Opportunistic Spectrum Access in Cognitive Radio Networks

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***Abstract*—Enabled by regulatory initiatives and radio technology advances, opportunistic spectrum access has the potential to mitigate spectrum scarcity and satisfy the increasing demand for spectrum. In this paper, we consider a scenario where secondary users can opportunistically access unused spectrum vacated by idle primaries. We introduce two metrics to protect primary performance, namely collision probability and overlapping time. We present two spectrum access schemes using different sensing, back-off, and transmission mechanisms. We show that they achieve indistinguishable secondary performance under given primary constraints. We provide closed form analysis on secondary user performance, present a tight capacity upper bound, reveal the impact of various design options, such as sensing, packet length distribution, back-off time, packet overhead, and grouping. Our work sheds light on the fundamental properties and design criteria on opportunistic spectrum access.**

I. INTRODUCTION

The breakneck proliferation of wireless devices and rapid growth of wireless services continue to stretch the limited spectral resource. In fact, most spectrum bands suitable for terrestrial wireless communication have already been allocated by the regulatory agencies to existing licensees. On the other hand, the current approach of static spectral allocation is highly wasteful. Measurement has shown that over 60% of the licensed spectrum below 6 GHz remains unused or underutilized [1], [2]. To exploit the reported "white space" in existing bands [1], cognitive radio networks have been considered as the viable technology to improve spectral efficiency. With primary licensee's consent, secondary users equipped with cognitive radios may be allowed to transmit on primary bands when the Primary users are inactive [3], [4], [5].

Serious challenges must be resolved in order for the cognitive radios to be acceptable. First, secondary users must not be disruptive to primary user communications. Secondly, an access mechanism is required to reduce contention between secondary users to efficiently share spectrum opportunities. However, coordination and synchronization among secondary users may be limited due to the decentralized nature of secondary user access, particularly if secondary users of different networks coexist. In addition, each secondary users may not be able to sense all channels due to the limitation on hardware or/and sensing capability, and thus algorithm to decide which channel to monitor is needed.

In this paper, we focus on non-intrusive spectrum access schemes that do not require primary users to alter their existing hardware or behavior. The activities of primary users on different frequency bands are modeled as independent M/G/1 queues, where secondary users can exploit a primary channel when it is idle. We introduce two protection metrics (collision probability and overlapping time), to which secondary users' access schemes must abide in order to protect the quality of service (QoS) of primary users. Such constraints can be imposed by primary licensees or spectral regulators.

Under the constraints, we investigate the capacity of secondary users under various sensing-based random access schemes. We propose two random access schemes for secondary users, namely, VX and KS schemes, with different sensing, transmission, and back-off mechanisms. We study extensively the throughput performance of secondary users with the VX scheme. We thoroughly investigate both the simple system setup with with one primary band and one secondary user, and the more general case in which there are multiple primary bands and multiple secondary users. Our results illustrate the fundamental properties and design criteria of opportunistic spectral access for cognitive radio networks.

The rest of the paper is organized as follows. We first describe a number of related works in the field in Section II before presenting our system model, performance metrics and problem formulation in Section III. We present two random access schemes, and analyze the throughput performance and optimum access parameters in Section IV. Simulation results are also given in Section IV to verify our analysis. We extend the results to multi-band competitive systems in Section V. We conclude the paper and discuss future work in Section VI.

II. RELATED WORK

To facilitate spectrum sharing, researchers have considered the design of a common control channel to exchange spectrum access and sensing information and facilitate collaborative sensing and spectrum reservation/sharing, e.g., in [6], [7], [8]. Centralized and decentralized spectrum auction and brokerage have been proposed for efficient spectrum sharing, e.g., in [9], [10], [11]. Co-existence of cognitive users in unlicensed band has also been studied [12], [13], [14].

Researchers have also considered sensing-based decentralized cognitive medium access schemes [15], [16], [17]. In [15], the authors model the states of primary bands as two-state Markovian process and maximize the transmission rate of secondary users in certain time slots. In [16], the authors design a CSMA/CA-based cognitive radio MAC protocol that uses channel statistics to determine the optimal access range and the number of channels to access. In [17], the authors develop a slotted transmission scheme of secondary user via periodic channel sensing based on Constraint Markov Decision Processes. In comparison, our model is more general. We do not assume exponential busy period of primary users (required in Markovian models), neither synchronization between multiple users or feedback from receivers. Our work introduces explicit guarantee on the performance of primary users and we provide closed form analysis on the capacity limit of secondary users under the primary constraints.

III. SYSTEM MODEL

Consider a system with N homogeneous primary bands (channels) and M secondary users (SUs). Primary users (PUs) are the legacy users of these bands and thus have higher priority over SUs. A primary band is called busy if it is used by one or more PUs, and idle otherwise.

We assume the arrival process of a PU is Poisson while the service time distribution can be arbitrary. This assumption holds in many situations. For example, in a data network, packet arrival process is Poisson while packet length distribution can be arbitrary. Similarly, the arrival process of a data session is often Poisson while the length of the data file can be heavy tail. In addition, the call arrival process for voice traffic can also be modeled as a Poisson process. When there are multiple PUs in a band, the busy and idle periods of the band can be modeled as the busy and idle periods of an M/G/1 queue (with multiple inputs). In this case, the idle time distribution is exponential while the busy time distribution is general. From the viewpoint of an SU, because the objective is to utilize the idle period, one can treat the aggregated primaries as one PU. Therefore, we assume, without loss of generality, there is one PU per band and the activities in different bands are independent and identically distributed (i.i.d.). We also assume that the system is stationary and ergodic.

Let V_1 and L_1 be random variables denoting the idle period and busy period of a primary band, respectively. Let $v_1 = E[V_1]$ and $l_1 = E[L_1]$. Let

$$\alpha = \frac{v_1}{v_1 + l_1}.$$

Note that α is the probability (or the percentage of time) that a primary band is idle. In other words, a primary band is idle α fraction of time, which could be exploited by SUs.

