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Operational Flexibility and Financial Hedging: Complements or Substitutes?

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We consider a firm that invests in capacity under demand uncertainty and thus faces two related but distinct types of risk: mismatch between capacity and demand and profit variability. Whereas mismatch risk can be mitigated with greater operational flexibility, profit variability can be reduced through financial hedging. We show that the relationship between these two risk mitigating strategies depends on the type of flexibility: Product flexibility and financial hedging tend to be complements (substitutes)—i.e., product flexibility tends to increase (decrease) the value of financial hedging, and, vice versa, financial hedging tends to increase (decrease) the value of product flexibility—when product demands are positively (negatively) correlated. In contrast to product flexibility, postponement flexibility is a substitute to financial hedging as intuitively expected. Although our analytical results assume perfect flexibility and perfect hedging and rely on a linear approximation of the value of hedging, we validate their robustness in an extensive numerical study.

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1. Introduction

This paper studies the relationship between financial hedging and operational flexibility when both are used to mitigate a firm's exposure to demand uncertainty. Most operations are exposed to two related yet distinct risks. *Mismatch risk* is an operational risk that refers to the expected cost of supply (capacity or inventory) differing from demand. To mitigate mismatch risk, firms may invest in various flexibilities that enable them to better adapt to volatile market conditions.

The other type of risk relates to *profit variability*. The finance literature identifies several rationales explaining why value-maximizing firms may benefit from reducing profit variability and should thus engage in financial hedging. Smith and Stulz (1985) show that financial hedging can reduce expected tax liabilities, bankruptcy costs, or compensation paid to risk-averse managers. Stulz (1990), Bessembinder (1991), and Froot et al. (1993) show that financial hedging can increase firm value by leading to more efficient capital investment outcomes. To capture these various market imperfections in a parsimonious and tractable fashion, we assume that the firm value is a concavely increasing exponential function of the pretax profit. Thus, even though the firm is an expected value maximizer, the nonlinearity of taxes, bankruptcy cost, etc. induces behavior similar to risk aversion.

Although there is a considerable amount of literature both on flexibility and on financial hedging, relatively little research examines their relationship, and most of it considers financial hedging against currency exposure rather than product demand exposure. This paper attempts to fill this gap by showing that, contrary to initial intuition, flexibility and financial hedging can be complements (as well as substitutes) in the firm's overall risk management strategy.

We consider a value-maximizing firm whose operating profit depends on an exogenous random shock, which we refer to as "demand vector." Given that the firm value is a concave function of pretax profit, profit variability reduces the expected firm value, so the firm has an incentive to use financial hedging. Specifically, the firm enters a financial hedging contract whose payoff depends on some commonly observable underlying variable(s) such as stock indices, commodity prices, or weather-related variables that are (imperfectly) correlated with the demand vector. We assume

that the firm uses an optimal hedging contact that is fairly priced.[1]

Flexibility is the ability to adapt to change and may take many forms. This paper addresses two of them: product flexibility and postponement flexibility. To study product flexibility, we follow Van Mieghem (1998) and consider a two-product firm that invests in a mix of one product-flexible and two product-dedicated resources while product demands are uncertain. We measure the firm's product flexibility by the ratio of the unit cost of the flexible resource relative to the unit cost of dedicated resources. As this ratio decreases from 2 to 1, the firm's product flexibility increases from none (only dedicated resources are acquired) to full (only flexible resource is acquired). We show that product flexibility and financial hedging tend to be complements (substitutes)—i.e., product flexibility tends to increase (decrease) the value of financial hedging, and, vice versa, financial hedging tends to increase (decrease) the value of product flexibility—when demands are positively (negatively) correlated. This is because product flexibility increases (decreases) profit variance when demands are positively (negatively) correlated. Although our analytical results assume perfect flexibility and perfect hedging and rely on a linear approximation of the value of hedging, we validate their robustness in an extensive numerical study.

The second type of flexibility is postponement flexibility. We consider a two-product newsvendor with no product flexibility and measure its postponement flexibility by the fraction of the unit product cost that is incurred after demand realization. Higher postponement flexibility means a smaller up-front unit capacity cost. This not only reduces the cost of excess capacity when demand is low but also stimulates the firm to invest in a larger capacity level, which in turn mitigates capacity shortage when demand is high. We show that, as intuitively expected, postponement flexibility and financial hedging are substitutes.

As an illustration of our model, consider the following example. A U.S. sportswear retailer sells at two ski resorts, one on each coast, where the amount of snowfall (and hence demand for its products) varies considerably each year. Most of the retailer's products have short life cycles but long lead times and must be ordered several months before the selling season. The retailer orders some "dedicated" units of a particular product that are shipped directly to each retail location. In addition, she orders some "flexible" units to be kept at a central warehouse and shipped to a particular location only after demand has been observed. The lower the cost of relying on the central warehouse, the more units will be stored centrally, and the higher the retailer's *product flexibility*.[2]

Alternatively, the retailer may reduce the mismatch cost by increasing its *postponement flexibility*. Instead of ordering all inventory ahead of the selling season, the retailer only reserves quick-response capacity and orders after demand is known. (In reality, the retailer may order some units ahead of the selling season and use quick-response capacity to reorder once demand is known with high accuracy. In that case, our results may overstate the value of flexibility).

Finally, to protect profit against the possibility of low snowfall, which hurts sales, the retailer enters into a *financial hedging* contract whose payoff is a decreasing function of the amount of snowfall at the ski resort(s). Such a contract mitigates uncertainty in the retailer's profit to the extent that demand is correlated with the amount of snowfall.

Although not yet on a large scale, firms are increasingly experimenting with weather-related derivatives. The modern market for weather derivatives was born in the mid 1990s in the United States with innovative, but ultimately failed, energy trader Enron in the vanguard. Japan followed several years later with Mitsui Sumitomo Insurance selling a contract to a retail ski shop to hedge against low snowfall (Kao 2006). Other weather conditions sometimes specified in weather derivatives include temperature, rainfall, and wind. In 2002, Mitsui Sumitomo Insurance issued a weather-derivative contract to a soft drink wholesaler based on the number of hours of sunshine. If the number of sunshine hours recorded in the July–September quarter fell below a certain predetermined threshold, Mitsui Sumitomo Insurance would pay the company a predetermined amount (Sumitomo Group 2002). Since that time, the weather-derivative market has expanded rapidly. According to a survey by Weather Risk Management Association and Pricewaterhouse-Coopers, the notional value of weather-derivative contracts transacted globally from April 2007 through March 2008 amounted to $32 billion. Whereas most of the volume comes from energy companies, the share of the retail sector amounted to 7% in 2006.[3]

2. Relation to the Literature

The operations literature on product flexibility is extensive and mostly assumes expected profit maximization (see, e.g., Fine and Freund 1990, Van Mieghem 1998). The literature addressing postponement flexibility is less abundant. Our model of postponement flexibility is similar to the one considered

[1] There exists empirical evidence that firms do use financial derivatives to hedge, although most nonfinancial firms hold derivatives positions that are small in magnitude relative to entity-level risks (Guay and Kothari 2003).

[2] The assumption here is that ex post inventory transshipment between the stores is not economical, e.g., because of high transportation and coordination costs.

[3] For further details, see http://www.wrma.org.

by Chod et al. (2006), who study its relationship with other types of flexibility. A notion closely related to flexibility is operational hedging, which refers to risk mitigation using operational instruments and typically focuses on mitigating exchange rate risk. A classic example is Huchzermeier and Cohen (1996), who study the option value of shifting production among countries based on currency fluctuations.

The seminal work on resource investment under risk aversion is Eeckhoudt et al. (1995). This study demonstrates that the optimal resource level of a single-resource newsvendor-type firm decreases in risk aversion for any concave utility function. Other papers studying single-resource investment under risk aversion include Lau (1980), Bouakiz and Sobel (1992), and Agrawal and Seshadri (2000). Van Mieghem (2007) studies how risk-averse managers can use resource diversification, sharing, and flexibility in newsvendor networks to mitigate risk. Tomlin and Wang (2005) consider product flexibility and dual sourcing in unreliable newsvendor networks under both loss aversion and conditional value at risk.

The operations management literature addressing financial hedging includes the following several papers. Gaur and Seshadri (2005) consider a risk-averse newsvendor that hedges its risk exposure with financial options and show that financial hedging results in a larger order quantity. Caldentey and Haugh (2006) extend the work of Gaur and Seshadri by allowing continuous trading in the financial market. Chen et al. (2007) incorporate risk aversion and financial hedging in multiperiod inventory and pricing models. Zhou and Rudi (2007) study the pricing of over-the-counter financial hedging contracts used by firms to protect against demand risk by modeling the interaction between a hedging firm and the contract issuer.

Recently, several papers have considered the joint use of financial hedging and risk pooling. Ding et al. (2007) and Zhu and Kapuscinski (2006) study risk-averse multinational firms that rely on risk pooling (allocation flexibility and transshipment, respectively) as well as financial hedging to mitigate risk. Our work differs from both Ding et al. (2007) and Zhu and Kapuscinski (2006) in that they consider financial hedging against exchange rate exposure whereas we study financial hedging against demand exposure. Furthermore, they do not explicitly address the question of complementarity/substitution between flexibility and financial hedging.

The issue of complementarity/substitution between product flexibility and financial risk management (FRM) has been addressed by Boyabatli and Toktay (2006b). They consider a firm that uses FRM to influence the distribution of its budget available for investment in production technology, which may be dedicated or flexible. In their work, the link between FRM and flexibility stems from the fact that flexible technology is more expensive and thus requires more financing, which can be generated externally at a cost and/or from an internal budget that is affected by FRM. The relationship between flexibility and FRM thus depends on factors such as the firm size. Specifically, flexibility and FRM tend to be complements for large firms. This is because with FRM, a large firm may be able to secure enough financing to purchase flexible technology without relying on costly external financing. In contrast, we consider a situation in which flexibility and financial hedging are used simultaneously to manage a firm's exposure to uncertain demand. Thus, the relationship between flexibility and financial hedging in our context depends on their joint impact on the operating profit variability, which we show depends on demand correlation.

