

# Optical generation of a precise microwave frequency comb by harmonic frequency locking

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A semiconductor laser under negative optoelectronic feedback is applied to the generation of a microwave frequency comb through the nonlinear dynamics. The laser system is operated in a harmonic frequency-locked pulsing state, where its power spectrum is a microwave frequency comb that consists of multiples of a locking frequency. Every frequency component of the comb can be simultaneously stabilized by simply injecting an external microwave modulation at any component of the comb. This phenomenon can be viewed as a kind of microwave injection locking of the laser dynamics. © 2007 Optical Society of America  
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Advances in semiconductor laser nonlinear dynamics have paved the way for its application in microwave photonics [1–3]. Under proper optical or electrical perturbations, a semiconductor laser can be found in different dynamical states that its optical intensity changes rapidly at a microwave frequency. The generated waveforms range from broadband to narrowband signals. Continuous broadband waveforms can be generated from the chaotic states [4], while pure narrowband signals can be obtained from the periodic oscillation states [5]. In this Letter, we focus on the optical generation of a microwave frequency comb, where a discrete broadband signal is produced from the frequency-locked state [3].

We consider a semiconductor laser subject to negative optoelectronic feedback [6]. The dynamics of the system is governed by two important frequencies. One of them is the loop frequency  $f_0$ , which is equal to the reciprocal of the feedback delay time. The other is the pulsing frequency  $f_p$ , which originates from the pulsation sustained by the feedback. The pulsing frequency usually emerges at the proximity of the relaxation resonance frequency or its subharmonic, depending on the bandwidth of the feedback loop [4,6]. The two frequencies  $f_0$  and  $f_p$  are mixed through the laser nonlinearity. The nonlinearity is attributed to the fact that the optical gain is a function of the electron and photon densities. A variety of nonlinear states can be obtained depending on the values of  $f_0$  and  $f_p$  [6]. For an arbitrary value of  $f_p/f_0$ , the frequency mixing usually generates a complicated spectrum that corresponds to quasi-periodic or chaotic pulsing. But if  $f_p/f_0$  equals an integer  $p$ , then the mixing can lock the two frequencies together. The result is a clean spectrum that consists of the linear combinations of  $f_p$  and a locking frequency  $f_l = lf_0$ , where  $l$  is an integer. The laser is said to be in an  $l:p$  harmonic frequency-locking state [3]. Furthermore, when  $p/l$  is also an integer, the frequency mixing can generate only the harmonics of  $f_l$ ; an evenly spaced microwave frequency comb is thus generated. The harmonic frequency locking states are part of the nonlinear dy-

namics. They are neighboring states of quasi-periodic and regular pulsing states in the parameter space [6]. The values of  $l$  and  $p$  are determined according to the period-adding route [3].

Just as in many other optical microwave generation systems, our system is susceptible to noise sources such as spontaneous emission and fluctuations in the operation parameters. As a result, each comb frequency has a certain linewidth. In this Letter, we demonstrate experimentally that these linewidths can be simultaneously narrowed through injecting a single, weak, and pure external microwave modulation. When the microwave is injected at one of the comb frequencies, the linewidths of the whole frequency comb are narrowed. The linewidth-narrowing effect could not be completely predicted prior to the experiments. Adding an external frequency to a nonlinear system can sometimes cause competition with the original frequencies and lead to complicated dynamics. These dynamics are observed in external-cavity lasers [2] and self-pulsing lasers [1].

The setup is shown in Fig. 1. A single-mode DFB laser at  $1.3\ \mu\text{m}$  is used. It is biased at 1.38 times its 29 mA threshold and is temperature stabilized. The output power is  $\sim 1\ \text{mW}$ . The relaxation oscillation frequency is roughly 2.3 GHz. Light is detected, after a variable delay and attenuation, by a photodiode of 12 GHz bandwidth. The electrical output is fed back to the laser through a 47 dB amplifier. The open-loop gain, including the contributions from the laser and the amplifier, has an electronic bandwidth from 0.5 to 2.5 GHz with 2 dB ripples. The feedback is negative in the sense that a current proportional to the detected optical power is subtracted from the laser bias. The feedback strength is denoted  $\xi$  as a normalized

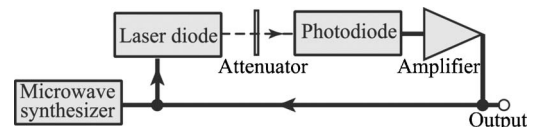


Fig. 1. Schematic of the experimental setup. Dashed line: optical path. Solid line: microwave path.

