Opportunistic Routing in Wireless Ad Hoc Networks: Upper Bounds for the Packet Propagation Speed

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Abstract—Classical routing strategies for mobile ad hoc networks operate in a hop by hop "push mode" basis: packets are forwarded on pre-determined relay nodes, according to previously and independently established link performance metrics (*e.g.,* using hellos or route discovery messages). Conversely, recent research has highlighted the interest in developing opportunistic routing schemes, operating in "pull mode": the next relay can be selected dynamically for each packet and each hop, on the basis of the actual network performance. This allows each packet to take advantage of the local pattern of transmissions at any time. The objective of such opportunistic routing schemes is to minimize the end-to-end delay required to carry a packet from the source to the destination.

In this paper, we provide upper bounds on the packet propagation speed for opportunistic routing, in a realistic network model where link conditions are variable. We analyze the performance of various opportunistic routing strategies and we compare them with classical routing schemes. The analysis and the simulations show that opportunistic routing performs significantly better. We also investigate the effects of mobility and of random fading. Finally, we present numerical simulations that confirm the accuracy of our bounds.

Index Terms—Opportunistic routing; Wireless; Ad hoc; Information propagation speed.

I. INTRODUCTION

Conventional routing strategies for mobile ad hoc networks operate in "push mode": depending on the destination, packets are forwarded on a per hop basis to pre-determined relay nodes, based on previously established link performance statistics. The next hop relays can be determined by a simple shortest path algorithm, or by more complicated optimizations, taking into account the channel conditions and the performance of the network links. In link state protocols, such as OLSR [8], the next relay is determined by a route calculation relying on measurements on the average link performance (*e.g.,* based on statistics of hello messages). In reactive protocols, such as AODV [18], routes between nodes are computed on demand with route request and route reply messages, as desired by the source nodes, and they are maintained as long as they remain active. Similarly, in DSR [15], even though the route computation is performed by broadcasting a route discovery

message, the actual forwarding of packets uses source routing. When the performance of a link deteriorates (triggering a link break event), the routes are updated with route maintenance mechanisms, while packets are possibly kept in cache to avoid losses. However, once the route is re-established, packets continue to be forwarded to the next hop which is determined by the route maintenance mechanism.

On the other hand, recent research has highlighted the interest in developing opportunistic routing schemes, where the next relay is selected dynamically for *each packet* and each hop. As a result, these opportunistic strategies operate in "pull mode", since the relays can be selected (eventually even self-selected) based on the actual network performance, in contrast to classical routing protocols. Therefore, each packet can take advantage of the local pattern of transmissions at each hop and at any time. The general aim of such opportunistic routing schemes is to minimize the end-to-end delay required to carry a packet from the source to the destination, and maximize in this sense the throughput in the network.

Several strategies have been proposed, based on geographic routing and/or time-space opportunistic routing. Geographic routing strategies [3], [6], [16] use the positions of the nodes to determine the route to the destination, while they try to optimize geographic criteria, such as the distance to the destination. In time-space opportunistic routing [4], [5], the selection of each relay takes advantage not only of the local topology but also of the current MAC and channel conditions. However, performance evaluations are often limited to comparative simulations (*e.g.,* [4], [19], [20]) or measurements (*e.g.,* [5]) as a complete understanding of what one can expect for optimal performance (*e.g.,* through theoretical bounds) is still missing. In this context, our objective in the present paper is to evaluate the maximum speed at which a packet of information can propagate in a multi-hop wireless network, using any possible opportunistic routing strategy.

In terms of related work on the information propagation speed in mobile ad hoc networks, the problem has been studied in unit disk graph models [13], [14], [17]. Kong and Yeh [17] showed that the information propagation latency scales linearly with the distance (*i.e.,* the information propagation speed tends to a constant) under a critical node density threshold, while the latency scales sub-linearly in the super-critical case where the network is percolated. The articles [13], [14] present the first analytical upper bounds on the achievable information propagation speed in unbounded and bounded networks, respectively. In contrast, here, we use a realistic interference

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model based on stochastic geometry.

In an interference-based model, the authors of [21] have showed that there is a unified upper bound on the maximum information propagation speed in large multi-hop wireless networks. This case is similar to our analysis of classical routing, the main difference being that we assume a fixed required signal-to-noise ratio for correct reception of packets (as is the case in current protocols), while [21] uses a capacity bound on the information transmission rate. However, our main focus here is opportunistic routing and the evaluation of upper bounds on the information propagation speed. Baccelli et al. [4] presents some analytical results on optimizing specific time-space opportunistic routing strategies, and comparing them to classical routing (in addition to a detailed simulation study). In this paper, we use the framework of [4] in order to compare our analysis with simulation measurements, but our objective is different: we wish to determine the best possible packet propagation speed using any opportunistic routing strategy. Our main contributions are the following:

- we propose a new probabilistic model of space-time paths of packets of information; we upper-bound the optimal performance, in terms of delay, that can be achieved using any opportunistic routing algorithm, in a realistic network model where link conditions are variable; we derive theoretical bounds on the packet propagation speed with generic opportunistic routing strategies and we investigate the effects of random fading and mobility;
- we verify the accuracy of our bounds in numerous scenarios using numerical simulations: we compare them with the performance of an optimized time-space opportunistic routing scheme [4];
- we also compare opportunistic and classical routing; the analysis and the simulations show that opportunistic routing performs significantly better, even when classical routing schemes are optimized based on an absolute knowledge of the statistics of the channel conditions.

In Section II, we describe the network model and we present our first result, *i.e.,* an upper bound on the propagation speed using a classical routing strategy. We then adopt a didactic approach. In Section III, we overview the methodology for the analysis of opportunistic routing. We present our main theorems and theoretical bounds on the packet propagation speed in Section IV. In Section V, we verify the accuracy of our bounds using numerical simulations, and we compare them with the performance of an optimized time-space opportunistic scheme, introduced in [4]; we also present a comparison with classical routing. We investigate the performance of opportunistic routing with node mobility in Section VI. We conclude and we discuss some possible directions for further research in Section VII.

II. MODEL AND CLASSICAL ROUTING

A. Network and propagation model

We use the model developed in [2]. We consider a network on an infinite 2-D map, with a constant density of ν nodes per square area unit, dispatched according to a Poisson distribution. We assume that time is slotted, and at each slot, each node has a packet to transmit with probability $\frac{\lambda}{\nu}$, with $\lambda < \nu$. Therefore, the distribution of the number of active transmitters per slot is Poisson; the rate of transmitters per square area unit and per slot is λ. Therefore, λ corresponds to the overall traffic density, including all generated and relayed data, as well as eventual protocol packets.

We assume that all nodes transmit at the same nominal power. We take a simple power attenuation function, with attenuation coefficient $\alpha > 2$: the signal level received at distance r from the transmitter is $W = \frac{\exp(F)}{r^\alpha}$, where F is a random fading of mean 0. Fading is an alteration of the signal which is due to factors other than the distance (obstacles, co-interferences with echoes, and so on). Therefore, F is a random variable, i.i.d. for each node. With this general fading distribution model, we do not need to distinguish in the analysis whether the fading is permanent (for a given node) or changes at every slot. In both cases, the total power of all transmissions at a given slot follows the same distribution: the traffic density is Poisson in time and space, while each packet is transmitted at the same nominal power. Whether, the fading is fixed in time for each node or not, this does not affect our analysis, since we are interested in the distribution of the signal-over-noise ratio (SNR).

A packet can be successfully decoded if its signal-over-noise ratio is greater than a given threshold K. By noise, we mean the sum of powers received from all other transmissions in the same slot.