Boyabatli and Toktay (2006a) is an extension of their other work (Boyabatli and Toktay 2006b), which endogenizes the cost of external financing by modeling the interaction between the firm and the creditor as a Stackelberg game. Finally, there exists some finance and economics literature (Mello et al. 1995, Chowdhry and Howe 1999, Hommel 2003) that integrates operational flexibility and financial hedging, where the latter is used to mitigate exchange rate risk.

3. Optimal Financial Hedging and Its Value

3.1. Firm Value

We consider a value-maximizing firm whose profit depends on uncertain demand and thus is itself uncertain. As discussed in the introduction, there exist several market imperfections such as taxes, the cost of financial distress, and costly external financing, as a result of which the firm value is a concave function of pretax profit (Smith and Stulz 1985). Rather than modeling these market imperfections explicitly, we assume for tractability and parsimony that the firm value v has the following form as a function of pretax profit x:

$$v(x) = \gamma^{-1} - \gamma^{-1}\exp(-\gamma x). \tag{1}$$

The concavity of the firm value implies that there exists a positive risk premium r that the firm is willing to pay to eliminate (pretax) profit variability; i.e., $\mathbb{E}v(x) = v(\mathbb{E}x - r)$, where $\mathbb{E}$ denotes the expectation operator with respect to the true (physical) probability measure. This risk premium

$$r = \gamma^{-1}\ln\mathbb{E}\exp(-\gamma(x - \mathbb{E}x) \tag{2}$$

depends on the distribution of x as well as on the concavity of the value function captured in the "curvature parameter" $\gamma = -v''(x)/v'(x)$. A higher value of parameter γ corresponds to a stronger effect of taxes, financial distress, and other agency costs that result in concavity of the firm value.[4, 5, 6]

3.2. Financial Hedging

Let $\Pi_o(\mathbf{D})$ be the firm's operating profit (of a general functional form) that depends on stochastic demand vector $\mathbf{D}$. To mitigate operating profit variability, the firm enters into a financial hedging contract that is designed to generate a positive cash flow when demand is low. In practice, demand depends on the firm's effort, which may create agency problems in contracting. Therefore, the hedging contract payoff is typically based on another "underlying" vector $\boldsymbol{\theta}$ that is exogenous and commonly verifiable, such as the amount of snowfall at a certain location during a certain period of time, a commodity price, or a stock index. As a result, financial hedging is typically imperfect: it counterbalances operating profit variability only partially.

Let $\Pi_h(\boldsymbol{\theta})$ denote the payoff of the financial hedging contract. Following Froot et al. (1993), we assume that this contract is "fairly priced," i.e., its expected payoff $\mathbb{E}\Pi_h(\boldsymbol{\theta})$ is zero.[7] In reality, financial hedging is likely to be costly; therefore, we will also examine how much a firm should be willing to pay for financial hedging. The firm chooses a hedging contract to maximize its expected value:

$$\max_{\Pi_h} \mathbb{E}v(\Pi_o(\mathbf{D}) + \Pi_h(\boldsymbol{\theta})) \quad \text{subject to } \mathbb{E}\Pi_h(\boldsymbol{\theta}) = 0.$$

We define $\Pi = \Pi_o + \Pi_h^*$ as the total profit when the optimal hedging contract Π_h^* is used.

Before developing any managerial insights, it is useful to establish several technical results. We start by characterizing the optimal hedging contract.

Proposition 1. *The optimal hedging contract is*

$$\Pi_h^*(\boldsymbol{\theta}) = f(\boldsymbol{\theta}) - \mathbb{E}f(\boldsymbol{\theta}),$$
$$\text{where } f(\boldsymbol{\theta}) = \gamma^{-1} \ln \mathbb{E}[\exp(-\gamma\Pi_o(\mathbf{D})) \mid \boldsymbol{\theta}]$$

and results in the following expected firm value:

$$\mathbb{E}v(\Pi(\mathbf{D}, \boldsymbol{\theta})) = \gamma^{-1} - \gamma^{-1}\exp(\gamma\mathbb{E}f(\boldsymbol{\theta})). \tag{3}$$

Proof. Maximizing the expected firm value $\mathbb{E}v(\Pi_o(\mathbf{D}) + \Pi_h(\boldsymbol{\theta}))$ is equivalent to minimizing

$$\begin{aligned}
&\mathbb{E}\exp(-\gamma(\Pi_o(\mathbf{D}) + \Pi_h(\boldsymbol{\theta}))) \\
&\quad = \mathbb{E}[\exp(-\gamma\Pi_o(\mathbf{D}))\exp(-\gamma\Pi_h(\boldsymbol{\theta}))] \\
&\quad = \mathbb{E}\{\mathbb{E}[\exp(-\gamma\Pi_o(\mathbf{D}))\exp(-\gamma\Pi_h(\boldsymbol{\theta})) \mid \boldsymbol{\theta}]\} \\
&\quad = \mathbb{E}\{\mathbb{E}[\exp(-\gamma\Pi_o(\mathbf{D})) \mid \boldsymbol{\theta}]\exp(-\gamma\Pi_h(\boldsymbol{\theta}))\} \\
&\quad = \mathbb{E}\exp(\gamma f(\boldsymbol{\theta}) - \gamma\Pi_h(\boldsymbol{\theta})).
\end{aligned} \tag{4}$$

We show by contradiction that (4) is minimized by $\Pi_h^*(\boldsymbol{\theta}) = f(\boldsymbol{\theta}) - \mathbb{E}f(\boldsymbol{\theta})$ among all $\Pi_h(\boldsymbol{\theta})$ such that $\mathbb{E}\Pi_h(\boldsymbol{\theta}) = 0$. Suppose that there exists a better hedging payoff $g(\boldsymbol{\theta})$ such that $\mathbb{E}g(\boldsymbol{\theta}) = 0$ and

$$\begin{aligned}
&\mathbb{E}\exp(\gamma f(\boldsymbol{\theta}) - \gamma g(\boldsymbol{\theta})) < \mathbb{E}\exp(\gamma f(\boldsymbol{\theta}) - \gamma\Pi_h^*(\boldsymbol{\theta})) \\
\Leftrightarrow\ &\mathbb{E}\exp(\gamma f(\boldsymbol{\theta}) - \gamma g(\boldsymbol{\theta})) < \exp(\mathbb{E}\gamma f(\boldsymbol{\theta})) \\
\Leftrightarrow\ &\mathbb{E}\exp(X) < \exp(\mathbb{E}X), \\
&\qquad \text{where } X = \gamma f(\boldsymbol{\theta}) - \gamma g(\boldsymbol{\theta}),
\end{aligned}$$

which contradicts Jensen's inequality. Hence, there cannot be such g, and Π_h^* is indeed optimal. □

The payoff of the optimal hedging contract depends on a conditional expectation and thus can be rather complex. However, as any continuous derivative, it can be arbitrarily closely approximated by a portfolio of simple options (Ross 1976). Obviously, the presence of financial hedging will impact the firm's operations, which we consider next.

3.3. Optimal Capacity

Suppose that the firm's operating profit depends, in addition to the demand vector $\mathbf{D}$, on a vector $\mathbf{K}$ of resource or capacity levels that must be chosen prior to uncertainty resolution (together with the financial hedging contract). Reflecting decreasing marginal returns, we assume that the operating profit $\Pi_o(\mathbf{D}, \mathbf{K})$ is jointly concave in the capacity vector $\mathbf{K}$, as is the case for a variety of operations models, including any newsvendor network (Van Mieghem and Rudi 2002). The next proposition characterizes the optimal capacity vector.

[4] Reflecting all the aforementioned motives for hedging by assuming a relatively simple concave objective function is not uncommon in the literature. For example, Brown and Toft (2002) assume that the deadweight cost of tax liabilities, financial distress, and external financing can be represented by an exponential function of the profit. Bessembinder and Lemmon (2002) reflect the same market imperfections by assuming that the firm maximizes the mean-variance criterion.

[5] In Online Appendix 2 (provided in the e-companion), we numerically examine the robustness of our main results under an alternative objective function that models the effect of corporate profit tax and costly external financing more explicitly.

[6] For all practical purposes, γ plays the same role as the coefficient of absolute risk aversion, although our firm is risk neutral.

[7] Technically, this is the case when the expectation is taken with respect to the risk-neutral probability measure. Thus, we are implicitly assuming that the risk-neutral and the true probability measures are equivalent, which is the case when all agents are risk-neutral.

PROPOSITION 2. *The optimal capacity vector* $\mathbf{K}^*$ *that maximizes the expected firm value* (3) *is uniquely characterized by the following first-order condition*:

$$\mathbb{E}\nabla_{\mathbf{K}}\Pi_o(\mathbf{D},\mathbf{K}) + \mathbb{E}\left[\mathbb{C}\mathrm{ov}\left(\nabla_{\mathbf{K}}\Pi_o(\mathbf{D},\mathbf{K}), \frac{v'(\Pi_o(\mathbf{D},\mathbf{K}))}{\mathbb{E}(v'(\Pi_o(\mathbf{D},\mathbf{K}))\mid\boldsymbol{\theta})}\,\middle|\,\boldsymbol{\theta}\right)\right] = \mathbf{0}. \quad (5)$$

PROOF. We assumed that $\Pi_o(\mathbf{D},\mathbf{K})$ is jointly concave in $\mathbf{K}$ for any $\mathbf{D}$, and it is well known that $\ln\mathbb{E}\exp(\cdot)$ is a convex function. Thus, the expected firm value (3) is strictly concave and the optimal capacity is the unique solution of the first-order necessary and sufficient condition, $\nabla_{\mathbf{K}}\mathbb{E}v(\Pi(\mathbf{D},\boldsymbol{\theta})) = \mathbf{0}$, which can be rewritten as (5). □

Note that the operating profit gradient $\nabla_{\mathbf{K}}\Pi_o(\mathbf{D},\mathbf{K})$ is a vector, and thus the covariance in (5) should be interpreted componentwise.