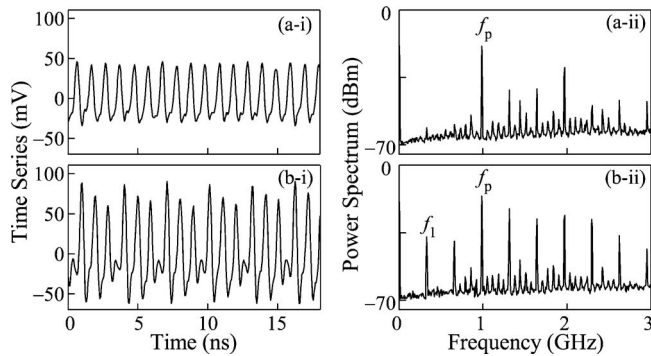


Fig. 2. Output in (i) the time domain, and (ii) the frequency domain. The loop frequency  $f_0$  is kept constant at 65.6115 MHz, while the feedback strength  $\xi$  is varied. (a) When  $\xi = -0.22$ , the laser undergoes regular pulsing. (b) When  $\xi = -0.23$ , it enters 5:15 harmonic frequency locking.

quantity [4]. The output of the system is monitored by a power spectrum analyzer and a 3 GHz real-time oscilloscope. A microwave frequency synthesizer is used in the latter part of the experiment to stabilize the frequencies.

The loop frequency is kept constant at  $f_0 = 65.6115$  MHz. Figure 2 shows the time series and the power spectrum under two different values of  $\xi$ . When  $\xi = -0.22$ , the time series in Fig. 2(a-i) shows a regular pulse train. It corresponds to the power spectrum in Fig. 2(a-ii) with a pulsing frequency of  $f_p = 982.5$  MHz, which is approximately half of the relaxation resonance [6]. The laser is in the regular pulsing state. When  $\xi = -0.23$ , the time series in Fig. 2(b-i) shows a clear modulation period in every three pulses. The power spectrum in Fig. 2(b-ii) reveals the development of a locking frequency  $f_l = 328.0575$  MHz  $= 5f_0$ . The pulsing frequency is slightly pulled and is locked at  $f_p = 984.1725$  MHz  $= 3f_l = 15f_0$ . The laser is in a 5:15 harmonic frequency-locked state. It generates a frequency comb with equal spacings of  $f_l$  that extends from dc to the microwave frequency of the order of the relaxation resonance. The rest of the Letter is focused on this 5:15 state at  $\xi = -0.23$ .

Detailed spectra of the components  $nf_l$  are shown as the black curves in Fig. 3 before an external microwave is injected. Broad linewidths are observed due to the intrinsic noise of the system. A stable external microwave at a modulation frequency  $f_m = mf_l$  is then applied. As long as  $m$  is an integer, the modulation directly injection locks the comb component at  $mf_l$  and reduces its linewidth. Because of the nonlinear interaction among the comb frequencies, linewidth narrowing takes place simultaneously for the whole comb. The colored curves of Fig. 3 shows the representative cases for  $f_m = f_l$  (red dotted curves),  $f_m = f_p = 3f_l$  (blue dashed curves), and  $f_m = f_p + f_l = 4f_l$  (green solid curves). Although only one frequency is directly applied at a time, the whole group of curves presented in the same color are narrowed simultaneously to below our 1 kHz 3 dB resolution limit. When the modulation strength increases, the signal generally improves in purity. However, strong modulation can push the comb components away from its original frequency through the nonlinear dynamics. Optimal stabilization requires iterative adjustments of  $f_m$  and  $f_0$  to maintain the frequency-locked state while the modulation strength increases. The optimal comb therefore has a slightly shifted  $f_l$ , although the comb is still composed of the exact harmonics of the shifted  $f_l$ . Phase control between the modulation and the feedback is achieved when the feedback delay time is fine tuned. The power level of the microwave modulation is kept between  $-37$  and  $-10$  dBm. Adjustments are made in Fig. 3 for clarity, so that the overlapping curves are recentered to the respective harmonic of the shifted  $f_l$ . In any case,  $f_l$  is shifted by no more than 1.07 MHz, and the comb remains strictly evenly spaced. A similar optimization method has been adopted for an optical injection system under modulation [7].

To quantify the stabilization, the suppression of the phase noise is estimated. Assuming that the noisy sidebands in Fig. 3 come mainly from small phase fluctuations, the statistical variance of the phase noise is approximated by integrating the normalized single sidebands within a frequency offset of

