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OPERATOR OF THIN PLATE REINFORCED WITH THIN-WALLED RIBS

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In the engineering practice structures have been used since long which have the shape of a thin plate reinforced with ring-shaped stiffening ribs. The methods of analysis of such plates have not, hitherto, reached full generality. It is, therefore, necessary to put up with approximate solutions obtained by the methods of variational

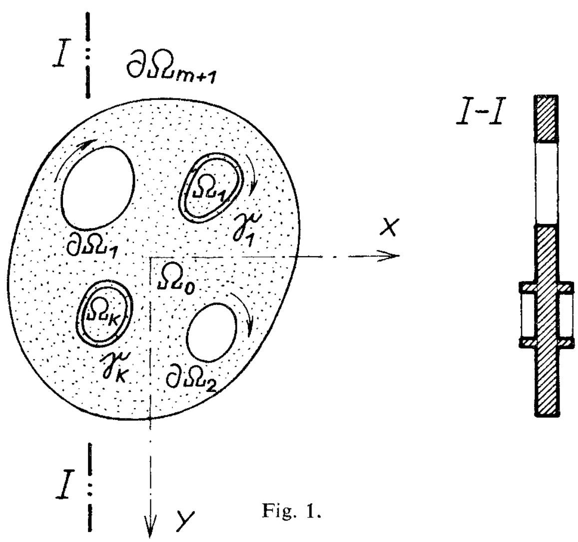

Fig. 1.

calculus. If this method using the methods of Hilbert spaces is applied, approximate solutions may be found which constitute a minimizing progression. If the operator of a thin plate reinforced with stiffening ribs is positively definite, then this progression converges in an energy space to the exact solution. In this paper it is proved that this operator is positively definite on a linear subset M. The middle plane forms a multiple connected region Ω in the plane Oxy with a Lipschitzian boundary $\partial\Omega$.

$\partial\Omega = U_{j=1}^{m+1} \partial\Omega_j$, where $\partial\Omega_j$ are closed sufficiently smooth curves (Lipschitzian boundary) and all curves $\partial\Omega_j$ for $j \neq m + 1$ lie in the interior of $\partial\Omega_{m+1}$.

The stiffening ribs are made of a material which is different from that of the plate, they are ring-shaped and of rectangular cross-section. The axis of every ring γ_k lies in the plane Oxy. The line $\Gamma = U_1^l \gamma_k$ divides the region Ω into $(l + 1)$ regions. It is supposed that all curves γ_k are sufficiently smooth.

Then $\bar{\Omega} = \bar{\Omega}_0 + \Omega_1 + \ldots + \Omega_l$. For the sake of simplicity it is supposed that the region Ω_k $(k = 1, 2, \ldots, l)$ is simply connected and that the region Ω_0 is multiple connected and defined by the set of curves $\partial\Omega = U_1^{m+1} \partial\Omega_j$. Furthermore it is assumed that the curves Γ and $\partial\Omega$ are not tangent to each other at any point.

The vertical continuous forces acting on the region Ω_k are denoted by $p_k(x, y)$. In addition to $p_k(x, y)$, a singular moment and singular forces may act on the region Ω_k, too. It is, furthermore, assumed that the plate is perfectly built-in on its contour $\partial\Omega$, which means that on the boundary $\partial\Omega$ kinematic conditions are given (the second boundary-value problem)

(1)
$$w = w_* = 0$$
$$\frac{\partial w}{\partial n} = \frac{\partial w_*}{\partial n} = 0 \quad \text{on} \quad \partial\Omega$$

The stiffening ribs are regarded as thin-walled rings of constant flexural and torsional rigidity. The behaviour of the ribs is given by the theory of small strains of thin curved bars.

Further, it is assumed that one of the principal central axes of inertia lies in the plane Oxy. In that case the principal axis of inertia in every cross-section of the rib coincides with the principal normal to the axis γ_k $(k = 1, 2, \ldots, l)$. The connection between regions Ω_0 and Ω_k is considered to follow the individual curves γ_k.