The focus here is on the non-intrusive spectrum access schemes of cognitive radio devices. The non-intrusiveness has two-folded meanings. On one hand, PUs are not required to change their existing transmission strategies and algorithms to coordinate with SUs. On the other hand, the access activities of SUs should guarantee that the impact incurred on the transmissions of PUs is insignificant. We also assume that the PU's channel access is not affected by SU's behavior. For example, PUs does not sense the channel before its transmission. If a PU and a SU transmit simultaneously, the PU does not retransmit or back-off. In other words, the idle and busy periods of a primary band are not affected by SUs.

An SU performs sensing before transmission and only transmits if the primary band is idle. We assume that SUs perform perfect sensing, i.e., the false alarm and missing probability of the sensing is zero. Additionally, we assume that the sensing of the channel takes an infinitely small amount of time to finish. Furthermore, due to the limit of radio front-end equipment, we assume that the SU cannot sense the channel when it is transmitting. We do not assume synchronization between primary and secondary users or control channel among SUs.

A. Primary Protection

In our system, collision only happens in the following scenario: a secondary user senses the channel idle and starts transmission, and the primary user returns to the band before the SU finishes its transmission. To capture the impact of collision on PUs and thus to protect PUs, we introduce the following metrics. The first constraint metric is the probability of the collision observed by the PU. As shown in Fig. 1, collision is defined as the event that the PU starts transmission when the SU is transmitting a packet on the channel. Let P_1^c and P_2^c be the collision probabilities observed by a PU and an SU, respectively. Under the assumption of stationary and ergodicity, we have:

$$\begin{aligned} P_1^c &= \lim_{T->\infty} \frac{\text{No. of collisions in [0,T]}}{\text{No. of busy periods of PU in [0,T]}} \\ P_2^c &= \lim_{T->\infty} \frac{\text{No. of collisions in [0,T]}}{\text{No. of packets transmitted for SU in [0,T]}}. \end{aligned} \tag{1}$$

This constraint is suitable for the situation where the average packet length of SUs, is close to that of PUs.

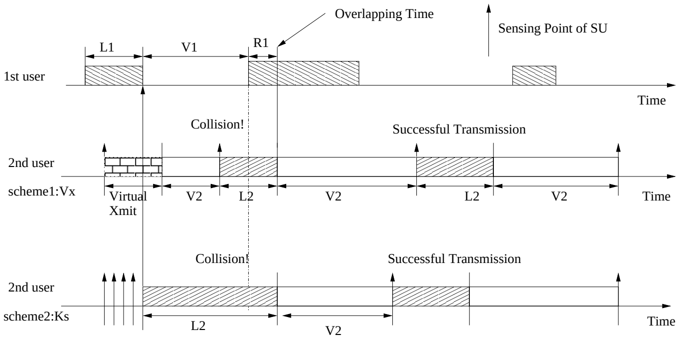

Fig. 1. Random Access Schemes of A Cognitive Radio User.

On the other hand, if a busy period of a PU consists of one or more service sessions (e.g., the whole duration of a telephone call or the total time to transfer a large file), then only a very small portion of the primary's traffic session is affected by collision. For example, suppose that the PU is having voice communication, the busy period (call session) may be thousands times longer than the duration of a IP packet transmitted by SUs. In this case, from the perspective of the PU, the collision probability metric is not appropriate, and the duration of the interruption caused by SUs is more important. Therefore, the second constraint metric is the percentage of overlapping time. One example of the overlapping time is illustrated in Fig. 1 with length $R1$. Specifically, we have the following definition of the percentage of overlapping time observed by the PU:

$$P_1^r = \lim_{T->\infty} \frac{\text{Length of overlapping time in [0,T]}}{T}. \tag{2}$$

To protect the transmission of PUs, the system can set the following constraints:

$$\begin{aligned} & P_1^c \leq \eta \\ \text{or,} \quad & P_1^r \leq r_0, \end{aligned} \tag{3}$$

where η and r_0 are performance thresholds predetermined by the network operator of the primary bands and/or the spectrum regulators. In fact, we will show that the above two constraints are closely related in the following sections.

B. Objective Function

We assume that there are always packets waiting for transmission for the SU. So the results obtained can be regarded as an upper bound on the capacity of SUs. We also assume that SUs have the knowledge of the statistics of the channels. In particular, each SU knows the average of busy period l_1 and idle period v_1 , i.e., the average channel occupancy behavior of the PUs. This knowledge can be obtained by the SU from historic information, measurement results, and/or database. SUs also have the information of the access constraints posed by the PUs or government regulators, i.e., η or r_0, which are predefined when they are admitted into the network.

The objective of the spectrum access is to maximize the achievable capacity (throughput) of the SU, denoted by C_2. More specifically, C_2 is defined as the time proportion that an SU transmits on the channel without collision. Given that α is the fraction of time the PU is idle, we have $C_2 \leq \alpha$. The resulting optimization problem is formulated as below:

$$\begin{aligned} & \max\{C_2\} \\ & s.t. \\ & P_1^c \leq \eta \\ \text{or,} \quad & P_1^r \leq r_0. \end{aligned} \tag{4}$$

In other words, an SU can decide its sensing scheme, access scheme, packet length and distribution, and back-off duration and distribution to maximize its capacity. For the rest of the paper, let L_2 be a random variable denoting the secondary transmission time, which is refereed to as packet length. Let V_2 be a random variable denoting the back-off time, which is also referred to as vacation. Let $l_2 = E[L_2]$ and $v_2 = E[V_2]$.

IV. ONE PRIMARY BAND, ONE SU

In this section, we first consider the case where there is only one SU and one primary band. We analyze the throughput performance of the SU under the constraints posed by the PU.