3.4. Contract Correlation

In general, the efficacy of financial hedging depends on the entire joint distribution of $\mathbf{D}$ and $\boldsymbol{\theta}$. However, we simplify our analysis by assuming that $(\mathbf{D},\boldsymbol{\theta})$ follows a multivariate normal distribution and, furthermore, that $\mathbf{D}$ and $\boldsymbol{\theta}$ have the same number of components; i.e., there is an underlying variable corresponding to each demand class. We also assume that the correlation coefficient between D_i and θ_i is the same for all i and is denoted by ρ. Thus, the efficacy of financial hedging depends on a single parameter ρ, which we assume, without loss of generality, to be nonnegative and refer to as "contract correlation." As contract correlation increases from 0 to 1, the efficacy of financial hedging increases from none (no financial hedging available) to perfect (financial hedging eliminates all profit variability). Finally, whenever we resort to numerical analysis, we assume the following structure of $(\mathbf{D},\boldsymbol{\theta})$:

$$\mathbf{D}\sim\mathcal{N}(\boldsymbol{\mu}_D,\boldsymbol{\Sigma}_D),\quad \boldsymbol{\theta}\sim\mathcal{N}(\boldsymbol{\mu}_D,\rho^2\boldsymbol{\Sigma}_D),\quad \text{and}$$
$$\mathbf{D}\mid\boldsymbol{\theta}\sim\mathcal{N}(\boldsymbol{\theta},(1-\rho^2)\boldsymbol{\Sigma}_D),$$

where $(\boldsymbol{\Sigma}_D)_{ii}=\sigma_D^2$ and $(\boldsymbol{\Sigma}_D)_{ij}=\rho_D\sigma_D^2$, $i=1,2$, $j=3-i$. (In this, $\mathbf{D}$ and $\boldsymbol{\theta}$ can be thought of as the states of a multidimensional Brownian motion at times 1 and $\rho\le 1$, respectively; i.e., $\boldsymbol{\theta}$ captures $\rho^2\times 100\%$ of the variability of $\mathbf{D}$.)

3.5. An Illustration: Optimal Hedging of Newsvendor Profit

Although the optimal hedging contract characterized in Proposition 1 does not assume any functional form of the operating profit, it is instructive to illustrate what this contract looks like when the firm's operating profit has the standard newsvendor form,

$$\Pi_o(K;D) = -cK + p\min(K,D), \quad (6)$$

where K is capacity, c is the unit capacity cost, and p is the unit contribution margin (price minus unit production cost). There are two special cases in which the optimal hedging contract simplifies considerably: (i) when the firm has ample capacity ($c=0$) and (ii) under perfect contract correlation ($\rho=1$). We characterize these special cases in the following two corollaries to Proposition 1.

COROLLARY 1. *When the firm has ample capacity, i.e., the operating profit is* $\Pi_o(D)=pD$, *the optimal hedging contract has a payoff that is linear in the underlying variable*:

$$\Pi_h(\theta) = -p\rho\frac{\sigma_D}{\sigma_\theta}(\theta-\mu_\theta).$$

The contract characterized in Corollary 1 corresponds to linear contracts such as a simple forward, swap, or futures contract.

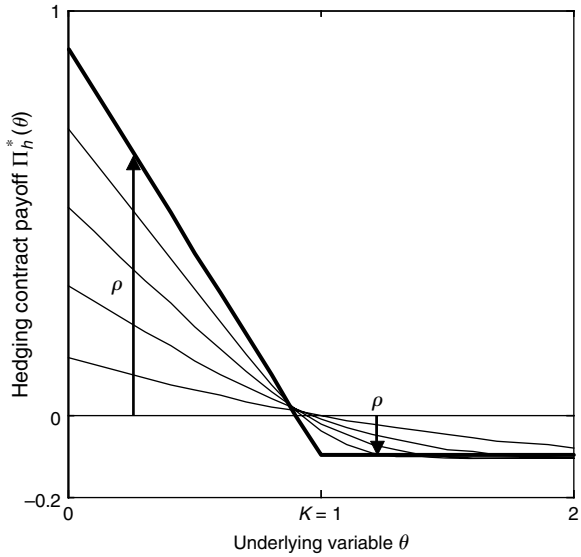
COROLLARY 2. *When the operating profit is given by* (6) *and contract correlation is perfect, the optimal hedging contract is to buy* $p\sigma_D/\sigma_\theta$ *European call options with spot price* $-\theta$ *and strike price* $-\mu_\theta-(\sigma_\theta/\sigma_D)(K^*-\mu_D)$, *where the optimal capacity* $K^*=\mu_D+\sigma_D\Phi^{-1}((p-c)/p)$ *and* Φ *denotes the standard normal cdf. The payoff of this hedging contract is*

$$\begin{aligned}\Pi_h(\theta) &= -\Pi_o(D)+\mathbb{E}\Pi_o(D)\\ &= -p\min(K,D)+p\mathbb{E}\min(K,D),\end{aligned}$$

where D *and* θ *are related by* $D=\mu_D+(\sigma_D/\sigma_\theta)(\theta-\mu_\theta)$.

When the contract correlation is less than perfect, the optimal hedging contract is more exotic as illustrated in Figure 1. This figure shows the payoff of

Figure 1 **Payoff of the Optimal Hedging Contract $\Pi_h^*(\theta)$ as a Function of the Underlying Variable θ for a Newsvendor**



Note. Newsvendor has $p=1$, $c=0.5$, $\mu_D=\mu_\theta=1$, $\sigma_D=\sigma_\theta=0.25$, $\gamma=1$, given capacity $K=1$, and contract correlation $\rho\in[0,1]$.

the optimal hedging contract of a newsvendor with a given capacity at different levels of contract correlation ρ. With zero contract correlation, there is no financial hedging, i.e., $\Pi_h(\theta) = 0$. As contract correlation increases and approaches 1, the optimal hedging contract approaches the option contract that is optimal when contract correlation equals 1 (as indicated by the bold graph).

3.6. The Value of Financial Hedging

In reality, financial hedging is likely to come at a cost, so it is useful to examine how much a firm should be willing to pay for it. We define the *value of financial hedging* $\Delta(\rho, \gamma)$ as the maximum amount of money a firm is willing to pay for the optimal hedging contract, i.e.,

$$\Delta(\rho, \gamma) \equiv \left\{\Delta\colon \max_{\mathbf{K}} \mathbb{E}v(\Pi - \Delta) = \max_{\mathbf{K}} \mathbb{E}v(\Pi_o)\right\}. \quad (7)$$

The value of financial hedging is similar to the risk premium (2) with two distinctions. First, financial hedging reduces but generally does not eliminate all profit variability. Second, financial hedging not only reduces profit variability but also affects the optimal capacity vector and thereby expected profit.

To derive some of our analytical results, we rely on the first-order (linear) approximation of the value of hedging; i.e., we approximate the value of hedging $\Delta(\gamma)$ by the first two terms of its Taylor expansion around 0:[8]

$$\Delta(\gamma) \approx \Delta(0) + \frac{d\Delta(0)}{d\gamma}\gamma. \quad (8)$$

Because $\Delta(\gamma)$ is a continuous function, all the comparative statics of the approximate value of hedging are guaranteed to hold for the exact value of hedging for any $\gamma \in (0, \bar{\gamma})$ for some $\bar{\gamma} > 0$.

As we show in Online Appendix 1 (provided in the e-companion),[9] the value of hedging $\Delta(\gamma)$ is roughly linear and thus can be estimated by the first-order approximation for a wide range of γ. Most importantly, an extensive numerical study (summarized in §5) demonstrates that our key managerial insights based on (8) remain valid even as γ becomes large. The next proposition shows that for small γ, the value of financial hedging depends on how much it reduces the operating profit variance.

Proposition 3. *The value of financial hedging can be written as*

$$\Delta(\rho, \gamma) = \tfrac{1}{2}\gamma[\mathbb{V}\mathrm{ar}\,\Pi_o(\mathbf{K}^0) - \mathbb{E}(\mathbb{V}\mathrm{ar}(\Pi_o(\mathbf{K}^0) \mid \boldsymbol{\theta}))] + o(\gamma),$$

where $\mathbf{K}^0$ *is the optimal capacity vector at* $\gamma = 0$, *and* $o(\gamma)/\gamma \to 0$ *as* $\gamma \to 0^+$.

Proof. Using Proposition 1, the value of financial hedging defined by (7) can be expressed as

$$\Delta(\rho, \gamma) = \gamma^{-1}\mathbb{E}\left[\ln \frac{\mathbb{E}\exp(-\gamma\Pi_o(\mathbf{K}^*(0)))}{\mathbb{E}[\exp(-\gamma\Pi_o(\mathbf{K}^*(\rho))) \mid \boldsymbol{\theta}]}\right],$$

where $\mathbf{K}^*(\rho)$ is the optimal capacity vector as a function of the contract correlation. Because $\Delta(\rho, 0) = 0$, the first-order Taylor expansion around $\gamma = 0$ gives

$$\Delta(\rho, \gamma) = \frac{d\Delta(\rho, 0)}{d\gamma}\gamma + o(\gamma).$$

Differentiating $\Delta(\rho, \gamma)$ with respect to γ yields

$$\begin{aligned}\frac{d\Delta(\rho, \gamma)}{d\gamma} &= \frac{1}{\gamma^2}\left[\mathbb{E}\left(\frac{\gamma\mathbb{E}[\Pi_o(\mathbf{K}^*(\rho))\exp(-\gamma\Pi_o(\mathbf{K}^*(\rho))) \mid \boldsymbol{\theta}]}{\mathbb{E}[\exp(-\gamma\Pi_o(\mathbf{K}^*(\rho))) \mid \boldsymbol{\theta}]}\right)\right.\\ &\quad - \mathbb{E}\left(\ln \frac{\mathbb{E}\exp(-\gamma\Pi_o(\mathbf{K}^*(0)))}{\mathbb{E}[\exp(-\gamma\Pi_o(\mathbf{K}^*(\rho))) \mid \boldsymbol{\theta}]}\right)\\ &\quad \left. - \frac{\gamma\mathbb{E}[\Pi_o(\mathbf{K}^*(0))\exp(-\gamma\Pi_o(\mathbf{K}^*(0)))]}{\mathbb{E}\exp(-\gamma\Pi_o(\mathbf{K}^*(0)))}\right]. \quad (9)\end{aligned}$$

