Optical Bragg Accelerators

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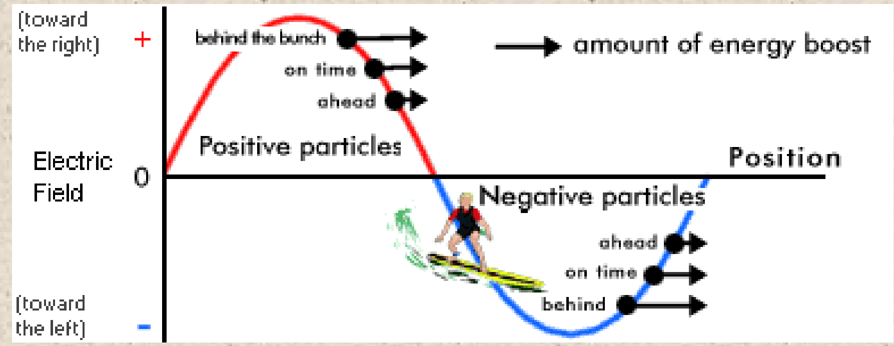
Outline

- Background
- Motivation
- Field Confinement
- Accelerator parameters
- Wake-field analysis
- Summary

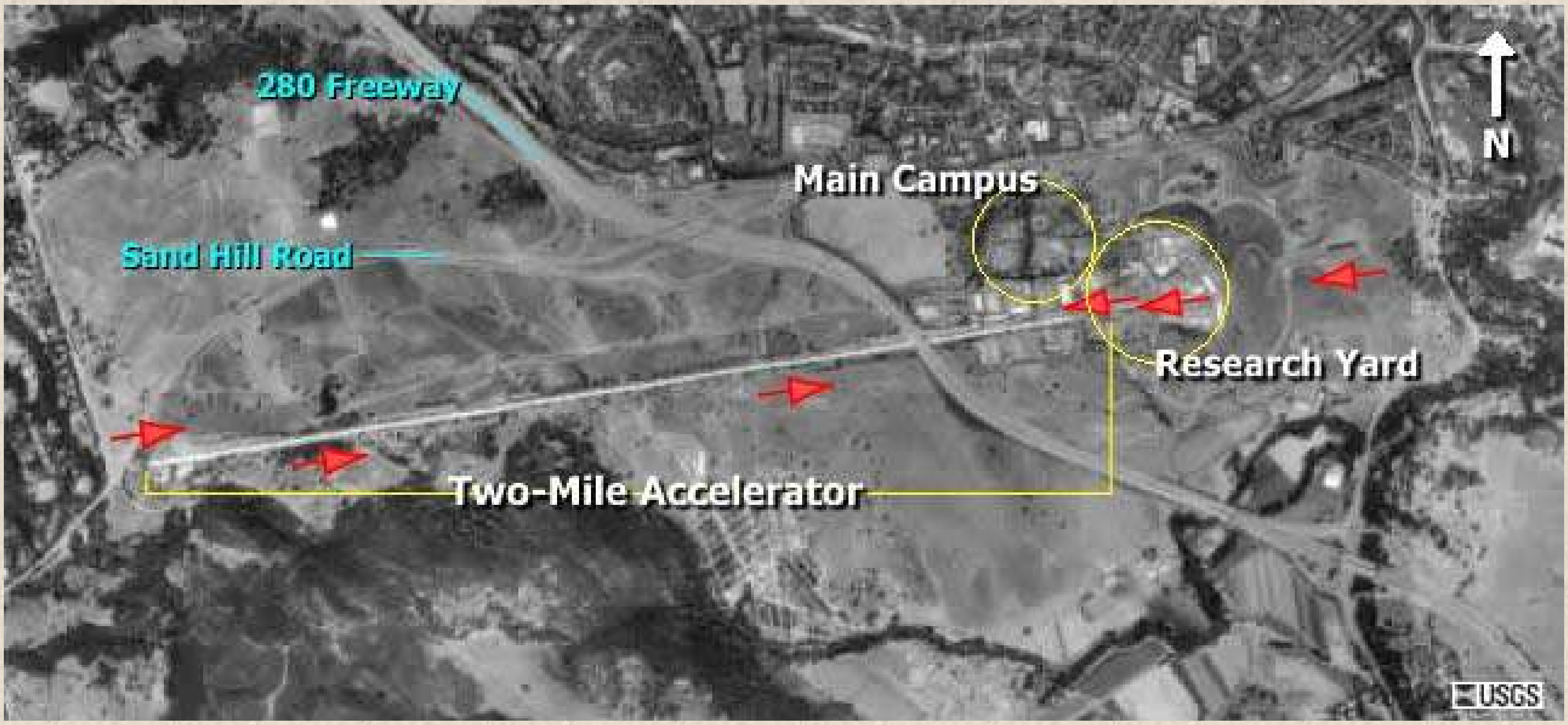
How Do Accelerators Work?



Charges "surf" on the longitudinal electric field

Phase velocity equals the speed of light!

Stanford Linear Accelerator



Motivation

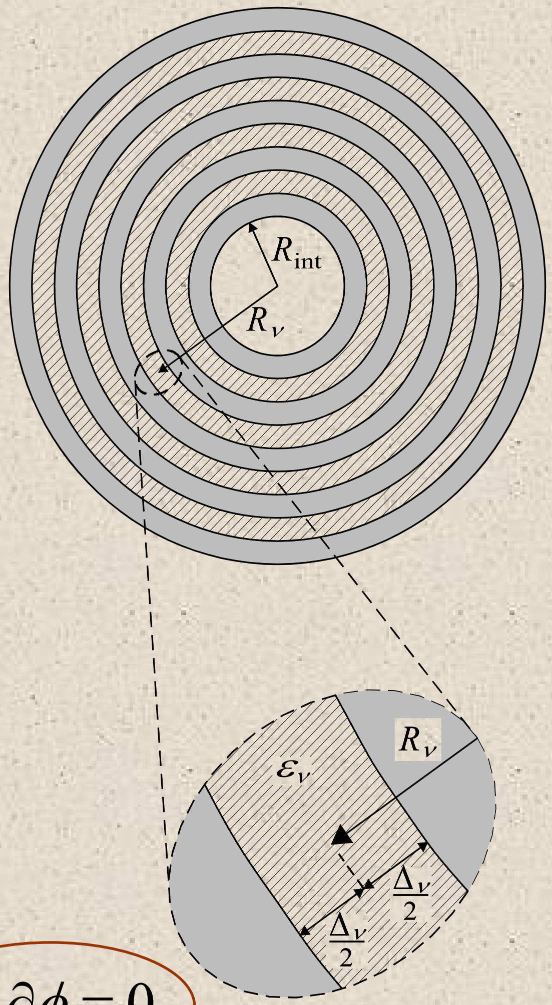
- Smaller and cheaper accelerators (table-top).
- Applications: X-ray, medical, materials.
- Availability of high power lasers.
- Dielectric materials can sustain higher fields than metals.
- Fabrication: harness communication technology.
- Need vacuum tunnel – confinement can not be achieved as in optical fibers – **Bragg waveguide!**

Objectives

Is a dielectric optical Bragg waveguide adequate for acceleration?

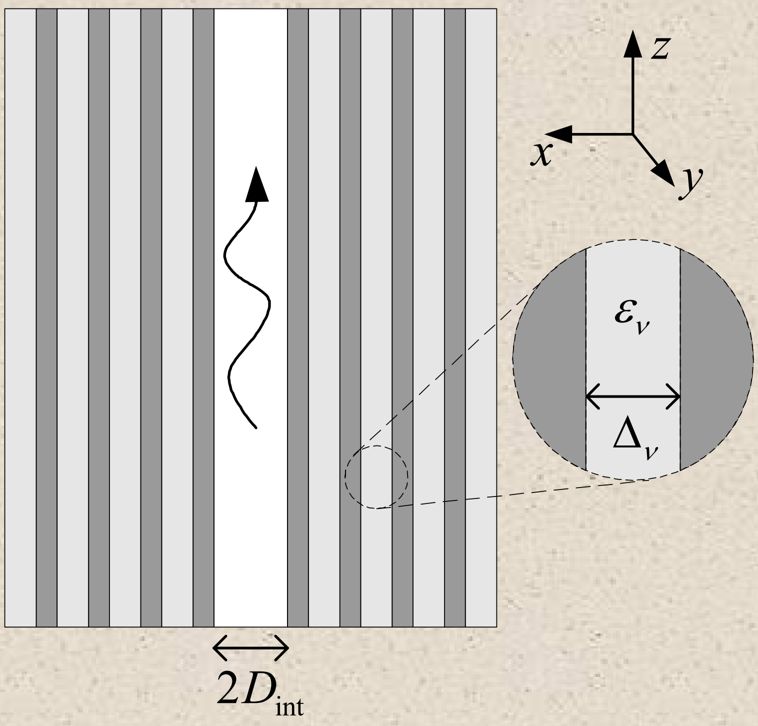
- Design of Bragg waveguide that supports propagation with phase velocity *c* for the driving laser frequency.
- Analyze accelerator parameters (interaction impedance, energy velocity, maximal field).
- Analyze wake-field due to a train of micro-bunches.

Hollow Bragg Fiber



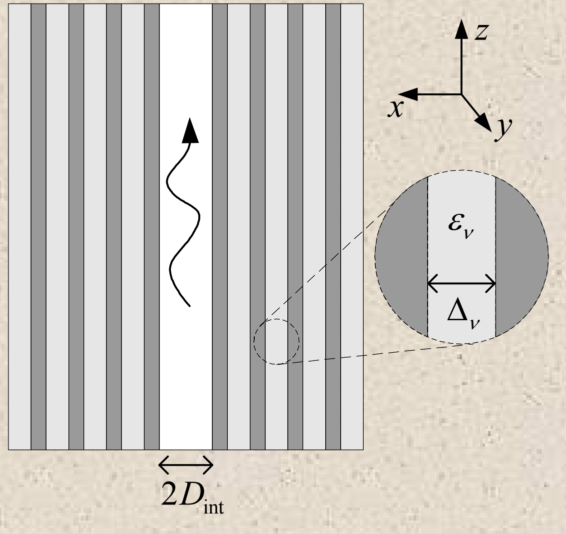
$\partial / \partial \phi \equiv 0$

Planar Bragg Waveguide



$\partial / \partial y \equiv 0$

Required EM Fields



Vacuum fields

$$E_z = E_0 e^{-j\frac{\omega}{c}z}$$

$$E_x = j\frac{\omega}{c} x E_0 e^{-j\frac{\omega}{c}z}$$

$$H_y = \frac{j}{\eta_0}\frac{\omega}{c} x E_0 e^{-j\frac{\omega}{c}z}$$

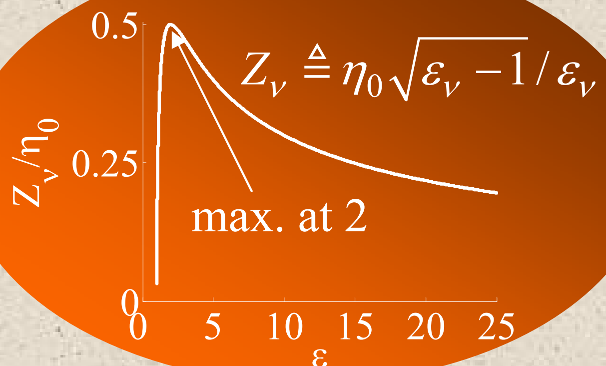
Fields in layer ν

$$k_\nu \triangleq \frac{\omega}{c}\sqrt{\varepsilon_\nu - 1}$$

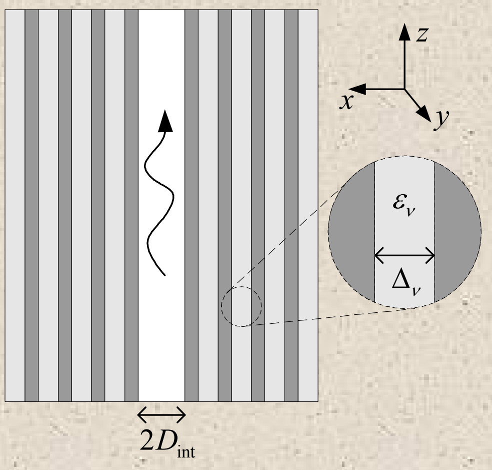
$$E_z = \left[A_\nu e^{-jk_\nu x} + B_\nu e^{+jk_\nu x}\right] e^{-j\frac{\omega}{c}z}$$

$$E_x = \frac{-1}{\sqrt{\varepsilon_\nu - 1}}\left[A_\nu e^{-jk_\nu x} - B_\nu e^{+jk_\nu x}\right] e^{-j\frac{\omega}{c}z}$$

$$H_y = \frac{-1}{Z_\nu}\left[A_\nu e^{-jk_\nu x} - B_\nu e^{+jk_\nu x}\right] e^{-j\frac{\omega}{c}z}$$



Bragg Reflection



Application to Bragg waveguides

- Yeh *et al.*, Opt. Commun. **19**, 427–430, (1976).
- Yeh *et al.*, JOSA, **68**, 1196–1201, (1978).