A. Random Access Schemes

The media access schemes (or protocol) we consider in this paper are illustrated in Fig. 1 and described as below:

- VX Scheme (Virtual-Xmit-if-Busy): The SU senses the channel. If the channel is idle, the SU transmits a packet of length L_2. Then, the SU starts a vacation of length V_2. If the channel is busy, the SU starts a so-called virtual transmission stage and then enters into the vacation stage afterward. Here, virtual transmission means that the SU does not actually transmit the packet but waits for a time interval which is equal to the packet length. After vacation, the SU senses the channel again.
- KS Scheme (Keep-Sensing-if-Busy): After a vacation, the SU senses the channel. If the channel is idle, the SU transmits a packet and then starts vacation. If the SU senses the channel busy, it keeps sensing until the channel is idle. Then, the SU transmits a packet and starts a random vacation of length V_2.

Since the sensing is perfect, given $L_2 = \tau$, the collision probability observed by the SU in the above random access schemes is

$$\begin{aligned} Pr\{Collision|L_2 = \tau\} &= 1 - \int_\tau^\infty \frac{1}{v_1} e^{-\frac{t}{v_1}} dt \\ &= 1 - e^{-\frac{\tau}{v_1}}. \end{aligned}$$

Note here we ignore the probability that one secondary packet collide with multiple PU's busy periods. This is reasonable when l_2 is much smaller than $l_1 + v_1$. However, we note that in all simulations, such events are not ignored. We show that the analysis and simulation match well for a wide range of values.

In the VX scheme, the transmission activity (including virtual transmission) of the SU is independent of the PU's occupancy of the channel, thus its analysis is simplified. In this paper, we obtain closed form analysis on the collision probability, the overlapping time, the capacity of the SU. The closed-form solutions provide insights on the system performance and facilitate the implementation of the MAC protocol. On the other hand, the analysis is more difficult in the KS scheme, since the transmission of the SU is somewhat dependent on the activities of the PU. Interesting enough, simulation results show that the throughput performance of the KS scheme is indistinguishable from that of the VX scheme under the same collision probability constraint.

B. Performance Analysis of VX Scheme Under Collision Probability Constraint

In the VX scheme, the probability that the SU actually transmits a packet is equal to the probability that the SU senses the channel idle. Due to the independence between the sensing activities of the SU and the activities of the PU, this probability is $\alpha = v_1/(v_1 + l_1)$. Let $f_{L_2}(\tau)$ be the probability density function of L_2. The average collision probability observed by the SU can be expressed as below:

$$P_2^c = \int_0^\infty (1 - e^{-\frac{\tau}{v_1}}) f_{L_2}(\tau) d\tau. \tag{5}$$

The virtual transmission stage in the VX scheme has the same statistic characteristics as the actual transmission. Thus, the number of "collision" events in time interval $[0, T]$ can be calculated as $\alpha \cdot P_2^c \cdot \frac{T}{l_2+v_2}$. Consequently, the collision probability for the PU can be calculated as

$$\begin{aligned} P_1^c &= \lim_{T->\infty} \frac{\alpha \cdot P_2^c \cdot \frac{T}{l_2+v_2}}{\frac{T}{l_1+v_1}} \\ &= P_2^c \frac{v_1}{l_2+v_2}. \end{aligned} \tag{6}$$

Let $\hat{L}_2$ denote the contribution of each transmission to the throughput of the SU. In particular, if a SU's packet of length L_2 is successfully transmitted, $\hat{L}_2 = L_2$, otherwise $\hat{L}_2 = 0$. Let $\hat{l}_2 = E[\hat{L}_2]$. We can regard $\hat{l}_2$ as the average length of effective packets. Then we have

$$\begin{aligned} \hat{l}_2 &= \int_0^\infty \frac{1}{v_1} e^{-\frac{t}{v_1}} \int_0^t \tau f_{L_2}(\tau) d\tau dt \\ &= \int_0^\infty \tau e^{-\frac{\tau}{v_1}} f_{L_2}(\tau) d\tau. \end{aligned} \tag{7}$$

The achievable capacity of the SU is obtained as the time proportion that the SU actually transmits packets (excluding the time when the SU performs virtual transmission) without collision, i.e.,

$$C_2 = \alpha \frac{\hat{l}_2}{v_2 + l_2} = \alpha \frac{\int_0^\infty \tau e^{-\frac{\tau}{v_1}} f_{L_2}(\tau) d\tau}{v_2 + l_2}. \tag{8}$$

Our objective is to find the optimum l_2, f_{L_2}, and v_2 to maximize C_2 in (8) under the collision probability constraint $P_1^c \leq \eta$.

According to (6), to satisfy the collision probability constraint $P_1^c \leq \eta$, we have

$$l_2 + v_2 \geq \frac{v_1 \int_0^\infty (1 - e^{-\frac{\tau}{v_1}}) f_{L_2}(\tau) d\tau}{\eta}. \tag{9}$$

Note that in (9), when $l_2 \ll v_1 + l_1$, we have $v_2 > 0$ since the right hand side in (9) can be approximated as l_2/η. The maximum throughput of the SU is achieved when equality holds in the above inequality. Thus, we have

$$C_2^{max} = \eta\alpha \frac{\int_0^\infty \frac{\tau}{v_1} e^{-\frac{\tau}{v_1}} f_{L_2}(\tau) d\tau}{\int_0^\infty (1 - e^{-\frac{\tau}{v_1}}) f_{L_2}(\tau) d\tau}. \tag{10}$$

For the VX scheme, we have the following result.

