cyclotron.

5b. Coupling at $n = 0.25$.

The coupling here would be between the vertical oscillation and azimuthal inhomogeneities in the magnetic field, the ion performing two revolutions for each vertical oscillation. A term of the form $z \cos \theta$ would give coupling at $n = 0.25$. It is possible to develop H_r, the radial component of $\vec{H}$, in a series such that we can have

$$H_r = \frac{zH_z}{r} n \, (1 + h \cos \theta) \tag{27}$$

where h is a small quantity. We then must investigate the solution of

$$\frac{d^2 z}{dt^2} = - \frac{e}{Mc} r \dot{\theta} H_r = - \omega_0^2 \, n \, (1 + h \cos \theta) \, z \tag{28}$$

Letting $\omega_0 dt = d\theta$, $\theta = 2\phi$, $n = 0.25$ the equation becomes

$$\frac{d^2 z}{d\phi^2} + (1 + h \cos 2\phi) \, z = 0 \tag{29}$$

which is a form of Hill's equation. An approximate solution can be obtained in the following form

$$z = \text{Const. } e^{\left(\pm \sqrt{\frac{2}{3}} \frac{h}{2\pi} \theta\right)} \Phi\left(\frac{\theta}{2}\right), \tag{30}$$

where $\Phi\left(\frac{\theta}{2}\right)$ is a periodic function. Hence z will change by a factor e when

$$\theta = 2\pi \left(\sqrt{\frac{3}{2}} \frac{1}{h}\right). \tag{31}$$

Investigation of the available data on the field of the 184" magnet indicates that it is possible that such an inhomogeneity exists near $n = 0.25$ with h of the order of 0.05. Hence, we may expect that in a period of 24 revolutions, ions could increase the amplitude of their vertical oscillation by a factor e.

The resonance time in this case is found to be

$$\tau_R = 4 \sqrt{\frac{\pi n_0}{\sigma}} \tag{32}$$

Letting $\Delta E_s = 5$ Kev, $\alpha = 0.0015 \, r^2$ it is found that $\tau_R = 2\pi(104)$. Hence, the effective resonance time is sufficiently long to permit a considerable increase in the amplitude of vertical oscillation. Only particles with an initial $A_z \ll 1$

inch could be expected to get through this resonance in the 184" synchro-cyclotron. Hence, this coupling could be expected to account for the loss of practically all of the beam that might have gotten past the $n = 0.2$ resonance.

There is insufficient data on the 37" magnet to investigate the effect of this coupling for that magnet. The effective resonance time, τ_R, is quite a bit shorter -- only 27 revolutions. Radioautographs of a C shaped probe do indicate that there is some blowing up of the beam at $r = 18$ inches near where $n = 0.25$.

Investigation of the resonance at $n = 0.1$ indicates that it is not dangerous. As we go to higher order resonances the coupling is probably much weaker so that the coupling period becomes longer. Hence, these resonances are probably unimportant.

5c. Coupling at $n = 0.1$.

The coupling here would be between the vertical and radial oscillations with $\omega_r = 3\,\omega_z$. Assuming a median plane of symmetry, the coupling term in the Hamiltonian (21) would be of the form $r_0^2\,\omega_0^2\,\mu_{2,6}\,\rho^2\,\zeta^6$, where $\mu_{2,6} \simeq \frac{1}{6.6!}\left(\frac{r^7}{H}\,\frac{\partial^7 H}{\partial r^7}\right)_{r_0} = \frac{\alpha_7}{6.6!}$. Actually, $\mu_{2,6}$ will contain other terms but unless the lower derivatives are extremely large, this will be the important one to consider. We then find for the condition $\tau_R > \tau_J$ that we must have

$$6480\,\frac{r_0^{\,6}}{A_r^2 A_z^4} > \alpha_7\,\sqrt{2\,\beta^2\,\frac{E_s}{\Delta E_s}}\,.$$

Assuming $A_r = 2$ inches, $A_z = 1$ inch, $r_o = 80$ inches, $\beta^{-2} = 5.7$, $\Delta E_s = 10$ Kev, we find that the seventh derivative of (H/H_o) should be less than $.1000/\text{inch}^7$. Magnetic field measurements at each inch of radius might be used to estimate what the actual situation is by replacing derivatives by differences. However, the magnetic measurements are not good enough to give really significant values for the seventh difference. Even so, it appears that there is a large safety factor at this frequency.

6. Variation of Ion Current with Voltage.

The maximum obtainable ion current as a function of dee voltage is plotted in Fig. 4. All other conditions -- such as condenser speed, etc. -- were adjusted at each voltage to give the maximum current. The slope of the line between 9 and 17.5 kv indicates that the current varies as the third power of the voltage. As pointed out in the paper by Richardson, et al,[5] there are many factors that influence the variation of current with voltage: such as

[5] J. Reginald Richardson, Byron T. Wright, E. J. Lofgren, and Bernard Peters, Phys. Rev. 73, 424, 1948.

1. Source current. The ion current that can be pulled out of the source will depend on the accelerating dee voltage and might be expected to vary as $V^{3/2}$.
2. Loss of ions due to collision decreases with increasing ion energy. Hence, if the ion has a shorter path at low energies there will be less collision loss. With higher dee voltages the path length before a given energy is attained will be shorter.
3. The catching efficiency also varies with dee voltage. The equation B-10 indicates for a fixed phase angle the efficiency will vary as $V^{1/2}$. Actually, the more detailed considerations of what ions are caught in phase stable orbits indicate that the ion current variation is even more sensitive to the voltage. This is related to the fact that with higher voltages the ions caught in phase stable orbits will reach larger radii in the first phase oscillation.

7. Variation of Ion Current with Condenser Speed.

The ion current obtainable for fixed dee voltages but varying condenser speeds is plotted in Fig. 6. The figure is in terms of I/I_{max} vs. C/C_{max}, where I_{max} is the maximum ion current for the given voltage and C_{max} is the condenser speed at which the beam disappears. This variation of C is equivalent to varying the synchronous phase angle.

The second paper by Bohm and Foldy gives a curve (B-Fig. 2) which represents the capture efficiency as a function of synchronous phase angle. However, this does not represent the above experimental data for the 184" synchro-cyclotron too well. This is primarily due to the fact that during a frequency modulation cycle of the 184" synchro-cyclotron the value of sin ϕ_s varies by a considerable amount. The increase of ϕ_s from its value during the first phase oscillation to its value at the maximum (near r = 35 inches) means that the range of $\dot{\phi}_m$, the maximum value of $\dot{\phi}$ during the phase oscillation, in stable orbits will shrink. Hence, although the $\dot{\phi}_m$ for a given ion may have been in a stable range during the first phase oscillation, it may be that at the maximum ϕ_s the $\dot{\phi}_m$ is no longer stable. If we know the maximum value of ϕ_s during the fm cycle, we can determine what range of $\dot{\phi}_m$ will be stable at that ϕ_s. Since $\dot{\phi}_m$ will not remain constant during the cycle it is necessary to be able to connect the value of $\dot{\phi}_m$ at r = 35 inches ($\dot{\phi}_{mf}$) with the value of $\dot{\phi}_m$ in the first phase oscillation ($\dot{\phi}_{ms}$). (In the following the second subscript $\underline{s}$ denotes the value of the quantity at the start in the first phase oscillation and the subscript $\underline{f}$ the final value -- or the value at r = 35 inches.) An approximate relation between these two values of $\dot{\phi}_m$ may be obtained as follows.

We know that during adiabatic changes the action integral as given in (A), $J = \oint I \dot{\phi} \, d\phi$ is invariant under such changes, where $I = E_s/\omega_s^2 K$ 33

and

$$\dot{\phi}^2 = \frac{eV\omega_s^2 K}{\pi E_o}\left[U(\phi_m) - U(\phi)\right],$$

or

$$\dot{\phi}^2 = \dot{\phi}_o^2 + \frac{eV\omega_s^2 K}{\pi E_o}\left[\cos\phi + (\phi - \pi/2)\sin\phi_s\right]. \qquad 34$$

We shall assume that the change in ϕ on going from the first phase oscillation to the situation where r = 35 inches is adiabatic. Now $\dot{\phi}$ varies between upper and lower limits and we assume that a crude approximation to the evaluation of J may be obtained by setting $\dot{\phi} = \dot{\phi}_m \sin \omega_{ph} t$ (See A-22).

Then

$$J = I\dot{\phi}_m^2 \oint \sin^2 \omega_{ph} t \, d t = \pi \frac{I\dot{\phi}_m^2}{\omega_{ph}} \qquad 35$$

Hence, we assume for this approximation that $I\dot{\phi}_m^2/\omega_{ph}$ is constant in time. Now we are interested in the limiting $\dot{\phi}_m$ retained when r = 35 inches. This will be for $\phi_m = \pi - \phi_s$. Hence

$$\dot{\phi}^2 = \frac{eV\omega_s^2 K}{\pi Mc^2}\left[\cos\phi + \cos\phi_s + (\phi + \phi_s - \pi)\sin\phi_s\right] . \qquad 36$$

Since $\dot{\phi}$ has its maximum value at $\phi = \phi_s$, we have

$$\dot{\phi}_{m,f}^2 = \left(2\frac{eV\omega_s^2 K}{\pi Mc^2}\left[\cos\phi_s + (\phi_s - \pi/2)\sin\phi_s\right]\right)_f = \left(2\frac{eV\omega_s^2 K}{\pi Mc^2} F_1(\pi/2, \phi_s)\right)_f , \qquad 37$$

where

$$F_1(\phi_o, \phi_s) = \left[\cos\phi_o + \cos\phi_s + (\phi_o + \phi_s - \pi)\sin\phi_s\right] . \qquad 38$$