The bending moments and forces acting on the k-th ring on the Ω_0 side are denoted by $m_{0k}(s)$ and $p_{0k}(s)$ respectively and the same forces acting on the Ω_k side by $m_{kk}(s)$ and $p_{kk}(s)$ for $k = 1, 2, \ldots, l$ where s is a parameter equal to the length of arc of the curve γ_k measured clockwise from an arbitrarily chosen initial point.

The k-th ring is then subjected to the total load

(2)
$$m_k(s) = m_{0k}(s) - m_{kk}(s)$$

(3)
$$p_k(s) = p_{0k}(s) - p_{kk}(s)$$

Under the loads (2), (3) the ring deforms together with the adjacent plates. The following kinematic conditions must be satisfied on the curve γ_k:

(4)
$$w_0 = w_k = \Delta_k \,, \quad \frac{\partial w_0}{\partial n} = \frac{\partial w_k}{\partial n} = \theta_{\tau k}$$

where w_0 is the deflection of the plate in the region Ω_0, w_k is the deflection of the plate in the region Ω_k $(k = 1, 2, \ldots, l)$, Δ_k is the deflection of the k-th ring, n is the normal external with respect to the region Ω_0, $\theta_{\tau k}$ is the angle twist of the elastic axis of the ring, i.e., the angle of rotation of the ring cross-section about the tangent τ along the curve γ_k. The direction of the tangent is parallel to the direction of growth of the length s.

Let further the following notation be used

(5) $$\theta_{nk} = - \frac{d\Delta_k}{ds} = - \frac{dw_0}{ds}$$

where θ_{nk} is the angle of bending of the ring, i.e., the angle of rotation of the tangent to the axis of the ring about the normal at the given of the axis.

Without proof, which is presented in [1], let the internal forces of the rings $L_{\tau k}$ and L_{nk} be expessed in terms of the deformation quantities $(\theta_{nk}, \theta_{\tau k})$.

They are

(6) $$L_{\tau k} = C_k \left(\frac{d\theta_{\tau k}}{ds} - \frac{1}{\varrho_k} \theta_{nk} \right) \quad L_{nk} = A_k \left(\frac{d\theta_{nk}}{ds} + \frac{1}{\varrho_k} \theta_{\tau k} \right)$$

where A_k is the flexural rigidity of the ring with respect to the axis n.
C_k is the tersional rigidity of the ring,
L_{nk} is the bending moment (about the axis n), and
$L_{\tau k}$ is the torsional moment in the section s.

It is easy to prove the following differential relationships between the internal forces of the ring and the load $p_k(s)$ $m_k(s)$. They are given in [1].

(7) $$\frac{dL_{\tau k}}{ds} = \frac{L_{nk}}{\varrho_k} - m_k(s) \quad \frac{dL_{nk}}{ds} = - \frac{L_{\tau k}}{\varrho_k} + V_{bk}$$

where $V_{bk} = -\int_0^s p_k(s)\, ds + \text{const.}$ – is the shearing force acting in the cross-section s of the ring with the axis γ_k.

The internal forces of the plate and the ring are defined in terms of the deflection function of the plate with stiffening ribs as follows:

Bending moments:

$$M_x = -D \left(\frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2} \right)$$

$$M_y = -D \left(\frac{\partial^2 w}{\partial y^2} + \mu \frac{\partial^2 w}{\partial x^2} \right)$$

$$L_{nk} = A_k \left[- \frac{\partial}{\partial s} \left(\frac{\partial w}{\partial s} \right) + \frac{1}{\varrho_k} \frac{\partial w}{\partial n} \right]$$

Twisting moments:

$$H_{xy} = -D(1-\mu)\frac{\partial^2 w}{\partial x\,\partial y}, \tag{8}$$

$$L_{\tau k} = C_k\left[\frac{\partial}{\partial s}\left(\frac{\partial w}{\partial n}\right) + \frac{1}{\varrho_k}\frac{\partial w}{\partial s}\right]$$

Shearing forces:

$$N_x = -D\frac{\partial}{\partial x}(\Delta w), \quad N_y = -D\frac{\partial}{\partial y}(\Delta w)$$

where $D = E\,h^3/12(1-\mu^2)$ is the cylindrical rigidity of the plate and $0 \leqq \mu \leqq \frac{1}{2}$ is Poisson's number.