Proposition 1. *For a primary channel of which the idle period V_1 obeys the exponential distribution, suppose that the probability of channel being idle as α, under the constraint that the collision probability of the primary user is less than η, there exists an upper bound on the achievable capacity of the SU which is expressed as below:*

$$C_2 \leq \eta \cdot \alpha. \tag{11}$$

Proof: We have

$$\begin{aligned} &\int_0^\infty \frac{\tau}{v_1} e^{-\frac{\tau}{v_1}} f(\tau) d\tau + \int_0^\infty e^{-\frac{\tau}{v_1}} f_{L_2}(\tau) d\tau \\ =& \int_0^\infty (\frac{\tau}{v_1} + 1) e^{-\frac{\tau}{v_1}} f_{L_2}(\tau) d\tau \\ \leq& \int_0^\infty (\frac{\tau}{v_1} + 1) \frac{1}{1 + \frac{\tau}{v_1}} f_{L_2}(\tau) d\tau \\ =& 1, \end{aligned}$$

where the inequality holds because $e^{-x} < \frac{1}{x+1}$ for $x > 0$. Thus,

$$\int_0^\infty \frac{\tau}{v_1} e^{-\frac{\tau}{v_1}} f_{L_2}(\tau) d\tau \leq 1 - \int_0^\infty e^{-\frac{\tau}{v_1}} f_{L_2}(\tau) d\tau.$$

Therefore, we have

$$\frac{\int_0^\infty \frac{\tau}{v_1} e^{-\frac{\tau}{v_1}} f_{L_2}(\tau) d\tau}{\int_0^\infty (1 - e^{-\frac{\tau}{v_1}}) f_{L_2}(\tau) d\tau} \leq 1$$

As a result,

$$C_2 \leq \eta\alpha.$$

■

Note that, through the derivation, we only require that the primary idle period be exponential and do not pose any requirement on the distribution of L_1, L_2, and V_2.

The result is somewhat surprising: C_2 cannot be larger than $\eta\alpha$ to satisfy the collision probability constraint, where α is the time fraction of spectrum vacancy. The intuition is as follows. Since the idle period is exponentially distributed, whenever an SU starts to transmit on an idle channel, it faces the same probability of collision. Therefore, the aggressiveness of an SU's transmission should be proportional to the collision probability allowed by PUs. When $\eta = 1$, the SU can fully utilize the idle periods, and thus reach the capacity limit α when l_2 goes to zero. On the other hand, if $\eta = 0$, the SU can never transmit and thus the capacity is zero. We note that in both access schemes, the aggressiveness is controlled by L_2 and V_2.

The other side of the story is more straightforward. If the idle period is deterministic, the capacity of SU, C_2, can reach α regardless of the value of η. An SU can simply track the beginning of the idle period and transmit until the end of the idle period. Deterministic and exponential distribution are the two extremes in terms of predictability/entropy. While one can maximize C_2 in the case of deterministic idle period, our conjecture is that the exponential idle period results in the worst opportunistic spectrum access capacity among all idle period distributions (for a given l_1 and v_1).

In the following, we consider two special cases, i.e., exponentially distributed L_2 and fixed L_2.

1) Exponentially distributed L_2: When the packet length of the SU, L_2, is exponentially distributed, we have

$$P_2^c = \frac{l_2}{l_2 + v_1}, \tag{12}$$

$$P_1^c = \frac{v_1}{v_2+l_2}\cdot\frac{l_2}{l_2+v_1}. \tag{13}$$

Following (7), we have

$$\hat{l}_2 = l_2\frac{v_1^2}{(l_2+v_1)^2}. \tag{14}$$

From (9), in order to satisfy the collision probability, for a given l_2, the optimal v_2 should be chosen such that

$$v_2 = max\{0, \frac{v_1 l_2}{\eta(v_1+l_2)} - l_2\}. \tag{15}$$

Therefore, for given l_2 and η, C_2^{max} is given as:

$$C_2^{max} = \eta\alpha\frac{v_1}{l_2+v_1}. \tag{16}$$

We can observe that the smaller l_2, the larger the C_2. This is intuitive. With smaller l_2, the collision probability is smaller, and the amount of transmission wasted is smaller when a collision happens. Therefore, more packets can be transmitted successfully with the collision constraint satisfied. We note that $C_2^{max} \to \eta\alpha$ when $l_2 \to 0$.

2) Fixed Packet Length of SU: If the SU uses fixed packet length, i.e., $L_2 = l_2$, we have

$$P_1^c = (1-e^{-\frac{l_2}{v_1}})\frac{v_1}{v_2+l_2}, \tag{17}$$

From (9), in order to satisfy the collision probability, for a given l_2, v_2 should be chosen such that

$$v_2 = max\{0, \frac{v_1(1-e^{-l_2/v_1})}{\eta} - l_2\}. \tag{18}$$

Following similar approach in (8), we have

$$C_2^{max} = \eta\alpha\frac{l_2 e^{-l_2/v_1}}{v_1(1-e^{-l_2/v_1})}. \tag{19}$$

Again, we can observe that $C_2^{max} \to \eta\alpha$ when $l_2 \to 0$.

3) Choosing fixed length packet: It is interesting to compare the capacity performance of the SU with random and fixed length packet (with the same mean). We have the following *Proposition* for the VX scheme.

Proposition 2. *For VX, let the largest packet length be* $l_{max} = (2 - \frac{l_2/v_1}{e^{l_2/v_1}-1})v_1$. *Under the constraint* $P_1^c \le \eta$ *and* $E[L_2] = l_2$, *the SU achieves the maximum throughput when it transmits fixed length packets, i.e.,* $L_2 = l_2$. *In other words,*

$$C_2(l_2) = \eta\alpha\frac{\int_0^\infty \frac{\tau}{v_1}e^{-\frac{\tau}{v_1}}f_{L_2}(\tau)d\tau}{\int_0^\infty (1-e^{-\frac{\tau}{v_1}})f_{L_2}(\tau)d\tau} \le \eta\alpha\frac{\frac{l_2}{v_1}e^{-l_2/v_1}}{1-e^{-l_2/v_1}}, \tag{20}$$

where $f_{L_2}(\tau)$ *is the probability density function of* L_2 *with finite support* $0 \le \tau \le l_{max}$.