The value of $\dot{\phi}^2$ at the start should be given by (B-6)

$$\dot{\phi}^2 = \dot{\phi}_o^2 + \frac{eV\omega_s^2 K}{\pi Mc^2}\left[\cos\phi - \cos\phi_o + (\phi - \phi_o)\sin\phi_s\right] , \qquad 39$$

where ϕ_o is the phase angle at the start of the acceleration and $\dot{\phi}_o$ is value of $\dot{\phi}$ at this angle. It is shown in B that the effective starting phase, ϕ, for all ions is $\pi/2$. Hence, setting $\phi = \phi_s$ to get $\dot{\phi}_{ms}$, we have

$$\dot{\phi}_{m,s}^2 = \left[\dot{\phi}_o^2 + \frac{eV\omega_s^2 K}{\pi Mc^2} F_1(\pi/2, \phi_s)\right]_s . \qquad 40$$

Using (35), (37), and (40) we then have

$$\dot{\phi}_{ms}^2 = \left(\frac{\omega_{ph}}{I}\right)_s \left(\frac{I}{\omega_{ph}}\right)_f \dot{\phi}_{m,f}^2 , \qquad 41$$

or

$$\dot{\phi}_{o,s}^2 = \left(\frac{\omega_{ph}}{I}\right)_s \left(\frac{I}{\omega_{ph}}\right)_f \left[\frac{2eV\omega_s^2 K}{\pi Mc^2} F_1(\pi/2, \phi_s)\right]_f - \left[\frac{eV\omega_s^2 K}{\pi Mc^2} F_1(\pi/2, \phi_s)\right]_s . \qquad 42$$

Substituting for I, ω_{ph} we get

$$\left(\frac{\pi Mc^2}{eV\omega_s^2 K}\right)_s \dot{\phi}_{o,s}^2 = 2\sqrt{\frac{\cos\phi_{ss} \cdot V_f K_s \cdot E_{sf}^3 \cdot \omega_{ss}^2}{\cos\phi_{sf} \cdot V_s K_f \cdot E_{ss}^3 \cdot \omega_{sf}^2}}\, F_1(\pi/2, \phi_{sf}) - F_1(\pi/2, \phi_{ss}) . \qquad 43$$

This will give the limiting stable $\dot{\phi}_o$ which can start in the first phase oscillation and just be retained when r = 35 inches. For small phase angles this may give a greater range of phase stability than is given by B-8. Hence, the limiting $\dot{\phi}_o$ is determined by the smaller of the two quantities either B-8 or 43 above, since the first one indicates ions that can be retained at small radii and the latter indicates those that can be retained at larger radii.

The calculation of $\dot{\phi}^2_{os}$ depends on the value of certain quantities in the first phase oscillation. Now K varies considerably in the first few inches. A reasonable value to take seems to be that at the radius where $\dot{\phi}$ goes from positive values to negative values since only those where $\dot{\phi}$ becomes negative will be retained. We are interested in ions whose initial $\dot{\phi}_o$ is near the maximum positive value. A rough calculation indicates that for $\phi_{ss} < \pi/4$ a reasonable value of r $\simeq$ 10 inches. On this basis a capture efficiency curve $L_1\ (\rho)$ was computed and plotted against the maximum phase angle an ion might have in its orbit (See Fig. 5) where the relation between the total range of $\dot{\phi}_o$ that may be captured into permanently stable orbits and $L_1\ (\rho)$ is given by

$$\Delta\omega_s = (\dot{\phi}_o)_{total} = \left(\sqrt{\frac{eV\omega^2_s K}{\pi Mc^2}}\right)_s 2\ L_1\ (\rho)\ . \qquad 43a$$

It will be noted that this capture efficiency drops to zero before $\sin\phi_{sf} = 1$. This is due to the fact that all ions captured start off with a certain minimum amount of phase oscillation -- since they start at a phase $\phi_o = \pi/2$ which is different from ϕ_{ss}. As $\phi_{sf} \to \pi/2$ the range of phase stability decreases rapidly toward zero. Hence, ions start with a $\dot{\phi}_o$ which may be stable when the ϕ_s for the ion is small at the start of the orbit -- but then as ϕ_s increases as the radius increases, the $\dot{\phi}$ falls outside of the range of stability. This capture curve is compared with experimental data in Fig. 6. The scales have been expanded so that the curves become zero at the same abscissa. There is good agreement in the general shape of the curves.

8. Efficiency or Acceptance Time.

An expression for the efficiency is given in B-9. With the changes in notation introduced in this paper and using 37 the efficiency, $\mathcal{E}$, will be given now by

$$\mathcal{E} = \frac{\Delta\omega_s}{\frac{1}{c}\left|\frac{d\omega_s}{dt}\right|} = \frac{1}{\left|\frac{d\ln\omega_s}{d\tau}\right|}\,\frac{\Delta\omega_s}{\omega_s} = \sqrt{\frac{eV\,K}{\pi Mc^2}}\cdot\frac{2L_1(\rho)}{\left|\frac{d\ln\,\omega_s}{d\tau}\right|}. \qquad 44$$

The optimum value of $L_1(\rho) \simeq 0.49$. At the start of acceleration $\left|\frac{d\ln\omega_s}{d\tau}\right| = 1.47$ and $K \simeq 2.10$, so that

$$\mathcal{E} \simeq 0.55\sqrt{\frac{eV}{Mc^2}} \qquad 45$$

If $V = 32$ kv, $\mathcal{E} = 0.0023$.

Some experimental data on the acceptance time is given in Fig. 7. This data was obtained by pulsing the arc for 2 μs and then measuring the ion current received on a probe at a radius of 75". The figure shows the ion current received when the arc was pulsed at different times in the frequency modulation cycle. A reasonable explanation of this curve would seem to be the following. Ions are created at the time the arc is pulsed. These ions can persist in the central region of the machine for a certain length of time, possibly undergoing radial phase oscillations. If during this persistence time, the accelerating frequency comes into the range for acceleration into stable orbits, a certain fraction will be accelerated. If not, the ions are presumably lost to the top or bottom of the tank. Hence, the maximum length of arc pulse which can create ions that can be accelerated into stable orbits will be the sum of the persistence time plus the time that the frequency is in the range for acceleration into stable orbits. This latter time is what has been called the "acceptance" time in these papers. The data in Fig. 7 indicates a persistence time of about 60 μs (from $t = 20\,\mu$s to $t \simeq 82\,\mu$s) and an acceptance time of about 40 μs (from $t = 82\,\mu$s to $t = 122\,\mu$s). The observed "efficiency" will be the ratio of the "acceptance time" to the length of the cycle -- so in this case where the cycle is 1/80 of a second, $\varepsilon = (40\mu s)/(1/80 \text{ sec.}) = .0032$ which is in reasonable agreement with the efficiency computed above. [The integrated current comes out to be 1.05 μa. This is also in fair agreement with the total current normally received.]

9. Efficiency as a Function of Rate of Frequency Modulation.

The efficiency depends, as is indicated in (44), on the rate of frequency modulation during the first phase oscillation. It also depends on which ions caught in the first phase oscillation can be retained in stable orbits. The maximum frequency of the oscillator in the 184" synchro-cyclotron under normal operation is 12.61 megacycles, (See Fig. 1). The frequency for catching deuterons is 11.47 megacycles for $H_c \doteq 15{,}000$ gauss. Hence, the frequency is changing fairly rapidly in the first phase oscillation. By making an appropriate adjustment it was possible to have the maximum frequency in the fm cycle only 11.63 mc so that the rate of frequency modulation was much slower during the first cycle. Hence, this factor would indicate a considerable increase in efficiency. The effect of this change in $\left|\frac{d \ln \omega_s}{d\tau}\right|$ is to start ions off with a small phase angle, ϕ_s, and subsequently introduce a large change in it. But the function $L_1(\rho)$ will be small for very small values of ρ (= sin ϕ). Thus the gain from other factors will be offset to some extent.

Data is not available to investigate thoroughly this effect. However, rough calculations indicate that the efficiency probably should be increased somewhat, say 20 percent in one of the cases investigated. The experimental data gave values of increase in efficiency ranging from 0 percent to 17 percent.

10. Tank Pressure.

The 184" cyclotron dee will not hold voltage if the pressure in the tank is much above 2×10^{-5} mm Hg. On the other hand the normal operating tank pressures are in the neighborhood of 1×10^{-5} mm Hg. Hence, it has not been possible because of this limited range to obtain much satisfactory experimental data on the variation of ion current with tank pressure.

11. Vertical Focusing.

Electrostatic focusing in cyclotrons has been considered by Rose[6] and Wilson.[7]

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[6] M. E. Rose, Phys. Rev. 53, 392, 1937.

[7] R. R. Wilson, Phys. Rev. 53, 408, 1937.

It is due to the vertical components of the electric field near the dee gap. The formula derived by Rose is valid at larger radii, where the ion crosses the accelerating gap in a relatively short time. If the acceleration is averaged over one period of rotation, the mean acceleration is given by

$$\overline{\left(\frac{d^2z}{dt^2}\right)} = -\frac{eV\omega^2 z}{4\pi(E-E_0)} \cos\phi \;. \qquad 46$$

This expression breaks down for $E - E_0 \simeq 0$, i.e., at small radii, since here the ion spends most or all of its time in the accelerating region. For very small radii we can assume that the ion is in an electrostatic field uniform in the median plane. If there are two dees or a dee and a dummy dee so that the vertical component of the electrostatic field will be symmetrical about the center of the gap and zero at the center, an approximate formula for the electrostatic focusing in the central region is

$$\overline{\left(\frac{d^2z}{dt^2}\right)} = -\frac{er^3 z\cos\phi}{8M}\left[\frac{\partial^5 V}{\partial z^2\,\partial x^3}\right]_{x=0,\; z=0} , \qquad 47$$

where x is measured in the median plane perpendicular to a center line between the dees. The derivative may be evaluated in two cases from the potential distribution given in Wilson's paper.