Using l'Hôpital's rule to evaluate (9) at $\gamma = 0$ gives $d\Delta(\rho, 0)/d\gamma = \frac{1}{2}[\mathbb{V}\mathrm{ar}\,\Pi_o(\mathbf{K}^0) - \mathbb{E}(\mathbb{V}\mathrm{ar}(\Pi_o(\mathbf{K}^0) \mid \boldsymbol{\theta}))]$ and the result follows. □

3.7. Operational Flexibility and Its Value

In addition to financial hedging, the firm's ability to handle demand risk depends on its operational flexibility. We denote the degree of the firm's operational flexibility by a continuous parameter $\phi \in [0, 1]$ and use subscripts N and F to refer to a nonflexible firm ($\phi = 0$) and a flexible firm ($\phi > 0$), respectively. (We give parameter ϕ a specific meaning in the subsequent sections, in which we consider two particular forms of flexibility: product flexibility and postponement flexibility). We define the *value of flexibility* $\Lambda(\rho, \gamma)$ as the maximum amount of money a nonflexible firm is willing to pay for a given level of flexibility in the presence of the optimal hedging contract, i.e.,

$$\Lambda(\rho, \gamma) \equiv \{\Lambda\colon \mathbb{E}v(\Pi_F(\mathbf{K}_F^*) - \Lambda) = \mathbb{E}v(\Pi_N(\mathbf{K}_N^*))\}. \quad (10)$$

3.8. Relationship Between Flexibility and Financial Hedging

To assess the relationship between financial hedging and flexibility, we examine (i) how a firm's flexibility affects the value of financial hedging and (ii) how financial hedging affects the value of flexibility. In the case of perfect financial hedging, these are two sides of the same coin, as formalized in the following lemma.

[8] A similar analytical approach is used by Pratt (1964), who relies on the first-order approximation of the utility function $u(x)$ in arguing that the risk premium for a "small risk" is approximately $-u''(x)/u'(x)$ times half the variance of this risk.

[9] An electronic companion to this paper is available as part of the online version that can be found at http://mansci.journal.informs.org/.

Lemma 1. *Perfect financial hedging increases the value of flexibility if and only if flexibility increases the value of perfect financial hedging, i.e.,*

$$\Lambda(1,\gamma) > \Lambda(0,\gamma) \;\Leftrightarrow\; \Delta_F(1,\gamma) > \Delta_N(1,\gamma).$$

Proof. It follows from (10) that

$$\Lambda(1,\gamma) = \Pi_F(\mathbf{K}_F^0;1,\gamma) - \Pi_N(\mathbf{K}_N^0;1,\gamma),$$

$$\Lambda(0,\gamma) = \gamma^{-1}\ln\mathbb{E}\exp(-\gamma\Pi_{oN}(\mathbf{K}_N^*)) - \gamma^{-1}\ln\mathbb{E}\exp(-\gamma\Pi_{oF}(\mathbf{K}_F^*)).$$

Note that in case of $\rho = 1$, profits are evaluated at $\mathbf{K}^0$ because a perfectly hedged firm chooses the risk-neutral capacity vector. It further follows from (7) that

$$\Delta_F(1,\gamma) = \Pi_F(\mathbf{K}_F^0;1,\gamma) + \gamma^{-1}\ln\mathbb{E}\exp(-\gamma\Pi_{oF}(\mathbf{K}_F^*)),$$

$$\Delta_N(1,\gamma) = \Pi_N(\mathbf{K}_N^0;1,\gamma) + \gamma^{-1}\ln\mathbb{E}\exp(-\gamma\Pi_{oN}(\mathbf{K}_N^*)).$$

Combining these facts implies $\Lambda(1,\gamma) - \Lambda(0,\gamma) = \Delta_F(1,\gamma) - \Delta_N(1,\gamma)$, and the result follows. □

In the next section, we present analytical results on the relation between financial hedging and two types of operational flexibility. All results that rely on particular parametric assumptions are then numerically tested for robustness in §5.

4. Financial Hedging and Operational Flexibility

4.1. Product Flexibility

We consider a newsvendor-like firm that chooses capacity of three resources while facing uncertain demand for its two products, as analyzed in Van Mieghem (1998). Two resources are dedicated to the two products, whereas the third resource is product flexible. For simplicity, we assume all parameters to be equal for the two products. Let $\mathbf{K} = (K_1, K_2, K_F)'$, $\mathbf{c} = (c_N, c_N, c_F)'$, and p denote the capacity vector, the vector of unit capacity costs, and the unit net revenue, respectively. The firm's operating profit equals

$$\begin{aligned}\Pi_o(\mathbf{D},\mathbf{K}) = \max_{\mathbf{x},\mathbf{y}\in\mathbb{R}_+^2}\ & p(x_1+x_2+y_1+y_2) - \mathbf{c}'\mathbf{K},\\ \text{subject to}\ & x_i + y_i < D_i,\ i=1,2,\\ & x_i < K_i,\ i=1,2,\\ & y_1+y_2 < K_F, \end{aligned} \tag{11}$$

where x_i and y_i represent the amounts of nonflexible and flexible capacity, respectively, that are used to satisfy demand for product i, $i = 1, 2$. The operating profit (11) depends on the realized demand as follows:

$$\Pi_o(\mathbf{D},\mathbf{K}) = -\mathbf{c}'\mathbf{K} + \begin{cases} p(D_1+D_2) & \text{if } \mathbf{D}\in\Omega_0(\mathbf{K}),\\ p(K_1+K_F+D_2) & \text{if } \mathbf{D}\in\Omega_1(\mathbf{K}),\\ p(D_1+K_2+K_F) & \text{if } \mathbf{D}\in\Omega_2(\mathbf{K}),\\ p(K_1+K_2+K_F) & \text{if } \mathbf{D}\in\Omega_3(\mathbf{K}),\end{cases}$$

where

$$\Omega_0(\mathbf{K}) = \{\mathbf{D}\ge\mathbf{0} : D_1+D_2 \le K_1+K_2+K_F,\ D_i \le K_i+K_F,\ i=1,2\},$$

$$\Omega_1(\mathbf{K}) = \{\mathbf{D}\ge\mathbf{0} : D_1 > K_1+K_F,\ D_2 \le K_2\},$$

$$\Omega_2(\mathbf{K}) = \{\mathbf{D}\ge\mathbf{0} : D_2 > K_2+K_F,\ D_1 \le K_1\},$$

$$\Omega_3(\mathbf{K}) = \{\mathbf{D}\ge\mathbf{0} : D_1+D_2 > K_1+K_2+K_F,\ D_i > K_i,\ i=1,2\}.$$

Because $\Pi_o(\mathbf{D},\mathbf{K})$ is jointly concave in $\mathbf{K}$ (Van Mieghem and Rudi 2002), the optimal capacity vector is characterized by Proposition 2. The assumption of symmetric product parameters together with the uniqueness of the solution imply that the optimal solution is also symmetric. Hence, we can simplify notation by letting $K_1 = K_2 \equiv K_N$.

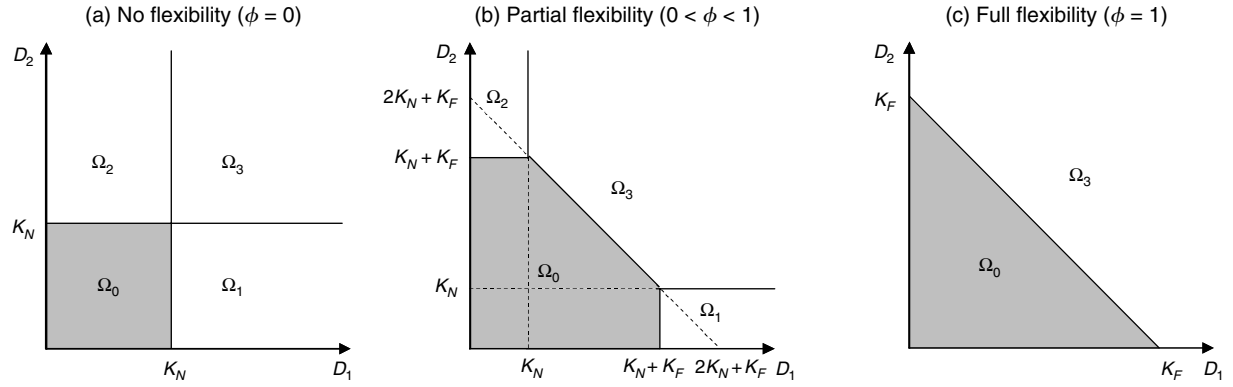
4.1.1. Level of Flexibility. To study the relationship between flexibility and financial hedging, we need an unambiguous measure of flexibility. Because the relative levels of flexible and nonflexible capacities depend on the relative costs of these two capacities, we can define the firm's product flexibility as

$$\phi \equiv (2c_N - c_F)/c_N.$$

To study the effect of flexibility, we vary $c_F \in [c_N, 2c_N]$ while keeping c_N constant. If $\phi = 0$, a unit of flexible capacity has the same cost as two units of nonflexible capacity, $c_F = 2c_N$; therefore, it is optimal to acquire only nonflexible capacity, $\mathbf{K}^* = (K_N^*, K_N^*, 0)'$, and the firm has *no product flexibility*. If $\phi = 1$, flexible and nonflexible capacities are equally expensive, $c_F = c_N$, and, therefore, it is optimal to acquire only flexible capacity, $\mathbf{K}^* = (0, 0, K_F^*)'$, and the firm has *full product flexibility*. In general, as ϕ increases from 0 to 1, the unit cost of flexible capacity c_F decreases from $2c_N$ to c_N, and the firm substitutes nonflexible capacity with flexible capacity.

The effect of flexibility is illustrated in Figure 2, which shows the partitioning of demand state space into four events: if $\mathbf{D}\in\Omega_0$, both demands are fully satisfied; if $\mathbf{D}\in\Omega_1$ (Ω_2), only demand for product 2 (1) is fully satisfied; and, finally, if $\mathbf{D}\in\Omega_3$, neither demand is fully satisfied (assuming flexible capacity is split between the two products proportionally to the residual demands after using product-specific capacities). As shown in the figure, product flexibility increases sales when one demand is "high" while the other one is "low." (Note that all panels in Figure 2 are plotted for the same total capacity $2K_N + K_F$, although flexibility may increase or decrease total capacity, depending on problem parameters.)