T – Unit cell Transition matrix of incoming and outgoing amplitudes of transverse waves

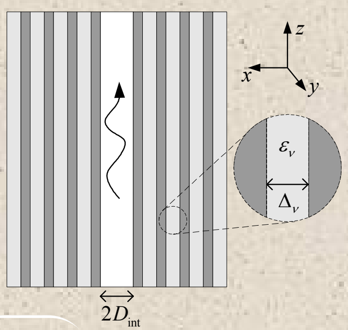
Eigen-value problem $\left|T - e^{-jKL} I\right| = 0$

L–periodicity, *K*–propagation coefficient

Dispersion relation $\cos(KL) = \frac{1}{2}(T_{11} + T_{22})$

Confinement condition $\left(\frac{T_{11} + T_{22}}{2}\right)^2 > 1$

Optimal Confinement



Confinement condition

$$\left(\frac{T_{11}+T_{22}}{2}\right)^2 = \left(\frac{(Z_1+Z_2)^2}{4Z_1Z_2}\cos(\chi_1+\chi_2) - \frac{(Z_1-Z_2)^2}{4Z_1Z_2}\cos(\chi_1-\chi_2)\right)^2$$

$$\chi_{1,2} \triangleq 2\pi\frac{\Delta_{1,2}}{\lambda_0}\sqrt{\varepsilon_{1,2}-1}$$

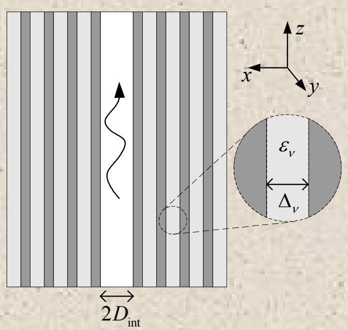
$$\left.\begin{array}{l}\chi_1-\chi_2=0\\ \chi_1+\chi_2=\pi\end{array}\right\} \Rightarrow \left|\frac{T_{11}+T_{22}}{2}\right|_{\max} = \frac{1}{2}\left(\frac{Z_1}{Z_2}+\frac{Z_2}{Z_1}\right)$$

$$\chi_{1,2} = \frac{\pi}{2}$$

$$\Delta_{1,2} = \frac{\lambda_0}{4\sqrt{\varepsilon_{1,2}-1}}$$

Quarter λ structure !!

Decay Coefficient



$$\left.\begin{aligned} \cos(KL) &= \frac{1}{2}(T_{11}+T_{22}) \\ \left|\frac{T_{11}+T_{22}}{2}\right|_{\max} &= \frac{1}{2}\left(\frac{Z_1}{Z_2}+\frac{Z_2}{Z_1}\right) \end{aligned}\right\}$$

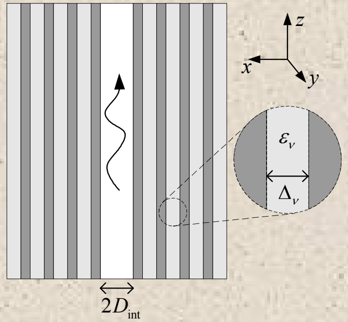
$$\left|e^{-jKL}\right|^n = \begin{cases} \left(\dfrac{Z_1}{Z_2}\right)^n & Z_1 < Z_2 \\ \left(\dfrac{Z_2}{Z_1}\right)^n & Z_1 > Z_2 \end{cases}$$

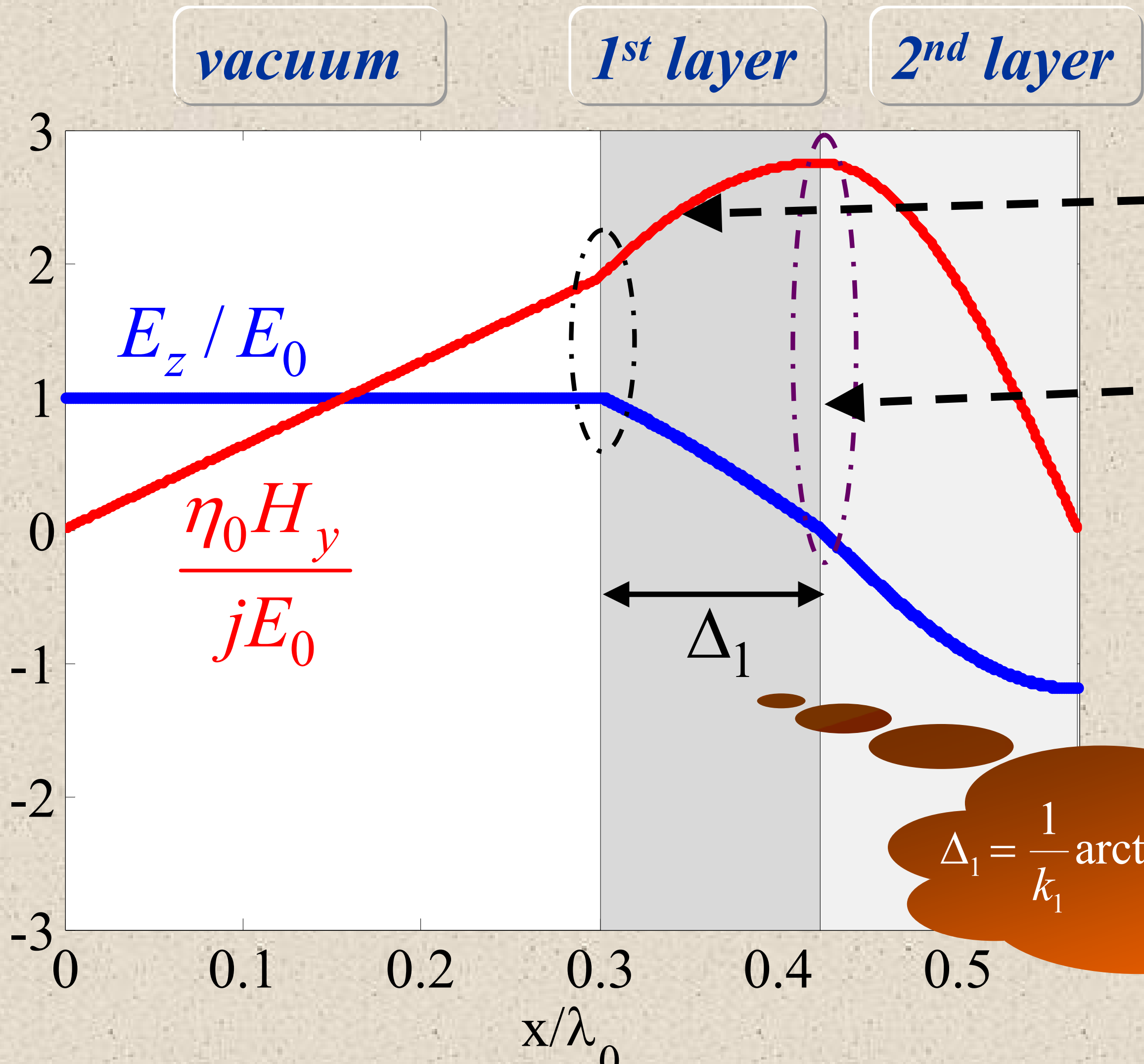
$$\left.\begin{aligned} Z_1 &> Z_2 \\ x &\simeq nL \end{aligned}\right\} \Rightarrow \left(\frac{Z_2}{Z_1}\right)^{2n} \simeq \left(\frac{Z_2}{Z_1}\right)^{2x/L} \triangleq \exp\left(-2\frac{x}{x_c}\right)$$

Given subsequently

$$x_c = \frac{\lambda_0}{4}\left(\frac{1}{\sqrt{\varepsilon_1 - 1}} + \frac{1}{\sqrt{\varepsilon_2 - 1}}\right)\left|\ln^{-1}\left(\frac{\varepsilon_1\sqrt{\varepsilon_2 - 1}}{\varepsilon_2\sqrt{\varepsilon_1 - 1}}\right)\right|$$

Field Confinement



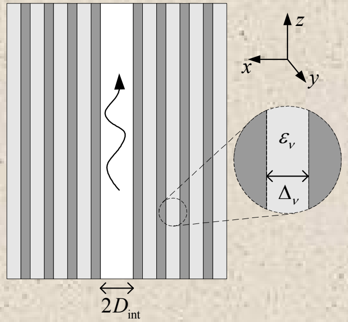


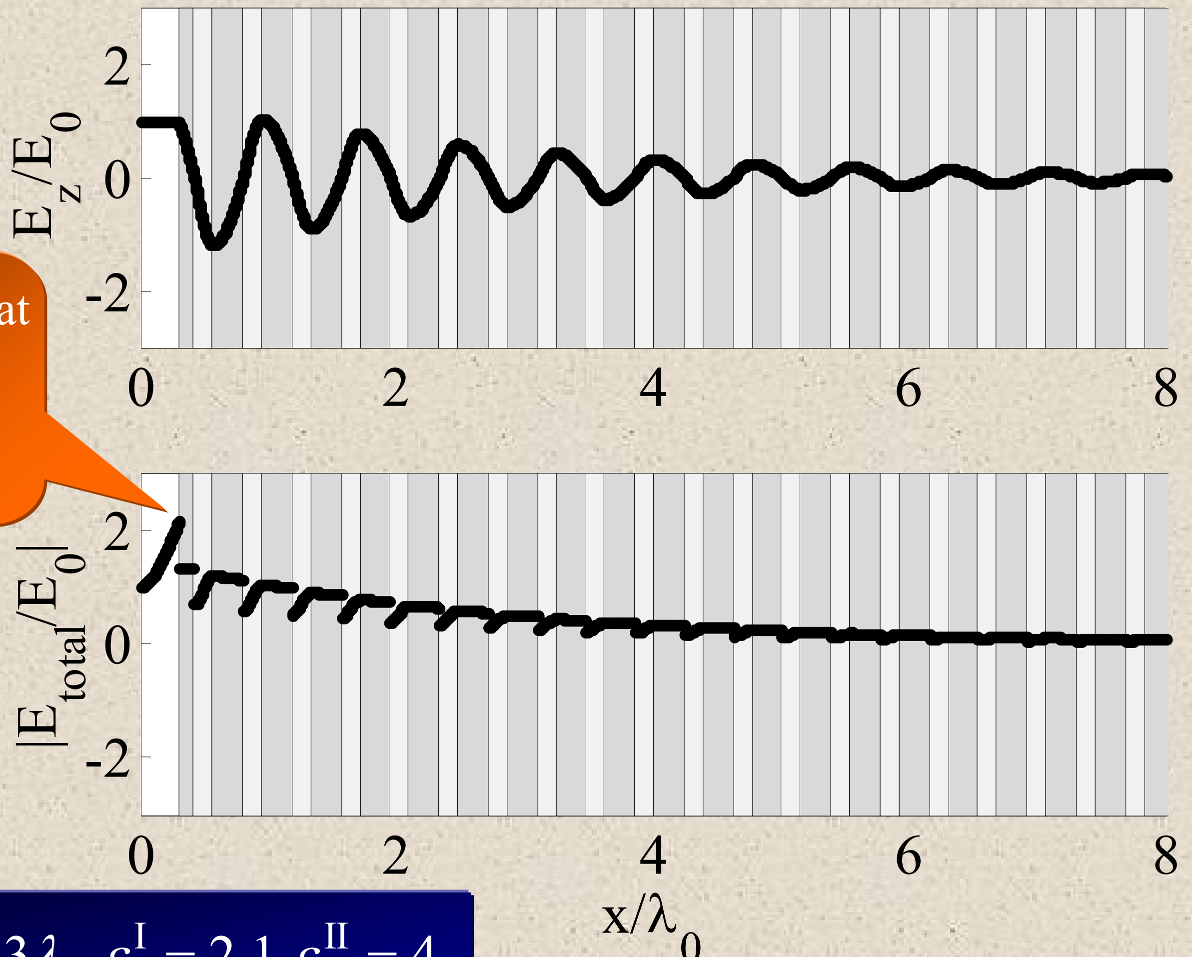
Dictated by vacuum fields.