Now it has been pointed out before that the effective value of ϕ near the beginning of the orbit is $\phi_0 \simeq 90^\circ$. Also, most of the ions that are caught have $\phi > 0$ at the start. Hence, $\phi > \pi/2$ in the early part of the acceleration and we can expect a defocusing electrostatic force for most of the ions with which we are concerned.

A magnetic focusing force is the only force that can be used to counteract this defocusing force. The magnitude of the magnetic focusing force is given by (18). The value of n at the center is zero. Hence, to introduce magnetic focusing as rapidly as possible small cones of mild steel were put in the central part of the magnet on the pole faces. At first cones with base diameter 4 inches and 1-7/8 inches high were used, later cones with base diameter 6 inches were used.

With the cones in place the field is essentially constant to the 1 inch radius, then falls off about linearly with radius, however with the cones removed the field is constant to the 8 inch radius then falls off linearly. With these cones removed, the ion current received on an internal target at the 80 inch radius is approximately 10 percent of the ion current obtainable with the cones in place. It, therefore, seems very desirable that such cones be used to produce vertical magnetic focusing at as small a radius as possible.

12. Oscilloscope Data.

It has been possible to examine the structure of the individual beam pulses that are received once each frequency modulation cycle, using a synchroscope and taking photographs of the oscilloscope pattern. Time markers were superposed on the beam pattern to permit determination of time intervals. Typical photographs are given in Fig. 9. A schematic picture is given in Fig. 8 -- with the notation which is to be used below. Data obtained from such photographs are given in Table 3.

The ions as they are accelerated oscillate in various fashions about a synchronous orbit of steadily increasing radius. The time it takes for the ions to reach the probe radius is called the "time of flight." The time of flight to any specified radius can be computed by determining when the oscillator frequency corresponds to the natural frequency of rotation at the specified radius. This computed time in all cases lies within or close to the limits t_1, t_2. The cases where it lies outside the limits are understandable in terms of the arc being pulsed at a slightly too early time so that ions are not accelerated immediately but must stay for a short time in the central region of the machine before acceleration.

The actual motion of the ions which affect these oscilloscope patterns can be broken down into three components.

1. Expansion of the synchronous radius.

2. Phase oscillations about this radius.

3. Free radial oscillations about this radius.

The most noticeable thing about the pictures is the series of small pulses that occur every 7-13 microseconds, depending on the radius. The separation time, t_3, of these pulses agrees well with the precessional period, T_{pr} (Eqn 18a). This precessional period will be apparent only for ions which have a free radial oscillation. The fact that the ions strike the probe in bunches -- not continuously -- indicates that the azimuthal angles at which the various ions have their maximum radial displacement are not distributed at random but are grouped together to a certain extent. Such a radial oscillation might be built up due to some sort of an azimuthal inhomogeneity at small radii where $n \simeq 0$, by the dee bias used, etc. If $n = 0$ we can expect that the amplitude of such oscillations can be built up; for when $n = 0$ displacements or forces acting at a certain azimuthal angle in the machine will be cumulative for successive revolutions; however, when $n > 0$, the azimuthal angle of maximum displacement will precess around the machine so these successive displacements will come at different parts of the orbit and hence will not be cumulative. Some such condition appears to exist in this machine. The ions receive some radial displacement in a definite direction at the start of their acceleration so that the azimuthal angles of maximum displacement are pretty well bunched together in the early part of the acceleration. Calculation indicates that such a bunching could persist for several phase oscillations.

If the envelope of the small peaks is drawn, two broad peaks may be seen on some photographs. The time separation of these peaks, t_4, agrees well with the period of phase oscillation, T_{ph} (See Eqn 13). The fact that these are separated by the full period T_{ph}, must mean that ions that strike the probe in the early group have an amplitude of free radial oscillation which is about as large as or larger than the amount that the radius increases in a period of phase oscillation.

It was also observed while making these oscilloscope pictures if the arc pulse was moved so that the arc was pulsed very late that these ions were received at the probe at a somewhat earlier time than the ions which start just before them. This effect probably is to be interpreted in the following manner. Ions that start late will have large phase oscillations. Hence, for most probe positions these late ions will be expected to strike the probe earlier than ions with smaller phase oscillations.

13. Width of Beam on Probe.

When the ions strike the probe they will not be scraped off by the very edge of the probe, but will hit at various small distances in on the probe due to the combined effect of radial oscillation and precession. For radii where there are a large number of revolutions in a precessional period an approximate analytical formula can be obtained to give the maximum width, h, in on the probe at which ions will strike,

$$h_{max} = A_r \left\{ \sin f\theta_2 \cdot \sin 2\pi f(1-f) + \cos f\theta_2 \left[\cos 2\pi f(1-f) - 1 \right] \right\} \qquad 48$$

where A_r is the amplitude of free radial oscillation, $f = \sqrt{1-n}$ and

$$\cos f\theta_2 = 1 - \frac{\Delta r_{pr}}{A_r} \qquad 49$$

A maximum value of Δr_{pr} may be obtained using (19a) and an energy gain per turn $\Delta E = eV$, the maximum possible increase in energy per turn. To get the distribution of the ions on the probe an energy gain per turn of $\Delta E = eV \sin \phi_s$ is more representative if the precessional period is a considerable fraction of the period of phase oscillation. (The relation (48) is valid only if $\Delta r_{pr}/A_r \ll 1$). A first approximation to the solution of (48) indicates roughly the following dependance of h_{max} on n, A_r, and Δr_p as

$$h_{max} \simeq \pi n \sqrt{2A_r \; \Delta r_{pr}} \; . \qquad 50$$

Values of h_{max} based on this formula for r = 20 inches, 40 inches and 70 inches and with both assumptions as to ΔE (with V = 40 kv) are given in the following table. The value of h_{max} for r = 81 inches was determined by a more detailed study of the orbits.

Table 1. Maximum Width for Ions to Hit on Probe

(all distances in inches)

r	Δr_{pr}*	h_{max}			Δr_{pr}**	h_{max}		
		A_r = 1"	A_r = 3"	A_r = 5"		A_r = 1"	A_r = 3"	A_r = 5"
20	1.56	---	0.10"	0.13"		---	---	---
40	0.33	0.06	.12	.16	1.1	---	0.23	0.30"
70	0.12	0.08	.14	.18	0.4	0.14	0.25	0.32
81	0.03	0.06	.08	---	0.1	0.11	0.16	---

* based on $\Delta E = eV \sin \phi_s$ 　　　　** based on ΔE = 40 Kev

Various experiments have been performed to determine the distribution of the beam on the probe. In one experiment a lead probe was inserted into the beam at a radius of 81 inches. Thin slices also were taken off the leading edge of the probe and specific activity counts were made. The number of counts at various distances is shown in Fig. 10. The activity is almost all confined to h < 0.16 inch -- only about 5 percent of the measured activity shown on the curve having a greater h. The counts for h > 0.20 inch can probably be attributed to ions which pass through the edge of the probe and then are scattered, or lose energy and precess with a large A_r.

14. Amplitude of Free Radial and Vertical Oscillation.

A method of accurately determining the amplitudes of radial oscillation has not been found. There are various things which might give some indication of it. The radioautographs of the C shaped probes indicate an upper limit on it of maybe 5 inches. The oscilloscope data indicates that A_r may be of the order of 3 inches for a considerable fraction of the ions. The specific probe activity as a function of the distances from the leading edge seems to indicate somewhat the same thing. However, none of these data are very precise.

The amplitude of vertical oscillation is somewhat easier to observe by making an analysis of the distribution of activity on special probes, e.g., a grid of fine wires. Radioautographs of these wire probes indicate that the amplitude of vertical oscillation is of the order of three-quarters of an inch.

15. Displacement of the Center of Rotation of the Ions from the Center of the Magnet.

The position of the center of rotation was determined in two ways. The first method was based on magnetic field measurements. The vertical field strength was measured as a function of azimuthal angle for various radii. The field measurements could then be analyzed to give

$$H_z(r,\theta) = H_z(r)\left[1+\sum h_i(r)\cos(\theta+\delta_i)\right] \qquad 51$$

If we then let a be the displacement of the center of rotation, using only the first order harmonic, the value of a will be given by

$$a \cong rh_1/n \qquad 52$$

The direction of displacement is indicated by δ_1 which is found to be about 105° away from the probe position. The magnitude of the displacement is about 0.6 inch at $r = 10$ inches and increases to about 0.8 inch at $r = 60$ inches. After 60 inches it increases somewhat due to the increase in $h_1(r)$. However, as soon as n starts to increase rapidly a drops and is about 0.2 inch from 81 to 82 inches.

This center was also determined by means of two extra non-current reading probes. The results of these measurements agree well with the results as calculated from the magnetic measurements.

If the energy of the ions is going to be determined from the radius of rotation, this effect should be taken into account.

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Appendix I. Typical operating conditions.

1. The magnetic field is 15,000 guass at the center of the gap.
2. The dee voltage (average over the cycle) is 20 kv on the dee or a total maximum possible energy gain per turn of 40 kv.
3. A d.c. bias voltage of 1 kv negative with respect to ground is applied to the dee.
4. A dummy dee is used which is held at ground.
5. The rf is modulated 100 times per second. This is accomplished by a variable capacitor which is rotated at 250 r.p.m. The capacitor has 24 teeth so that it then gives $\frac{250.24}{60} = 100$ cycles per second. The total frequency range is from 12.6 to 9.0 megacycles.
6. The pressure in the main vacuum tank and in the rotating capacitor tank is about 10^{-5} mm Hg.
7. The arc voltage of 150 volts is applied for 100 microseconds during each rf modulation cycle. The arc current during this 100 μ sec. pulse is about 20 amps.