Figure 2 Product Flexibility Increases the Probability That Both Demands Will Be Fully Met, Which Happens When $\mathbf{D} \in \Omega_0$ (Shaded Region)



4.1.2. Product Flexibility and Financial Hedging. To assess whether flexibility and financial hedging are complements or substitutes, we examine whether flexibility increases or decreases the value of financial hedging. In the next proposition, we compare the values of perfect financial hedging under no flexibility and under full flexibility. We use subscripts N and F to denote the cases of no flexibility ($\phi = 0$) and full flexibility ($\phi = 1$), respectively.

Proposition 4. *When demand correlation is positive* (*negative*), *there exists* $\bar{\gamma} > 0$ *such that full product flexibility increases* (*decreases*) *the value of perfect financial hedging, i.e.,* $\Delta_F(1, \gamma) \geq (\leq)\ \Delta_N(1, \gamma)$ *for any* $\gamma \in (0, \bar{\gamma})$.

Proof. Proposition 3 implies that at sufficiently small γ and $\rho = 1$, $\Delta_F \geq \Delta_N$ if and only if

$$\begin{aligned}
&\mathbb{V}\mathrm{ar}\,\Pi_{oF}(K_F^0) \geq \mathbb{V}\mathrm{ar}\,\Pi_{oN}(K_N^0)\\
&\Leftrightarrow\ \mathbb{V}\mathrm{ar}\,\min(D_1+D_2, K_F^0) \geq \mathbb{V}\mathrm{ar}\sum_{i=1}^{2}\min(D_i, K_N^0)\\
&\Leftrightarrow\ \mathbb{V}\mathrm{ar}\,\min(2\mu+Z_1\sigma_D\sqrt{2+2\rho_D}, 2\mu+z^0\sigma_D\sqrt{2+2\rho_D})\\
&\quad \geq \mathbb{V}\mathrm{ar}\sum_{i=1}^{2}\min(\mu+Z_i\sigma_D, \mu+z^0\sigma_D)\\
&\Leftrightarrow\ (2+2\rho_D)\mathbb{V}\mathrm{ar}\,\min(Z_1, z^0)\\
&\quad \geq 2\mathbb{V}\mathrm{ar}\,\min(Z_1, z^0)+2\mathbb{C}\mathrm{ov}[\min(Z_1, z^0), \min(Z_2, z^0)]\\
&\Leftrightarrow\ \rho_D \geq \frac{\mathbb{C}\mathrm{ov}[\min(Z_1, z^0), \min(Z_2, z^0)]}{\mathbb{V}\mathrm{ar}\,\min(Z_1, z^0)},
\end{aligned}$$

where $z^0 = \Phi^{-1}((p - c_N)/p)$, and $\mathbf{Z} = (\mathbf{D} - \boldsymbol{\mu}_D)/\sigma_D$ is a bivariate standard normal vector with correlation coefficient ρ_D. Thus, the last inequality compares the correlation of two standard normal random variables with the correlation of the same standard normal random variables right censored at z^0. (If X is a random variable, we say that the distribution of $\min(X, x)$ is right censored at x.) It is straightforward to verify that the inequality holds if and only if $\rho_D > 0$ for any z^0. □

It follows from Proposition 3 that for sufficiently small γ, product flexibility increases the value of perfect financial hedging if and only if it increases the operating profit variance, i.e.,

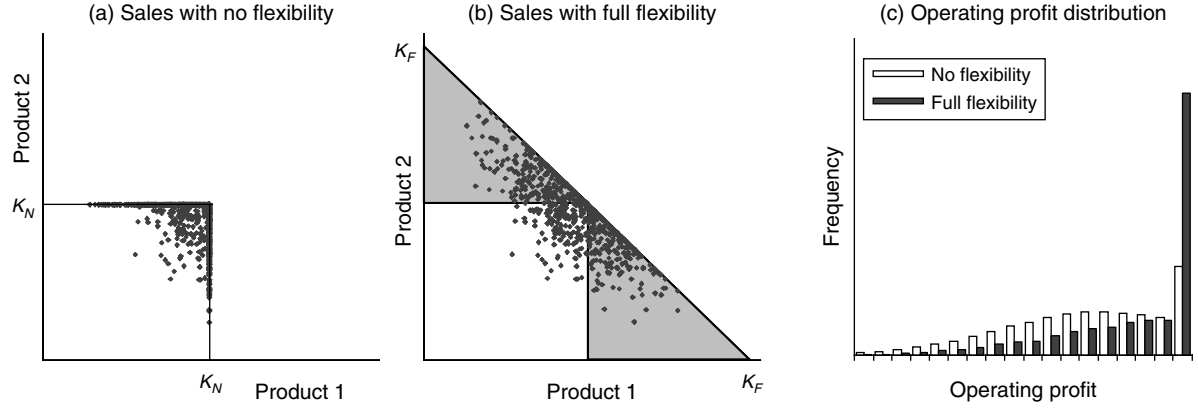
$$\Delta_F(1, \gamma) \geq \Delta_N(1, \gamma) \ \Leftrightarrow\ \mathbb{V}\mathrm{ar}\,\Pi_{oF}(K_F^0) \geq \mathbb{V}\mathrm{ar}\,\Pi_{oN}(K_N^0).$$

Proposition 4 shows that this is the case if and only if demands are positively correlated. To gain some intuition for this effect, consider Figures 3 and 4, which illustrate the effect of flexibility on operating profit distribution under negative and positive demand correlation, respectively. When demands are negatively correlated, relative to the dedicated capacity constraints, the flexible capacity constraint allows more uneven product sales (sales falling into shaded triangles in Figure 3(b)), which reduces operating profit variance. To put it differently, although product flexibility increases the upside profit variability while decreasing the downside profit variability, negative demand correlation makes the latter effect more pronounced (Figure 3(c)). The "operational hedging" effect of flexibility is strongest when demands are perfectly negatively correlated, in which case, full product flexibility eliminates operating profit variability entirely (given our symmetry assumption).

When demands are positively correlated, relative to the dedicated capacity constraints, the flexible capacity constraint allows more symmetrical product sales (sales falling into shaded triangles in Figure 4(b)), which increases operating profit variance. In other words, with positive demand correlation, the effect of product flexibility on increasing the upside variability is more pronounced than its effect on reducing the downside variability (Figure 4(c)).

Our numerical study (presented in §5) shows that the relationship between product flexibility and the value of hedging characterized in Proposition 4 continues to hold when hedging is imperfect and tends to hold when the curvature parameter γ is large. Specifically, we show that as γ increases, the demand

Figure 3 **Effect of Product Flexibility on Sales and Operating Profit Distribution Under *Negative* Demand Correlation** $\rho_D = -0.5$, $\mu_D = 1$, $\sigma_D = 0.25$, $p = 1$, $c = 0.5$, **and Total Capacity** $K_F + 2K_N = 2$



correlation threshold above which $\Delta_F > \Delta_N$ slightly decreases; i.e., the range of demand correlations for which full flexibility increases the value of hedging includes positive as well as slightly negative values. We also show that the value of hedging may not be monotone in ϕ over the entire range of $\phi \in [0, 1]$.

An alternative approach to assessing the same relationship is to ask how financial hedging affects the value of flexibility. The next result simply mirrors Proposition 4.

Corollary 3. *When demand correlation is positive* (*negative*), *there exists* $\bar{\gamma} > 0$ *such that perfect financial hedging increases* (*decreases*) *the value of full product flexibility, i.e.,* $\Lambda(1, \gamma) \geq (\leq)\ \Lambda(0, \gamma)$ *for any* $\gamma \in (0, \bar{\gamma})$.

Proof. The result follows directly from Lemma 1 and Proposition 4. □

4.2. Postponement Flexibility

We consider again a two-product newsvendor-like firm with identical cost, revenue, and demand parameters for both products. Whereas capacity is chosen under demand uncertainty, actual output is determined after demand has been observed. Let K be the capacity dedicated to each product, and let c_K be the unit capacity cost. The output of product i, denoted as Q_i, is constrained by capacity as well as realized demand, i.e., $Q_i = \min(K, D_i)$. The output is produced at a unit output cost c_Q and sold at a predetermined price p. The firm's operating profit is thus

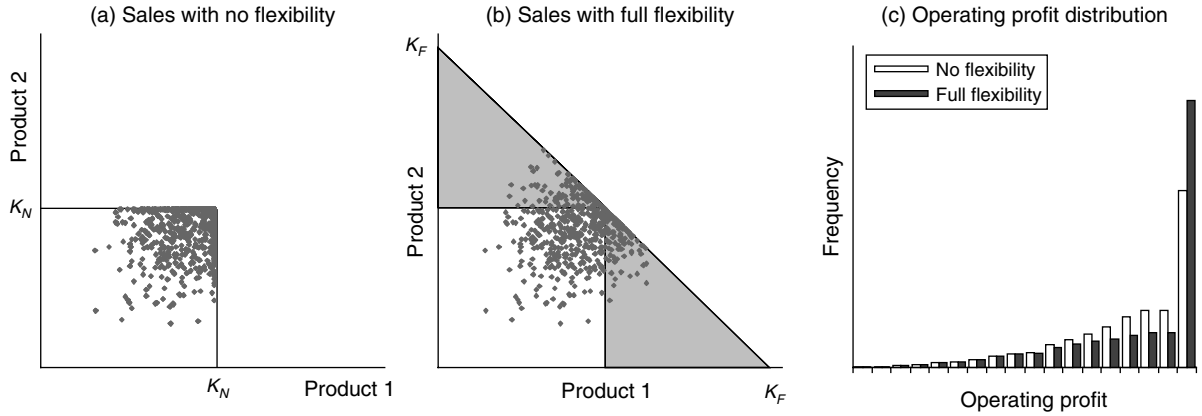
$$\Pi_o(K, \mathbf{D}) = (p - c_Q) \sum_{i=1,2} \min(K, D_i) - 2c_K K$$

$$= (p - c) \sum_{i=1,2} D_i - G(\mathbf{D}),$$

where $c = c_K + c_Q$ is the total unit cost, and $G(\mathbf{D}) = c_K \sum_{i=1,2} (K - D_i)^+ + (p - c) \sum_{i=1,2} (D_i - K)^+$ is the mismatch cost. Because the expected operating profit is concave in K, the optimal capacity is uniquely determined by the first-order condition stated in Proposition 2.