Let us denote $W(\lambda)$ the total power received by a node at a random slot, when transmissions are distributed according to a 2-D Poisson process with intensity of λ transmitters per slot and per square area unit. Quantity $W(\lambda)$ is then a random variable. According to [2], the Laplace transform of $W(\lambda)$ can be calculated exactly, assuming *w.l.o.g.* that all transmitters emit at unit nominal power. The Laplace transform $\tilde{S}(\theta, \lambda) = E(e^{-W(\lambda)\theta})$ has the following expression:

$$\tilde{S}(\theta, \lambda) = \exp\left(-\lambda\pi\Gamma(1 - \frac{2}{\alpha})E(e^{\frac{2}{\alpha}F})\theta^{\frac{2}{\alpha}}\right), \quad (1)$$

where the expectation $E(.)$ indicates the average with respect to the random fading factor F.

We note that the random variable $W(\lambda)$ is invariant by translation, *i.e.,* it does not depend on the node location. Moreover, we notice from (1) that $W(\lambda)$ follows a Levy-Stable distribution ([22]). For general α, there is no closed formula for the probability function $P(W(\lambda) < x)$. However, we have the following series expansion and asymptotic behavior [12] (we denote $\gamma = \frac{2}{\alpha}$ and $C = \pi\Gamma(1 - \gamma)E_F(e^{\gamma F})$):

- $P(W(\lambda) < x) = \sum_{n \geq 0}(-C\lambda)^n \frac{\sin(\pi n\gamma)}{\pi} \frac{\Gamma(n\gamma)}{n!} x^{-n\gamma}$,
- $-\log P(W(\lambda) < x) = \Theta(x^{\frac{\gamma}{\gamma-1}})$ when $x \to 0$.

Therefore, a silent node will correctly receive (with SNR at least K) a packet from another node at distance r with probability $p(r) = P(W(\lambda) < \frac{e^F}{K} r^{-\alpha})$. By substitution in the series expansion, we obtain the probability $p(r)$ as a function of the distance r:

$$p(r) = \sum_{n \geq 0}(-C\lambda)^n \frac{\sin(\pi n\gamma)}{\pi} \frac{\Gamma(n\gamma)}{n!} E(e^{-\gamma F}) K^{n\gamma} r^{2n}. \quad (2)$$

Notice that $p(\frac{rK^{-\frac{1}{\alpha}}}{\sqrt{\lambda}})$ is an invariant.

When we take $\alpha = 4$ (*i.e.*, in the case corresponding to the reflection-absorption model of wave propagation over an infinite plane), we obtain the following closed formula:

$$p(r) = 1 - E\left(\text{erf}(\frac{1}{2}\lambda\pi^{\frac{3}{2}}K^{\frac{1}{2}}e^{-\frac{F}{2}}r^2)\right),$$

where erf() is the error function; the expectation $E(.)$ indicates the average with respect to the random fading factor F.

B. Classical Routing Bound

Based on the model and the previous analysis, we can establish a first upper bound on the packet propagation speed, when a classical routing strategy is employed, *i.e.*, when packets are forwarded in "push mode" to the next relay on a hop by hop basis. For the analysis, we consider that the distribution of the signal to noise ratio is exactly known and that classical routing is optimized to achieve the fastest propagation speed under this distribution.

We notice that the propagation delay is caused by the fact that packets must be retransmitted several times until a correct reception occurs. For instance, the probability of successful transmission in one hop of length r is $p(r)$, which we computed from the signal distribution in Section II. Therefore, the average number of retransmissions needed until the packet is received correctly is $\frac{1}{p(r)}$, and the corresponding average delay is (at best) $\frac{1}{p(r)}$ slots. Clearly, a compromise can be achieved between the length of each hop in a route and the average delay that will result from the necessary packet retransmissions. Indeed, the maximum speed at which a packet can propagate in one hop will be equal to $\max_{r\geq 0}\{rp(r)\}$. Hence, this quantity is an upper bound on the packet propagation speed. We formalize and refine this result in the following theorem.

Theorem 1: In classical routing (*i.e.*, when all packets from a source to a destination follow the same route) the packet propagation speed is bounded from above by the quantity $(1-\frac{\lambda}{\nu})\max_{r\geq 0}\{rp(r)\}$.

Proof: We assume *w.l.o.g.* that the source is at position 0 and the destination at position $\mathbf{z}$. Let us suppose that the chain of relays between the source and the destination is made of n nodes. We denote $\mathbf{z}_i$ the position of node i, with $\mathbf{z}_1 = 0$ and $\mathbf{z}_n = \mathbf{z}$. The probability of correct reception between node i and node $i+1$ is $p(|\mathbf{z}_{i+1} - \mathbf{z}_i|)(1 - \frac{\lambda}{\nu})$; the term $(1 - \frac{\lambda}{\nu})$ corresponds to the probability that the receiver is idle (*i.e.*, it is not transmitting simultaneously). Therefore, the delay for a correct transmission is on average $\left(p(|\mathbf{z}_{i+1} - \mathbf{z}_i|)(1 - \frac{\lambda}{\nu})\right)^{-1}$. As a result, when $|\mathbf{z}| \to \infty$ and $n \to \infty$, we can deduce from the strong law of large numbers that the optimal packet speed is almost surely:

$$(1 - \frac{\lambda}{\nu})\frac{|\mathbf{z}|}{\sum_i \frac{1}{p(|\mathbf{z}_{i+1}-\mathbf{z}_i|)}}.$$

This quantity is smaller than $(1 - \frac{\lambda}{\nu})\frac{\sum_i |\mathbf{z}_{i+1}-\mathbf{z}_i|}{\sum_i \frac{1}{p(|\mathbf{z}_{i+1}-\mathbf{z}_i|)}}$ (from the triangle inequality), which in turn is smaller than $(1 - \frac{\lambda}{\nu})\max_{r\geq 0}\{rp(r)\}$. ∎

III. METHODOLOGY

As we discussed in the previous section, in optimal classical routing, each node selects as next relay the node that offers the best compromise between its routing delay towards the destination and its probability to receive the packet. This compromise can be achieved according to the average performance, sampled over a link state approach (for example with OLSR [8]). On the other hand, opportunistic routing consists in selecting the best chain of relays in terms of actual delay (for each packet, at each hop and time slot).

In wireless networks, the quality of a signal reception can vary greatly due to the variation of the Signal-over-Noise Ratio (SNR). In particular, variations occur in time but also in space. Indeed, the closer is the receiver, the better is the SNR, as we saw in Section II. These variations provide substantial possibilities for improvement of the routing performance, when opportunistic strategies are employed. In the following sections, we evaluate the maximum speed at which a piece of information can propagate in the network with opportunistic routing, and compare it with classical routing. We establish generic upper bounds, but we do not analyze specific algorithms. In terms of algorithms, our upper-bound would be attainable if all SNR variations in the network were known in advance. Our aim is to compute the fastest possible packet propagation without predictive knowledge. Equivalently, we aim to find the "foremost" path in time ([7]) that connects a source to a destination; this is achieved because our analysis unfolds all possible paths.

We note here that, in the upper bound derivations, we do not consider the delays experienced by packets in the queues. This does not affect the validity of our analysis, since we are interested in upper bounds for the packet propagation speed.

We investigate the performance of two generic strategies, which we call "store-forward", and "store-hold-forward". In the first strategy, nodes attempt to forward packets immediately. The second strategy is more general: each relay has the choice to either immediately transmit the packet or to wait for better signal propagation conditions (requiring a *store-hold-and-forward* routing model). The reception of each packet can be performed on the basis of a self-selection rule (see [4]).