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Figure Captions

Table 2. Data on 184" Synchro-Cyclotron. Deuteron Energy, etc.

r (inches)	$\frac{H(r)}{H_c^*}$	n	K	$\frac{1}{\beta^2}$	$E-E_o$ (Mev)	$\frac{\Omega \cdot 10^{-6}}{\text{sec.}}$	$\frac{d \ln \omega_s}{d\tau}$	$\Delta E_s \left(\frac{100}{C}\right)$ kev	$\frac{V}{\bar{V}}$
0	1.0000	0.0000	6.37		0	11.465	1.43	3.7	0.955
5	.9981	0.0026	3.83	1084.7	0.8	11.439	1.45	6.2	.960
10	.9957	0.0040	2.10	273.25	3.4	11.396	1.47	11.5	.963
15	.9937	0.0060	1.74	121.46	7.6	11.347	1.48	14.1	.972
20	.9913	0.0111	1.78	69.66	13.5	11.285	1.49	14.0	.980
25	.9883	.014	1.66	45.21	21.0	11.206	1.49	15.2	.990
30	.9856	.016	1.53	31.87	30.1	11.122	1.49	16.7	1.002
35	.9829	.020	1.49	23.79	41.1	11.027	1.46	17.0	1.014
40	.9798	.028	1.53	18.57	52.6	10.927	1.42	16.4	1.027
45	.9763	.031	1.48	14.98	65.9	10.813	1.38	16.7	1.043
50	.9732	.032	1.41	12.40	80.4	10.699	1.32	17.1	1.058
55	.9698	.040	1.44	10.50	96.2	10.576	1.25	16.2	1.070
60	.9658	.055	1.52	9.037	113.2	10.442	1.15	14.4	1.086
65	.9614	.055	1.46	7.912	127.9	10.319	1.07	14.2	1.097
70	.9576	.051	1.38	7.007	150.0	10.166	0.97	14.0	1.110
75	.9540	.057	1.38	6.272	170.0	10.028	0.87	12.9	1.122
76	.9533	.056	1.36	6.144	174.1	10.001	0.85	12.8	1.125
77	.9526	.057	1.36	6.013	178.5	9.972	.83	12.5	1.128
78	.9519	.066	1.41	5.893	182.6	9.945	.82	12.0	1.129
79	.9510	.081	1.51	5.779	186.8	9.915	.80	11.0	1.130
80	.9499	.113	1.72	5.674	190.8	9.885	.78	9.5	1.131
81	.9482	.176	2.19	5.575	194.7	9.848	.76	7.3	1.135
82	.9456	.292	3.27	5.486	198.4	9.804	.73	4.7	1.138
83	.9413	.424	5.00	5.423	201.1	9.747	.70	---	1.140
84	.9351	.914	6.67	5.372	203.3	9.672	.65	---	1.142
85	0.9219	1.080	----	5.393	202.4	9.540	0.64	---	1.145

* H_c = 15,000 gauss (4 inch cones in center)

Table 3. Oscilloscope Pattern Data
(Dee Voltage ≃ 19 kv)

Plate No.	Dee Bias (kv)	Radius (inches)	Arc Pulse Duration (μ s)	C sec.$^{-1}$	t_1 μs	t_2 μs	Time of Flight μ s	t_5 μs	No. of Pips	t_3 μs	T_{pr}= Precessional Period μs	t_4 μs	T_{ph} Phase Period μs	$(\delta r)_{ph}$ inches
48		25"	5	92	156	192	158	36	4	12.0	12.9	--	41	3.9
51		34"	5	92	235	294	256	59	7	9.8	9.5	--	47	3.7
54		45"	5	92	450	535	473	85	12	7.7	6.1	45	47	2.8
83		25"	50	92	112	166	158	41	4	13.1	12.9	--	41	
80		34"	115	92	168	238	256	71	8	10.1	9.5	41	47	
88		45"	50	119	270	357	318	80	12	7.2	6.1	47	47	
70		34"	5	52	440	536	450	96	9	12.5	9.5			
73		34"	5	108	240	310	216	70	7	12.0	9.5			
76		34"	5	140	180	205	167	25	3	11.5	9.5			
64	0.5	34"	5	92	250	285	256	45	4	10.2	9.5			
66	2.0	34"	5	92	250	320	256	65	7	10.2	9.5			
68	4.5	34"	5	92	225	290	256	100	7	10.3	9.5			