4.2.1. Level of Flexibility. We measure the firm's postponement flexibility as the ability to postpone some of its decisions that impact cost until demand is

Figure 4 **Effect of Product Flexibility on Sales and Operating Profit Distribution Under *Positive* Demand Correlation** $\rho_D = 0.5$, $\mu_D = 1$, $\sigma_D = 0.25$, $p = 1$, $c = 0.5$, **and Total Capacity** $K_F + 2K_N = 2$



known. Formally, we define postponement flexibility as the fraction of the total unit cost c that is incurred after demand is observed:

$$\phi \equiv c_Q/c.$$

To study the effect of flexibility, we vary parameter $\phi \in [0, 1]$ while keeping the total product cost c fixed. With zero flexibility ($\phi = 0$), all costs are incurred before demand is known, as in a pure *make-to-stock* production environment. With full flexibility ($\phi = 1$), the firm does not need to reserve any capacity before observing demand, so output is always equal to demand. This corresponds to a pure *make-to-order* scenario in which the cost of capacity excess, $c_K \sum_{i=1,2}(K - D_i)^+$, as well as the cost of capacity shortage, $(p - c)\sum_{i=1,2}(D_i - K)^+$, are completely eliminated.

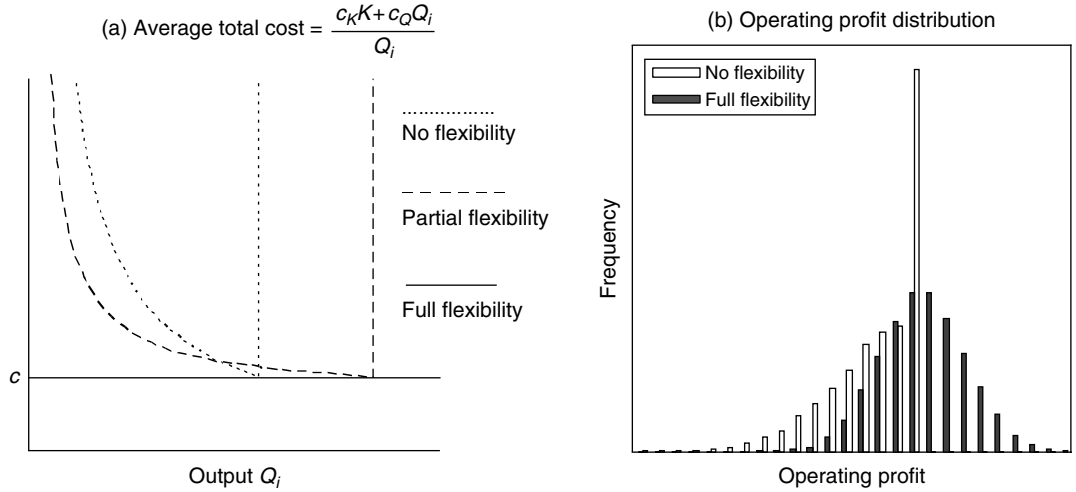
In general, greater flexibility corresponds to a lower unit cost of capacity, $c_K = (1-\phi)c$, and thus to a lower expected cost of capacity excess $c_K \mathbb{E}\sum_{i=1,2}(K - D_i)^+$. Furthermore, because greater flexibility means a lower unit cost of capacity excess c_K without affecting the unit cost of capacity shortage, $p - c$, it results in a higher capacity level and thus a lower expected cost of capacity shortage $(p-c)\mathbb{E}\sum_{i=1,2}(D_i - K)^+$. (Although greater flexibility always reduces the *expected* mismatch cost, it might result in a greater *realized* mismatch cost. This is because the lower unit cost of capacity results in a larger optimal capacity level, and thus, might result in a larger excess capacity.)

4.2.2. Relationship to Other Flexibilities. As shown in Figure 5(a), postponement flexibility decreases the average total cost at low output levels while making large outputs feasible. This makes postponement flexibility closely related to volume flexibility, which is typically defined as the ability to operate profitably at different output levels (Sethi and Sethi 1990). Postponement flexibility is also similar to how flexibility is often interpreted in the economics literature. In the seminal work on the topic, Stigler (1939) considers a plant to be flexible if it has a relatively flat average cost curve and thus incurs relatively smaller losses when deviating from the minimum average cost output.

4.2.3. Postponement Flexibility and Financial Hedging. The relationship between flexibility and financial hedging depends, again, on how flexibility impacts operating profit variability. The impact of postponement flexibility on operating profit variability is a result of two opposite effects. The fact that greater postponement flexibility corresponds to a lower unit cost of excess capacity mitigates the downside variability. At the same time, greater postponement flexibility results in a higher capacity level and thereby increases the upside variability. This is illustrated in Figure 5(b), which shows the operating profit distribution with zero and full postponement flexibility. Proposition 5 characterizes the effect of full postponement flexibility on the value of perfect financial hedging for sufficiently small γ. The subscripts N and F denote the cases of no postponement flexibility ($\phi = 0$) and full postponement flexibility ($\phi = 1$), respectively.

Proposition 5. *There exists $\bar{\gamma} > 0$ such that full postponement flexibility decreases the value of perfect financial hedging, i.e., $\Delta_F(1, \gamma) \le \Delta_N(1, \gamma)$ for any $\gamma \in (0, \bar{\gamma})$.*

Figure 5 Average Total Cost and Operating Profit Distribution



Notes. (a) The average total cost of product i as a function of output under zero, partial and full postponement flexibility, respectively. (b) The operating profit distribution under zero and full postponement flexibility, respectively, for $p = 1$, $c = 0.5$, $\mu_D = 1$, $\sigma_D = 0.25$, $\rho_D = 0$, $\gamma = 1$, and no hedging.

Proof. Proposition 3 implies that at sufficiently small γ and $\rho = 1$, $\Delta_F \le \Delta_N$ if and only if

$$\begin{aligned}
&\mathbb{V}\mathrm{ar}\,\Pi_{oF} \le \mathbb{V}\mathrm{ar}\,\Pi_{oN}(K_N^0)\\
&\Leftrightarrow\ \mathbb{V}\mathrm{ar}\left((p-c)\sum_{i=1,2} D_i\right)\\
&\qquad \le \mathbb{V}\mathrm{ar}\left(p\sum_{i=1,2}\min(K_N^0, D_i)\right)\\
&\Leftrightarrow\ (p-c)^2\sigma_D^2\,\mathbb{V}\mathrm{ar}(Z_1+Z_2)\\
&\qquad \le p^2\sigma_D^2\,\mathbb{V}\mathrm{ar}(\min(z^0, Z_1)+\min(z^0, Z_2))\\
&\Leftrightarrow\ \Phi^2(z^0)\,\mathbb{V}\mathrm{ar}(Z_1+Z_2)\\
&\qquad \le \mathbb{V}\mathrm{ar}(\min(z^0, Z_1)+\min(z^0, Z_2)),
\end{aligned}$$

where $z^0 = \Phi^{-1}((p-c)/p)$ and $\mathbf{Z} = (\mathbf{D} - \mu_D)/\sigma_D$ is a bivariate standard normal vector with correlation coefficient ρ_D. It is straightforward to verify that the last inequality holds for any z^0 and ρ_D. □

As shown in Figure 5(b), full postponement flexibility reduces the left tail of the profit distribution (by eliminating excess capacity in low-demand states), but it also increases its right tail (by eliminating the capacity constraint). According to Proposition 5, the former effect is always dominating in the sense that full postponement flexibility always reduces the operating profit variance. As a result, full postponement flexibility diminishes, at least for sufficiently small γ, the value of perfect financial hedging. As we show in §5, the result continues to hold for large values of γ, imperfect hedging, and partial flexibility.

To look at the same issue from another perspective, we can show that perfect financial hedging reduces the value of full postponement flexibility.

Corollary 4. *There exists $\bar{\gamma} > 0$ such that perfect financial hedging decreases the value of full postponement flexibility, i.e., $\Lambda(1, \gamma) \le \Lambda(0, \gamma)$ for any $\gamma \in (0, \bar{\gamma})$.*

Proof. The result follows directly from Lemma 1 and Proposition 5. □

5. Robustness Analysis

Our main results, namely Proposition 4 for product flexibility and Proposition 5 for postponement flexibility, assume that financial hedging is perfect and the curvature parameter γ does not exceed a threshold. In this section, we examine the robustness of these two propositions to both of these assumptions via an extensive numerical study. Unless stated otherwise, the figures illustrating our numerical results are representative of all parameter combinations that we examined.