Let further M_n, $H_{n\tau}$, N_n be respectively the bending moment, the twisting moment and the shearing force taken with respect to the unit length of the edge of the plate, on the cross-section of which the external normal is n.

Between the quantities $(M_n, H_{n\tau}, N_n)$ and $(M_x, M_y, H_{xy}, N_x, N_y)$ the following transposition formulas hold

$$\begin{aligned} M_n &= M_x\cos^2(n,x) + M_y\cos^2(n,y) + 2H_{xy}\cos(n,x)\cos(n,y) \\ H_{n\tau} &= (M_y - M_x)\cos(n,x)\cos(n,y) + H_{xy}[\cos^2(n,x) - \cos^2(n,y)] \\ N_n &= \pm[N_x\cos(n,x) + N_y\cos(n,y)] \end{aligned} \tag{9}$$

where n is the external normal to $\partial\Omega$;

the sign $+$ is used on the external boundary curve of the plate $\partial\Omega_{m+1}$

the sign $-$ is used on the internal boundary curve of the plate $\partial\Omega_j$, where $j \neq m+1$.

Let us further define some concepts and quote some theorems of functional analysis.

Definition 1. *By a region we understand an open connected set the boundary of which is constituted by a finite number of piecewise smooth simple curves, either finite and closed or infinite.*

Definition 2. *Let an arc (curve) $\partial\Omega$ be given in a parametrical form by the equations $x = x(s)$, $y = y(s)$, $\alpha \leqq s \leqq \beta$ where s is the parameter of length. We say that the arc (curve) $\partial\Omega$ is sufficiently smooth if the functions $x(s)$ and $y(s)$ have three continuous derivatives in $\langle\alpha, \beta\rangle$ and for no s it is at the same time $x'(s) = y'(s) = 0$. If the curve is a closed one it is required that in addition to the relationships $x(\alpha) = x(\beta)$; $y(\alpha) = y(\beta)$ the relationships for all three derivatives are satisfied, i.e., we have $x'(\alpha) = x'(\beta)$ up to $y'''(\alpha) = y'''(\beta)$ where by the derivative at the point α we understand the derivative from the right and at the point β the derivative from the left.*

Definition 3. *$\varepsilon(\Omega)$ is the set of all functions having derivatives of all orders continuous in the region Ω and continuously prolongable to $\bar{\Omega} = \Omega + \partial\Omega$. $\mathscr{D}(\Omega)$ is the linear space of all functions from $\varepsilon(\Omega)$ having a compact carrier in Ω. We say that the continuous boundary $\partial\Omega$ of the defined region Ω is, Lipschitzian if the functions $a_i(x)$ for $i = 1, 2, \ldots, n$, describing the i-th continuous part of the boundary are Lipschitzian.*

Definition 4. *The set of all metrizable functions w in Ω for which*

$$\|w\|_{L_2(\Omega)} = \left(\int_{\Omega} (w(x))^2 \, \mathrm{d}\Omega\right)^{1/2} < \infty$$

will be denoted by $L_2(\Omega)$.

$L_2(\Omega)$ is a separable Banach space. The proof is given in [4].

Definition 5. *Let $u, v \in L_2(\Omega)$, then the intergral*

$$\int_{\Omega} u(x)\, v(x) \, \mathrm{d}x$$

is definite and finite. The equations

$$(u, v)_{\Omega} = \int_{\Omega} u(x)\, v(x) \, \mathrm{d}x$$

defines the quantity $(u, v)_{\Omega}$ which is called the scalar (inner) product of functions u and v in the region Ω.

Definition 6. *Let k be a natural number. Then $W_2^{(k)}(\Omega)$ is the linear space of all functions $w \in L_2(\Omega)$ which have generalized derivatives $D^{\alpha}w$ up to and including the k-th order and for which $D^{\alpha}w \in L_2(\Omega)$. The following notation will be used*

$$|\alpha| = \sum_{i=1}^{2} \alpha_i\,; \quad D^{\alpha}w = \frac{\partial^{|\alpha|}w}{\partial x_1^{\alpha_1}\, \partial x_2^{\alpha_2}}$$

The norm is defined by the equation

$$\|w\|_{W_2^{(k)}(\Omega)} = \Big(\sum_{|\alpha|=0}^{k} \|D^{\alpha}w\|^2_{L_2(\Omega)}\Big)^{1/2}$$

The space $\mathring{W}_2^{(k)}(\Omega)$ is the closure of functions from $\mathscr{D}(\Omega)$ in the norm $W_2^{(k)}(\Omega)$.