We decompose paths into segments, using language theory and symbolic combinatorial methods (as described in [10] for generating functions), and we evaluate the Laplace transforms of the path probability density. From the Laplace transforms and complex analysis based on the saddle point method, we are able to establish an upper bound on the average number of paths arriving to a point $\mathbf{z}$ before a time t, where $\mathbf{z}$ is a 2D space vector. Using this approach, we prove our main theorems and theoretical bounds on the packet propagation speed.

We will show that, with the store-forward strategy, the propagation speed drastically collapses (*i.e.*, equals zero) when the node density is below a certain value, known as the percolation threshold [9] (our work gives a lower bound for this value). The store-hold-and-forward strategy does not collapse, since in fact the variations in the signal to noise ratio always guarantee connectivity; however, as expected, the propagation speed tends to zero when the node density

diminishes. In Section VI, we will investigate the performance of opportunistic routing when nodes move according to a basic random walk model. We will show that, in this case, the propagation speed does not collapse to zero when the density is small; it tends to a constant value, which depends on the random walk parameters.

A. Path Probability Density and Laplace Transform

Formally, a path is a space-time trajectory of the packet between the source and the destination. We assume that time zero is when the source transmits, and we will check at what time t the packet is received at coordinate $\mathbf{z} = (x, y)$. We will only consider simple paths, *i.e.,* paths which never return twice through the same node. As we will discuss in Section IV, this does not affect the validity of our final results.

Let $\mathcal{C}$ be a simple path. Let $Z(\mathcal{C})$ be the terminal point. Let $T(\mathcal{C})$ be the time at which the path terminates. Let $p(\mathcal{C})$ be the probability of path $\mathcal{C}$. In the following, we will in fact consider a path as a discrete event in a continuous set and, therefore, the probability weight should be converted into a probability density. We call $p(\mathbf{z}, t)$ the average number of paths that arrive at $\mathbf{z}$ before time t:

$$p(\mathbf{z}, t) = \lim_{r \to 0} \frac{1}{\pi r^2} \sum_{|\mathbf{z} - Z(\mathcal{C})| < r, T(\mathcal{C}) < t} p(\mathcal{C}) \ .$$

We now express the probability $q(\mathbf{z}, t)$ that there exists at least one path that arrives *at the destination node* before time t ($p(\mathbf{z}, t)$ is not conditioned on the existence of a node at $\mathbf{z}$).

Lemma 1: The probability $q(\mathbf{z}, t)$ that there exists at least one path that arrives to a destination node, located at $\mathbf{z}$, before time t, satisfies:

$$q(\mathbf{z}, t) \leq A p(\mathbf{z}, t),$$

where $p(\mathbf{z}, t)$ is the average density of paths arriving at $\mathbf{z}$ before time t, and A is a finite positive number.

Proof: See appendix. ∎

In the next sections, we calculate when the probability $q(\mathbf{z}, t)$ becomes 0. Therefore, we compute when the probability of reaching a given distance in space, before a given amount of time tends to zero; the smallest ratio of distance over time with this property provides us with an almost sure bound on the propagation speed. For the calculations, we make use of Laplace transforms.

Let ζ be a space vector with components expressed in inverse distance units, and θ a scalar in inverse time units. We denote $w(\zeta, \theta)$ the path Laplace transform:

$$
\begin{aligned}
w(\zeta, \theta) &= E(\exp(-\zeta \cdot Z(\mathcal{C}) - \theta T(\mathcal{C}))) \\
&= \sum_{\mathcal{C}} p(\mathcal{C}) \exp(-\zeta \cdot Z(\mathcal{C}) - \theta T(\mathcal{C})),
\end{aligned}
$$

defined for a domain definition for (ζ, θ). Notice that $\zeta \cdot Z(\mathcal{C})$ is the dot product of two vectors, and that this product is a pure scalar without units.

By virtue of the inverse Laplace transform, we have:

$$p(\mathbf{z}, t) = \left(\frac{1}{2i\pi} \right)^3 \int \int w(\zeta, \theta) e^{\zeta \cdot \mathbf{z} + t\theta} d\zeta \frac{d\theta}{\theta},$$

where the integration domains are planes parallel to the imaginary plane in the definition domain. In this case, quantity

$p(\mathbf{z}, t)$ is the average density of paths that arrive at $\mathbf{z}$ before time t.

In the following, we split the path into segments $\mathcal{C} = (s_1, s_2, \ldots, s_k)$, such that $p(\mathcal{C}) = p(s_1)p(s_2) \cdots p(s_k)$. Note that each segment is a space-time vector.

Based on the decomposition, we can compute the Laplace transform of the path, using the Laplace transforms of the individual segments. For the example above, the path $\mathcal{C}$ is described as a cartesian product of the segments $s_1, s_2, ...$; therefore, the Laplace transform of the path $\mathcal{C}$ can be expressed as the product of the Laplace transforms of the segments. Equivalently, a union (*i.e.,* a choice to use one *OR* another segment to obtain the path) translates into a sum of Laplace transforms. Similarly, if we express a path as an arbitrary sequence of the same type of segments s (*i.e.,* using regular expression notation: $\mathcal{C} = s^*$), the path Laplace transform has the expression: $w(\zeta, \theta) = \frac{1}{1 - l(\zeta, \theta)}$, where $l(\zeta, \theta)$ is the Laplace transform of segments s. This is the equivalent of the formal identity $\frac{1}{1-y} = 1 + y + y^2 + y^3 + ...$, which represents the Laplace transform of an arbitrary sequence of random variables with Laplace transform y.

More generally, we can use notation from language theory to express a path with any regular expression which characterizes all permitted combinations of different types of segments. Again, we can automatically deduce the Laplace transform of the path, based on the expressions for individual segments, and using the above constructions/translations (see [10]). We will show how to use this methodology in detail in Section IV.

IV. OPPORTUNISTIC ROUTING

We first develop our methodology in the simplified framework of the store-forward strategy; we then apply the same techniques to prove our main theorem on the packet propagation speed with the more general store-hold-forward strategy.

A. Opportunistic Store-forward

This is the most basic opportunistic routing scheme, since, in this strategy, nodes always attempt to transmit the packets immediately.

a) Routing path segmentation: The routing path is made only of "emission segments". In other words, a routing path is a sequence of emission segments taken in the alphabet $\{s_e\}$ and is of the form s_e^*. An emission segment s_e is a space time vector; the time component corresponds to the duration of one slot and the space component describes the distance traveled by the packet in one emission. For instance, according to the model in Section II, we can calculate the probability of such a segment as a function of this distance.

Our aim is to find the earliest path that arrives to a given destination at coordinate $\mathbf{z} = (x, y)$. We note again that we consider only simple paths, *i.e.,* paths that never loop on the same node. However, in a store and forward strategy, a simple path may not be the earliest path that arrives to the destination $\mathbf{z}$, since the earliest path may in fact loop on a node A (thus potentially encountering more favorable transmission conditions). In an equivalent simple path, node A would need to hold the packet during a certain time before retransmitting

it. Anyhow, the equivalence between earliest path and earliest simple path will be true for the next more interesting strategy: store-hold-forward. The store-forward strategy is only developed as a methodology introduction.

b) Path Laplace transform: Let $w(\zeta, \theta)$ be the path Laplace transform, *i.e.,* the Laplace transform of quantity $p(\mathbf{z}, t)$:

$$w(\zeta, \theta) = E(\exp(\zeta \cdot \mathbf{z} - \theta t)).$$

In the following lemma, we compute $w(\zeta, \theta)$.

