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Opinion formation in a social network: The role of human activity

Andrzej Grabowski

Central Institute for Labour Protection - National Research Institute, 00-701 Warsaw, Poland

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ABSTRACT

The model of opinion formation in human population based on social impact theory is investigated numerically. On the basis of a database received from the on-line game server, we examine the structure of social network and human dynamics. We calculate the activity of individuals, i.e. the relative time devoted daily to interactions with others in the artificial society. We study the influence of correlation between the activity of an individual and its connectivity on the process of opinion formation. We find that such correlations have a significant influence on the temperature of the phase transition and the effect of the mass media, modeled as an external stimulation acting on the social network.

1. Introduction

Studying the statistical properties of social (e.g. friendship) networks remains a challenge. It is possible to assess the form of degree distribution with a survey, as in the case of the web of human sexual contacts [1]. However it is much more difficult to learn about other important properties of networks, because there is no data on their entirety. A survey often provides data on a small sample only.

Progress in information technology has made it possible to investigate the structure of the social networks of interpersonal interactions maintained over the Internet, e.g. e-mail networks [2], web-based social networks of artificial communities [3] and blog networks [4]. However there is still an unexplored area of research. In recent years on-line games have become increasingly popular and have attracted an increasing number of players, who interact in the large virtual world of Massive Multiplayer Online Role Playing Games (MMORPGs).

MMORPG is a network game in which players enter a virtual world as characters they have invented — gaining virtual life. This virtual world takes the form of a game server connected to the Internet, on which accounts are registered for users who log in through special game client programs. The rules allow players to create more than one character on one account, with each of those characters having its own personality (further in the text we refer to such characters as individuals). Thousands of people can play on one server – they become a virtual society – so they share the common culture, area, identity, and interactions in the network of interpersonal relationships.

The aim of our work is to investigate the influence of human social activity on dynamic phenomena in a social network. In the present work, we use data on a social network consisting of 6×10^3 individuals. It is a giant component of a network of individuals who interact in the large virtual world of the aforementioned MMORPG [5]. On the basis of playing time, we calculate the activity A of individuals, i.e. the relative time devoted daily to interactions with others. On the basis of human activity calculated as above, we investigate the process of opinion formation in a real social network. For modeling of the

E-mail address: angra@ciop.pl.

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process of opinion formation, in the case of two possible states (i.e., the positive or negative answer to a certain question), the Ising-based models are used by many authors [6–14]. The Ising model was first introduced to describe interactions between fermions which are arranged in an array and are placed in an external field [15], and since then still attracts great interest among physicists. It was found that cooperative behavior, which arises from direct interactions between components of the system can be successfully described in different systems (eg. biological, economical and social systems) using Ising-like models.

2. Statistical properties of the network

Basic network measures of the whole network and the Giant Component (GC) [16] are presented in Table 1. This network consists of 28 011 individuals, but for many of them, the number of connections k equals zero. This means that those characters have no friends on their lists. Most individuals with $k = 0$ are abandoned characters who do not appear in the virtual world and who have therefore lost all their contacts with those still active, or are new characters. GC contains almost all individuals whose degree is greater than zero (6065 characters); only 252 individuals with $k > 0$ do not belong to GC (i.e. only 4%).

The average path length $\langle l \rangle$ in GC is similar to that in a random graph. A high value of the clustering coefficient and a short average path length $\langle l \rangle$ are characteristic features of social networks [16]; they are typical for small-world networks. The degree distribution of the network has the form $P(k) \sim k^{\eta-1}\exp(-\xi k^{\eta})$, where $\xi = 1.40 \pm 0.05$ and $\eta = 0.35 \pm 0.02$ ($R^2 = 0.99$). Hence, in the network under investigation, degree distribution decays as a stretched exponential for large k (see Fig. 1(a)).

The local clustering coefficient $C(k)$ is negatively correlated with the degree of the node k, showing the existence of the power law $C(k) \sim k^{-\alpha}$ with $\alpha = 0.44 \pm 0.02$ ($R^2 = 0.98$). A slightly lower value of the exponent α has been observed in other social networks, 0.33 [3] and 0.35 in a network consisting of over one million nodes [17]. The power-law relation $C(k)$ is similar to the relationship in hierarchical networks [18]. However it has been recently shown [20] that most observed degree dependence of the clustering coefficient follows from degree mixing patterns.