Perfect reflection condition.

$$\Delta_1 = \frac{1}{k_1}\arctan\left[\left(\frac{Z_1}{\eta_0}\frac{\omega_0}{c}D_{int}\right)^{-1}\right] \qquad (Z_1 > Z_2)$$

Field Confinement – E_z Profile



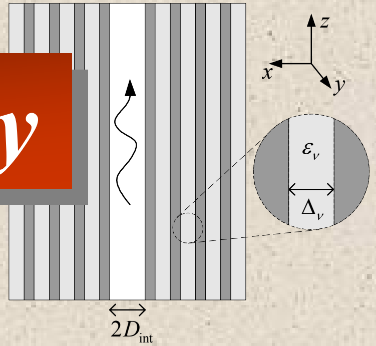


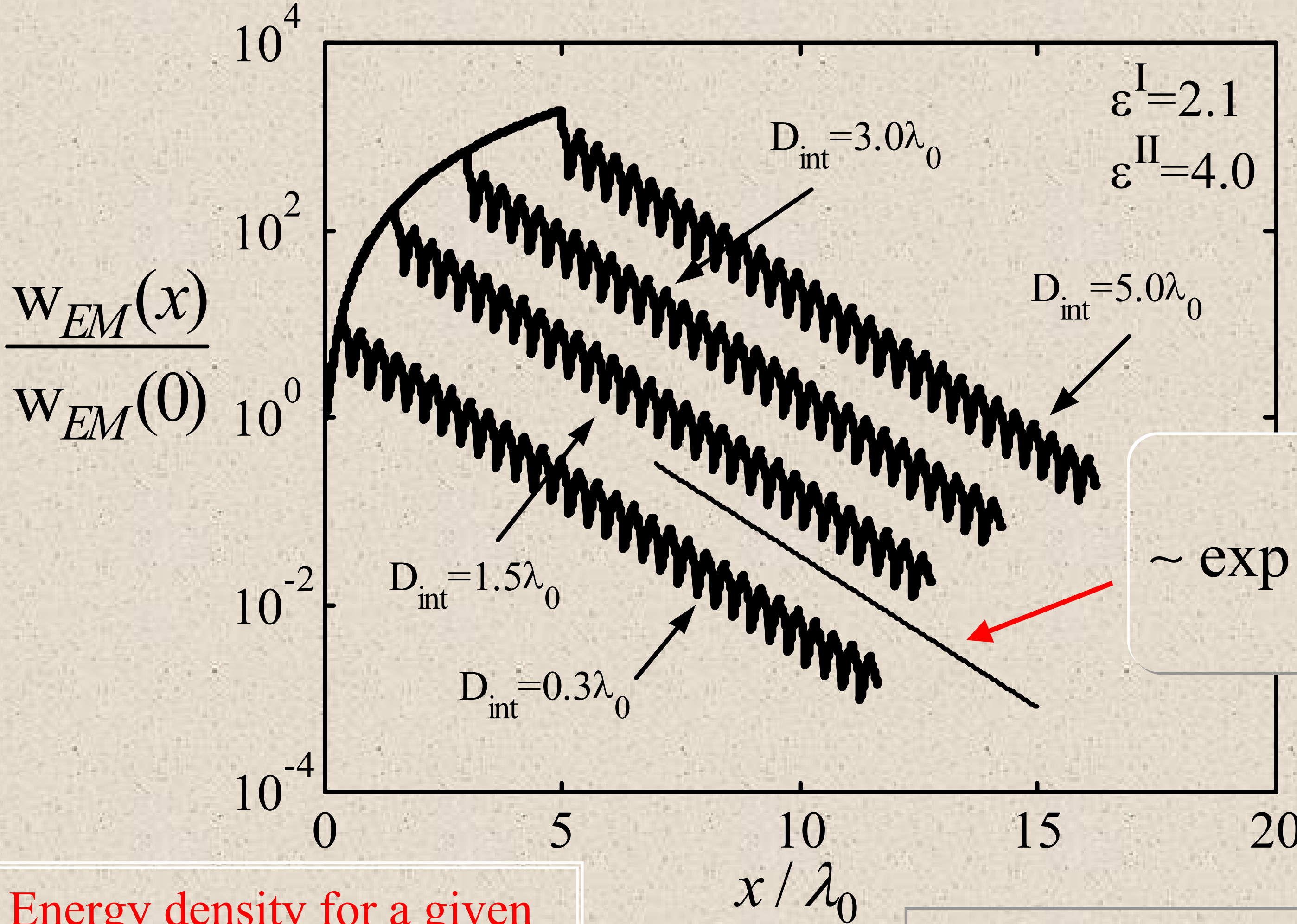
E_z either peaks or diminishes at every discontinuity

Maximum is at vacuum-dielectric interface

$D_{int} = 0.3\lambda_0, \varepsilon^{I} = 2.1, \varepsilon^{II} = 4$

Field Confinement – Energy Decay



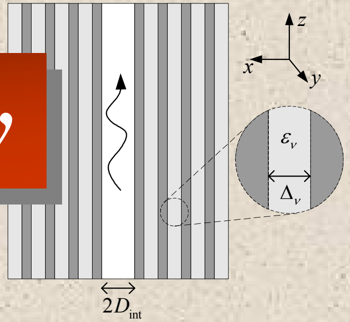


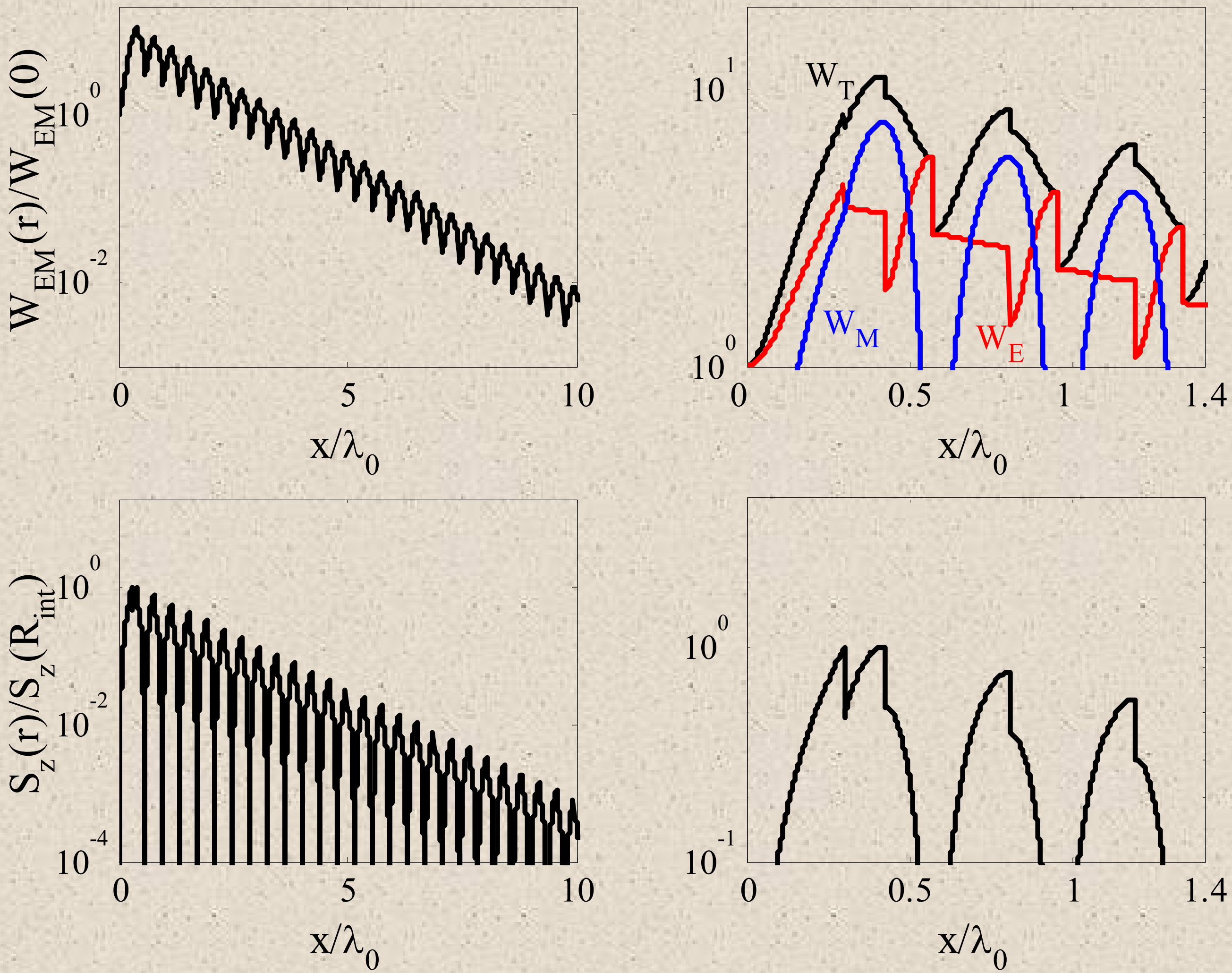
$$\sim \exp\left(-2\frac{x}{x_c}\right)$$

Energy density for a given E_0 increases for larger D_{int}/λ_0 !! Breakdown.

$$x_c = \frac{\lambda_0}{4}\left(\frac{1}{\sqrt{\varepsilon_1 - 1}} + \frac{1}{\sqrt{\varepsilon_2 - 1}}\right)\left|\ln^{-1}\left(\frac{\varepsilon_1\sqrt{\varepsilon_2 - 1}}{\varepsilon_2\sqrt{\varepsilon_1 - 1}}\right)\right|$$

Field Confinement – Energy Decay





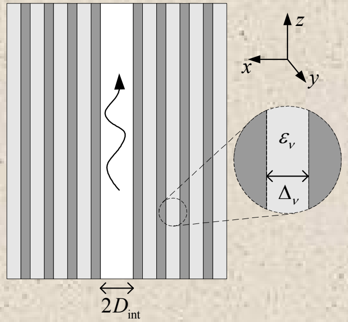
$D_{int} = 0.3\lambda_0$

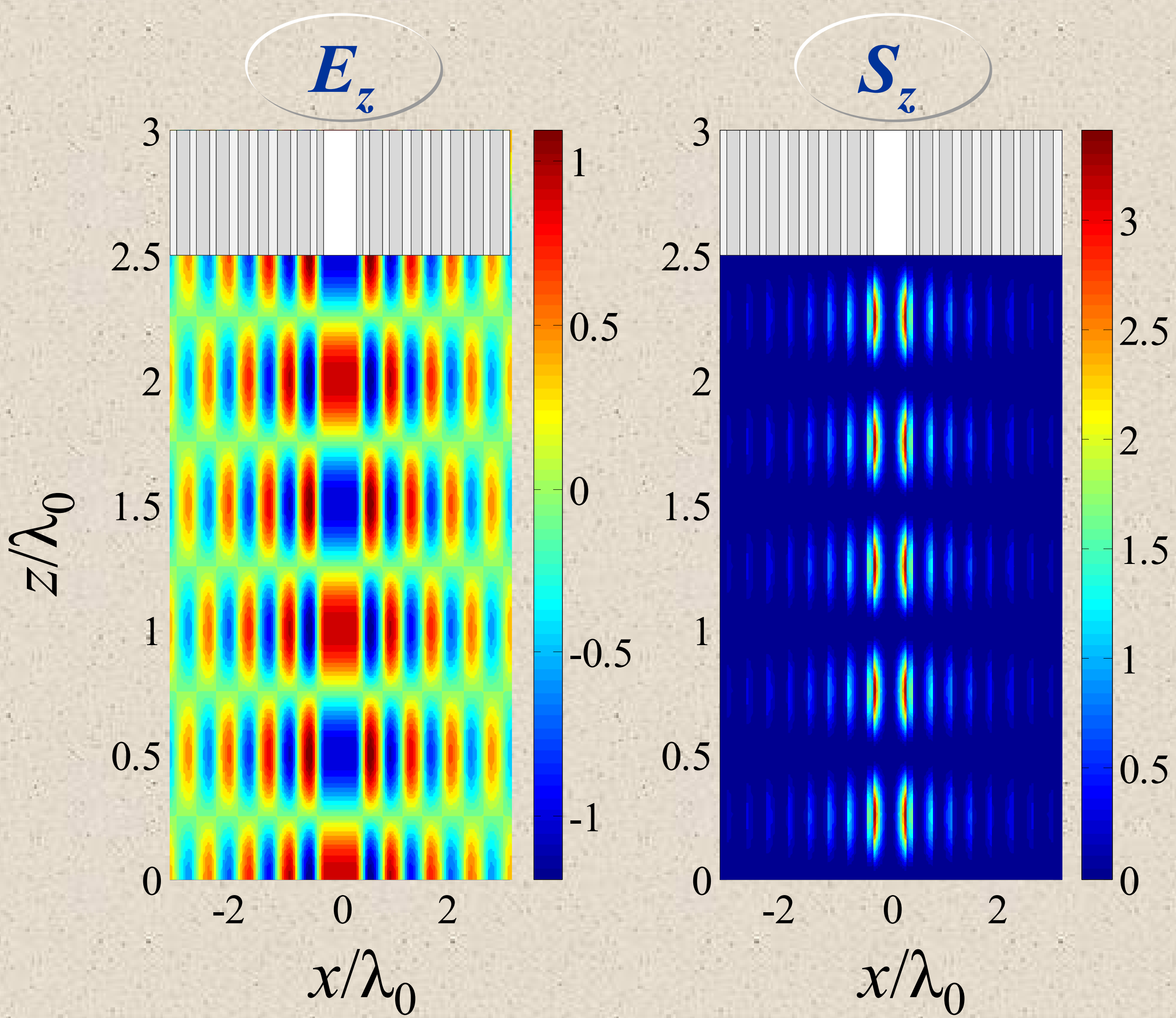
$\varepsilon^{I} = 2.1$

$\varepsilon^{II} = 4$

- Continuous W_M
- Discontinuous W_E
- W_M – zero-points
- E_z – zero-points
- S_z – zero-points