Lemma 2: In the store-forward strategy, the path Laplace transform has the expression:

$$w(\zeta, \theta) = \frac{1}{1 - (\nu - \lambda)\Psi_p(|\zeta|)e^{-\theta}},$$

where $\Psi_p(x) = 2\pi \int_0^\infty p(r)I_0(xr)rdr$, and $I_0()$ is a modified Bessel function of order 0 (see [1]).

Notice that $I_0(x) = \sum_{k \geq 0} (\frac{x}{2})^{2k} \frac{1}{(k!)^2}$. Developing with the expression for $p(r)$ in (2), we get:

$$\Psi_p(\rho) = \pi \sum_{k \geq 0} \frac{1}{\gamma \Gamma((k+1)\gamma)k!} \frac{E(e^{(k+1)\gamma F})}{(CK^\gamma)^{k+1}} \left(\frac{\rho}{2}\right)^{2k}.$$

We also note that, when we take $F = 0$, $K = 1$ and $\alpha = 4$, we have the specific formula:

$$\Psi_p(\rho) = \frac{2}{\pi} H\left([\frac{1}{2}, \frac{1}{2}], \frac{\rho^4}{64\pi^3}\right) + \frac{\rho^2}{2\pi^2} H\left([1, \frac{3}{2}], \frac{\rho^4}{64\pi^3}\right),$$

where $H([p, q], x)$ are hypergeometric functions.

Proof: In the store-forward model, a path is only made of successful emission segments s_e. An emission segment is a space time vector $(\mathbf{z}, 1)$ where $\mathbf{z}$ is a space vector and we assume that 1 is the slot time unit. An emission segment is successful if it ends on a mobile node (with density ν), if the receiver is not transmitting simultaneously (with probability $1 - \frac{\lambda}{\nu}$) and if the transmission is successful (with probability $p(|\mathbf{z}|)$). Therefore, the density probability of emission segments is $p(|\mathbf{z}|)\nu(1 - \frac{\lambda}{\nu})$ in space (which corresponds to the previously stated conditions) and is a Dirac measure on 1 in time (*i.e.,* the duration is always one slot).

We denote the space vector $\mathbf{z} = (r\cos\phi, r\sin\phi)$, where r is the segment length and $\phi \in [0, 2\pi]$ is the direction. Consequently, the emission segment Laplace transform $g_e(\zeta, \theta) = E(\exp(-\zeta \cdot \mathbf{z} - \theta t))$ is obtained by averaging on r and ϕ:

$$
\begin{aligned}
g_e(\zeta, \theta) &= e^{-\theta} \int_0^\infty \nu(1 - \frac{\lambda}{\nu})p(r)rdr \int_0^{2\pi} e^{-|\zeta| \cdot r\cos\phi}d\phi \\
&= 2\pi e^{-\theta}(\nu - \lambda) \int_0^\infty p(r)I_0(|\zeta|r)rdr.
\end{aligned}
$$

Since the path is equivalent to a sequence emission segment, expressed as s_e^* with the language wording (see Section III), we have: $w(\zeta, \theta) = \frac{1}{1 - g_e(\zeta, \theta)}$. ∎

c) Maximum propagation speed: Recall that $q(\mathbf{z}, t)$ is the probability that there exists at least one path that arrives to the destination node before time t. We will prove that $q(\mathbf{z}, t) = O(\exp(-\rho_0|\mathbf{z}| + \theta_0 t))$, for some (ρ_0, θ_0). This implies that $q(\mathbf{z}, t)$ vanishes very quickly when t is smaller than the value such that $-\rho_0|\mathbf{z}| + \theta_0 t = 0$, *i.e.* when $\frac{z}{t} = \frac{\theta_0}{\rho_0}$. Therefore

(as shown in [13]), quantity $\frac{\theta_0}{\rho_0}$ is an asymptotic propagation speed upper-bound. Namely for all $v > \frac{\theta_0}{\rho_o}$:

$$\lim_{|\mathbf{z}| \to \infty} q(\mathbf{z}, \frac{|\mathbf{z}|}{v}) = 0.$$

Let $D(\rho, \theta) = (\nu - \lambda)\Psi_p(\rho)e^{-\theta}$. The path Laplace transform has a denominator $1 - D(|\zeta|, \theta)$. The key of the analysis is the set $\mathcal{K}$ of pairs (ρ, θ) such that $D(\rho, \theta) = 1$, called the *Kernel*. We show that a path Laplace transform of this form implies the following asymptotic estimate of the path density.

Lemma 3: When $|\mathbf{z}|$ and t tend both to infinity we have:

$$p(\mathbf{z}, t) = O\left(\exp(-\rho_0|\mathbf{z}| + \theta_0 t)\right),$$

where (ρ_0, θ_0) is the element of the kernel $\mathcal{K}$ that minimizes $-\rho|\mathbf{z}| + \theta t$.

Proof: See appendix. ∎

We can now prove the following theorem concerning the maximum packet propagation speed.

Theorem 2: In the store-forward strategy, the packet propagation speed is upper-bounded by the smallest ratio $\frac{\theta}{\rho}$ of the elements of $\mathcal{K} = \{(\rho, \theta) : D(\rho, \theta) = 1\}$, where:

$$D(\rho, \theta) = (\nu - \lambda)\Psi_p(\rho)e^{-\theta},$$

with $\Psi_p(\rho) = 2\pi \int_0^\infty p(r)I_0(\rho r)rdr$, and $I_0()$ is a modified Bessel function of order 0.

The Kernel $\mathcal{K}$ is made of the tuples (ρ, θ) with $\theta = \log((\nu - \lambda)\Psi_p(\rho))$. The minimum ratio $\frac{\theta_0}{\rho_0}$ is attained for the element of the Kernel such that: $\frac{\Psi_p'(\rho_0)}{\Psi_p(\rho_0)} = \frac{\log((\nu - \lambda)\Psi_p(\rho_0))}{\rho_0}$, where $\Psi_p'(\rho)$ is the derivative of $\Psi_p(\rho)$ with respect to variable ρ. Thus, $\theta_0 = \rho_0 \frac{\Psi_p'(\rho_0)}{\Psi_p(\rho_0)}$.

Proof: The Kernel of the path Laplace transform is the root of the denominator, *i.e.,* the set of pairs (ρ, θ) such that $D(\rho, \theta) = 1$. Therefore, following the asymptotic analysis of the average number of journeys (from Lemma 3) and Lemma 1, the propagation speed upper bound is given by the minimum ratio $\frac{\theta}{\rho}$ of $(\rho, \theta) \in \mathcal{K}$. ∎

B. Store-hold-forward Strategy

This strategy differs from the store-forward strategy by the fact that nodes can either transmit the packet immediately, or hold it and attempt to transmit on a later slot.

Lemma 4: In the store-hold-forward strategy, the path Laplace transform has the expression:

$$w(\zeta, \theta) = \frac{1}{1 - (\nu - \lambda)\Psi_p(|\zeta|)e^{-\theta} - e^{-\theta}}.$$

Proof: In this case, a path is made of an arbitrary sequence of emission and "hold segments". A hold segment is a space-time vector expressing the situation where the packet stays in a node's memory during one slot. Since we assume that all nodes do not move, a hold segment is the vector $(0, 1)$, where 0 is the space component and 1 corresponds to the slot duration. Hence, the hold segment Laplace transform is simply $g_h(\zeta, \theta) = e^{-\theta}$.