Definition 7. *Let H be a Hilbert space, $M \subset H$ a linear subset. Introduce the scalar product w, u by $[w, u] = (Aw, u)$ for $u, w \in M$, A being a positive definite operator on M. The complete envelope of M with the scalar product w, u is a Hilbert space which is denoted by H_A and called the energetic space of the operator A.*

Definition 8. *The concept of a minimizing progresion. A progression* $\{w_n\}$, $w_n \in H_A$ *is refferred to as a minimizing progression if*

$$\lim_{n\to\infty} F(w_n) = \inf_{H_A} F(w)$$

where $F(w) = [w, w] - 2(w, f)$, *a*, f *being a fixed element of the space* H.

The total potential energy of the system of the plate with stiffening ribs is (See [1])

$$E = \ni(w) - \int_\Omega pw \, \mathrm{d}\Omega \tag{10}$$

where

$$\ni(w) = \frac{D}{2}\int_\Omega \left\{(\Delta w)^2 + 2(1-\mu)\left[\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right]\right\} \mathrm{d}\Omega + \tag{11}$$

$$+ \sum_{k=1}^{l} \frac{1}{2}\int_{\gamma_k} \left\{C_k\left[\frac{\mathrm{d}}{\mathrm{d}s}\left(\frac{\partial w}{\partial n}\right) + \frac{1}{\varrho_k}\frac{\partial w}{\partial s}\right]^2 + A_k\left[\frac{1}{\varrho_k}\frac{\partial w}{\partial n} - \frac{\mathrm{d}}{\mathrm{d}s}\left(\frac{\partial w}{\partial s}\right)\right]^2\right\} \mathrm{d}s =$$

$$= \frac{D}{2}\int_\Omega \left\{(\Delta w)^2 + 2(1-\mu)\left[\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right]\right\} \mathrm{d}\Omega +$$

$$+ \sum_{k=1}^{l} \frac{1}{2}\int_{\gamma_k} \left(\frac{L_{\tau k}^2}{C_k} + \frac{L_{nk}^2}{A_k}\right) \mathrm{d}s = \sum_{k=1}^{l} \frac{1}{2}\int_{\gamma_k} \left(\frac{L_{\tau k}^2}{C_k} + \frac{L_{nk}^2}{A_k}\right) \mathrm{d}s +$$

$$+ \frac{D}{2}\int_\Omega \left[\left(\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}\right)^2 + (1-\mu^2)\left(\frac{\partial^2 w}{\partial y^2}\right)^2 + 2(1-\mu)\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2\right] \mathrm{d}\Omega$$

where $w(x, y)$ is the deflection of the plate with stiffening ribs, ϱ_k is the curvature at the given point of the curve γ_k. Let E be the total potential energy of the system. Then the principle of virtual work holds which implies

$$\delta E = \delta \ni(w) - \delta A = 0 \tag{12}$$

where δA is the virtual work of external forces on the virtual displacement δw.

The equation of equilibrium:

a) the equation of Sophie Germain

$$D\,\Delta^2 w = p(x, y) \tag{13}$$

b) two equations of equilibrium (7) for an element of the stiffening rib are represented by Euler's equations for the functional (10).

Let $M = \mathscr{D}(\Omega)$. It is known that

$$\overline{M} = L_2(\Omega) \tag{14}$$

Let Δ^2 be a biharmonic operator, defined on a linear subset M, and let operators b, c be introduced as products of adjoint operators B^*, C^* in the sense of the scalar product in $L_2(\Gamma)$ with operators B, C. Operators B, C are differential operators given on functions $w(x, y) \in M$ where $(x, y) \in \Gamma$ and

$$B = L_{\tau k}/\sqrt{(DC_k)} \tag{15}$$

$$C = L_{nk}/\sqrt{(DA_k)}$$

(D, C_k, A_k are constants).