Field Confinement – t=0 Picture



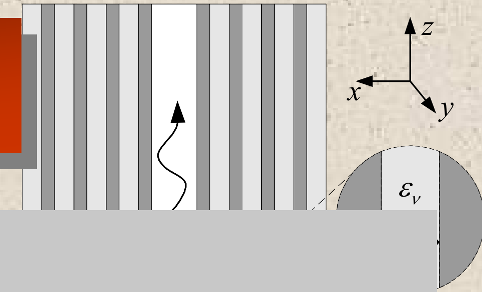


$D_{int} = 0.3\lambda_0$

$\varepsilon^{I} = 2.1$

$\varepsilon^{II} = 4$

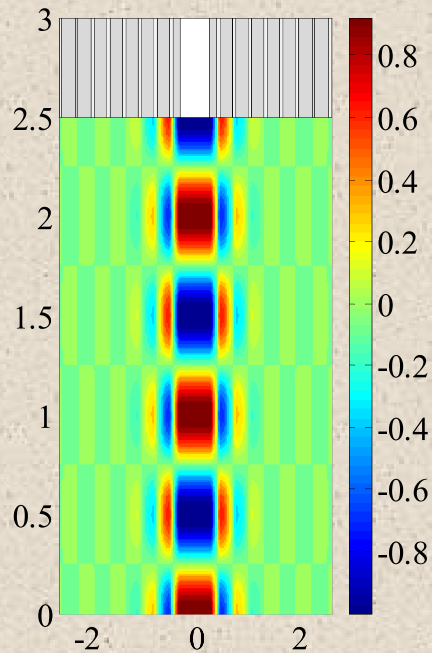
Field Confinement – The Movie

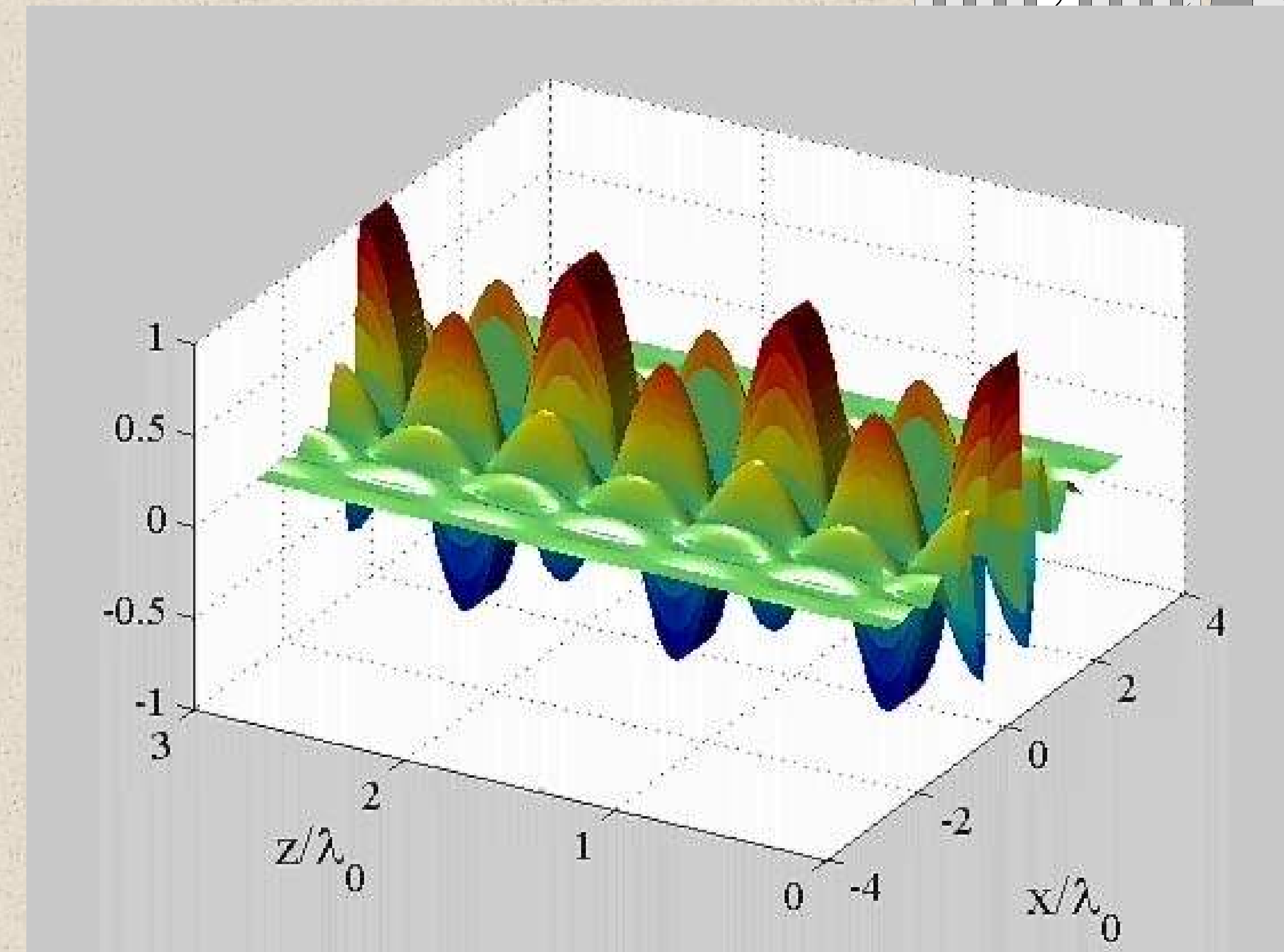


$D_{\mathrm{int}} = 0.3\lambda_0$

$\varepsilon^{\mathrm{I}} = 2.1$

$\varepsilon^{\mathrm{II}} = 16$

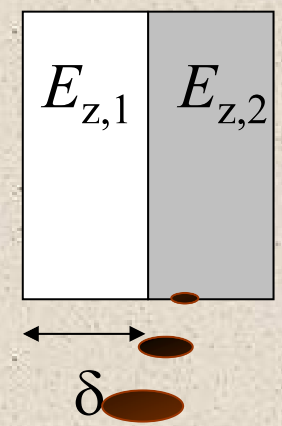




Field Confinement – Cylindrical Case

Generalization: if the impedance in the present layer is larger than in the next layer ($Z_1 > Z_2$), then the condition of minimum energy density is ensured provided the width of the present layer is chosen such that

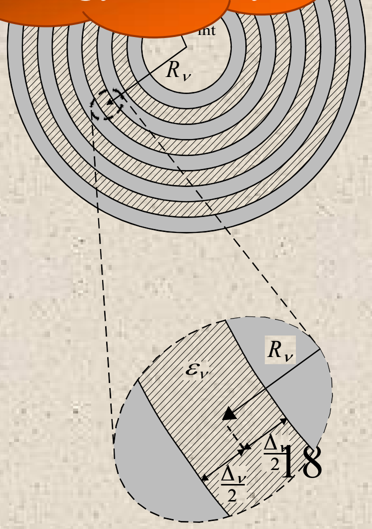
$$E_{z,1}(\delta) = 0.$$



In a similar way, if the impedance in the present layer is smaller than in the next one ($Z_1 < Z_2$) then the width is determined such that the azimuthal magnetic field vanishes

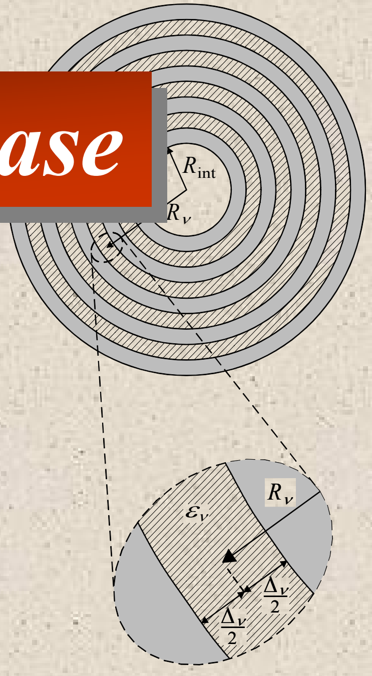
$$\dot{E}_{z,1}(\delta) = 0$$

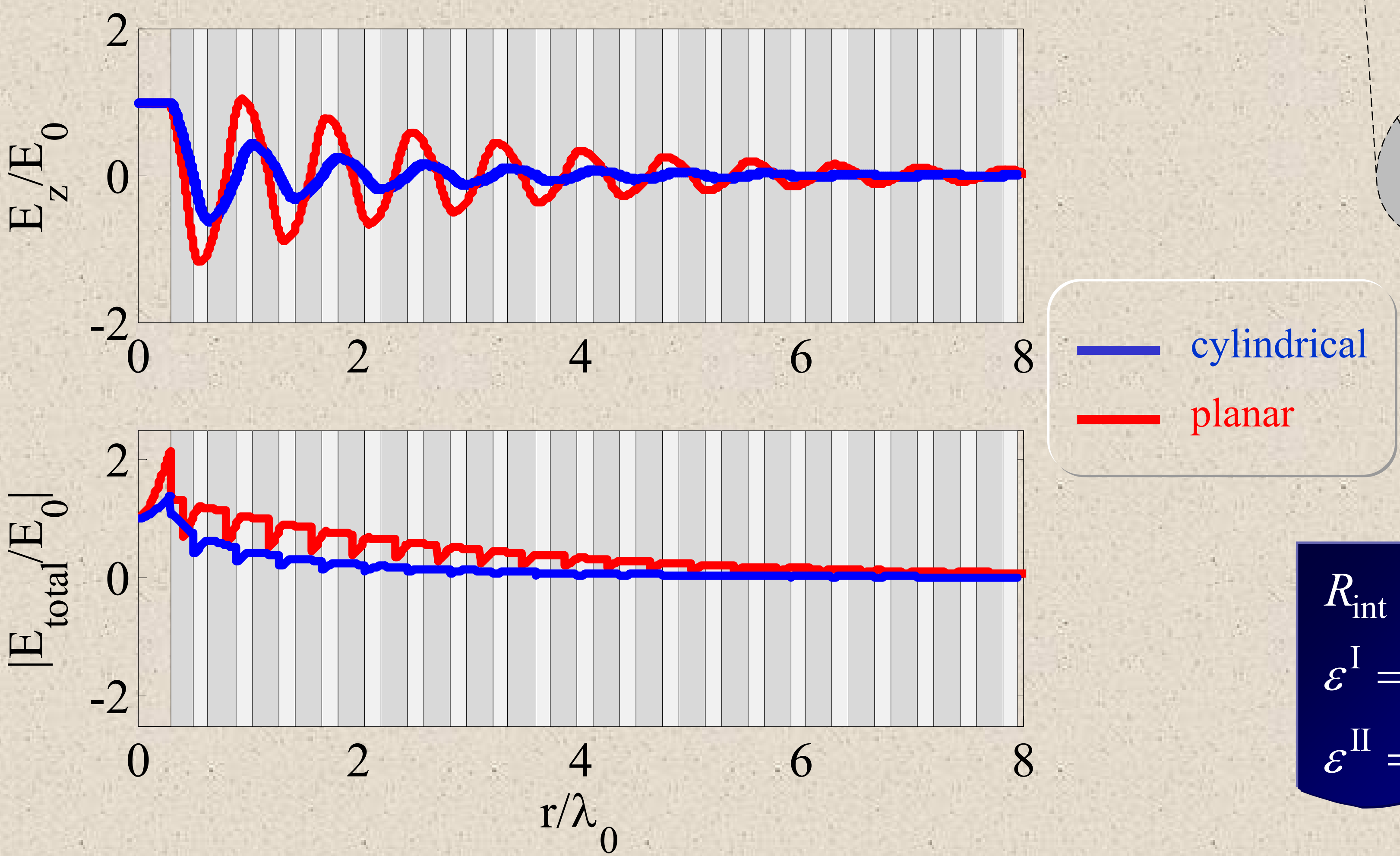
Minimal energy density



$$\begin{cases} E_{z1}(\delta) = 0 & \text{if} \quad Z_2 < Z_1 \\ \dot{E}_{z1}(\delta) = 0 & \text{if} \quad Z_2 > Z_1 \end{cases}$$