Proof: Define $X = \frac{L_2}{v_1}$ and its expectation $m_X = \frac{l_2}{v_1}$. From our assumption of $f_{L_2}(\tau)$, we know that the probability density function of X, $f_X(x) = 0$ if $x \notin [0,\ 2 - \frac{l_2/v_1}{e^{l_2/v_1}-1}]$. Next, we define an auxiliary function:

$$g(x) = (e^{m_X}-1)xe^{-x} + m_X e^{-x},\ 0 < x < 2 - \frac{m_X}{e^{m_X}-1},$$

whose second order derivative is

$$g''(x) = (x(e^{m_X}-1) - (2e^{m_X} - 2 - m_X))e^{-x}.$$

For $0 < x < 2 - \frac{m_X}{e^{m_X}-1}$, $g''(x) \le 0$; thus, $g(x)$ is concave over the support of $f_X(x)$. Therefore, by Jensen's inequality, we have:

$$\begin{aligned} E[g(X)] &= \int_0^\infty g(x)f_X(x)dx \\ &\le g(E[X]) = g(m_X) = m_X, \end{aligned}$$

or,

$$\int_0^\infty [xe^{-x}(1-e^{m_X}) + e^{-m_X}m_X e^{-x}]f_X(x)dx \le e^{-m_X}m_X.$$

Simple manipulations of this equation leads to the result of the proposition:

$$\frac{\int_0^\infty xe^{-x}f_X(x)dx}{\int_0^\infty (1-e^{-x})f_X(x)dx} \le \frac{m_X e^{-m_X}}{1-e^{-m_X}}.$$

■

Proposition 2 specifies a maximum packet length. First, packet size is limited in practice. Second, we note that, if $L_2 > v_1$, the collision probability will be very high and undesirable for both the PU and SU. Since $l_{max} = (2 - \frac{l_2/v_1}{e^{l_2/v_1}-1})v_1 > v_1$, l_{max} is a reasonable length constraint.

4) Simulations: In our simulations, we set $v_1 = 1$, $\eta = 0.1$, and $l_1 = 0.5$, leading to the channel idling probability $\alpha = 0.667$ which approximately equals the proportion of white space according to measurement.

In Fig. 2, we present results for the VX scheme when the SU adopts exponentially distributed packets and fixed length packets, respectively. For 10^6 busy PU periods, we vary the average SU packet length l_2 from 0.1 to 1.0. Parameter v_2 of the SU is obtained according to (15) and (18) to satisfy the collision probability constraint. With no assumption about the distribution of L_1 and V_2 in the analysis, different distributions are tested to verify the analytical results. Here, we include the cases L_1 being exponentially distributed (denoted by E) and fixed (denoted by F). The distributions of V_2 are exponential (denoted by E) and uniform over $[0, 2v_2]$ (denoted by U). In the legend, X/Y means that L_1 follows X distribution and V_2 follows Y distribution. Simulation result matches our analysis very well, both in terms of collision probability and capacity for different distributions of L_1 and V_2.

In Fig. 3, we compare the throughput of VX and KS. Without a selection criterion of v_2 for the KS, for each given l_2, we set v_2 for the KS scheme using the same formula for the VX scheme with $\eta = 0.1$. We then adjust the value of v_2 in the VX scheme using the actual collision probability obtained from the simulation results of the KS scheme, and compare the throughput. Hence, the comparison is fair because they achieve the same collision probability. We observe a higher collision probability for the more aggressive KS scheme. Also, for both access schemes, the SU with fixed length packet always achieves larger throughput.

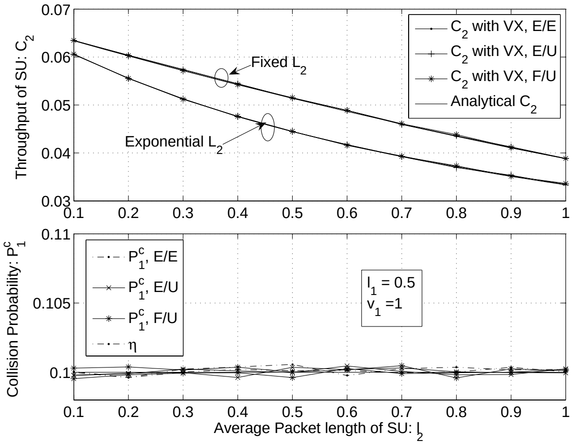

Fig. 2. Throughput of SU in VX scheme.

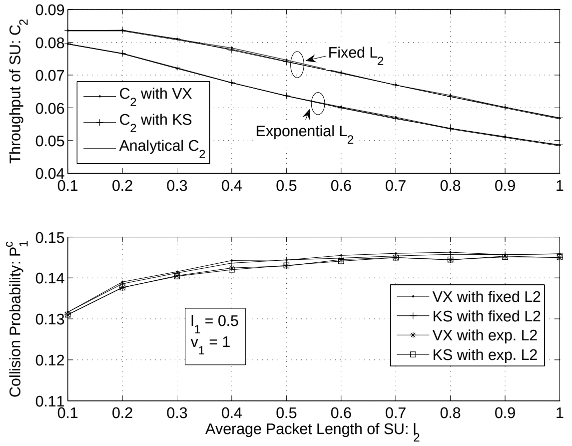

Fig. 3. Compare VX and KS schemes.

5) *Observations:*

- VX and KS schemes have indistinguishable throughput for the SU under the same collision probability constraint. Therefore, insistent sensing in KS scheme does not help. The main reason is that the idle period of channel, V_1, is exponentially distributed and the SU has to guarantee the collision probability.
- For VX scheme, an upper-bound of the throughput of the SU is $C \leq \eta\alpha$. We conjecture that this upper-bound is valid for any access schemes that exploit the idle time of a memoryless channel without coordination from the PU under the collision probability constraint.
- For a large range of packet length l_2, fixed length packet achieves the best capacity over other packet length distributions.