Then

$$b = B^*B\,, \quad c = C^*C \tag{16}$$

Further, it is assumed that $p(x, y) \in L_2(\Omega)$, $m_k(s) \in L_2(\Gamma)$, $V_{bk}(s) \in L_2(\Gamma)$.

Definition 9. *Let* $H = \{u \in W_2^{(2)}(\Omega)$, *the traces* u *and* $\partial u/\partial n$ *in* γ_k *are such that* $L_{\tau k}(u)$, $L_{nk}(u) \in L_2(\gamma_k)\}$ *represents a Hilbert space which scalar product*

$$(u, v) = \sum_{|\alpha| \leqq 2} \int_\Omega D^\alpha u\, D^\alpha v\, \mathrm{d}\Omega + \sum_{k=1}^{l} \int_{\gamma_k} [L_{\tau k}(u)\, L_{\tau k}(v) + L_{nk}(u)\, L_{nk}(v)]\, \mathrm{d}s \tag{17}$$

Theorem 1. *Operator* $\tilde{\Delta}^2 = \Delta^2 + b + c$ *is symmetric on the linear subset* $M \subset H$.

Proof. The bilinear form $(\tilde{\Delta}^2 w_1, w_2)$ on the linear subset M is defined by

$$(\tilde{\Delta}^2 w_1, w_2) = (\Delta^2 w_1, w_2)_\Omega + D(bw_1, w_2)_\Gamma + D(cw_1, w_2)_\Gamma = \tag{18}$$

$$= \int_\Omega \Delta^2 w_1 w_2\, \mathrm{d}\Omega + D \int_\Gamma bw_1 w_2\, \mathrm{d}s + D \int_\Gamma cw_1 w_2\, \mathrm{d}s$$

where w_1, $w_2 \in M$ and hence it is possible to show that this bilinear form has all the required properties of a new scalar product in H. Thereby the operator $\tilde{\Delta}^2$ in M is defined, too, as follows from Riesz's theorem on the representation of linear functionals.

Then, according to (18) the operator $\tilde{\Delta}^2$ is formally equal to the sum of three operators

$$\tilde{\Delta}^2 = \Delta^2 + b + c\,, \tag{19}$$

where Δ^2 is the biharmonic operator on M and operators b, c are given by formulas (16).

Operator $\tilde{\Delta}^2$ will be called the operator of a thin plate reinforced with stiffening ribs.

Operator Δ^2 is symmetric on the linear subset M. The proof is given in [3]. It is easy to verify that operators b and c are also symmetric on the linear subset M, because

$$(20)\qquad (bw_1, w_2)_\Gamma = \int_\Gamma bw_1 w_2 \, ds = (B^*Bw_1, w_2)_\Gamma = (Bw_1, Bw_2)_\Gamma = \int_\Gamma Bw_1 Bw_2 \, ds$$

$$(21)\qquad (bw_2, w_1)_\Gamma = \int_\Gamma bw_2 w_1 \, ds = (B^*Bw_2, w_1)_\Gamma = (Bw_2, Bw_1)_\Gamma = \int_\Gamma Bw_2 Bw_1 \, ds$$

Then it follows from (20) and (21)

$$(22)\qquad (bw_1, w_2)_\Gamma = (bw_2, w_1)_\Gamma$$

Similarly the following equality holds for the operator c:

$$(23)\qquad (cw_1, w_2)_\Gamma = (cw_2, w_1)_\Gamma$$

This means, however, that the operator $\tilde{\Delta}^2$ is symmetric on the linear subset M. It is easy to prove that for $w \in M$ the following identity results after taking into account (11) and (19) as well as the symmetry of operators Δ^2, b, c on the linear subset M:

$$(24)\qquad \ni(w) = \frac{D}{2}\int_\Omega (\Delta w)^2 \, d\Omega + \sum_{k=1}^{l} \frac{1}{2}\int_{\gamma_k} \left\{ C_k \left[\frac{d}{ds}\left(\frac{\partial w}{\partial n}\right) + \frac{1}{\varrho_k}\frac{\partial w}{\partial s}\right]^2 + \right.$$
$$\left. + A_k \left[\frac{1}{\varrho_k}\frac{\partial w}{\partial n} - \frac{d}{ds}\left(\frac{\partial w}{\partial s}\right)\right]^2 \right\} ds = \frac{D}{2}(\Delta^2 w, w)_\Omega + \frac{D}{2}(bw, w)_\Gamma +$$
$$+ \frac{D}{2}(cw, w)_\Gamma = \frac{D}{2}(\tilde{\Delta}^2 w, w)$$

or

$$(25)\qquad (\tilde{\Delta}^2 w, w) = \frac{2}{D} \ni(w), \quad \text{where} \quad w \in M$$