Field Confinement – Cylindrical Case



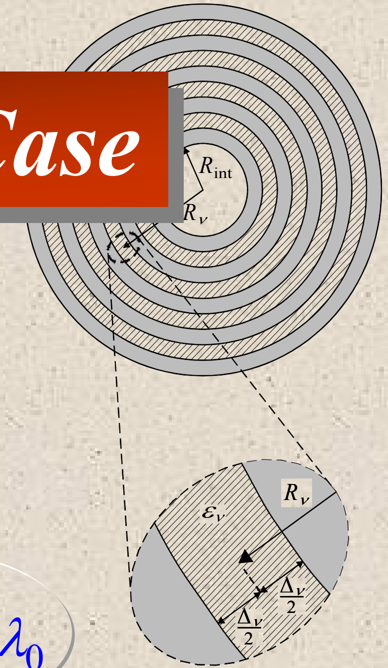


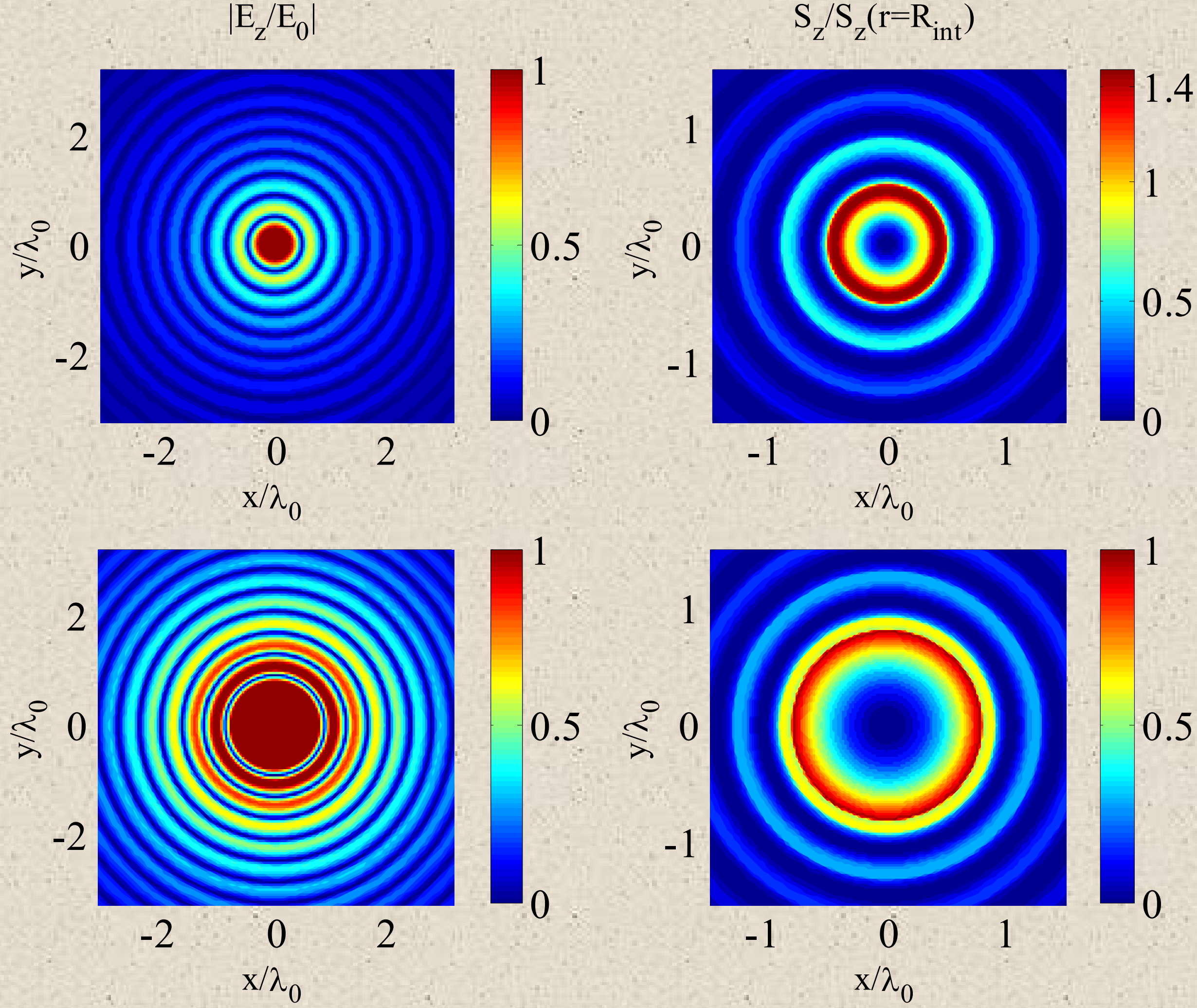
$R_{\text{int}} = 0.3\lambda_0$

$\varepsilon^{\text{I}} = 2.1$

$\varepsilon^{\text{II}} = 4$

Field Confinement – Cylindrical Case





$R_{int} = 0.3\lambda_0$

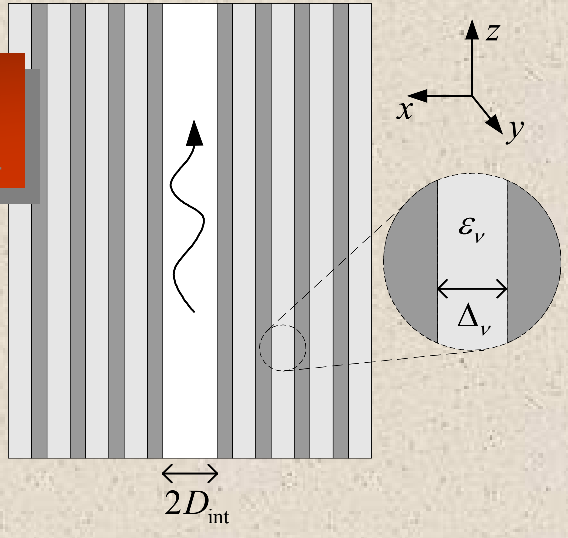
$R_{int} = 0.8\lambda_0$

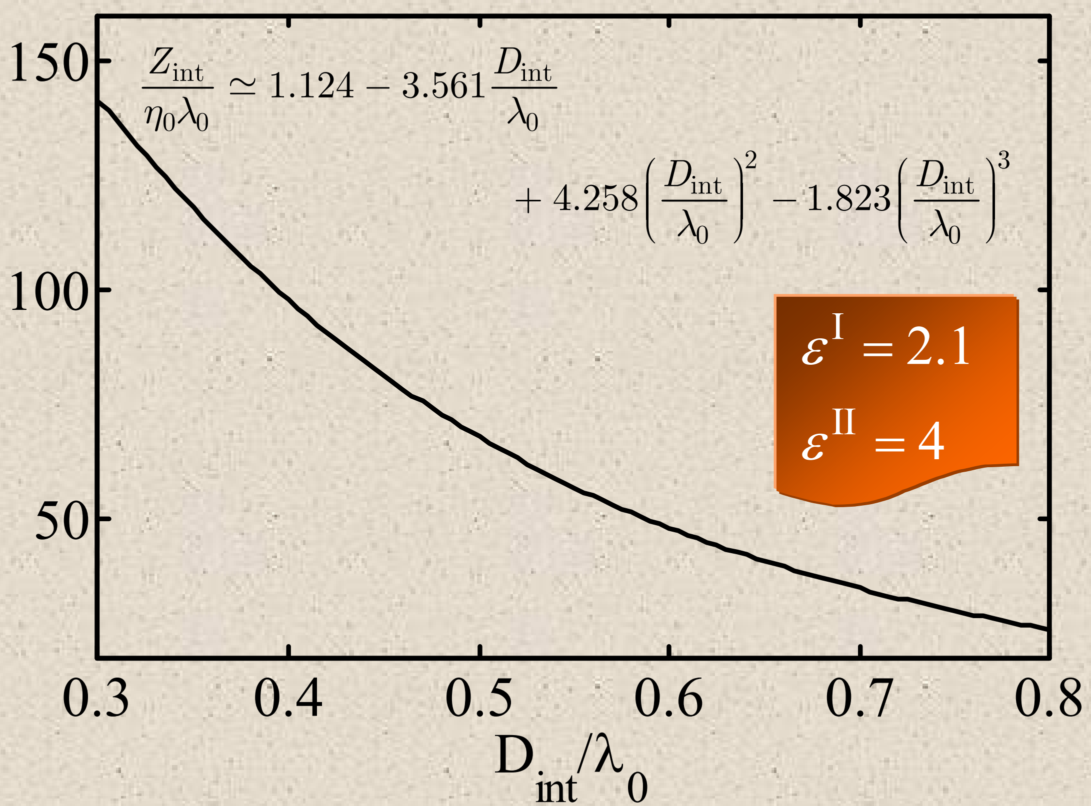
$\varepsilon^{I} = 2.1$

$\varepsilon^{II} = 4$

Accelerator Parameters – Z_{int}

A measure for the amount of power flowing in the system for a given accelerating field.





$\varepsilon^{I} = 2.1$

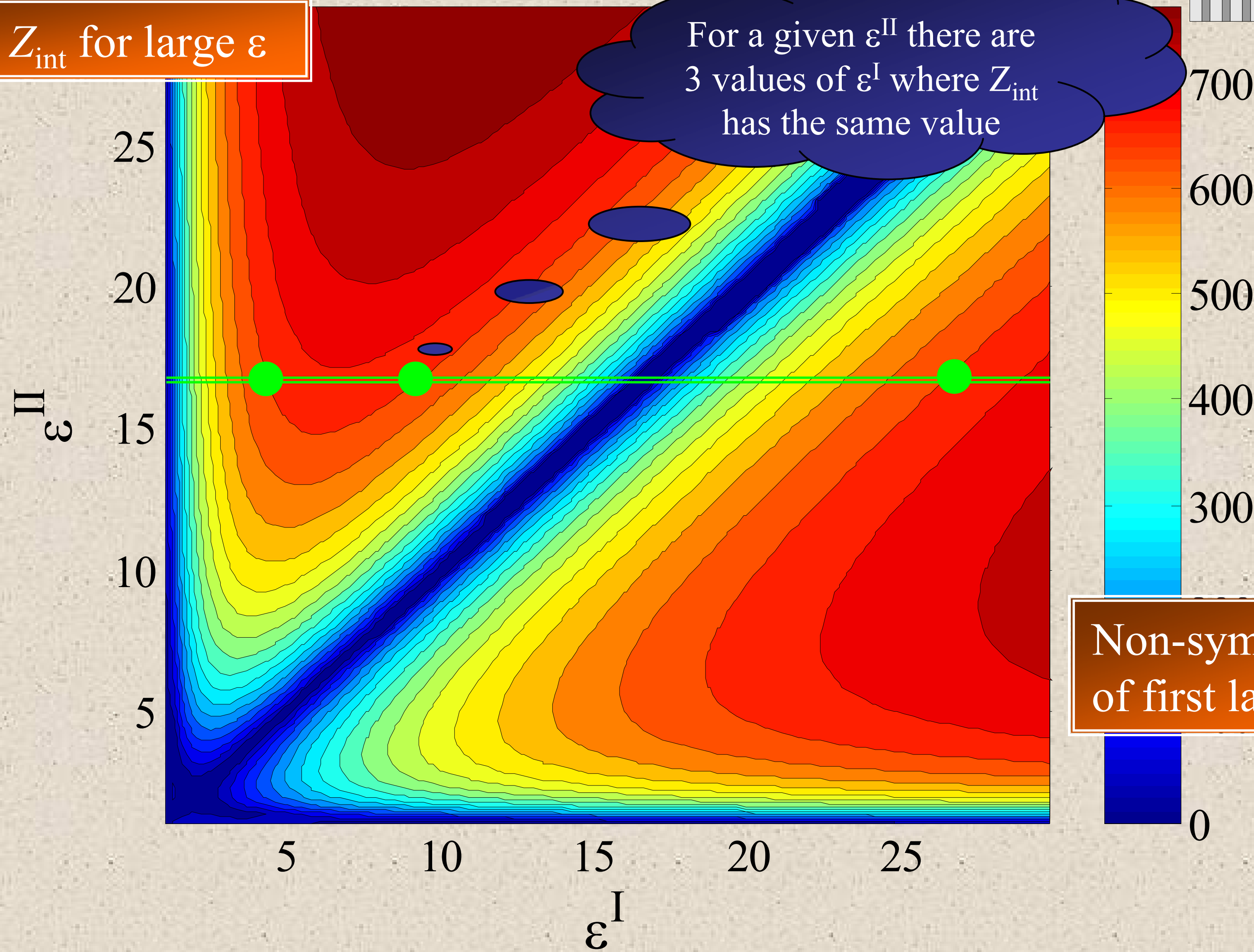
$\varepsilon^{II} = 4$

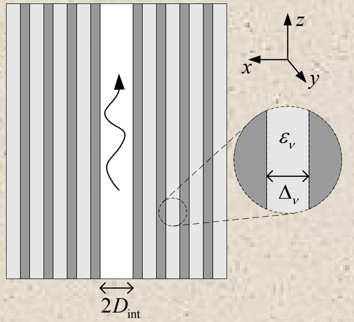
$$Z_{int}\,[\Omega m] \triangleq \frac{(\lambda_0 E_0)^2}{P}$$