A path is now a sequence in $\{s_e + s_h\}^*$, since each node can either emit *or* hold the packet. Therefore, the path Laplace transform is: $w(\zeta, \theta) = \frac{1}{1 - (\nu - \lambda)\Psi_p(|\zeta|)e^{-\theta} - e^{-\theta}}$. ∎
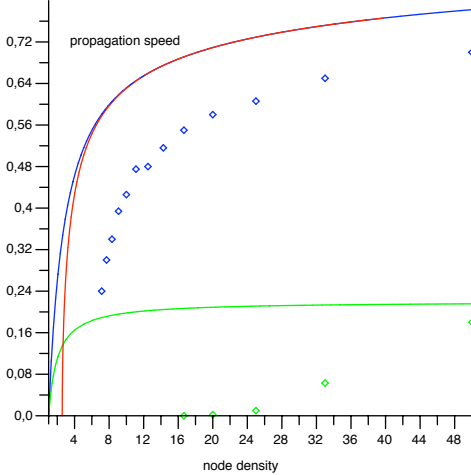
Fig. 1. Store-forward (red - top), store-hold-forward (blue - top) packet propagation speed upper-bounds (in meters per slot) versus the node density ν, compared to classical routing (green - bottom). The network traffic density is fixed ($\lambda = 1$). The dots correspond to measured values obtained via simulation of a classical and an opportunistic protocol.

Equivalently to Theorem 2, we have the theorem for the packet propagation speed, by substituting the new path Laplace transform.

Theorem 3: In the store-hold-forward strategy, the packet propagation speed is upper-bounded by the smallest ratio $\frac{\theta}{\rho}$ of the elements of $\mathcal{K} = \{(\rho, \theta) : D(\rho, \theta) = 1\}$, where:

$$D(\rho, \theta) = (1 + (\nu - \lambda)\Psi_p(\rho)) \, e^{-\theta},$$

with $\Psi_p(\rho) = 2\pi \int_0^\infty p(r) I_0(\rho r) r \, dr$, and $I_0()$ is a modified Bessel function of order 0.

The Kernel is made of tuples (ρ, θ) such that: $\theta = \log(1 + (\nu - \lambda)\Psi_p(\rho))$, and the minimum ratio $\frac{\theta_0}{\rho_0}$ is attained on:
$\frac{\theta_0}{1 - \exp(-\theta_0)} = \rho_0 \frac{\Psi_p'(\rho_0)}{\Psi(\rho_0)}$.

V. SIMULATIONS

In this section, we present simulations illustrating the packet propagation speed upper bounds proved in Theorem 1 concerning conventional routing, and Theorems 2 and 3 concerning opportunistic routing strategies (store-forward and store-hold-forward respectively).

First, in Figure 1, we plot the theoretical propagation speed bounds as a function of the node density ν, obtained from the theorems when the traffic density is fixed: $\lambda = 1$. The bounds express the maximum speed in meters per slot at which a packet can propagate in the network. The traffic load *per node* equals $\frac{\lambda}{\nu}$, hence as ν increases, the load of the nodes is smaller and the packets can propagate faster.

For this numerical example, we take a required signal to noise ratio $K = 1$ and a power attenuation coefficient $\alpha = 4$; we do not consider yet the effect of random fading. However, we note that our analysis provides bounds for any combination of values for λ, ν, K and α.

Notice that the upper bound for the store-forward strategy collapses to a zero speed for a value of $\nu \approx 2.47...$ (below

the percolation point, *i.e.,* when the network becomes disconnected); however, the collapse has an infinite slope at the critical point, meaning that the speed bound increases abruptly for slightly larger values of the node density. On the other hand, the store-hold-forward strategy upper bound remains non-zero until the node density ν reaches its minimal value: $\nu = \lambda$ (recall that $\lambda \leq \nu$). In this case, the speed bound decreases with a sharp but finite slope. This illustrates the fact that the variations in the signal to noise ratio always guarantee connectivity, as long as the nodes can hold the packets for the transmission possibilities to change. We also notice that, when the node density increases, *i.e.,* when the per node traffic density diminishes, the two opportunistic speed bounds converge.

In Figure 1, we also compare our theoretical bounds with measured values obtained via simulation of a classical and an opportunistic routing scheme (dots). The network simulator is self-developed. For the measurements, we perform the simulations following the framework of [4].

We implement the network model described in Section II. According to the model, nodes are randomly distributed following a Poisson distribution. For the simulations, we consider a finite square network domain, such that the node density is ν nodes per square area unit. We also consider that time is slotted and the overall traffic density is λ packets per slot per square area unit; in practice, each node independently transmits a packet with probability $\frac{\lambda}{\nu}$ at each slot (we assume that all nodes always have a packet to send). For the propagation speed measurements, we select a source and a destination, positioned at opposite locations of the network; we obtain the propagation speed by measuring the average packet delays for different sources and destinations, and taking the ratio of the source-destination distance over the delay. All nodes in the network except for the source-destination pair generate background traffic. We implement the signal attenuation and interference model, exactly as described in Section II; a packet can be successfully decoded if its signal over noise ratio is greater than a certain threshold K, while all background traffic is considered as noise.

For the measurements, routing is optimized and the relays are determined by a centralized algorithm, which has an absolute knowledge of the network state at any time. Therefore, we do not simulate specific protocol message exchanges (we consider that the protocol overhead is included in the overall traffic density λ). We compare a classical routing algorithm and and an opportunistic routing algorithm, following the simulation framework in [4]:

- The *classical routing* strategy is based on a Dijkstra algorithm. We fix a maximum transmission range, which is optimized according the channel conditions, as discussed in Section II-B. The packets are then forwarded following the shortest path (in hops) from the source to the destination but using the optimized MAC protocol described in [3].
- We also simulate an *opportunistic routing* algorithm, presented in [4], which is based on time-space opportunistic radial routing. At each hop and each slot, the next relay is the node that is closest to the destination, among the
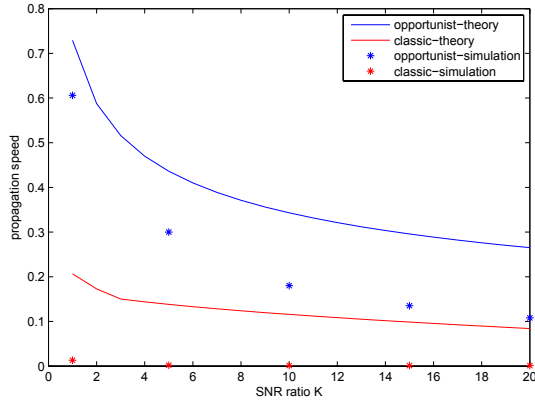
Fig. 2. Propagation speed versus required signal to noise ratio K, for $\nu = 25$, for $\alpha = 4$, and $\lambda = 1$. Comparison of theoretical bounds on opportunistic routing (blue - top) and classical routing (red - bottom), as well as simulations (dots).
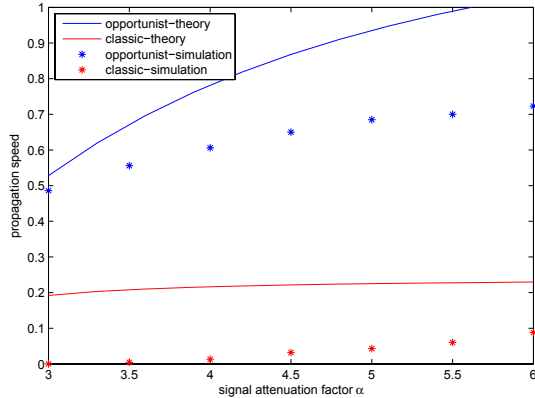

Fig. 4. Theoretical propagation speed bounds (opportunistic routing in blue - top, and classical routing in red - bottom) versus the traffic density λ, for $\alpha = 4$, $K = 1$, and $\frac{\lambda}{\nu} = 0.2$.