C. Overlapping Time Constraint

The upper bound of the SU throughput $C_2 \leq \eta\alpha$ may appear pessimistic, but not surprising. Consider applications with $l_2 \ll l_1$. For example, when the primary has many packets in each busy session (e.g., a large file transfer) or the primary has voice traffic on the channel, one secondary packet may overlap with a very small proportion of PU transmission in a busy period. Most PU packets will be successful while only a small portion suffers from collision. Thus, the collision probability constraint on P_1^c is overly pessimistic because it counts the whole busy period as collision. To be more practical, we introduce the overlapping time constraint as defined in (2), and study the corresponding throughput performance of the SU.

Our objective is to relate the overlapping time constraint to the previously discussed collision probability constraint. The problem is to calculate P_1^r in (2). Denote L_v as the random length of overlapping time given that there is a collision. We have:

$$P_1^r = P_1^c \frac{E[L_v]}{l_1 + v_1}. \tag{21}$$

Under the assumption that $l_2 \ll l_1 + v_1$, the probability that the packet from the SU collides with more than one busy period of the PU is negligible. Therefore, $L_v = min\{L_1, L_2^{residual}\}$, where $L_2^{residual}$ is the remaining transmission time of the SU's packet when the PU returns to the channel. For brevity, we present two special cases here.

First, when L_2 is exponentially distributed, $L_2^{residual}$ follows the same distribution as L_2. If L_1 is exponential with mean l_1, then L_v is also exponential with mean $\frac{l_1 l_2}{l_1 + l_2}$. Consequently, we have

$$P_1^r = P_1^c \frac{l_1 l_2}{(l_1 + l_2)(l_1 + v_1)}. \tag{22}$$

From (22), we can observe that P_1^r increases linearly with respect to P_1^c and the increasing factor is much smaller than 1. This is reasonable since the overlapping part is only a very small proportion of the whole busy period if $l_2 \ll l_1$.

If $L_2 = l_2$ (fixed length packet) and L_1 is exponentially distributed, we have

$$E[L_v] = l_1 - \frac{l_1^2 (e^{-l_2/v_1} - e^{-l_2/l_1})}{(v_1 - l_1)(1 - e^{-l_2/v_1})}. \tag{23}$$

Consequently, the proportion of the overlapping time is

$$P_1^r = P_1^c \frac{l_1[v_1(1 - e^{-l_2/v_1}) - l_1(1 - e^{-l_2/l_1})]}{(l_1 + v_1)(v_1 - l_1)(1 - e^{-l_2/v_1})}. \tag{24}$$

Based on (21), (22), and (24), we can easily convert the optimization problem under the overlapping time constraint to a problem under the corresponding collision probability constraint. Therefore, most of previous results in solving the collision probability constraint apply to this case.

In our simulations, we set $l_1 = 0.5$, $v_1 = 1$, and $l_2 = 0.05 = 0.1 l_1$. The average vacation time v_2 is set to satisfy the collision probability constraint $P_1^c \leq \eta$. Fig. 4 shows the resulting P_1^r with respect to η for exponentially distributed L_2 and fixed L_2. We can observe that, in both cases, P_1^r increases linearly with η and the analytical results agree with simulations. Also observe that for fixed L_2, the fraction of overlapping time is smaller than that with exponentially distributed L_2 for any given η. This again shows the advantage of fixed SU packet length. Our results are much more optimistic by using overlapping time constraint. In particular, the achievable throughput for the SU is much closer to the limit of the available spectrum holes, α. For example, for exponentially distributed L_2, if the constraint is posed as $P_1^r \leq 0.018$, the throughput of the SU is $C_2 = 0.4$. When the constraint is $P_1^r \leq 0.015$, the throughput of using fixed length packet is $C_2 = 0.65$, very close to the maximum $\alpha = 0.67$.

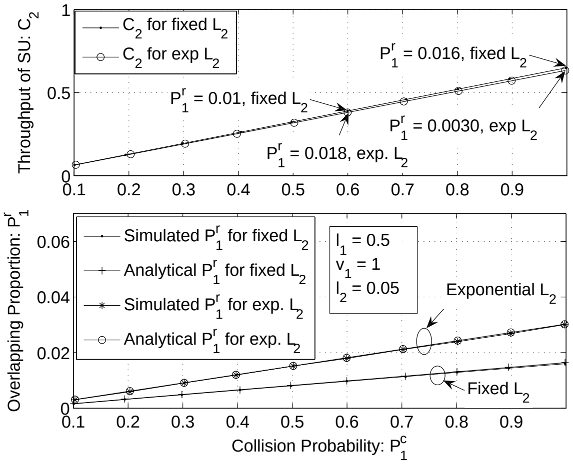

Fig. 4. Throughput of SU under different constraints.