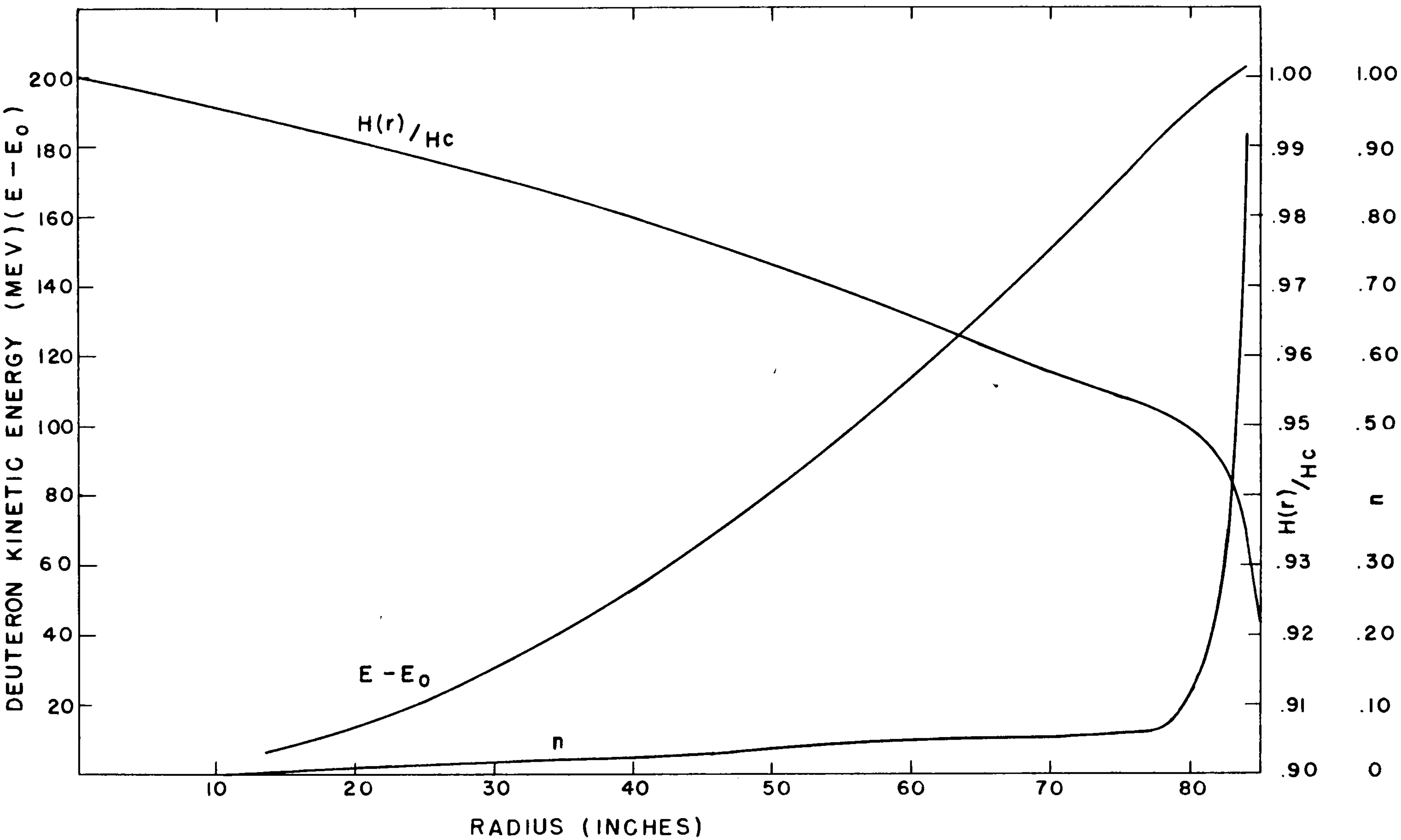

FIG. 1a

DEUTERON ENERGY, MAGNETIC FIELD AND n AS A FUNCTION OF THE RADIUS

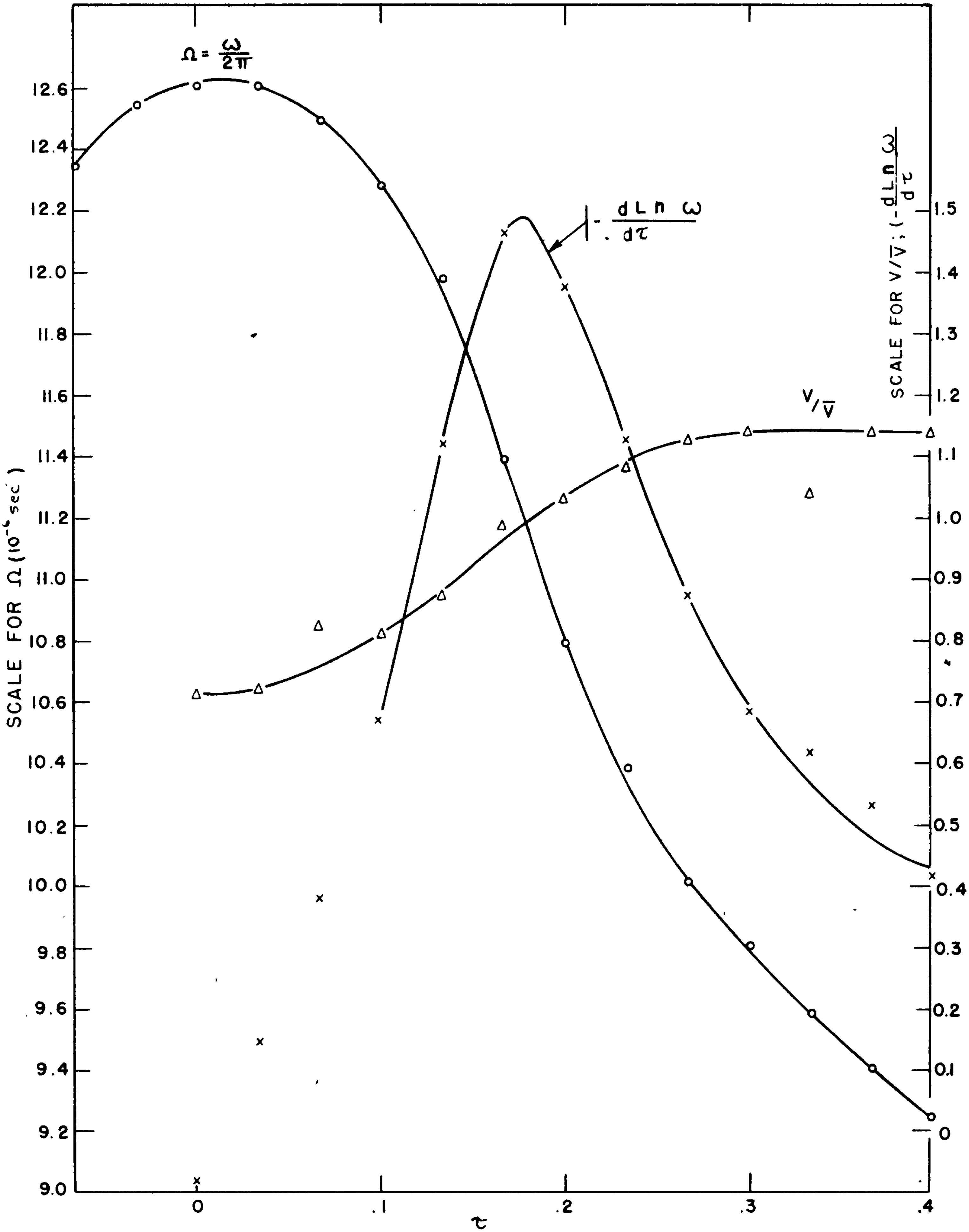

FIG. 1b

VARIATION OF MODULATION FREQUENCY AND APPLIED VOLTAGE WITH TIME

1349