$$P \triangleq \int_{-\infty}^{\infty} dx\, S_z(x)$$

Accelerator Parameters – Z_{int}

High Z_{int} for large ε

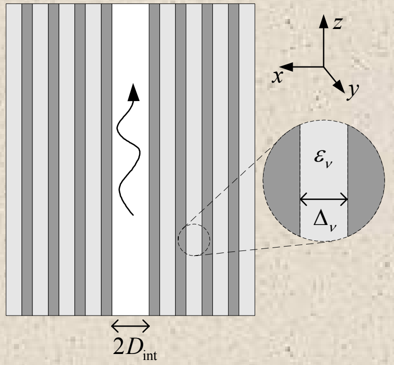




$D_{int} = 0.3\lambda_0$

Non-symmetry: role of first layer

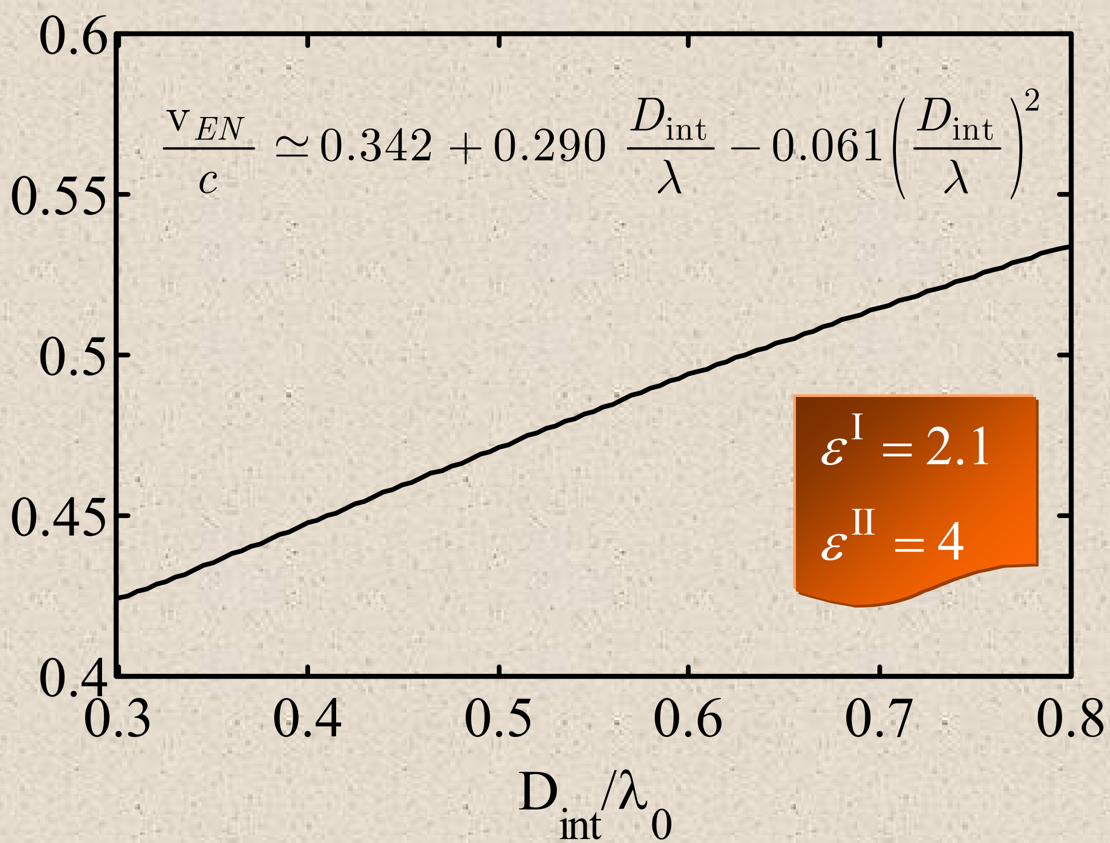
Accelerator Parameters – v_{EN}



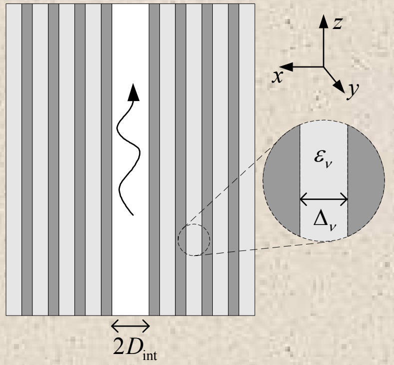
Equals the group velocity.

$$\frac{\mathrm{v}_{EN}}{c} \triangleq \frac{P}{cW}$$

$$W \triangleq \int_{-\infty}^{\infty} dx\, w_{EM}(x)$$



Accelerator Parameters – E_{max}



Planar

$$E_z = E_0 e^{-j\frac{\omega}{c}z}$$

$$E_x = j\frac{\omega}{c} x E_0 e^{-j\frac{\omega}{c}z}$$

$$\frac{E_{max}}{E_0} = \sqrt{1 + \left(2\pi \frac{D_{int}}{\lambda_0}\right)^2}$$

2 [GV/m]

1 [GV/m]

$$\frac{E_{max}}{E_0} = 2$$

$$D_{int} \simeq 0.28\lambda_0$$

$$R_{int} \simeq 0.55\lambda_0$$

Cylindrical

$$E_z = E_0 e^{-j\frac{\omega}{c}z}$$

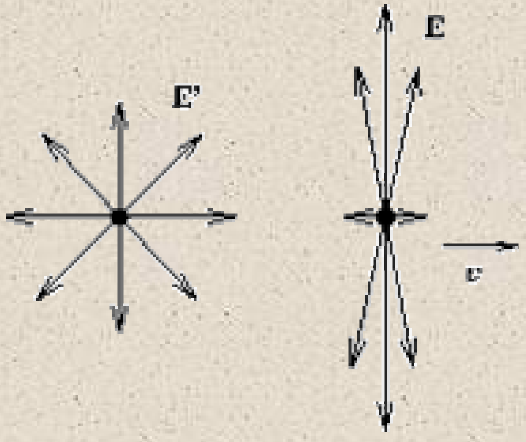
$$E_r = j\frac{\omega}{c}\frac{1}{2} r E_0 e^{-j\frac{\omega}{c}z}$$

$$\frac{E_{max}}{E_0} = \sqrt{1 + \left(\pi \frac{R_{int}}{\lambda_0}\right)^2}$$

Wake-Fields – Fields of Moving Charges

Charge in free space

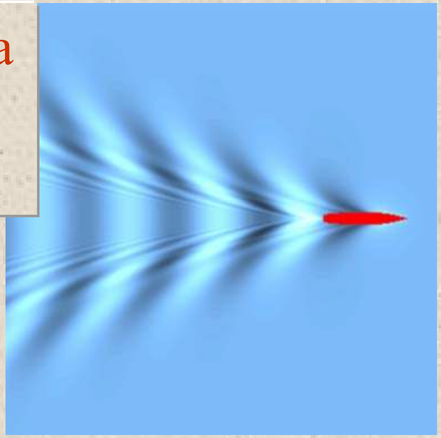
At high speed the field shrinks in the direction of motion.



Cerenkov radiation

A charge that exceeds the speed of light in the material (*c*/*n*) emits radiation.

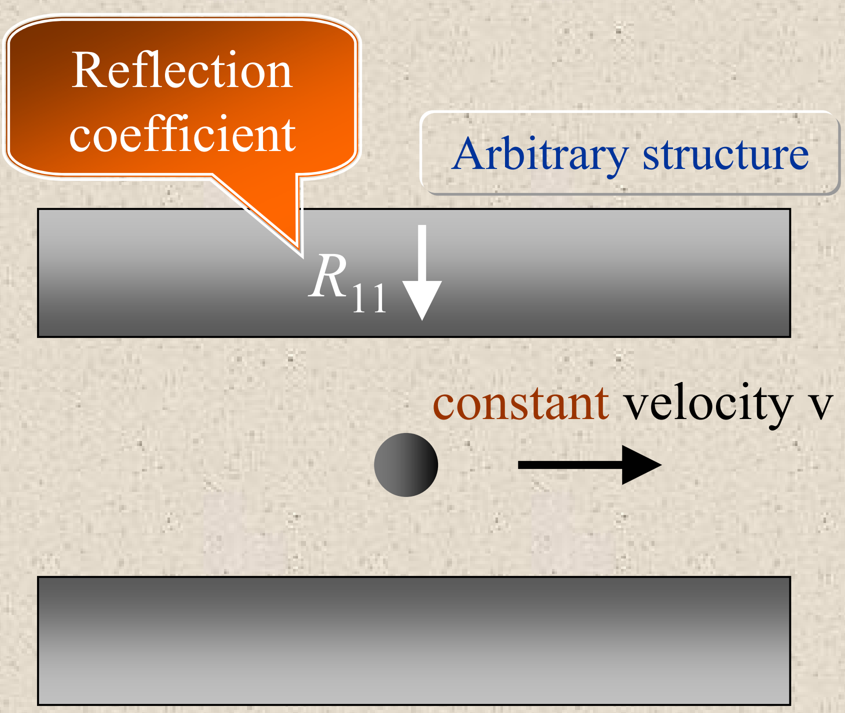
Similarly to a ship moving in water



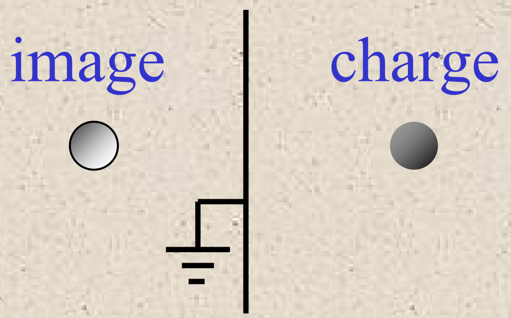
Wake-Field – General Approach

Primary field: free-space field of the moving charge.

Secondary field: structure effect.

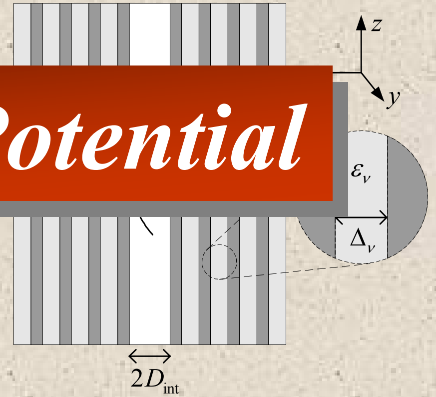


Static analogy



Schächter *et al.*, Phys. Rev. E, **68**, 036502 (2003).