Fig. 3. Propagation speed versus signal attenuation factor α, for $\nu = 25$, $K = 1$, and $\lambda = 1$. Comparison of theoretical bounds on opportunistic routing (blue - top) and classical routing (red - bottom), as well as simulations (dots).

nodes that capture the packet successfully (for a detailed description, see [4]).

The plots confirm the accuracy of our theoretical bounds on the opportunistic packet propagation speed. They also show that radial time-space routing achieves a close to optimal performance. Our classical routing bound is too optimistic (albeit still valid). This is expected since we proved the bound in a simpler framework: the optimal performance is achieved when each hop has a length exactly equal to the optimal transmission range (which is obviously not true in practice). However, when the node density increases, the performance of the simulations converges to the theoretical bound. It is important to note that, in all cases, the opportunistic routing performance is significantly better.

In Figure 2, we illustrate the behavior of the upper bounds and the simulation measurements, for different values of the signal to noise ratio K. We fix the node density to $\nu = 25$ and the traffic density to $\lambda = 1$; this means that each node has a packet to transmit with probability $\frac{\lambda}{\nu} = 0.04$ at each slot. We also take $\alpha = 4$. From now on, we refer to opportunistic routing in general, since for the given densities both strategies analyzed in the paper show the same performance. Interestingly enough, the simulated classical routing protocol
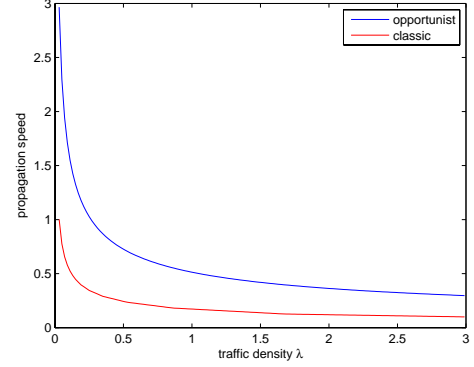
almost collapses under the given traffic conditions, while opportunistic routing yields a packet propagation speed well above 0.

In Figure 3, we plot the theoretical upper bounds and the simulation measurements, for different values of the power attenuation factor α. We fix the node density to $\nu = 25$ and the traffic density to $\lambda = 1$; for the required SNR ratio, we take $K = 1$. Again, we notice that opportunistic routing offers a significant improvement.

Remark: The derived upper bounds assume a given overall traffic density (in packets per slot per square area unit), denoted λ; this traffic density includes the protocol overhead as well. Since we are interested in fundamental performance limits, in Figures 1, 2 and 3, we evaluate and compare the propagation speed using classical and opportunistic routing for the same overall traffic density λ (implying a similar protocol overhead in both cases). In practice, depending on the protocols in use, the actual overhead may vary; however, comparing specific protocol solutions is outside the scope of this paper. Moreover, we did not consider the delays experienced by packets in the queues, because we are interested in upper bounds on the best possible information propagation speed. In practice, the propagation speed is scaled down because of these delays.

Furthermore, it is worth noting that our analysis of the packet propagation speed can be used as an estimate of the network capacity in number of packets that can be transported per square area unit and per slot. In fact, if we fix the per node traffic density $\frac{\lambda}{\nu}$ and we vary the network traffic density λ, the analysis shows that the packet propagation speed follows an inverse square root law; equivalently, the network capacity is $O(\frac{1}{\sqrt{\lambda}})$ bit-meters per second, in accordance with the result of [11]. This is illustrated in Figure 4, where we fix the probability that a node has a packet to transmit in a slot: $\frac{\lambda}{\nu} = 0.2$, and we plot the theoretical packet propagation speed versus the traffic density λ; we observe a square root decrease for both classical and opportunistic routing propagation speeds. Therefore, in Figure 4, we can compare classical and opportunistic routing under variable overall traffic conditions. Conversely, from the scaling results of Gupta and Kumar [11],
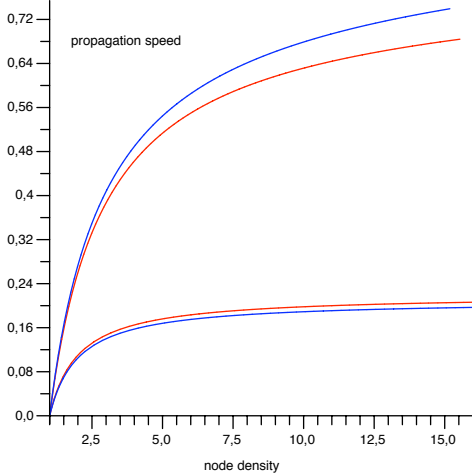
Fig. 5. Classical (bottom) and opportunistic (top) routing upper bounds of the packet propagation speed versus the node density ν, with random fading (blue) and no fading (red).

we could deduce that the average information propagation speed must scale at most in $O(\frac{1}{\sqrt{n}})$, where n is the number of nodes (distributed uniformly at random) in the network. However, since our speed analysis is more specific, we derive a precise upper-bound on the packet propagation speed (not just a scaling law on the average speed) and we can differentiate between classical and opportunistic routing.

A. Effect of Fading

In this section, we investigate the impact of random signal fading on the routing performance. To fix ideas, we assume that the random factor F (introduced in Section II) is a random variable uniformly distributed on an interval $[-a, a]$, for some $a > 0$. Then, when we take a power attenuation coefficient $\alpha = 4$, we get closed formulas for $p(r)$ and $\Psi_p(r)$, based on hypergeometric functions (for other values of α or different fading distributions, we can use the infinite series expansions).

In Figure 5 we compare the propagation speed bounds with fading ($a = 1$), as opposed to no fading ($a = 0$); the bounds are derived from Theorems 1 and 3, for classical and opportunistic routing respectively. We assume a signal-over-noise ratio $K = 1$. Interestingly enough, the upper-bound on classical routing decreases when compared to the no-fading case, while the upper-bound on opportunistic routing increases. This hints to the fact that opportunistic routing can take advantage of variations of the signal-over-noise ratio and improve the packet propagation speed; on the other hand, these variations cause the performance of conventional routing strategies to deteriorate.

VI. Node Mobility

In this section, we will adapt the analysis from Section IV to account for node mobility. Every node follows an i.i.d. random trajectory, so that the nodes are distributed with constant Poisson density ν. The mobility model is the random walk: at

each slot a mobile node changes direction with probability τ. The motion direction angles are uniformly distributed between 0 and 2π. The nodes keep a constant speed between direction changes, which we denote by s.

A. Path and Movement Decomposition

Again, we consider a store-hold-forward routing scheme. However, the nodes can now move while they hold the packets. We will decompose the path in the following three kinds of segments:
1) *emission* segments s_e;
2) *move-to-turn* segments s_t;
3) *move-to-emit* segments s_m.

Emission segments are defined in the same manner as in Section IV. The move-to-turn and move-to-emit segments substitute the hold segments. The "move-to-turn" segment corresponds to the straight line that a mobile node follows until it changes direction; it represents one step of the random walk. The "move-to-emit" segment is similar, but now the mobile node emits the packet before the next change of direction; in other words, it corresponds to an incomplete random step.

More precisely, the move-to-turn segment is a space-time vector $(k \cdot \mathbf{u}s, k)$ where $\mathbf{u}$ is a unitary vector (marking the direction of the movement) and k is the number of slots during which the mobile has moved without turning. The segment has a duration of k slots with probability $\tau(1-\tau)^{k-1}$ and $k > 0$. The Laplace transform of a move-to-turn segment is:

$$
\begin{aligned}
g_t(\zeta, \theta) &= \frac{1}{2\pi} \sum_{k>0} \tau(1-\tau)^{k-1} \int_0^{2\pi} E(e^{\cos(\phi)|\zeta|ks - k\theta}) d\phi \\
&= \sum_{k>0} \tau(1-\tau)^{k-1} E(I_0(|\zeta|ks)) e^{-k\theta}.
\end{aligned}
$$

The expectation $E(.)$ indicates the average value with respect to the speed factor s.