In the network under investigation, the greater the k, the greater the average degree of nearest neighbors k_{NN}. Hence, the network is assortatively mixed by degree; such a correlation has been observed in many social networks [19]. In social networks it is entirely possible, and it is often assumed in sociological literature that similar people attract one another. The relation $k_{\mathrm{NN}}(k)$ can be approximated with the power-law relation $k_{\mathrm{NN}}(k) \sim k^{0.18\pm0.01}$ ($R^2 = 0.93$). A similar value of the exponent (0.2) has been found in other social networks [17]. However, it was recently shown that such correlations can be explained by modularity and a heavy-tailed degree distribution [21].

On-line games like MMORPGs offer a great opportunity to investigate human dynamics [22], because much information about individuals is registered in databases. Knowing the accumulative time spent by the user in the virtual world T_{G} and the lifespan T_{L} (i.e. the number of days from the time the individual was created to the date of last logging), we can calculate the average time devoted daily to interactions in the virtual environment. By dividing this average time by 24 h we obtain the activity A of a character. Thus, the activity A_{i} denotes the probability that an i-th character exists in the virtual world.

$$A = \frac{T_{\mathrm{G}}}{24 T_{\mathrm{L}}}. \tag{1}$$

The value of activity relates to the whole lifespan of an individual. However the activity is not biased by the lifespan of an individual, because the average time devoted daily for playing the game is approximately independent on T_L. It should be noted that we have found similar results (e.g. lack of correlations between A and T_L) analyzing the behavior of 5×10^6 players in the web-service www.xfire.com. The activity of a character is positively correlated with its degree, and the results can be approximated with the power law

$$A(k) \sim k^{\eta} \tag{2}$$

where $\eta = 0.35 \pm 0.02$ (see Fig. 1(a)). The activity distribution $P(A)$ is exponential: $P(A) \sim \exp(-\mu A)$, where $\mu = 12.0 \pm 0.2$; $R^2 = 0.98$ (see Fig. 1(b)).

3. The process of opinion formation

Let us describe the effect of social activity on the phenomenon of opinion formation. Each individual is influenced by the local field h_i, which depends on interactions with k_i neighbors and external stimulation I:

$$h_i(t) = -S_i(t)A_i\left(\sum_{j}^{k_i} A_j S_j(t) + I\right) \tag{3}$$

where $S_i = \pm 1$ – state of i-th individual, k_i – number of neighbors of i-th individual, A_i – activity of i-th individual. The external stimulation I may be regarded as a global preference towards one of the opinions stimulated by mass-media, government policy etc.

Table 1
Average properties of the whole network and the giant component (GC) and comparison with a random graph (RG) with the same number of nodes N and the same average connectivity $\langle k \rangle$.

	N	C	$\langle l \rangle$	$\langle k \rangle$	k_{max}
Network	28 011	0.02	–	1.4	64
RG	28 011	0.006	29	1.4	–
GC	6 065	0.1	4.8	6.4	64
RG	6 065	0.006	4.8	6.4	–

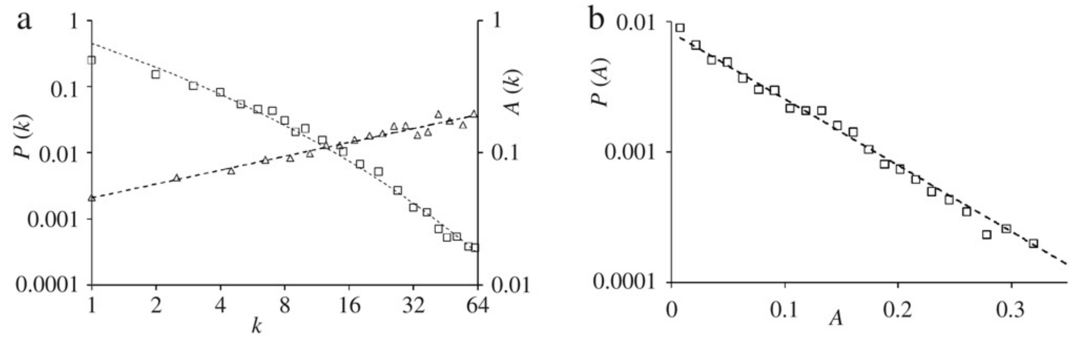

Fig. 1. (a) The degree distribution $P(k)$ (squares) and the relation between the degree of an individual and its activity $A(k)$ (triangles)—results can be approximated with the power law (dashed line). (b) The activity distribution and fit to the exponential form (dashed line).