Wake-Field – Magnetic Vector Potential



Moving line charge

q – charge per unit length

$$J_z(x,z,t) = -q\,\mathrm{v}\,\delta(x)\delta(z-\mathrm{v}t)$$

$$\beta \triangleq \mathrm{v}/c$$

$$\gamma \triangleq 1/\sqrt{1-\mathrm{v}^2/c^2}$$

$$\Gamma \triangleq \frac{|\omega|}{c\gamma\beta}$$

$$\Lambda \triangleq \frac{|\omega|}{c}\sqrt{\varepsilon_1 - \beta^{-2}}$$

Primary potential

$$A_z^{(p)}(x,z,t) = -\frac{q\mu_0}{4\pi}\int_{-\infty}^{\infty} d\omega\, e^{j\omega(t-z/\mathrm{v})}\frac{1}{\Gamma}e^{-\Gamma|x|}$$

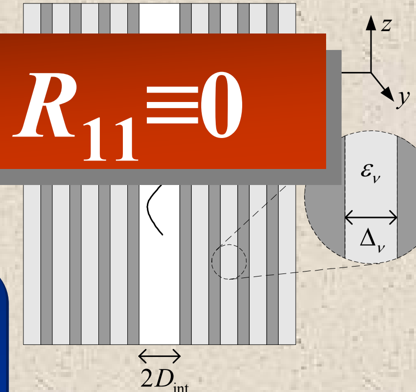
Secondary potential

$$A_z^{(s)}(x,z,t) = -\frac{q\mu_0}{4\pi}\int_{-\infty}^{\infty} d\omega\, e^{j\omega(t-z/\mathrm{v})}\frac{1}{\Gamma}\begin{cases} B_0\cosh(\Gamma x) & |x| < D_{\text{int}} \\ -\dfrac{\varepsilon_1}{\gamma^2\beta^2\left(\varepsilon_1-\beta^{-2}\right)}\left(C_0 e^{-j\Lambda x} + D_0 e^{j\Lambda x}\right) & x > D_{\text{int}} \end{cases}$$

All waves have $k_z=\omega/\mathrm{v}$!!

$$R_{11} \triangleq \frac{D_0}{C_0}e^{2j\Lambda D_{\text{int}}}$$

Wake-Field – Decelerating Force $R_{11} \equiv 0$



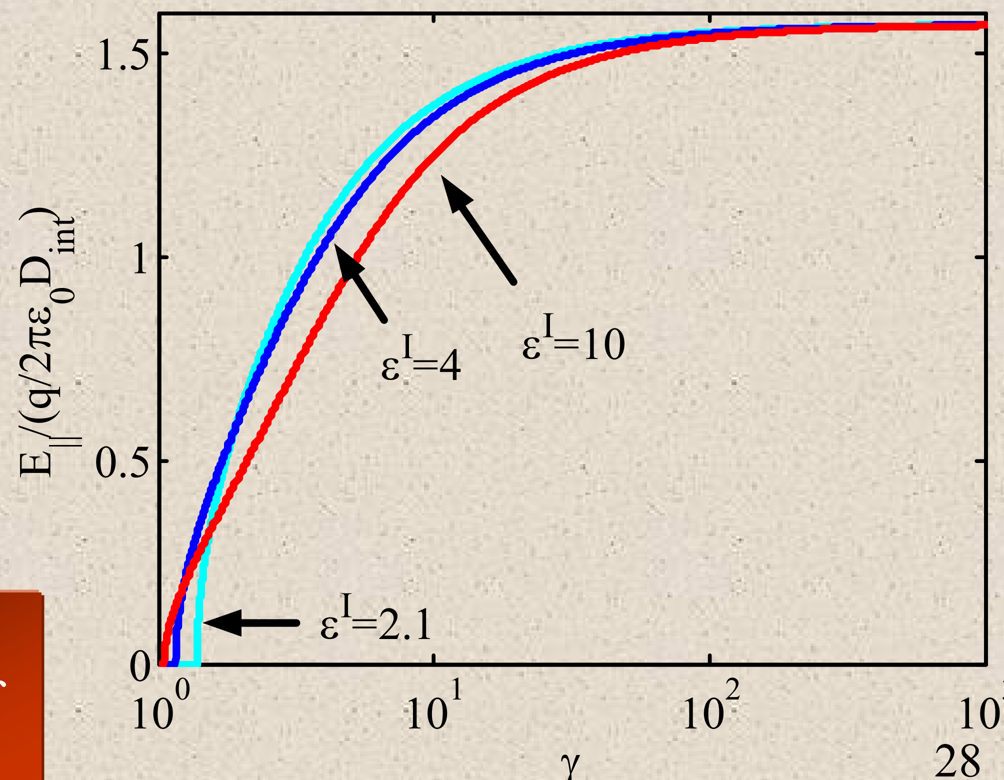
$$E_{\|} \triangleq E_z^{(s)}(x=0, z=vt, t) = \frac{-q}{2\pi\varepsilon_0 D_{\text{int}}} \mathrm{Re}\left[j \ln\left(1 + j\frac{\gamma\sqrt{\beta^2\varepsilon_1 - 1}}{\varepsilon_1} \right) \right]$$

Ultra-relativistic regime $\gamma \to \infty$

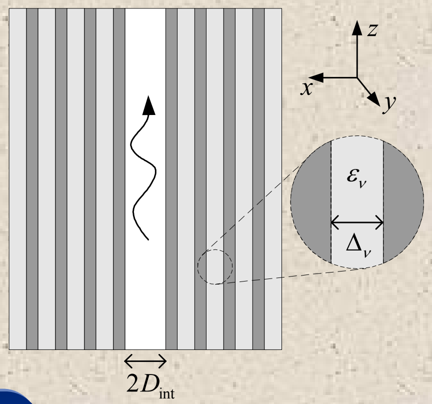
$$E_{\|} = \frac{q}{2\pi\varepsilon_0 D_{\text{int}}} \times \frac{\pi}{2}$$

Static line-charge field at D_{int}

In fact – independent of structure!!



Wake-Field On Axis



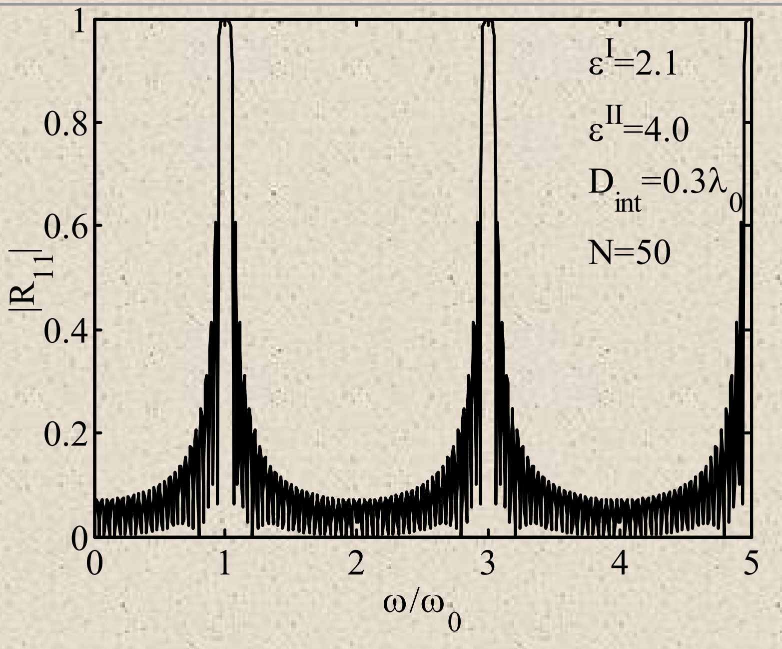
Ultra-relativistic wake-field on axis $\gamma \to \infty$

$$E_z^{(s)}(\bar{\tau}) = \frac{q}{2\pi\varepsilon_0 D_{\text{int}}} \times \frac{1}{2}\int_{-\infty}^{\infty} d\bar{\omega}\, e^{j\bar{\omega}\bar{\tau}} \frac{1+R_{11}(\bar{\omega})}{(1+j\bar{\omega})-(1-j\bar{\omega})R_{11}(\bar{\omega})}$$

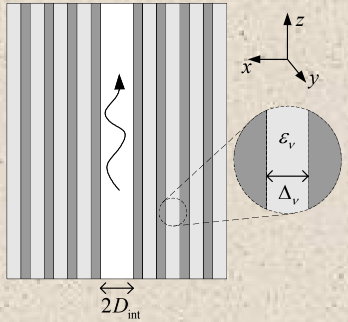
$$\bar{\omega} \triangleq \frac{\omega}{c} D_{\text{int}} \frac{\sqrt{\varepsilon_1 - 1}}{\varepsilon_1}$$

$$\bar{\tau} \triangleq \left(t - \frac{z}{c}\right)\frac{c}{D_{\text{int}}}\frac{\varepsilon_1}{\sqrt{\varepsilon_1 - 1}}$$

Reflection coefficient (analytic expression)