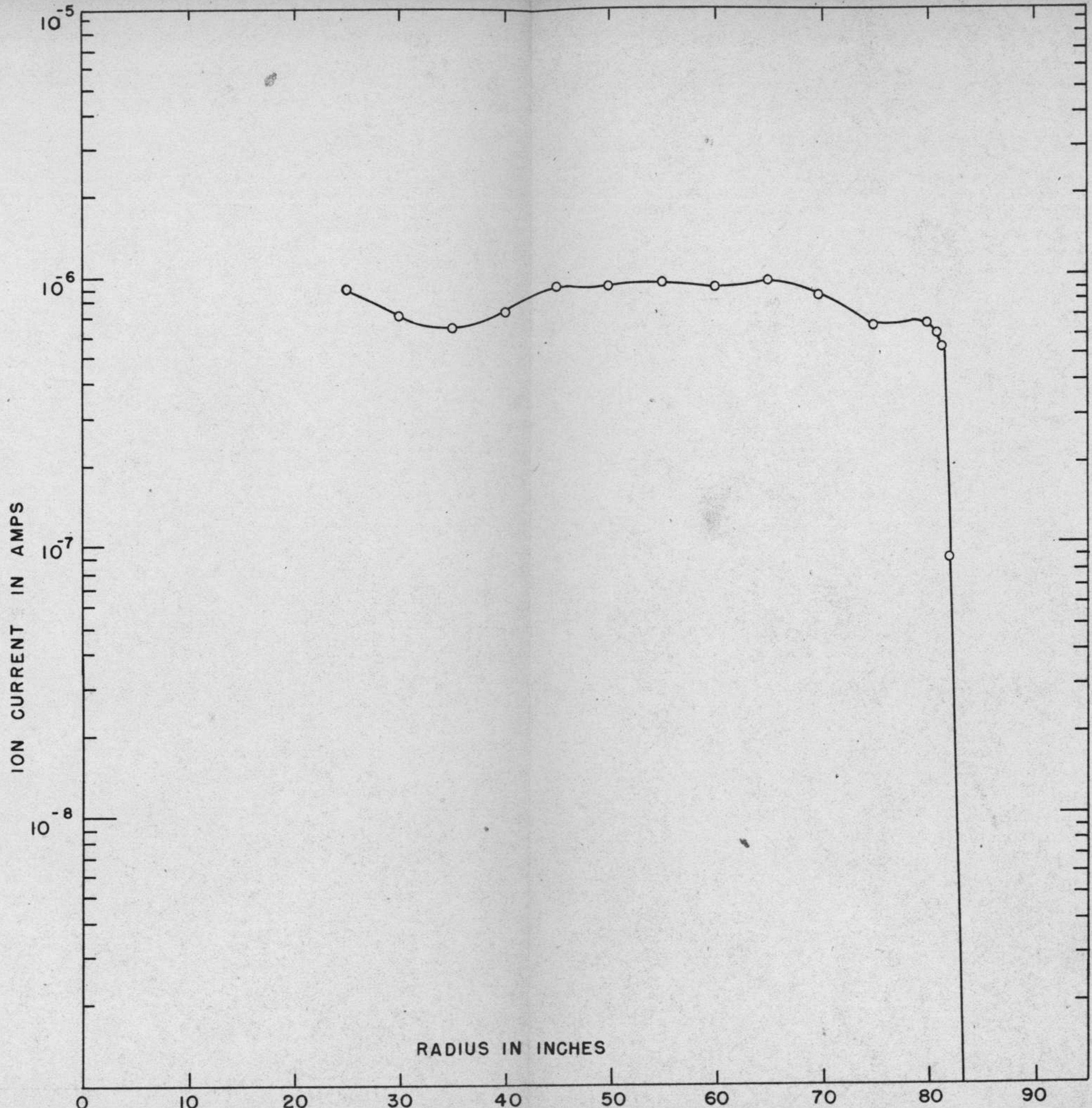

FIG. 2
ION CURRENT AS FUNCTION OF RADIUS

13497-2

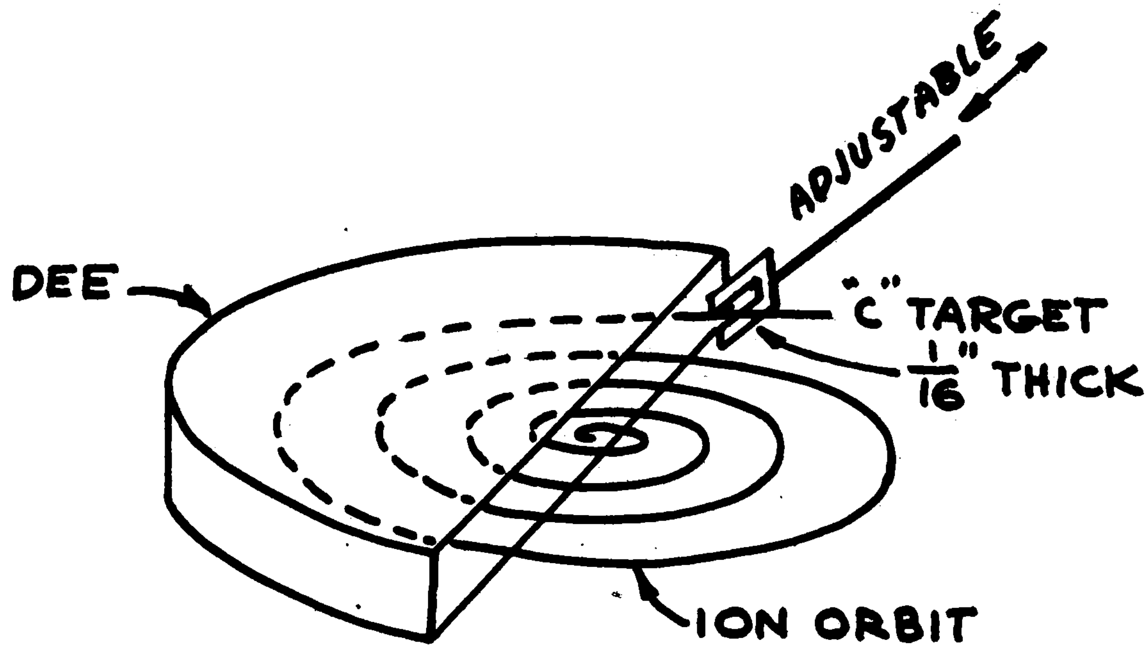

FIG. 3a

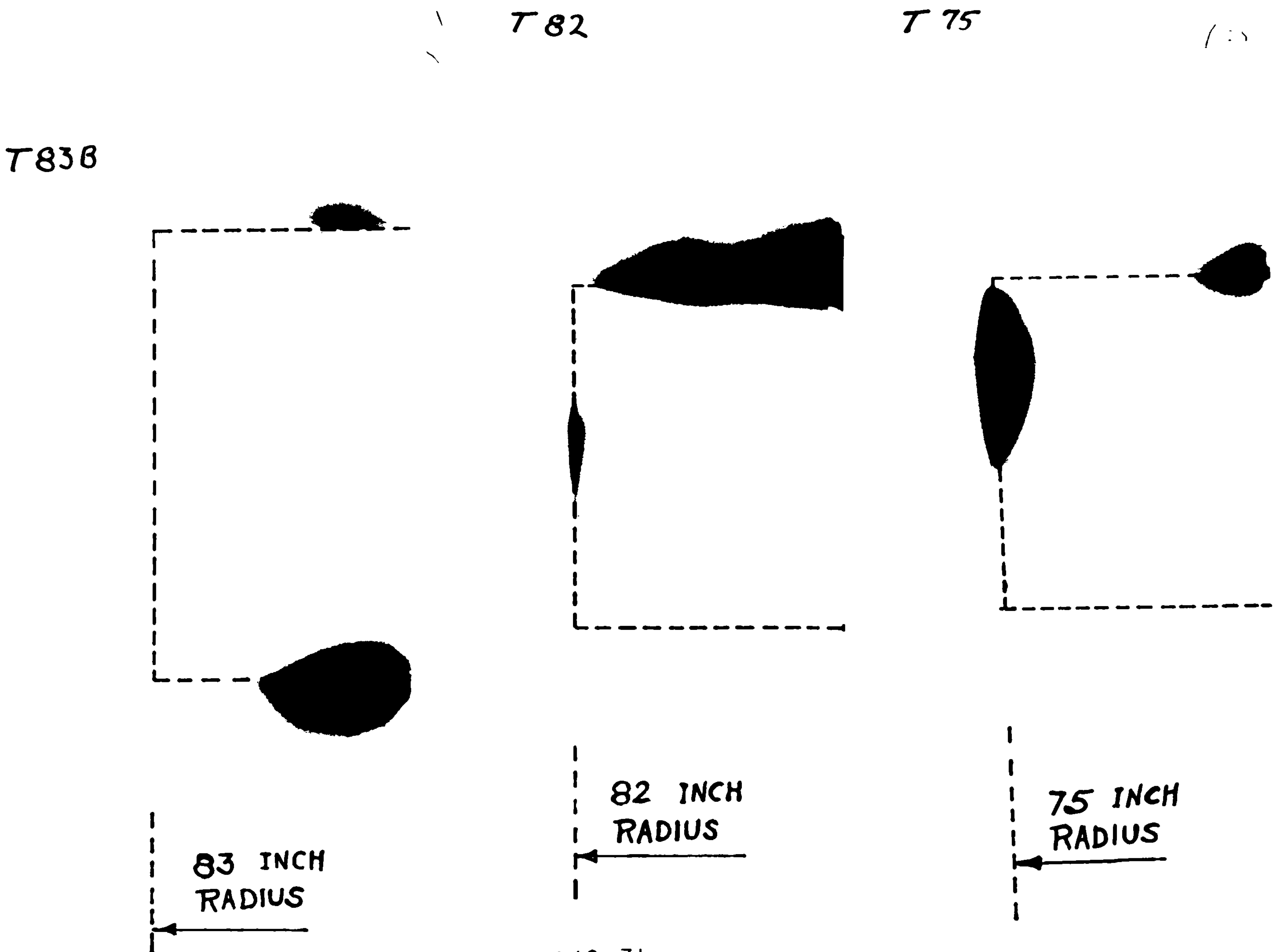

FIG. 3b

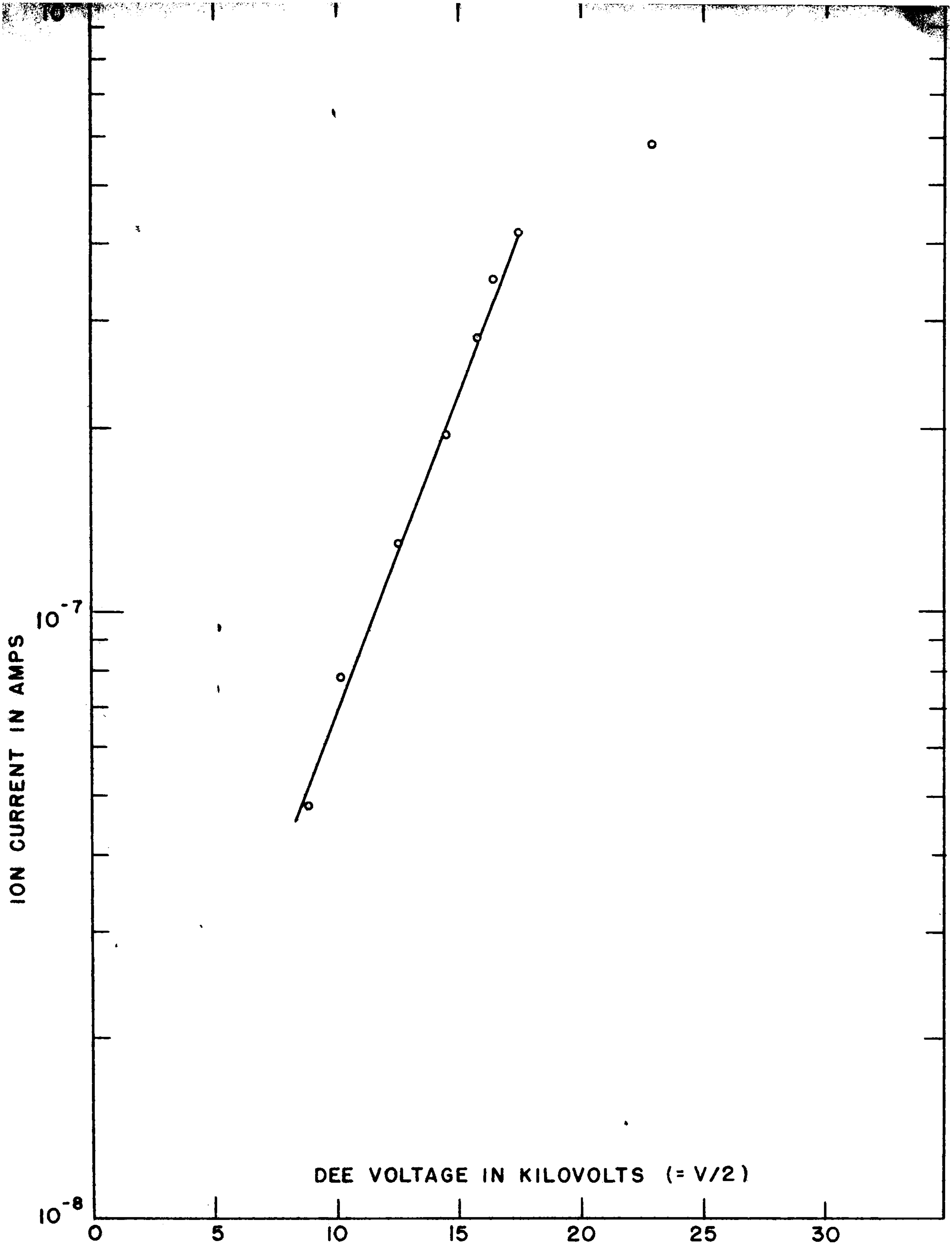

FIG. 4
ION CURRENT AS FUNCTION OF DEE VOLTAGE