D. Packet Overhead

Our results have thus far favored smaller l_2 for better throughput C_2. This requires quick actions by SU. However, packet overhead must be considered in practical transmission by the SU. Therefore, we consider the effect of packet overhead here by assuming that a fixed length overhead is added to a payload of length L_2. Let l_0 be the length of the overhead. The collision probability observed by the SU is:

$$\begin{aligned} P_2^c &= Pr\{l_0 + L_2 > V_1\} \\ &= \int_0^\infty (1 - e^{-\frac{l_0+t}{v_1}}) f_{L_2}(t) dt. \end{aligned} \tag{25}$$

Correspondingly, the PU collision probability constraint is

$$P_1^c = P_2^c \cdot \frac{v_1}{l_2 + v_2 + l_0} \leq \eta. \tag{26}$$

The average length of successfully transmitted payload of the SU is

$$\begin{aligned} \hat{l}_2 &= \int_0^\infty t f_{L_2}(t) \int_{t+l_0}^\infty \frac{1}{v_1} e^{-\frac{\tau}{v_1}} d\tau dt \\ &= e^{-\frac{l_0}{v_1}} \int_0^\infty t e^{-\frac{t}{v_1}} f_{L_2}(t) dt. \end{aligned} \tag{27}$$

To satisfy the collision probability constraint, we have the optimal v_2 for the VX scheme as:

$$v_2 = max\{0, \frac{v_1 P_2^c}{\eta} - l_2 - l_0\} \tag{28}$$

Thus, the effective throughput of the SU is

$$\begin{aligned} C_2 &= \alpha \frac{\hat{l}_2}{l_2 + v_2 + l_0} \\ &\leq \alpha\eta \frac{\int_0^\infty \frac{t}{v_1} e^{-\frac{l_0+t}{v_1}} f_{L_2}(t) dt}{\int_0^\infty (1 - e^{-\frac{l_0+t}{v_1}}) f_{L_2}(t) dt}. \end{aligned} \tag{29}$$

Using a similar approach as in the proof of *Proposition 1*, we can show that $C_2^{max} < \alpha\eta$. In this case, small packet size may not always be desirable. Next, we present the optimal packet size when L_2 is exponential and fixed, respectively.

1) Exponentially distributed L_2: We start with

$$P_1^c = \alpha \left(1 - \frac{v_1}{l_2 + v_1} e^{-\frac{l_0}{v_1}}\right) \frac{l_1 + v_1}{l_2 + v_2}. \tag{30}$$

The maximum effective throughput (excluding the overhead) of the SU is:

$$C_2^{max} = \alpha\eta \frac{l_2 v_1}{(l_2 + v_1)^2} \frac{e^{-l_0/v_1}}{1 - \frac{v_1}{l_2+v_1} e^{-l_0/v_1}}. \tag{31}$$

The optimal average packet length l_2 is:

$$l_2^* = v_1 \sqrt{1 - e^{-\frac{l_0}{v_1}}}. \tag{32}$$

2) Fixed Length L_2: For fixed payload length of the SU,

$$P_1^c = (1 - e^{-\frac{l_0+l_2}{v_1}}) \frac{v_1}{v_2 + l_2 + l_0}, \tag{33}$$

$$C_2^{max} = \eta\alpha \frac{l_2}{v_1} \frac{e^{-(l_0+l_2)/v_1}}{1 - e^{-(l_2+l_0)/v_1}}. \tag{34}$$

The optimal l_2 is the solution of

$$1 - \frac{l_2^*}{v_1} - e^{-\frac{l_2^*+l_0}{v_1}} = 0, \tag{35}$$

In Fig. 5, we illustrate the numerical results of the throughput performance of the VX scheme with exponentially distributed L_2 and fixed L_2. In the calculation, we set $l_1 = 0.5$, $v_1 = 1$, $l_0 = 0.05$, and $\eta = 0.1$. Clearly, the throughput of the SU with overhead is smaller than that without overhead. The optimal l_2 for both exponentially distributed L_2 and fixed L_2 demonstrate the trade-off between overhead cost and packet collision. The optimal values of l_2 in this simulation are found to be $l_2^* = 0.21$ and $l_2^* = 0.26$ for exponential L_2 and fixed L_2, respectively.

V. RANDOM ACCESS SCHEME FOR MULTI-BAND COMPETITIVE SYSTEM

Now consider the system with multiple primary channels and multiple SUs. Denote the number of channels N, and the number of SUs M. Let only one PU own each channel. Each SU can only transmit in one channel at a time. Multiple SUs compete for available spectrum in N channels. We consider the VX scheme only in this section. All SUs adopt the same access parameters, and thus they can be viewed as homogeneous. They do not communicate with one other. Additionally, as perfect sensing is assumed, an SU can detect

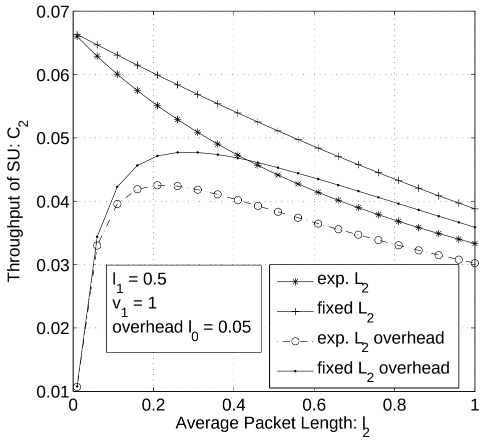

Fig. 5. Throughput of SU with Packet Overhead.

transmissions by both the PU and other SU's in a channel. Assuming instantaneous sensing, as in Section IV, collision in channels can only happen between the returning incumbent PU and a transmitting SU.

Two sensing strategies are considered:

- Random-Sensing: After a random vacation time V_2, each SU randomly selects a channel, and then detects whether the channel is busy. If it is, then SU enters the Virtual Transmission stage. If the channel is idle, the SU transmits its packet before taking a vacation.
- All-Channel-Sensing: After a vacation, each SU senses all channels. If there is no idle channel, the SU enters the Virtual Transmission stage. Otherwise, the SU randomly selects an idle channel for packet transmission.

With the Random-Sensing strategy, the SU only needs to monitor one band at each instant. By comparison, the All-Channel-Sensing strategy requires that each SU monitor all channels. Thus, the former is much easier to implement than the latter.

We present Monte-Carlo simulations on the performance of the two strategies. We set $l_1 = 0.5$, $v_1 = 1$, and $l_2 = 0.1$. Due to limited paper length, we only present results for exponentially distributed L_2 here. The aggregated SU throughput is defined as the sum throughput of all SUs in one particular channel. Similarly, the aggregated collision probability is the collision probability observed by the PU. For comparison, we introduce a One-Band-One-Secondary system (OBOS), where the SU has the same average packet length l_2 and the collision constraint of the PU is equal to the aggregated collision probability in the multi-band competitive system.