Wake-Field – t=0 Picture

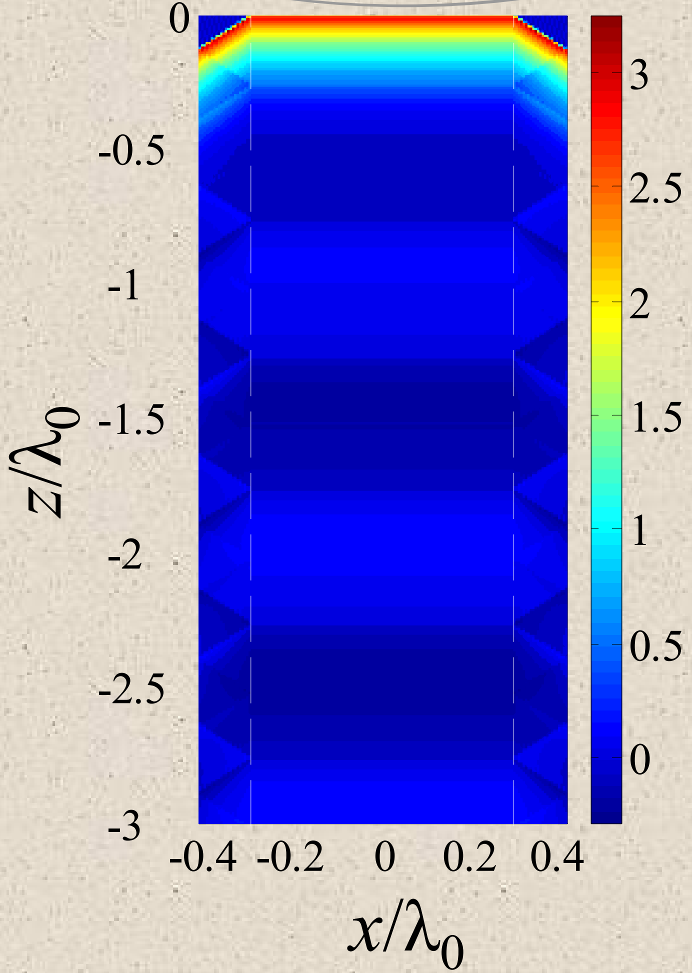


$D_{int} = 0.3\lambda_0$

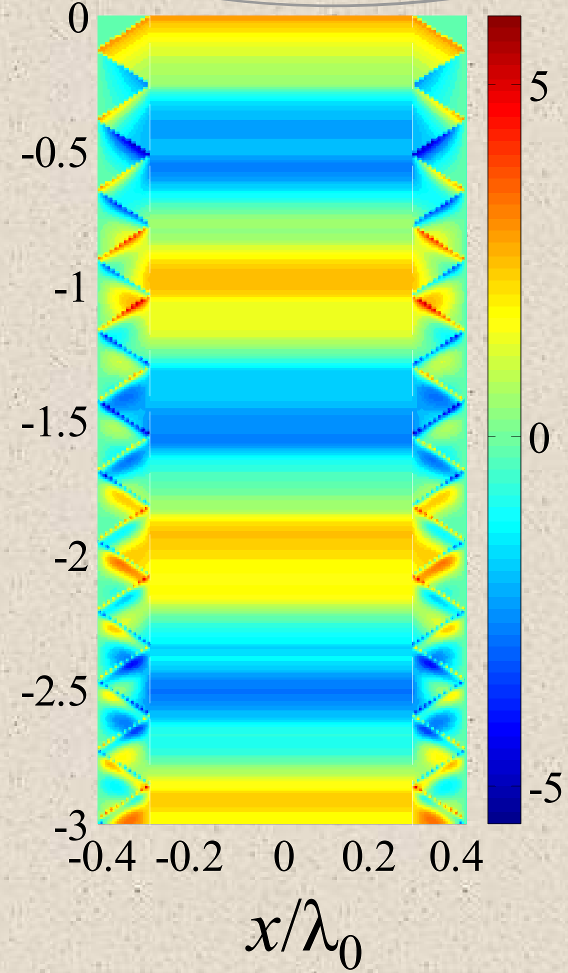
$\varepsilon^{I} = 2.1$

$\varepsilon^{II} = 4$

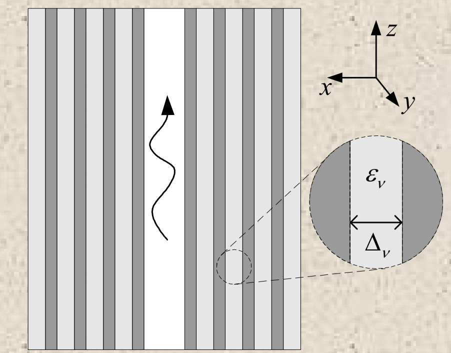
Bragg Structure



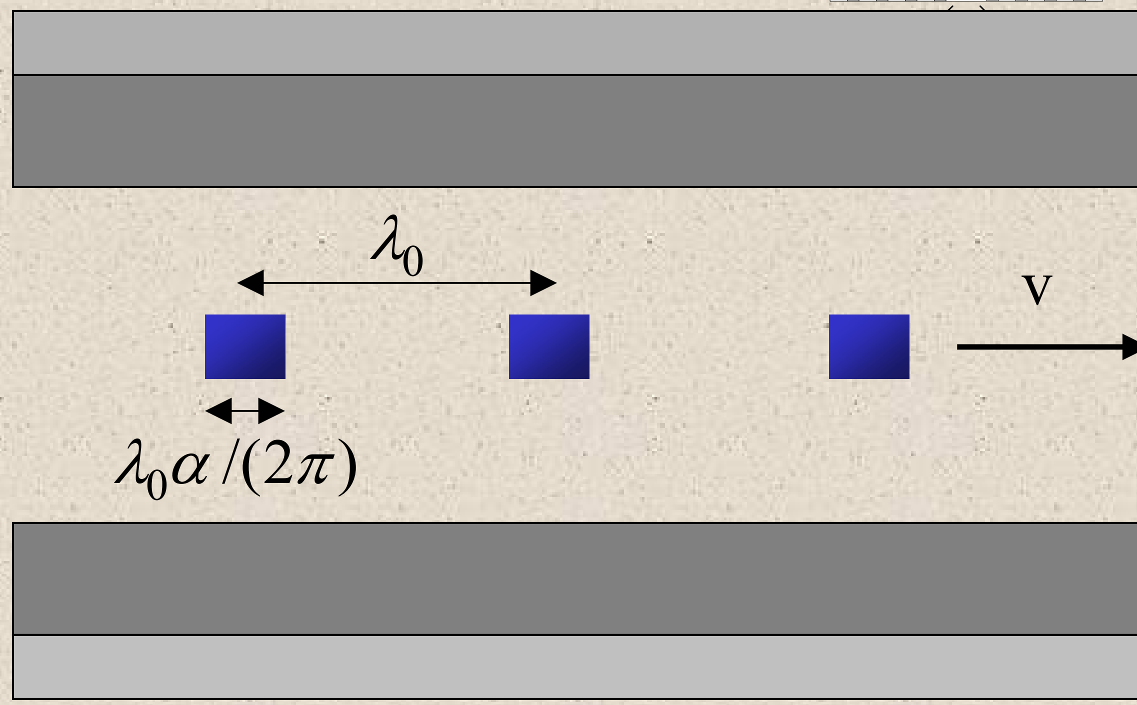
Dielectric Loaded



Train of Micro-Bunches



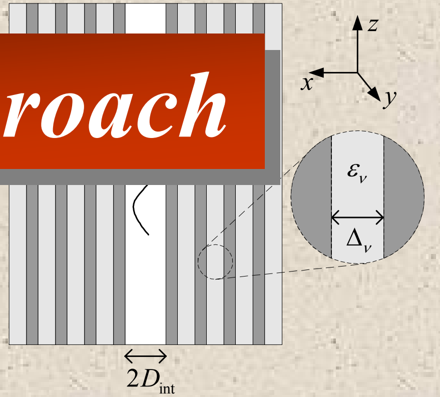
Total charge -q is split into M+1 micro bunches.



Only secondary fields contribute to the emitted power.

Causality entails that each micro-bunch affects only the trailing micro-bunches.

Emitted Power – Qualitative Approach



one line-charge

$$P = \frac{\mathrm{v}q^2}{2\pi\varepsilon_0 D_{\text{int}}} \times \frac{\pi}{2}$$

Total power neglecting mutual effects.
$q = N_{el} q_{el}$, $\alpha = 0$

$$P = \frac{\mathrm{v}q_{el}^2 N_{el}^2 / (M+1)^2}{2\pi\varepsilon_0 D_{\text{int}}} \times \frac{\pi}{2} \times (M+1) = \frac{\mathrm{v}q_{el}{}^2}{2\pi\varepsilon_0 D_{\text{int}}} \times \frac{N_{el}^2}{(M+1)} \times \frac{\pi}{2}$$

$\sim 1/(M+1)$

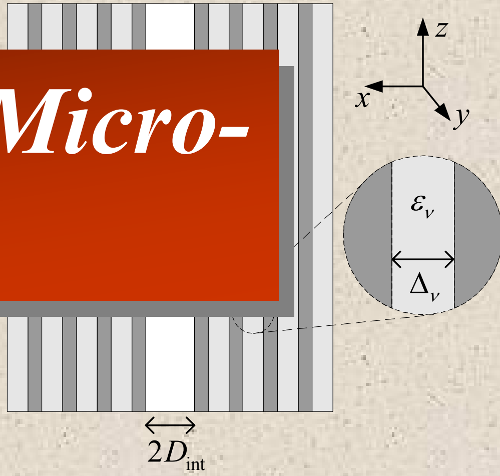
One micro-bunch (line-charge)

$$P = \frac{\mathrm{v}q_{el}^2}{2\pi\varepsilon_0 D_{\text{int}}} \times \frac{\pi}{2} \times N_{el}^2$$

Randomly distributed

$$P = \frac{\mathrm{v}q_{el}^2}{2\pi\varepsilon_0 D_{\text{int}}} \times \frac{\pi}{2} \times N_{el}$$

Emitted Power from a Train of Micro-Bunches – Exact Expression

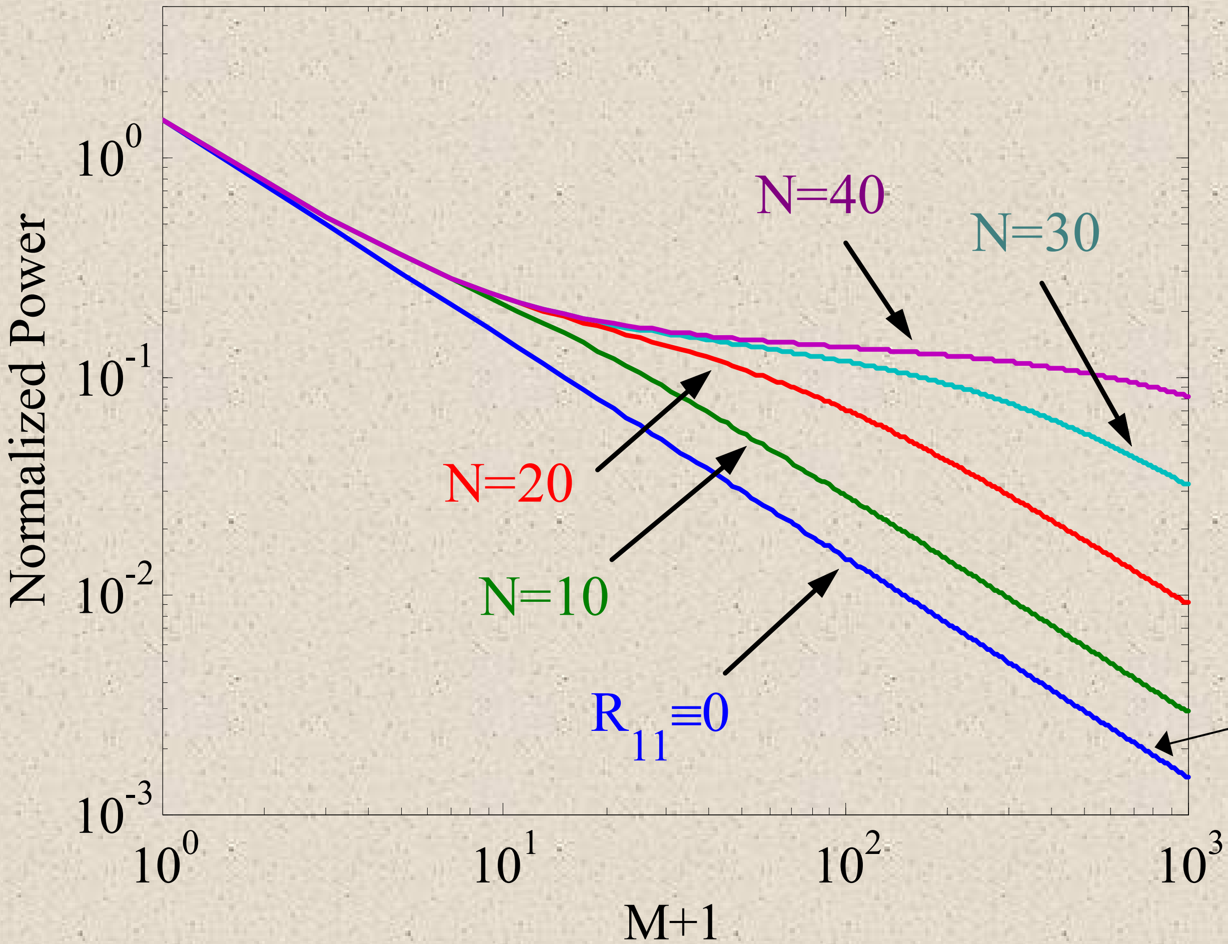


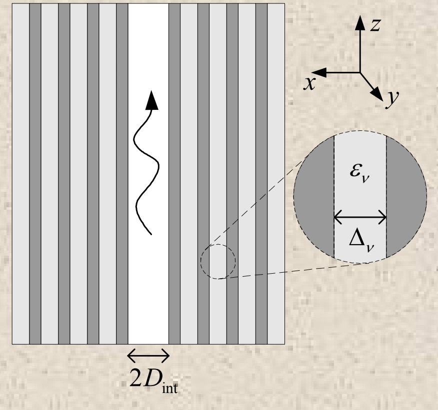
$$P = \frac{\mathrm{v}q^2}{2\pi\varepsilon_0 D_{\rm int}} \frac{1}{2} \int_{-\infty}^{\infty} d\bar{\omega} \frac{1 + R_{11}(\bar{\omega})}{(1 + j\bar{\omega}) - (1 - j\bar{\omega}) R_{11}(\bar{\omega})}$$

$$\times \operatorname{sinc}^2\left[\frac{\alpha}{2}\frac{\bar{\omega}}{\bar{\omega}_0}\right] \frac{\operatorname{sinc}^2\left[\pi \frac{\bar{\omega}}{\bar{\omega}_0}(M+1)\right]}{\operatorname{sinc}^2\left[\pi \frac{\bar{\omega}}{\bar{\omega}_0}\right]}$$

Emitted Power

$$P=\frac{\mathrm{v}q^2}{2\pi\varepsilon_0 D_{\mathrm{int}}}\frac{1}{2}\int_{-\infty}^{\infty}d\bar{\omega}\frac{1+R_{11}(\bar{\omega})}{(1+j\bar{\omega})-(1-j\bar{\omega})R_{11}(\bar{\omega})}$$
$$\times\,\mathrm{sinc}^2\left[\frac{\alpha}{2}\frac{\bar{\omega}}{\bar{\omega}_0}\right]\frac{\mathrm{sinc}^2\left[\pi\frac{\bar{\omega}}{\bar{\omega}_0}(M+1)\right]}{\mathrm{sinc}^2\left[\pi\frac{\bar{\omega}}{\bar{\omega}_0}\right]}$$





~1/(M+1)

Summary

- Detailed design of Bragg acceleration structures – theoretical feasibility was shown!
- Structure parameters suitable for acceleration purposes (interaction impedance, energy velocity, maximal field).
- Interaction impedance over 10 times larger than that of PBG [Lin, Phys. Rev. STAB, **4**, 051301 (2001)].
- Better materials can dramatically improve performance.
- Analysis of Wake-field – power decreases with the number of micro-bunches, and increases with the number of layers.