Similarly, the move-to-emit segment is a space-time vector $(k \cdot \mathbf{u}s, k)$, where k is the number of slots during which the mobile has moved. However, we now have a duration of k slots with probability $(1-\tau)^{k-1}$, since there is no change of direction. This leads to a Laplace transform equal to:

$$
g_m(\zeta, \theta) = \sum_{k>0} (1-\tau)^{k-1} E(I_0(|\zeta|ks) e^{-k\theta}).
$$

Notice that $g_t(\zeta, \theta) = \tau g_m(\zeta, \theta)$.

A path is made of segments according to the following rules that describe the node movement as well as the packet transmissions:
1) an s_e segment is followed by any segment;
2) an s_t segment is either followed by an s_t segment or an s_m segment;
3) an s_m segment is always followed by an s_e segment.

Therefore, a path is a word in the alphabet $\{s_e, s_t, s_m\}$, following the regular expression $s_e^*(s_t^* s_m s_e s_e^*)^*(1 + s_t^* s_m)$; this expression decomposes a path according to the three previous rules. According to the approach described in Section III, we can directly deduce the path Laplace transform from the regular expression:

$$
\begin{aligned}
w(\zeta, \theta) &= \frac{1}{1 - g_e(\zeta, \theta)} \left(1 + \frac{g_m(\zeta, \theta)}{1 - g_t(\zeta, \theta)}\right) \frac{1}{1 - \frac{g_m(\zeta, \theta)}{1 - g_t(\zeta, \theta)} \frac{g_e(\zeta, \theta)}{1 - g_e(\zeta, \theta)}} \\
&= \frac{1 - (1-\tau) g_m(\zeta, \theta)}{1 - g_e(\zeta, \theta) - \tau g_m(\zeta, \tau) - (1-\tau) g_m(\zeta, \tau) g_e(\zeta, \theta)}
\end{aligned}
$$

Notice that, when the speed is $s = 0$, we have $g_m(\zeta, \theta) = \frac{e^{-\theta}}{1-(1-\tau)e^{-\theta}}$, and we find, as expected, the result of the previous section, where there is no mobility.

With the new path Laplace transform, we can again apply the saddle point technique, and derive an upper bound on the packet propagation speed. If we assume that the destination node is not moving, the analysis follows directly the methodology of Section IV.

If we consider the fact that the destination is also moving according to the same mobility model as the relay nodes, this does not affect the analysis. The destination's movement implies that we need to multiply the quantity $w(\zeta, \theta)$ with the Laplace transform of the journey of a moving node. The journey can be described by the regular expression $s_t^* s_m$, *i.e.,* an arbitrary sequence of random steps (the last step is incomplete). This yields a Laplace transform: $\frac{g_m(\zeta,\theta)}{1-g_t(\zeta,\theta)}$. Notice that this modification does not change the saddle point optimization since it only adds to the kernel tuples (ρ, θ') such that $\theta' < \theta$, for some (ρ, θ) belonging in the original kernel. In other words, the packet propagation speed upper bound remains unchanged, whether we take into account the destination's mobility or not. This is expected because of the random walk model: the average drift of the destination node is zero.

B. Analysis

We will investigate in detail the realistic case where the speed is small, that is: $s \ll 1$. Indeed, s is expressed in meters per slot, and wireless transmissions are expected to occur faster than physical node motions.

Using the expansion $I_0(\rho) = 1 + \left(\frac{\rho}{2}\right)^2 + O(\rho^4)$, we get:

$$
\begin{aligned}
g_m(\zeta, \theta) &= \sum_{k>0}(1-\tau)^{k-1}e^{-k\theta}(1 + k^2\left(\frac{|\zeta|}{2}\right)^2 E(s^2) \\
&\quad + O(|\zeta|^4 E(s^4)) \\
&= \frac{e^{-\theta}}{1-(1-\tau)e^{-\theta}} + \left(\frac{|\zeta|}{2}\right)^2 \sigma_2 \frac{(1+(1-\tau)e^{-\theta})e^{-\theta}}{(1-(1-\tau)e^{-\theta})^3} \\
&\quad + O(\sigma_4)
\end{aligned}
$$

where σ_2 and σ_4 are the second and fourth moments of the speed s, respectively.

We take $\rho = |\zeta|$ to simplify the notation. The denominator of the modified path Laplace transform is:

$$
e^\theta - 1 - (\nu - \lambda)\Psi_p(\rho) - g(\rho, \theta)\sigma_2 + O(\sigma_4),
$$

where:

$$
g(\rho, \theta) = (\tau + (1-\tau)(\nu-\lambda)\Psi_p(\rho)e^{-\theta})\left(\frac{\rho}{2}\right)^2 \frac{(1+(1-\tau)e^{-\theta})e^{-\theta}}{(1-(1-\tau)e^{-\theta})^2}.
$$

Denoting $f(\rho) = 1 + (\nu - \lambda)\Psi_p(\rho)$, we have a kernel set (ρ, θ) such that:

$$
\theta = \log(f(\rho)) + \sigma_2 \frac{g(\theta, \rho)}{f(\rho)} + O(\sigma_2^2 + \sigma_4).
$$

As a result, when we apply the saddle point analysis of Section IV, the maximum packet propagation speed is:

$$
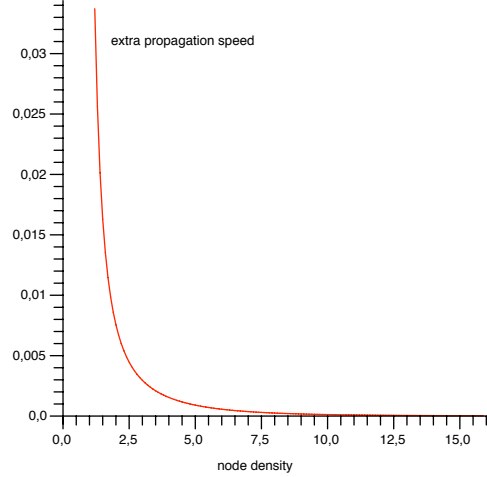\frac{\log f(\rho_0)}{\rho_0} + \sigma_2 \frac{g(\rho_0, \log f(\rho_0))}{f(\rho_0)\rho_0} + O(\sigma_2^2 + \sigma_4),
$$

Fig. 6. Improvement in the propagation speed upper bound (with opportunistic routing) versus the node density ν, with traffic density $\lambda = 1$, mobile speed $s = 0.1$ and direction change probability $\tau = 0.1$.

where ρ_0 is the root of $\frac{f'(\rho)}{f(\rho)} - \frac{\log f(\rho)}{\rho}$.

When $\nu \to \lambda$, then we have $f(\rho) \to 1$ and $\theta \to 0$. Therefore, the residual propagation speed upper bound tends to:

$$
\log\left(1 + (2-\tau)\frac{\rho_0^2}{4}\frac{\sigma_2}{\tau}\right),
$$

where ρ_0 is now the root of $\rho \frac{\Psi_p'(\rho)}{\Psi_p(\rho)} - 1$.

Notice that the propagation speed upper bound is larger than zero, as long as $\frac{\sigma_2}{\tau}$ tends to a positive constant. This is equivalent to say that the random walk has a constant standard deviation rate per time unit. Conversely, we showed in Section IV that, when the nodes do not move, the propagation speed tends to zero when the node density decreases.