The value of a local field acting on individuals is proportional to their activity; less active individuals (i.e. individuals who spend less time in the virtual world) have fewer opportunities to interact with others than more active ones. Similarly, we take into account the neighbors' activity. The greater the activity of neighbors, the greater their influence on an individual.

The probability that an individual changes their state S_i depends on the local field and temperature T:

$$S_i(t+1) = \begin{cases} S_i(t); & \dfrac{\exp(-h_i/T)}{\exp(-h_i/T)+\exp(h_i/T)} = \dfrac{1}{1+\exp(2h_i/T)} \\ -S_i(t); & \dfrac{\exp(h_i/T)}{\exp(-h_i/T)+\exp(h_i/T)} = \dfrac{1}{1+\exp(-2h_i/T)}. \end{cases} \quad (4)$$

The parameter T may be interpreted as a *social temperature* describing a degree of randomness in the behavior of individuals. With an increase in temperature T there is an increase in the probability that the individual will have a state opposite to the local field. Note that Eq. (4) is analogous to the Glauber dynamics.

4. Results

Computations were performed for the initial conditions corresponding to a paramagnetic phase ($\langle S \rangle = \frac{1}{N}\sum_i S_i = 0$ for $t = 0$) using synchronous dynamics. To investigate the influence of human activity on the process of opinion formation we made computations for four different distributions of activity, real ($\eta \approx 0.35$, see Eq. (2)), uniform $A_i = const$ and for weak ($A \sim k^{0.1}$) and strong ($A \sim k^1$) correlations between degree and activity. To obtain more comparable results, the average activity was the same in all cases.

The relation between $\langle S \rangle$ and temperature T is shown in Fig. 2. For low temperatures, dominant opinion emerges in the community in all cases. When the temperature exceeds a certain critical value T_C there is an abrupt disappearance of dominant opinion in the community. It is visible that the stronger the correlations between degree and activity, the greater the temperature of phase transition T_C. In the case of real distribution of activity, the value of T_C is approximately ten times greater than in the case of uniform distribution of activity. The network under investigation is assortative mixed, hence nodes with large degree (hubs) are highly interconnected. Strong interactions between hubs in the case of real distribution of activity (activity is positively correlated with degree) cause the value of local field h_i to be greater than in the case of uniform distribution of activity. Therefore the probability that an individual will have a state opposite to the state of neighbors, is lower. As a result, the stronger the correlations between activity and degree, the wider the range of temperatures for which dominant opinion can emerge. The phenomenon of the phase transition in social systems was found by other authors [8,23–28].

To study the influence of hubs on the process of opinion formation, we calculate the time evolution of average degree $\langle k_D \rangle$ of individuals with dominant opinion (i.e. opinion which will be dominant at the end of simulation). In the beginning of simulation ($t = 0$) $\langle k_D \rangle = \langle k \rangle$ because initially the system is in paramagnetic state. In the first stage, there is an increase in $\langle k_D \rangle$. Next, the $\langle k_D \rangle$ decreases (see Fig. 3(a)). It indicates that, initially, hubs take a common state. Next, the hubs talk less active individuals with low degree into taking their opinion.

Correlations between activity and degree influence the time development of $\langle S \rangle$. The greater the value of the exponent η, the faster the increase in $\langle S \rangle$ (see Fig. 3(b)). Thus, the dominant opinion emerges faster in the case of real distribution