5.1. Product Flexibility

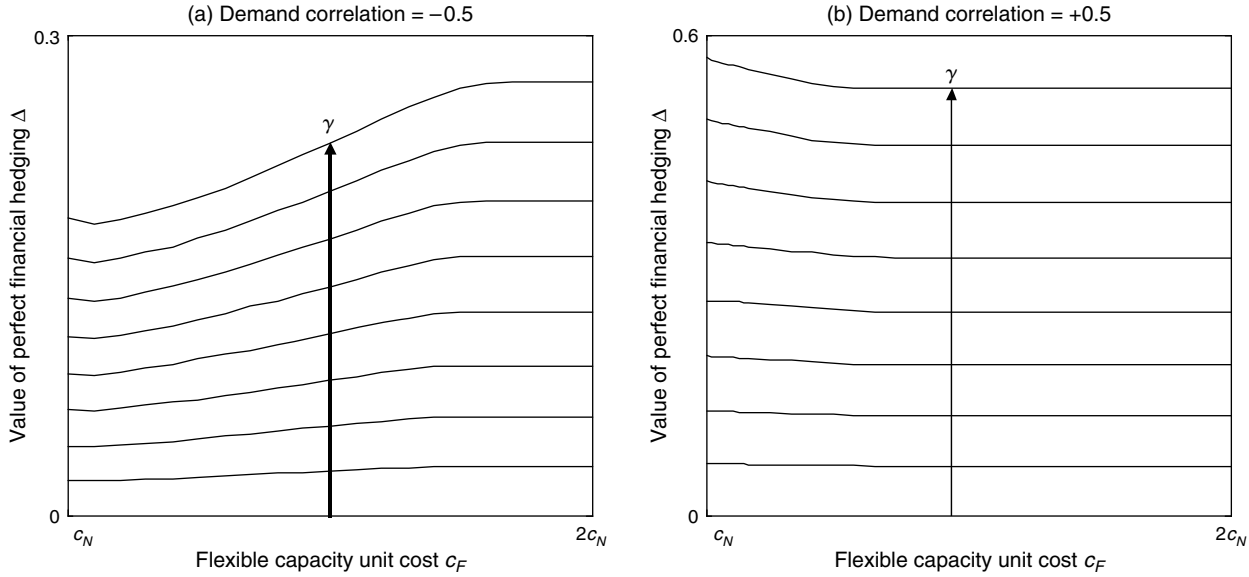
5.1.1. Numerical Study Design. Our numerical study relies on optimization via simulation with 10,000 demand scenarios and the following base-case parameter values: $\mu_D = 1$, $\sigma_D = 0.5$, and $p = 1$. The remaining parameter values are varied as follows: $c_N \in \{0.25, 0.5, 0.75\}$ (because our results are qualitatively similar at different values of c_N, all our figures are plotted only for $c_N = 0.5$), flexibility $\phi \in [0, 1]$ (i.e., c_F ranges from $2c_N$ to c_N), contract correlation $\rho \in [0, 1]$, and demand correlation $\rho_D \in [-1, 1]$. Most important, we vary the curvature parameter γ from 0 to 4. In the absence of financial hedging, as γ increases from 0 to 4, the total capacity, $K_F^* + 2K_N^*$, declines by roughly 75% (50%, 25%) when demand correlation $\rho_D = 0.5$ (0, −0.5) at any level of flexibility. Such a strong decline in the optimal capacity investment indicates that, in our setting, $\gamma = 4$ represents a rather extreme curvature of the value function.[10]

5.1.2. The Effect of Large γ. The reason Proposition 4 assumes γ to be "sufficiently small" is that it relies on the first-order approximation of the value of hedging $\Delta(\gamma)$ around $\gamma = 0$ (which can be made arbitrarily accurate by making γ sufficiently small). In general, the accuracy of this approximation, and therefore, our confidence that Proposition 4 holds for moderate and large values of γ, depends on the curvature of $\Delta(\gamma)$. As we show in Online Appendix 1, the value of hedging $\Delta(\gamma)$ is roughly linear and thus can be estimated by the first-order approximation for a wide range of γs. In particular, as γ varies from 0 to 4, the approximation accuracy decreases from 100% to roughly 90%–95%, depending on the parameter values. This is a first indication that the range of γs for which Proposition 4 holds is rather large. To verify whether this is indeed the case, we examine the relationship between flexibility and hedging as γ increases from 0 up to 4. We focus on the case of perfect hedging first.

Figure 6, which plots the value of perfect hedging as a function of flexibility at different levels of γ, confirms that, at least for $\rho_D \in \{-0.5, 0.5\}$, Proposition 4 continues to hold under large values of γ: A fully flexible firm values perfect hedging more (less) than a nonflexible firm when demands are positively (negatively) correlated. Note that at $\rho_D = -0.5$, the relationship between the value of hedging and flexibility is not monotone over the entire range of ϕ. Namely,

[10] When profit is normally distributed, in which case the firm value becomes $\mathbb{E}u(\Pi) = \mathbb{E}\Pi - (\gamma/2)\mathrm{Var}(\Pi)$, and when $\gamma = 4$, the certainty equivalent of a profit with mean \$1 and the coefficient of variation 0.2 is 92 cents, whereas the certainty equivalent of a profit with mean \$10 and the same coefficient of variation is only \$2. Thus, $\gamma = 4$ corresponds to a "moderate curvature" of the value function when stakes are low but an extreme curvature when stakes are high. In other words, the range of "realistic values" of parameter γ depends on the problem parameters. This is why our choice of γ is based on its impact on the capacity decision. See Rabin (2000) for more discussion.

Figure 6 Value of Perfect Hedging as a Function of Product Flexibility, $c_F \in [c_N, 2c_N]$, for $\gamma \in [0, 4]$ and $\rho_D \in \{-0.5, 0.5\}$



when flexibility is nearly perfect (c_F is very close to c_N), the value of hedging slightly increases in ϕ. At $\rho_D = 0.5$, in contrast, the relationship between the value of hedging and flexibility is monotone over the entire range of ϕ.[11]

To verify whether Proposition 4 holds for any ρ_D, consider the value of perfect hedging as a function of demand correlation $\rho_D \in [-1, 1]$ as plotted in Figure 7(a). Proposition 4 stipulates that at sufficiently small γ, full flexibility and perfect hedging are complements if and only if demand correlation $\rho_D > 0$. Figure 7(a) shows that as γ increases, the demand correlation threshold determining the relation between hedging and flexibility slightly decreases. Specifically, when $\gamma = 2$, full flexibility increases the value of perfect hedging (i.e., $\Delta_F > \Delta_N$) if and only if demand correlation $\rho_D > -0.05$.[12]

The relative magnitude of the effect of product flexibility on the value of hedging is illustrated in Figure 7(b), which mirrors Figure 7(a) except that the value of hedging is expressed as a percentage of its level at zero flexibility, Δ_N. As flexibility increases from 0 to 1, the value of perfect hedging drops by 100% (70%, 40%, 20%, 6%) at $\rho_D = -1$ ($-0.8, -0.6, -0.4, -0.2$) and increases by 2% (6%, 7%, 6%, 4%) at $\rho_D = 0$ (0.2, 0.4, 0.6, 0.8). Whereas Figure 7(b) is based on $\gamma = 2$, the *relative* effect of flexibility on the value of perfect hedging at other values of γ is similar. (Recall that $\Delta(\gamma)$ is nearly linear and $\Delta(0) = 0$. Thus, $\Delta_F(\gamma)/\Delta_N(\gamma)$ does not vary much as a function of γ.)

5.1.3. The Effect of Imperfect Contract Correlation $\rho < 1$. To examine the effect of imperfect contract correlation, we replicated all numerical experiments but with ρ varying between 0 and 1. The numerical analysis confirmed that the main insight from Proposition 4 remains intact when hedging is imperfect. As shown in Figure 8, a fully flexible firm values imperfect hedging more (less) than a nonflexible firm when demand correlation is positive (negative).

5.1.4. The Magnitude of the Value of Financial Hedging. To assess economic significance of financial hedging, we calculated its value relative to the value of unhedged profit (defined as the cash equivalent of the uncertain operating profit in the absence of hedging). At $\gamma = 2$, the value of perfect hedging ranges between roughly 10% and 70% of the value of unhedged profit, depending on other parameters, in particular, demand correlation. This indicates that at $\gamma = 2$, the economic significance of financial hedging is indeed considerable.

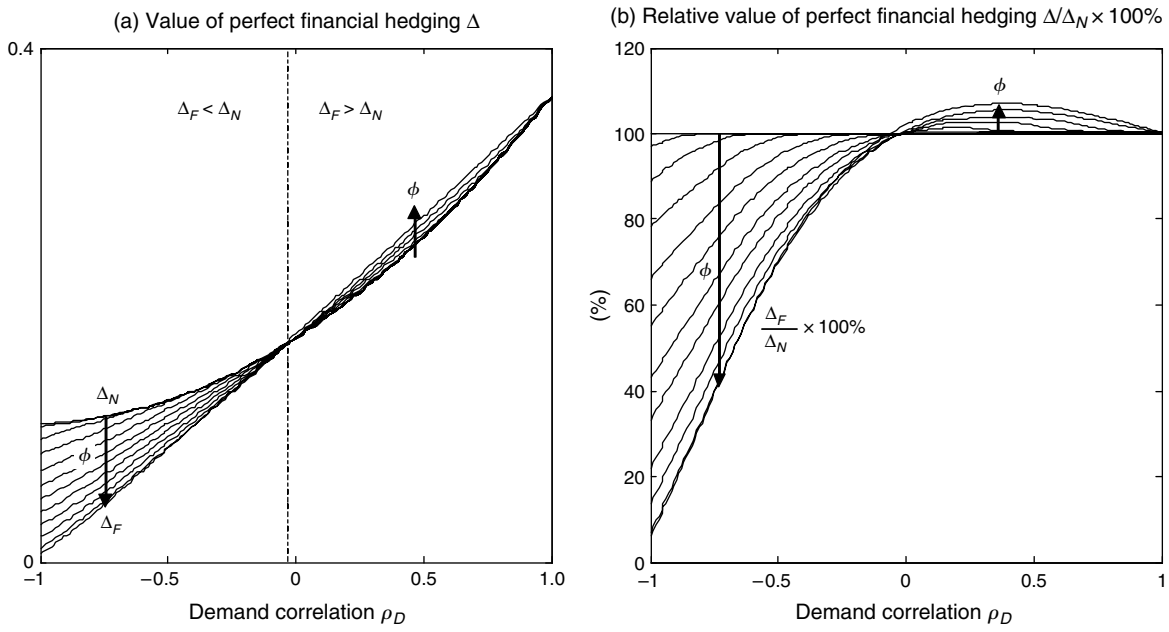
5.2. Postponement Flexibility

5.2.1. Numerical Study Design. Our numerical examination of postponement flexibility relies on the following base-case parameter values: $\mu_D = 1$, $\sigma_D = 0.5$, and $p = 1$. The remaining parameter values are varied as follows: $\rho_D \in \{-0.5, 0, 0.5\}$, $c \in \{0.25, 0.5, 0.75\}$ (because our results are qualitatively similar at different values of ρ_D and c, all our figures are plotted only for $\rho_D = 0$ and $c = 0.5$), $\phi \in [0, 1]$,

[11] Note that at $\rho_D = 0.5$, after c_F reaches approximately $1.25c_N$, a further increase in c_F has no impact on the value of hedging. This is because the flexible capacity becomes so expensive that $K_F^* = 0$, and further increase in c_F is irrelevant.