Fig. 6 illustrates the aggregated throughput of M SUs and the collision probability of the PU when $N = 1$. We can see that, for fixed l_2 and under the same collision probability, the aggregated throughput of M SUs is the same as the throughput of the SU in an OBOS system. In other words, given the same collision probability constraint, the system with multiple SUs has no throughput loss/gain. This is reasonable, because there is no collision between SUs under perfect sensing. Since SUs are homogeneous, each SU achieves an equal fraction ($\frac{1}{M}$) of the total throughput. We can also observe that collision probability caused by one individual SU with $M > 1$ is less than the collision probability with $M = 1$. This is due to the lower probability of the channel being idle from the perspective of one SU. Additionally, each SU contributes proportionally to the collision probability of the PU, demonstrated by the almost linear increase of P_1^c with respect to M.

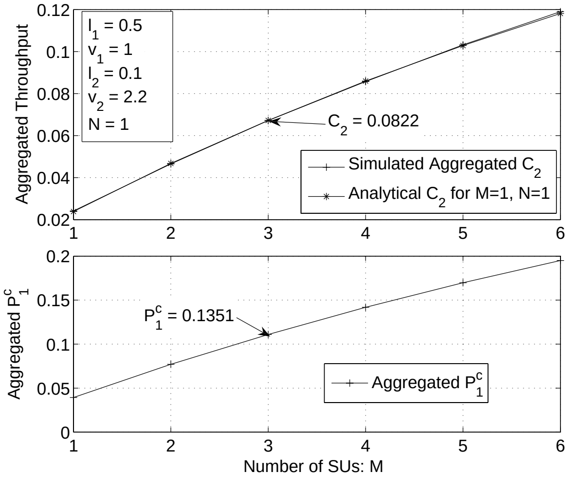

Fig. 6. Aggregated Throughput of SUs in VX scheme.

Next, we test a more general case where there are M SUs and N primary bands with $M = 3N$. Note that the primaries' activities are i.i.d., and all SUs behave in the same way, the performance is the same for all channels. Therefore, we only show the results for one of N channels here. The aggregated throughput of SUs and collision probability in each channel for Random-Sensing and All-Channel-Sensing strategies are shown in Fig. 7. The results show that, the aggregated throughput of SUs for both sensing strategies matches very well with the throughput in the OBOS system under the condition that they have the same collision probability for each channel. If we adjust the values of l_2 and v_2, such that the aggregated collision probabilities caused by Random-Sensing and All-Channel-Sensing strategies are the same, then they will have the same throughput. This indicates that, All-Channel-Sensing strategy does not improve the total spectral efficiency, despite the added complexity. This is mainly due to the memoryless characteristics of the idle time, rather than the limitation that each SU can access one channel each time. We also observe that, without dividing the available bands explicitly among multiple SUs, the autonomous random access performs the same as the coordinated method of organizing SUs into separate groups, each assigned a group of spectral bands. From the perspective of each secondary user, the total throughput it can achieve is $\frac{N}{M}N = \frac{N}{3}$ times the aggregated throughput in each channel.

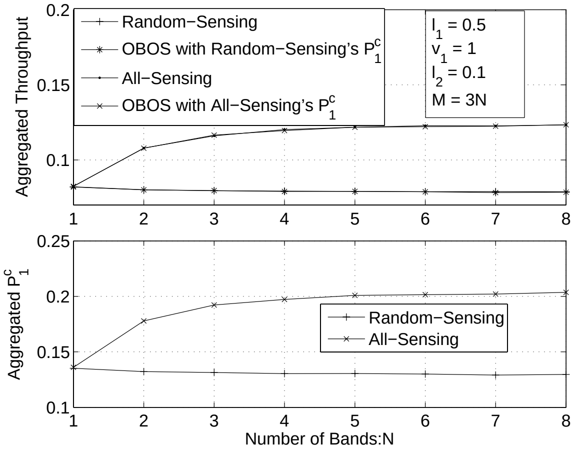

Fig. 7. Aggregated Throughput of SUs with Multiple Primary Bands.

As a summary for multi-band multi-user systems, we have:

- For the same collision probability constraint, the system with multiple SUs has no loss/gain in terms of total throughput.
- Under the same collision probability constraint, sensing all the frequency bands does not improve the total throughput of SUs.
- Dividing SUs into groups to access partitioned bands has the same throughput as the strategy of allowing each SU to randomly access all bands.

VI. CONCLUSIONS AND FUTURE WORK

In this paper, we study the data capacity of cognitive radio users in opportunistic spectral access under stringent intrusion constraints on collision probability and the overlapping collision time. Two random access schemes are proposed for cognitive radio devices to exploit the spectral opportunities in primary bands. We obtain closed-form expressions for the collision probability of the PU and the capacity of the SU. We show that the proposed random access schemes have similar capacity performance. Furthermore, we find that the collision probability and the overlapping time constraints are closed related. SUs can utilize a significant amount of spectral vacancy in the primary band under overlapping time constraint when appropriate. In addition, we consider the overhead cost in the SUs' packets and demonstrate that an optimal trade-off can be obtained for exponential and fixed packet length. Finally, we investigate the aggregated throughput performance and collision probability in a multi-band multi-secondary system.

Our work provides a better understanding on the fundamental properties and performance limit of opportunistic spectrum access. Future works in this direction may involve the extension to systems with inaccurate sensing devices. Imperfect sensing will induce collisions between different SUs and hidden nodes problems in networks. Advanced signal processing techniques in physical layers can also be integrated in the design of media access schemes.

VII. ACKNOWLEDGMENT

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