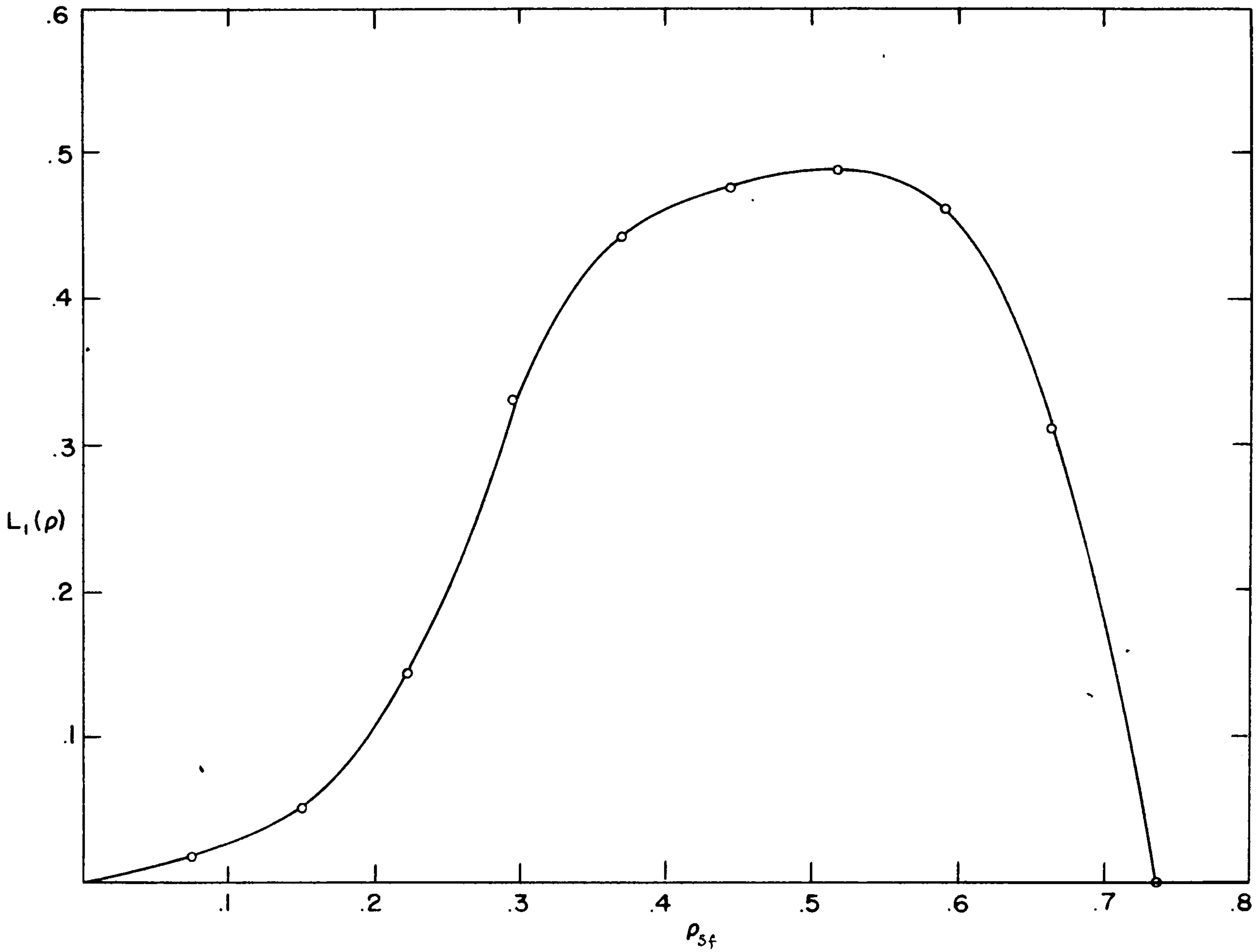

FIG. 5

THEORETICAL EFFICIENCY FOR CONSTANT DEE VOLTAGE AND VARYING CONDENSER SPEED

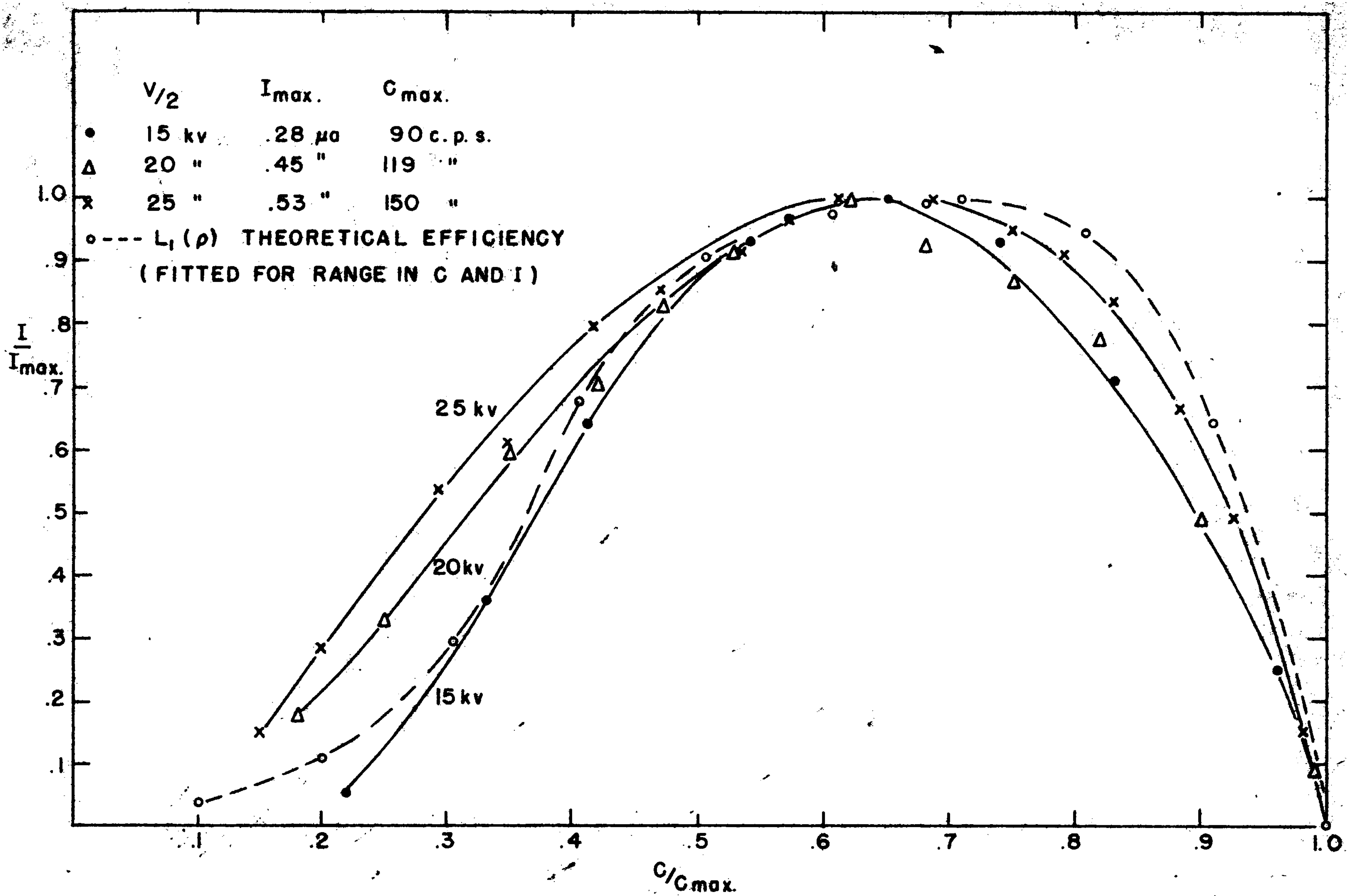

FIG. 6

VARIATION OF ION CURRENT WITH CONDENSER SPEED (OR SYNCHRONOUS PHASE ANGLE)