[12] Although not apparent from the graph, when $\rho_D < -0.05$ and $\phi \approx 1$, Δ marginally increases in ϕ, as illustrated in Figure 6(a).

Figure 7 **Value and Relative Value of Perfect Hedging as Functions of Demand Correlation $\rho_D \in [-1, 1]$ for Product Flexibility $\phi \in [0, 1]$ and $\gamma = 2$**



$\rho \in [0, 1]$, and $\gamma \in [0, 4]$. In the absence of financial hedging and at $\phi = 0$ (i.e., when risk-neutral critical fractile equals 0.5), as γ increases from 0 to 4, the optimal capacity declines by 72% (50%, 25%) when demand correlation $\rho_D = 0.5$ $(0, -0.5)$, indicating again that $\gamma = 4$ represents very strong curvature of the value function.

5.2.2. The Effect of Large γ and Imperfect Contract Correlation. Our numerical experiments confirmed that Proposition 5 remains robust when γ is large or when financial hedging is imperfect. In particular, the value of hedging Δ decreases monotonically in postponement flexibility at any level of $\gamma \in [0, 4]$ (Figure 9(a)) and any level of $\rho \in [0, 1]$ (Figure 9(b)). As flexibility ϕ increases from 0 to 1, the value of perfect hedging declines by approximately 25% (30%, 50%) when demand correlation $\rho_D = 0.5$ $(0, -0.5)$, at any $\gamma \in [0, 4]$.

To summarize this section, all numerical experiments confirmed that our key results regarding the relation between the two types of flexibility and financial hedging, namely Propositions 4 and 5,

Figure 8 **Value of Hedging as a Function of Product Flexibility, $c_F \in [c_N, 2c_N]$, for Contract Correlation $\rho \in [0, 1]$, Demand Correlation $\rho_D \in \{-0.5, 0.5\}$, and $\gamma = 2$**

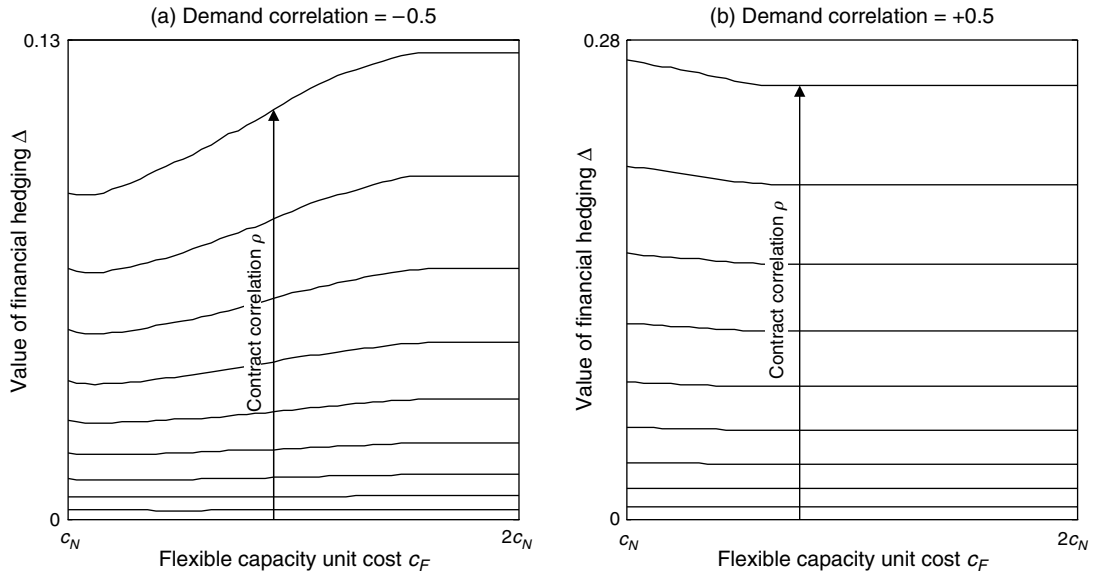
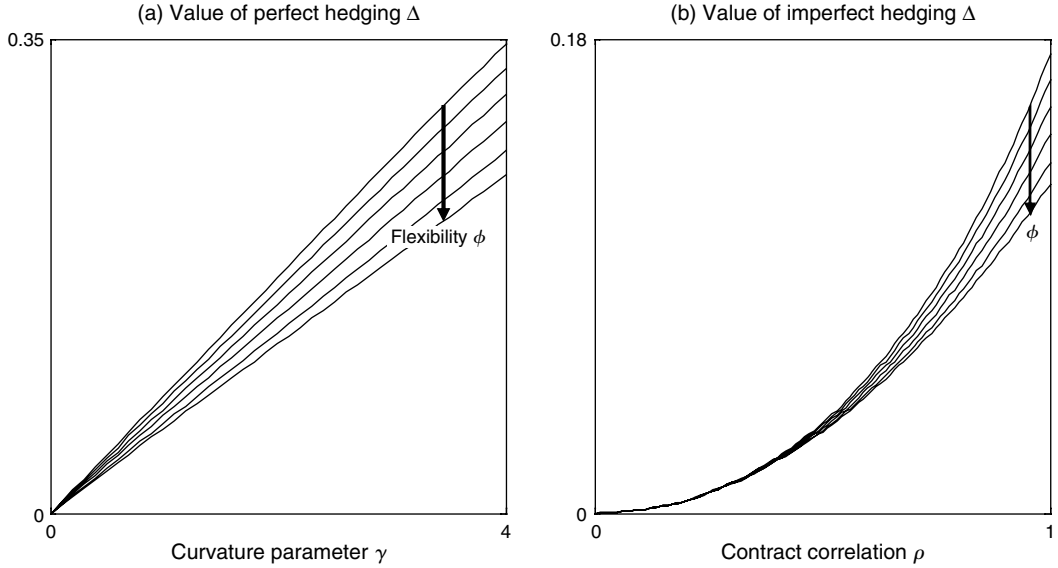


Figure 9 **(a) Value of Perfect Hedging as a Function of Curvature Parameter $\gamma \in [0, 4]$ for Postponement Flexibility $\phi \in [0, 1]$; (b) Value of Hedging as a Function of Contract Correlation $\rho \in [0, 1]$ for Postponement Flexibility $\phi \in [0, 1]$ and $\gamma = 2$**



remain qualitatively robust for large γ as well as for imperfect contract correlation.

6. Discussion and Limitations

The key contribution of this paper is to provide a better understanding of the relationship between flexibility and financial hedging when both are used to mitigate demand risk. We show that the type of flexibility matters. Product flexibility and financial hedging tend to be complements (substitutes) when product demands are positively (negatively) correlated. Thus, when demands are positively (negatively) correlated, (i) product-flexible firms should be willing to spend more (less) on financial hedging, and (ii) in the presence of financial hedging, firms should be willing to invest more (less) in product flexibility.

In contrast to product flexibility, postponement flexibility follows intuition: postponement flexibility and financial hedging are substitutes.

Our results also provide empirical predictions.

Prediction 1. *When demands are positively* (*negatively*) *correlated*, *product flexibility is positively* (*negatively*) *related to the firm's propensity to hedge*.

Prediction 2. *Postponement flexibility is negatively related to the firm's propensity to hedge*.

Although most empirical measures of product flexibility used in the extant literature rely on primary data such as changeover times (e.g., Upton 1995), there are exceptions. For example, the recent empirical analysis of product flexibility in the U.S. automotive industry by Goyal et al. (2006) relies entirely on secondary data found in industry reports (the number of models versus the number of assembly lines per plant). As for postponement flexibility, potential measures could be based on capital intensity or average capacity utilization (the larger the capital intensity, or the higher the average capacity utilization, the lower the postponement flexibility). A survey of the empirical literature on flexibility that discusses various methodological issues regarding validity and reliability of flexibility measurements can be found in Vokurka and O'Leary-Kelly (2000). As far as empirical measures of financial hedging are concerned, there are public data on firms' derivative positions as well as a large body of empirical literature on the subject (e.g., Guay and Kothari 2003). Any empirical test of these predictions would obviously need to control for industry-specific factors, such as demand uncertainty, that influence firms' flexibility as well as their hedging activities.

Our model has several limitations. Most important, it assumes that market imperfections such as taxes, bankruptcy costs, and agency costs result in an exponential value function. An important property of the exponential value function, besides its tractability, is that its curvature parameter, $\gamma = -v''(x)/v'(x)$, is independent of its argument x. This means that the firm's willingness to pay for financial hedging is independent of its current wealth or its expected profit. As a result, flexibility affects the value of hedging by affecting the shape of the operating profit distribution, but its effect on the mean profit is irrelevant. With all other objective functions, the relationship between flexibility and the value of hedging will be driven not only by the fact that flexibility increases or decreases profit variability (variability effect) but also by the fact that flexibility shifts the entire profit distribution to the right (wealth effect).

We examine how the existence of a wealth effect impacts the relationship between flexibility and financial hedging in Online Appendix 2, in which we consider a concave, piecewise-linear objective function. We explain how such objective function can model corporate tax and costly external financing. We then show, using a numerical analysis, that although the existence of the wealth effect can tilt the relation between flexibility and financial hedging either way, depending on the model parameters, the key insight from our base-case model remains intact: product flexibility and financial hedging are more likely to be complements (substitutes) when product demands are positively (negatively) correlated, ceteris paribus.

The exponential value function could be alternatively interpreted as the exponential utility function of a risk-averse manager-owner, as is commonly done in the operations literature (Bouakiz and Sobel 1992, Van Mieghem 2007, Chen et al. 2007, etc.). Assuming risk neutrality, however, makes the model applicable to publicly held corporations. Furthermore, it considerably simplifies the pricing of the hedging contract and hence our analysis. Finally, our model of product flexibility assumes symmetrical product parameters, which undervalues the revenue-maximizing option embedded in product flexibility.

Another limitation of our study is that it examines product and postponement flexibilities separately. In reality, these two as well as other dimensions of flexibility affect the value of financial hedging jointly. To assess this joint impact in practice, a simulation study fitting the firm's specific operational structure is likely to be required.

7. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at http://mansci.journal.informs.org/.

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