Lemma 1. *Let the following relationships be true for the linear operator A defined on $M \subset H$:*

(P) $(Aw_n, w_n) \to 0 \Rightarrow \|w_n\| \to 0$, *for all progressions* $\{w_n\}_1^\infty$, *where* $w_n \in M$. *Then A is a positive definite operator on M.*

Proof. Let A be not a positive definite operator. Then there exists a progression $\{u_n\}_1^\infty$, $u_n \in M$, which is such that $\|u_n\| = 1$, $(Au_n, u_n) < 1/n$. This, however, is in contradiction with (P).

Using the preceding definitions, Lemma 1 and Theorem 1, the following theorem may be proved:

Theorem 2. *Operator $\tilde{\Delta}^2$ is positive definite on the linear subset M.*

Proof. It is sufficient to prove (P) for the operator $A = \tilde{\Delta}^2$. According to Definition 9 the scalar product in H is given by the relation (17).

Then, if formulas (11), (29) are considered, it is

$$(\tilde{\Delta}^2 w_n, w_n) = \frac{2}{D} \ni (w_n) \to 0 \Rightarrow \sum_{|\alpha|=2} \int_{\Omega} (D^{\alpha} w_n)^2 \, d\Omega \to 0 \tag{26}$$

$$\sum_{k=1}^{l} \int_{\gamma_k} [L_{\tau k}^2(w_n) + L_{nk}^2(w_n)] \, ds \to 0$$

For the region Ω with a Lipschitzian boundary the following inequality is true for functions $w_n \in \mathring{W}_2^{(2)}(\Omega)$. (See [4].)

$$\sum_{|\alpha|=2} \int_{\Omega} (D^{\alpha} w_n)^2 \, d\Omega \geqq \text{const} \, \|w_n\|^2_{\mathring{W}_2^{(2)}(\Omega)} \tag{27}$$

The following relation results from the inequalities (27) and formulae (26): $(w_n, w_n) \to$ $\to 0$ (i.e., in the sense of the norm generated by the scalar product (17)).

In this way it has been proved that the operator $\tilde{\Delta}^2$ has the property (P). From Lemma 1 the positive definiteness of operator $\tilde{\Delta}^2$ follows immediately. According to the above Definition 7, the linear subset M may be considered another Hilbert space, $H_{\tilde{\Delta}^2}$.

In the set M the new scalar product is given by (17).

The basic variational problem consists in finding an element w from the set M which realizes the minimum of the functional $F(w) = [w, w] - 2(w, f)$ where f is a fixed element of the space H.

Such a problem has in general no solution. It has a solution if the functional $F(w)$ extends to cover the whole space H.

The following important theorem is valid:

Theorem 3. *If the operator $\tilde{\Delta}^2$ is positive definite, then each minimizing progression for the functional $F(w) = [w, w] - 2(w, f)$ converges in $H_{\tilde{\Delta}^2}$ to the element which realizes the minimum of the functional $F(w)$,* [2].

Hence, if the operator $\tilde{\Delta}^2$ is positive definite, the progression of approximate solutions according to Ritz of the basic variational problem in the metrics of $H_{\tilde{\Delta}^2}$ converges to the exact solution of the problem.

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Súhrn

OPERÁTOR TENKEJ DOSKY ZOSÍLENEJ TENKOSTENNÝMI REBRAMI

JÁN LOVÍŠEK

V tejto práci sa definuje riešenie tenkej dosky zosílenej tuhostnými rebrami. Je zavedený diferenciálny operátor $\tilde{\Delta}^2$ (operátor tenkej dosky zosílenej tuhostnými rebrami). Tento operátor je symetrický a pozitívne definitný na lineárnom podpriestore $M \subset W_2^{(2)}(\Omega)$.

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