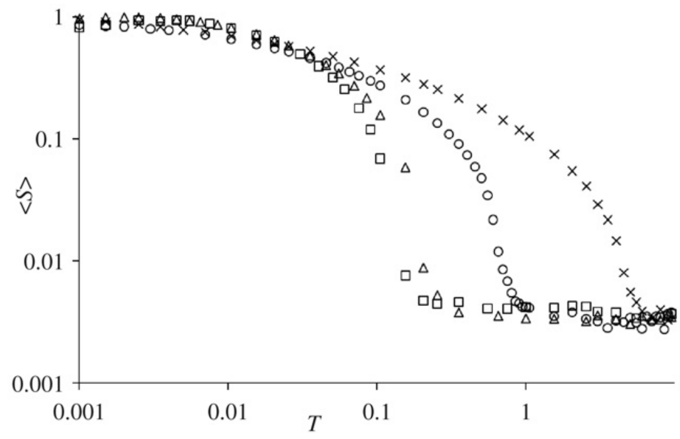

Fig. 2. Relationship between $\langle S\rangle$ and temperature T for different distributions of activity: original (circles), average (squares), weak (triangles) and strong (crosses) correlations between degree and activity. Values of other parameters are $I = 0$. Results are averaged over 100 independent simulations. Number of time steps of each simulation is 10^4.

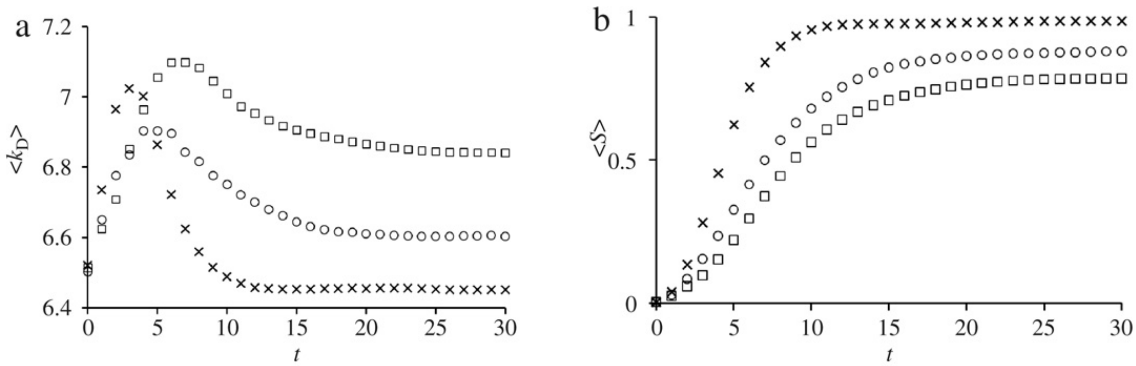

Fig. 3. The time evolution of average degree $\langle k_D\rangle(t)$ of individuals with dominant opinion (a) and $\langle S\rangle(t)$ (b) for different distributions of activity: original (circles), average (solid line) and strong (crosses) correlations between degree and activity. Values of other parameters are $T = 0$ and $I = 0$.

of activity, than in the case of uniform distribution of activity. However, the difference becomes smaller for higher temperatures.

The presence of external stimulation can simulate the influence of mass media on the phenomenon of opinion formation in the community. In our work, we investigate the possibility of change in the states of individuals due to external stimulation I, i.e., we assume that initially all individuals have states opposite to external stimulation.

Fig. 4 illustrates the relative number of individuals with the states conforming with external stimulation (denoted by N^+) as a function of I. It can be noticed that for I exceeding the critical value I_C most individuals have state $S_i = +1$ (the rapid change in N^+ is better visible for $T > 0$, see Fig. 4(b)). This means that a certain intensity of the influence of mass media can change the opinion of the community to the state constrained by the media. The value of I_C depends significantly on the correlation between A and k. The presence of those correlations means that the system is more robust against the external stimulation, (e.g. for real distribution of activity I_C) and is approximately three times larger than in the case of uniform distribution of activity. The origin of such behavior of the system is similar to the case of temperature. Highly interconnected and very active nodes with a high degree have strong support of the other hubs, which is even greater for stronger correlations between A and k. Therefore, the greater the value of η, the greater should be the value of I to change the opinion of the hubs.