In Figure 6, we plot the difference of the propagation speed bounds when nodes move as opposed to when they stay still. We see that mobility causes a small improvement in the propagation speed. However, this improvement enables the propagation speed to remain larger than zero, even when the node density tends to its minimal value (*i.e.*, $\nu \to \lambda$).

VII. CONCLUSION

We characterized the optimal performance, in terms of delay, that can be achieved using any opportunistic routing algorithm, in a realistic network model where link conditions are variable. We derived analytical upper bounds on the packet propagation speed with generic opportunistic routing strategies in Theorems 2 and 3. Our analysis is sufficiently general to provide bounds depending on the node and traffic densities in the network, as well as the signal propagation conditions. Such theoretical bounds are useful in order to evaluate and/or optimize the performance of specific opportunistic routing algorithms.

Furthermore, we compared opportunistic and classical routing; we showed that opportunistic routing performs significantly better, even when classical routing schemes are optimized based on an absolute knowledge of the statistics of the

channel conditions. We presented numerical simulations that confirm the accuracy of our bounds in numerous scenarios: we simulated a classical routing algorithm and an optimized time-space opportunistic routing scheme.

We also investigated the effects of random fading and node mobility. We showed that random fading in fact improves the performance of opportunistic routing strategies, which can take advantage of the random signal variations; conversely, the performance of classical routing deteriorates. Similarly, opportunistic store-hold-forward schemes can take advantage of the node mobility.

An interesting direction for further research consists in designing lower bounds for the packet propagation speed. Moreover, it should be possible to refine our classical routing analysis, in order to derive bounds regarding the relative gain of opportunistic strategies.

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APPENDIX

Proof of Lemma 1

We denote $f(\mathbf{z}, t)$ the density of paths starting from the origin at time 0 and ending on $\mathbf{z}$ at time t. Therefore, the average number of paths starting at $(0,0)$ and ending in a space area B at time t is $\int_B f(\mathbf{z}, t)d\mathbf{z}$, with the integral being multi-dimensional.

Furthermore, the number $n(\mathbf{z}, t)$ of paths that start on $(0,0)$ and arrive on a node at point $\mathbf{z}$ at time t is exactly:

$$\begin{aligned} n(\mathbf{z}, t) &= \int d\mathbf{z}' p(|\mathbf{z}' - \mathbf{z}|)f(\mathbf{z}', t-1) \\ &\leq Af(\mathbf{z}, t-1), \end{aligned}$$

with $A \geq \int_0^\infty e^{ar}p(r)2\pi r dr$.

Notice that A is finite if $f(\mathbf{z}, t)$ grows at most exponentially, *i.e.*, if $\frac{f(\mathbf{z}', t)}{f(\mathbf{z}, t)} \leq e^{a|\mathbf{z}' - \mathbf{z}|}$ (from (2) in Section II, we already know that $p(r)$ has a super-exponential decay with r). This condition is true, as it is shown in Section IV (from Lemma 3 and since $f(\mathbf{z}, t) \leq p(\mathbf{z}, t)$).

Let $q(\mathbf{z}, t)$ be the probability that there exists a path that arrives to point $\mathbf{z}$ before time t. We have $q(\mathbf{z}, t) \leq N(\mathbf{z}, t)$ where $N(\mathbf{z}, t)$ is the average number of paths that end on a node located at $\mathbf{z}$ before time t. We have $q(\mathbf{z}, t) \leq A \int_0^t f(\mathbf{z}, t)dt \leq Ap(\mathbf{z}, t)$.

Proof of Lemma 3

We use the methodology introduced in [13] to prove the following more detailed result; the Lemma follows.

When $|\mathbf{z}|$ and t tend both to infinity we have:

$$p(\mathbf{z}, t) = (1 + O(\frac{1}{\sqrt{t}}))\frac{\exp(-\rho_0|\mathbf{z}| + \theta_0 t)}{2\pi\theta_0\sqrt{\frac{D_\theta D_\rho}{\rho_0}}\nabla_2 D(t, |\mathbf{z}|)},$$

where (ρ_0, θ_0) is the element of $\mathcal{K}$ that minimizes $-\rho|\mathbf{z}| + \theta t$. We denote $D_\rho = \frac{\partial}{\partial \rho}D$, $D_\theta = \frac{\partial}{\partial \theta}D$ and $\nabla_2 D(x, y) = x^2\frac{\partial^2 D}{\partial \rho^2} + y^2\frac{\partial^2 D}{\partial \theta^2} + 2xy\frac{\partial^2 D}{\partial \rho \partial \theta}$.

Proof: We extract $p(\mathbf{z}, t)$ using the inverse Laplace transform:

$$(\frac{1}{2i\pi})^3 \int\int w(\zeta_0+i\zeta, \theta_0+i\theta)e^{\langle(\zeta_0,\theta_0),(\mathbf{z},t)\rangle+i\langle(\zeta,\theta),(\mathbf{z},t)\rangle}d\zeta\frac{d\theta}{\theta_0+i\theta},$$

where the integration domain consists of real planes.

For positive (ρ, θ), the function $1 - D(\rho, \theta)$ has a simple root at:

$$\theta(\rho) = \log((\nu - \lambda)\Psi_p(\rho)).$$

Notice that $(\rho, \theta(\rho))$ describes the kernel set $\mathcal{K}$.

Therefore, the residues analysis gives: $p(|\mathbf{z}|, t) = I(\mathbf{z}, t) + R(\mathbf{z}, t)$, where:

$$I(\mathbf{z}, t) = \frac{1}{(2i\pi)^2}\int\frac{\exp\langle(\zeta, \theta(|\zeta|)), (\mathbf{z}, t)\rangle}{\theta(\rho)D_\theta(\rho, \theta(\rho))}d\zeta,$$

with $D_\theta = \frac{\partial}{\partial \theta}D$.

Quantity $R(\mathbf{z}, t)$ is the integral of $\frac{\exp(\langle(\zeta,\theta),(\mathbf{z},t)\rangle)}{(1-D(|\zeta|,\theta))\theta}$. Therefore, $R(\mathbf{z}, t) = O(e^{-Bt}I(\mathbf{z}, t))$ for some $B > 0$. The evaluation of $I(\mathbf{z}, t)$ is obtained via the saddle point technique.

Let ζ_0 be the value that minimizes $(\zeta, \mathbf{z} + \theta(|\zeta|)t)$. We have $\zeta_0 = -\frac{\rho_0}{|\mathbf{z}|}\mathbf{z}$ with ρ_0 that minimizes $-\rho|\mathbf{z}| + \theta(\rho)t$. Let θ' and θ'' be the first and second derivatives of $\theta(\rho)$ respectively. We already know that $\theta' = \frac{|\mathbf{z}|}{t}$. Since $D(\rho, \theta(\rho)) = 1$, by derivation with respect to ρ we have $D_\rho + D_\theta\theta' = 0$ and, by second derivation, $\nabla_2 D(1, \theta') + D_\theta\theta'' = 0$ at $\rho = \rho_0$.

We get (*cf.* [13]):

$$
\begin{aligned}
I(\mathbf{z}, t) &= \frac{\exp(-\rho_0|\mathbf{z}|+\theta_0 t)}{(2\pi)^2} \int\int \frac{\exp(\frac{1}{2}(\theta''x^2+\frac{\theta'}{\rho_0}y^2))}{\theta D_\theta}dxdy \\
&\quad \times(1 + O(t^{-1/2})) \\
&= \frac{\exp(-\rho_0|\mathbf{z}|+\theta_0 t)}{2\pi\theta_0 D_\theta\sqrt{\frac{\theta'\theta''}{\rho_0}}}(1 + O(t^{-1/2})) \ .
\end{aligned}
$$

■

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