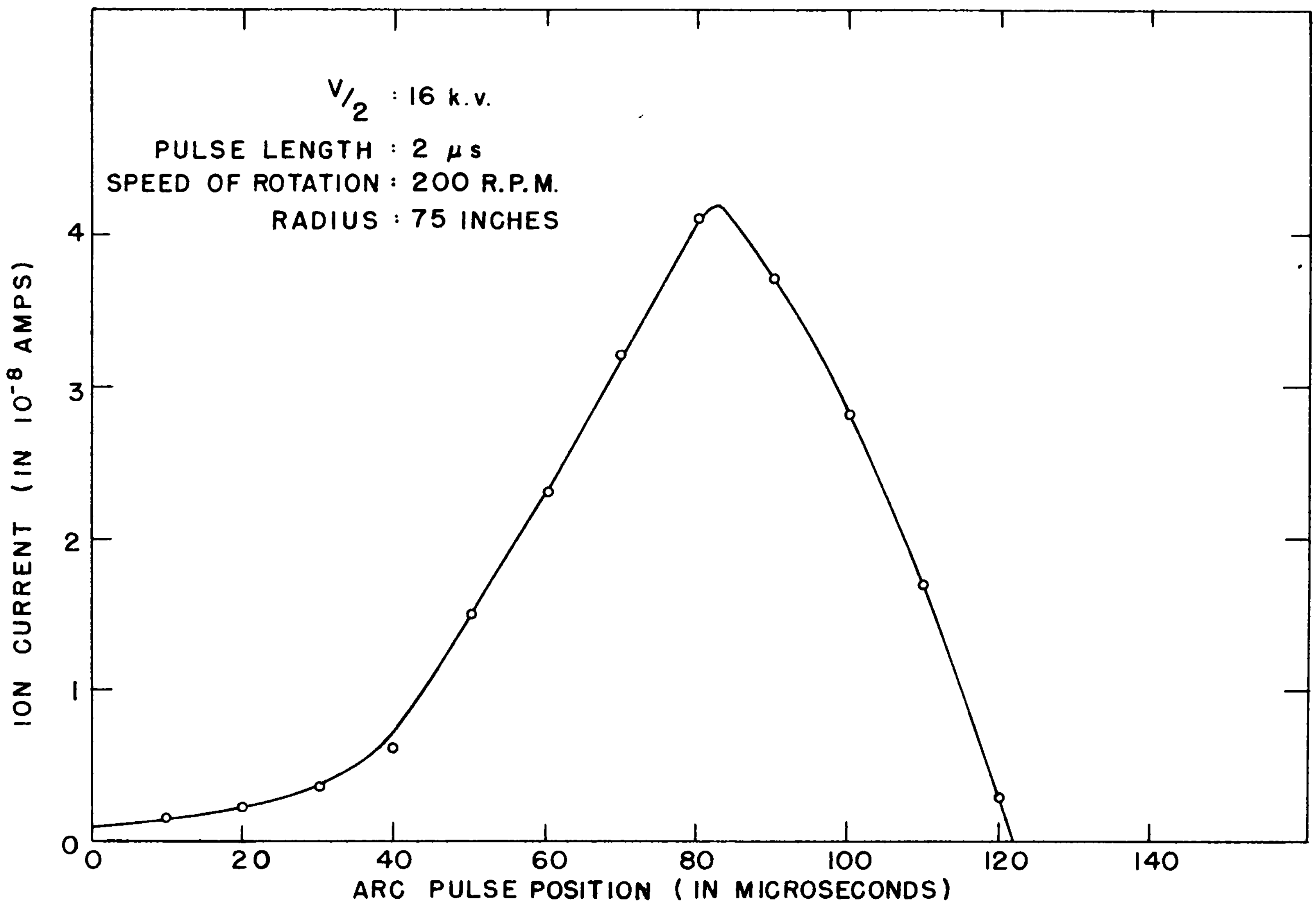

FIG. 7

ION CURRENT RECEIVED AS A FUNCTION OF ARC PULSE POSITION

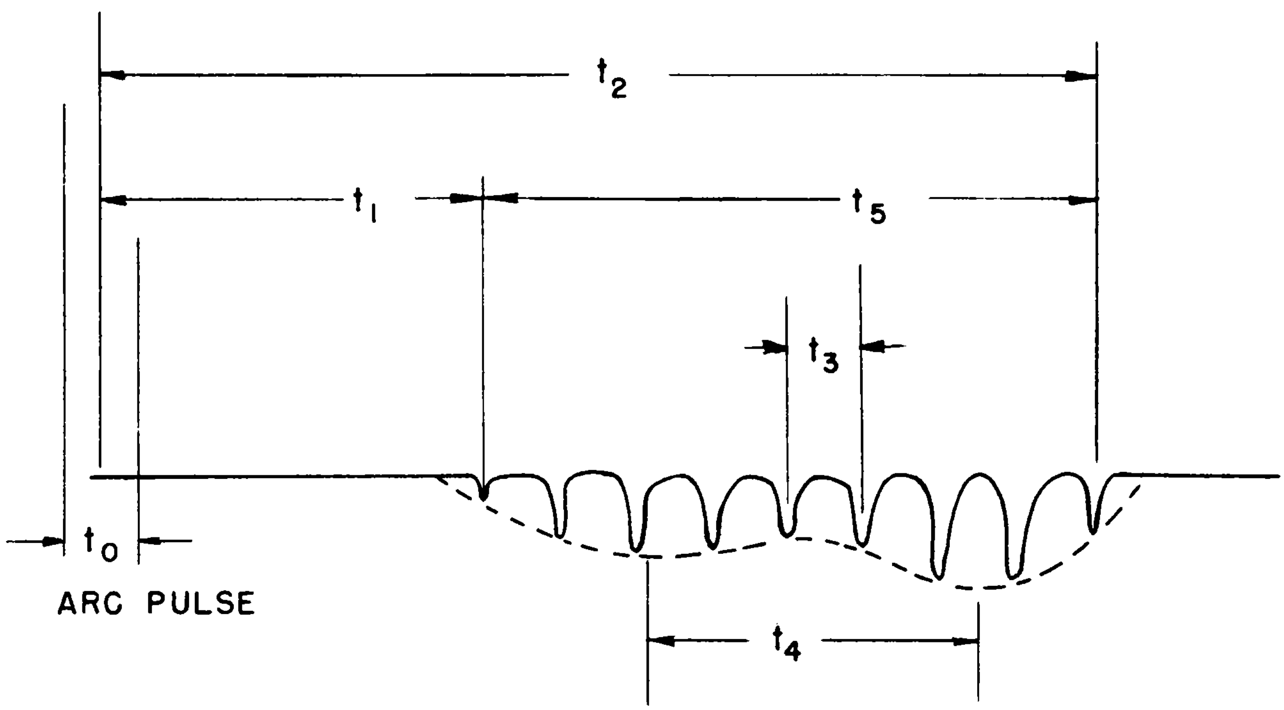

FIG. 8

SCHEMATIC DRAWING OF OSCILLOSCOPE PATTERN

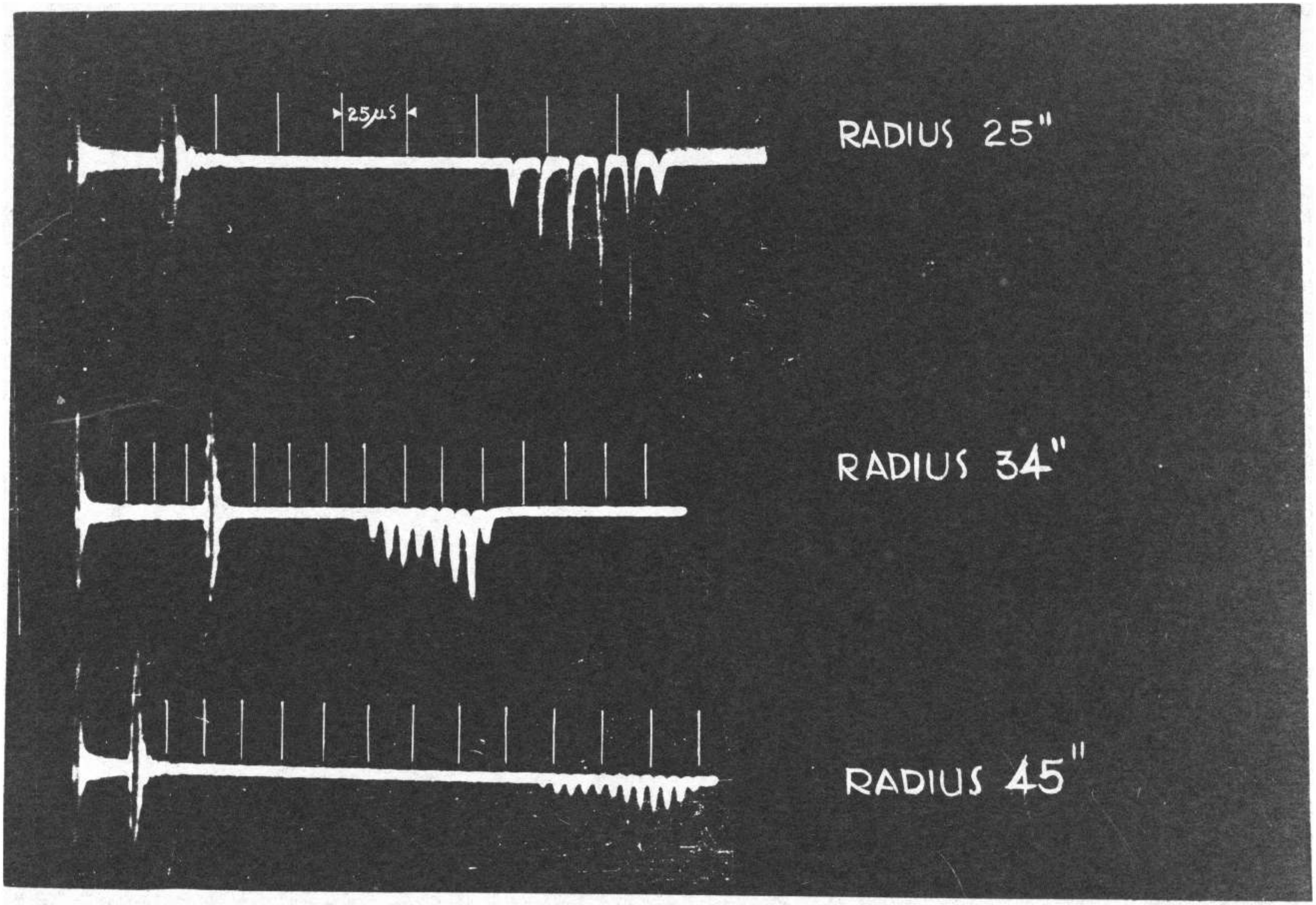

FIG. 9

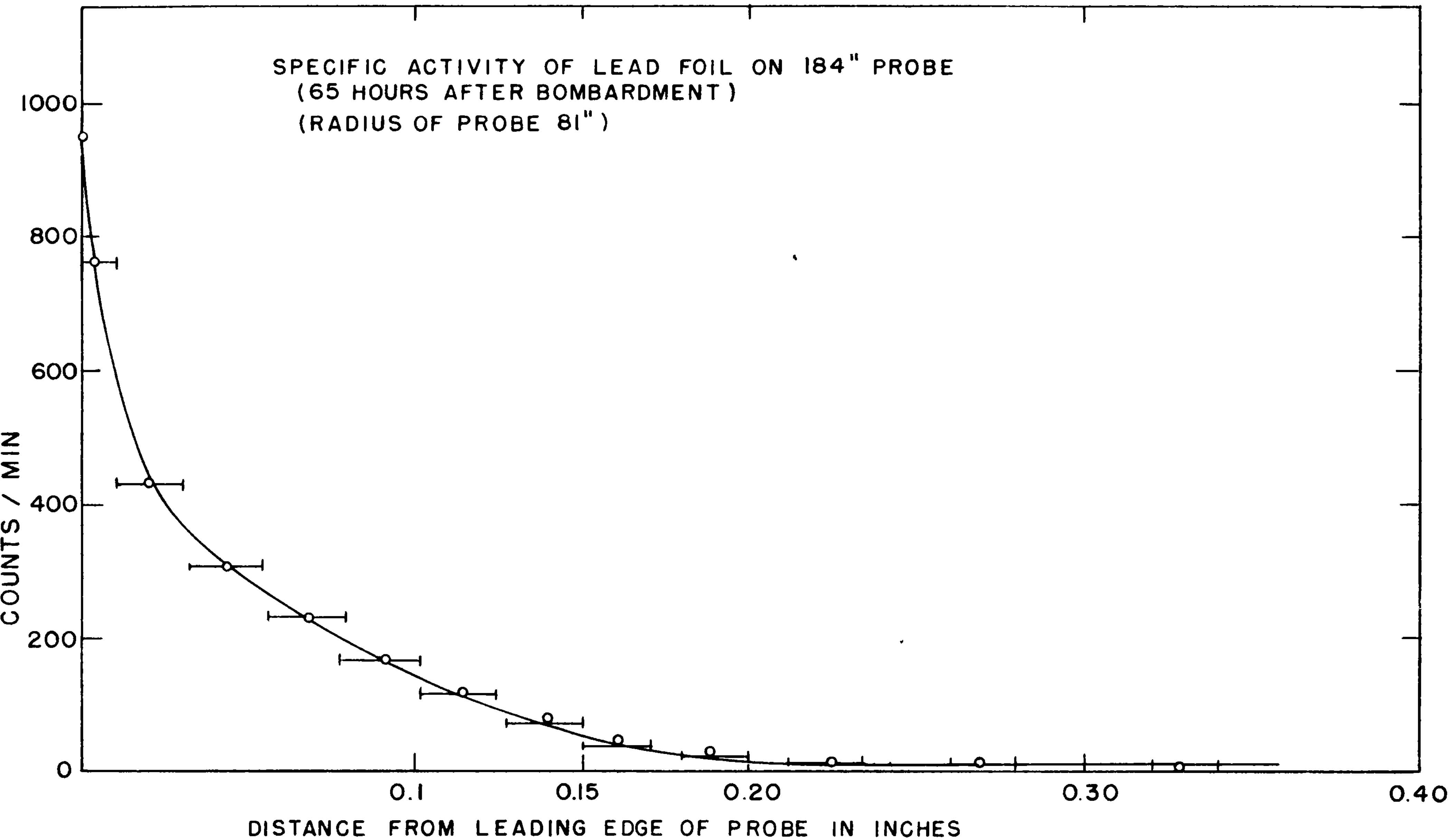

FIG. 10

WIDTH OF BEAM ON PROBE