Contrary to the case without external stimulation, the time evolution of $\langle k_D\rangle$ has an opposite character. At first, individuals with a low degree take the opinion confirming to external field. Those individuals are weakly connected with others, therefore the probability that they change their state from $\langle S_i\rangle = -1$ to $\langle S_i\rangle = +1$ is high even for low values of I (see Eq. (4)). Next, the individuals with higher degree change their state as a result of the influence of those with a lower degree. Such a behavior of the individuals and limited size of the network ($k \leq 64$) is responsible for the shape of $N^+(I)$ relationship at $T = 0$ for the uniform distribution of activity. For $I < 0.08$ the external field is too low to change the state any individual. For $0.08 < I < 0.15$ the value of external stimulation is large enough to change the sign of the local field of individuals with $k = 1$ (and as a consequence their states, too). For $I > 0.15$ the combined influence of the external field and individuals with $k = 1$, is large enough to change the state of those with $k = 2$. Next, the individuals with a higher degree change their state. In the case of non-uniform distributions of activity, the shape of the $N^+(I)$ curve is more complicated because the influence

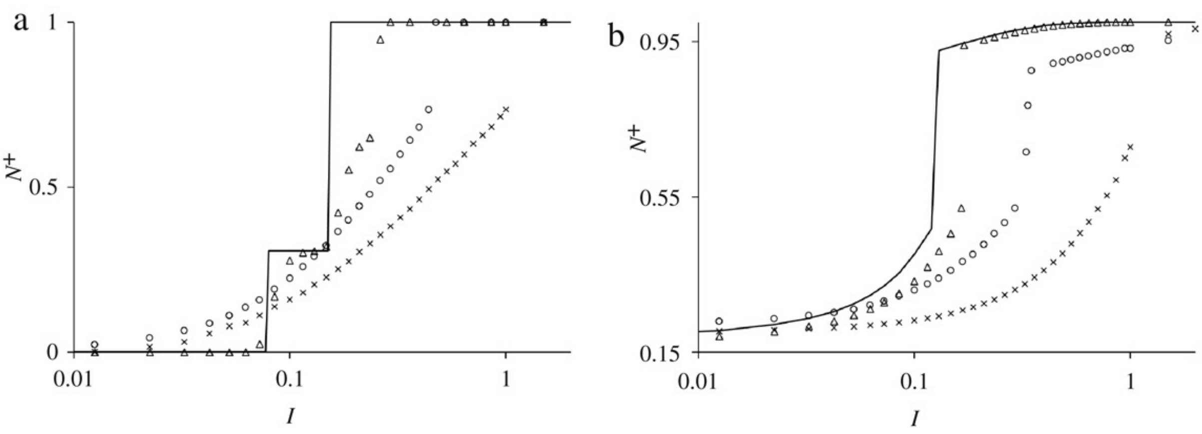

Fig. 4. The relation between the relative number of individuals with the states conforming with external stimulation (N^+) and the value of external stimulation I for different distributions of activity: original (circles), average (solid line), weak (triangles) and strong (crosses) correlations between degree and activity. Values of other parameters are $T = 0$ (a) and $T = 0.02$ (b). Results are averaged over 100 independent simulations. Number of time steps of each simulation is 10^4.

of individuals on local field h_i also depends on the activity of individuals (note that for uniform distribution of A the value of h_i depends on the degree of a node and the state of neighbors, only).

5. Conclusions

In this paper, we have studied the process of opinion formation in the real social network. In our model, we have taken into account the experimental data on social activity A of individuals, i.e. the relative time devoted daily to interactions with others.

We have found that the correlations between activity of an individual and its degree, have a strong influence on the process of opinion formation. In the case of real distribution of activity, the critical value of temperature T_C is approximately ten times greater than in the case of uniform distribution of activity. Therefore, the presence of correlations between A and k means that the dominant opinion can emerge even if the temperature is high. Moreover, those correlations speed up the process of opinion formation.

We have investigated the effect of the mass media on the process of opinion formation, modeled as external stimulation acting on the social network. Similarly, as in the case of temperature, the influence of external stimulation I is affected by the social activity of individuals. The influence of the mass media, in certain circumstances, can provoke critical rebuilding of opinions in the population. However, the stronger the correlation between A and k, the more robust is the system against influence of external stimulation. We have found that if the system starts from a paramagnetic state and $I = 0$ in the initial stage of the process, the hubs agree with the common opinion. Then, they talk less active individuals with low degree into taking their opinion. The opposite scenario is observed if in $t = 0$ the system is in a ferromagnetic state and $I > 0$. At first, the state of individuals with a low degree is changed by the external stimulation. Next, the combined influence of the external field and individuals with a low degree, causes individuals with a higher degree to change their state to the state constrained by the media.

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