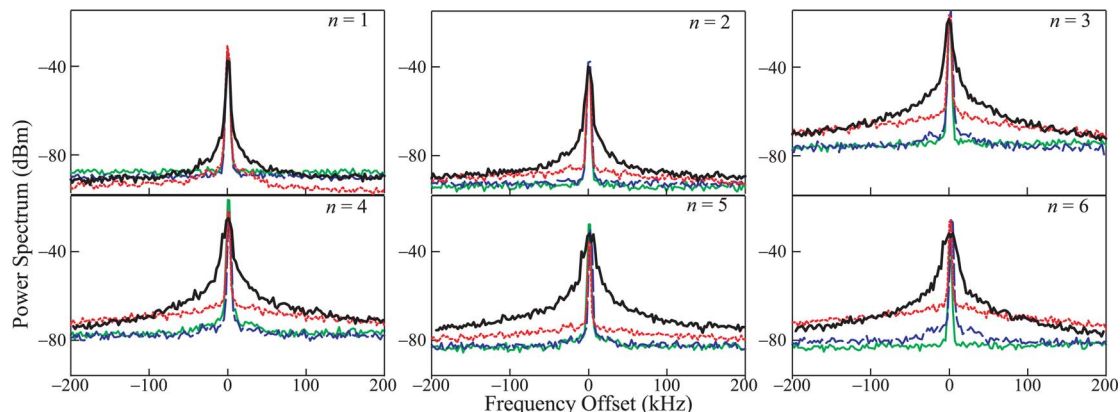


Fig. 3. (Color online) Power spectra of the  $n$ th harmonic of  $f_l$  with and without external microwave modulation. Broad linewidths are recorded as the black solid curves when there is no modulation. Linewidth narrowing is observed for all the frequency components when a modulation is applied at  $f_l$ ,  $f_p (=3f_l)$ , or  $4f_l$ . The resulting spectra are shown, respectively, as the red dotted curves, the blue dashed curves, and the green solid curves. The measurements are limited by the 3 dB resolution bandwidth of 1 kHz.

2–500 kHz. The phase noise suppression for the harmonics of  $f_l$  is shown in Fig. 4 when modulation is applied at  $mf_l$  for  $m=1,2,\dots,6$ . For each value of  $m$ , the stabilization is coarsely optimized at  $f_p$ , which receives noise suppression between 17 and 26 dB. Modulation is also tested at  $f_m=f_0$  for the completeness of the study, but it fails to stabilize the comb with our best effort. The result is not surprising because  $f_0$  is not a dominant component according to Fig. 2(b-ii). Extension of the stabilization method is also tested on an  $l:p$  state with a nonintegral  $p/l$  [3]. Modulation at neither  $f_l$  nor  $f_p$  can stabilize the frequencies generated. Careful examination reveals that  $f_p/f_l$  of the  $l:p$  state is not exactly rational. When the system is tuned in trying to approach a truly rational ratio, it jumps into other nonlinear states due to resonated frequency mixings, preventing us from obtaining the exact  $l:p$  states with a nonintegral  $p/l$ . In other words, frequency-locked states with a noninteger rational  $f_p/f_l$  cannot be found in our system. We are only able to find and to stabilize  $l:p$  states of integral  $p/l$  ratios.

The feedback loop of our system resembles an optoelectronic oscillator (OEO) [8]. In a conventional OEO, a laser source is used to pump an optoelectronic modulator that is connected in an electronic feedback loop. Because the laser is external to the loop, the OEO can be explained without considering the laser dynamics. However, in our case, the semiconductor laser is part of the feedback loop. Although it can be viewed as a special kind of OEO [9], the laser dynamics and nonlinearities have to be considered. The dynamics of the whole system has to be described by the nonlinear laser rate equations [4,6]. When the delay time is varied sufficiently in our experiment, the state changes from the frequency locking state to other states with irrational  $f_p/f_0$  such as chaotic pulsing [3]. These phenomena cannot be explained by a simple oscillator theory without considering the laser nonlinearity. The system is different from a simple OEO. Furthermore, the system is com-

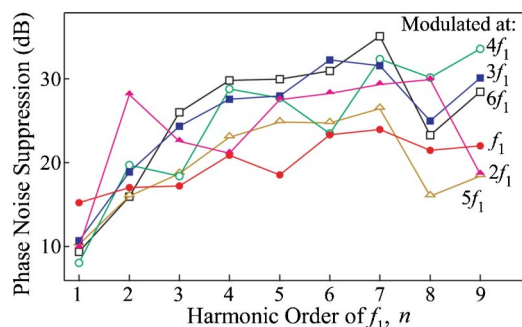


Fig. 4. (Color online) Phase noise suppressions of the comb components at  $nf_l$  under different external modulations.

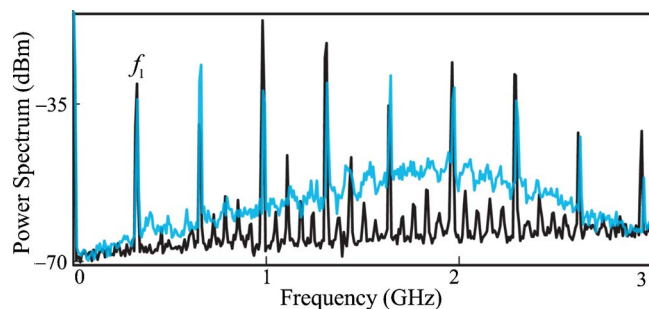


Fig. 5. (Color online) Comparison of microwave frequency combs generated by frequency locking (black, dark curve,  $-12$  dBm modulation at  $f_l$ ) and gain switching (blue, light curve,  $-2$  dBm modulation at  $f_l$ ). Note the substantially higher modulation power required and worse noise performance for gain switching.

pared with gain switching in Fig. 5 [10]. The black (dark) curve shows the comb generated by the frequency-locking state, which is stabilized by a weak modulation of  $-12$  dBm at  $f_l$ . The blue (light) curve shows a similar comb that is generated by gain switching the laser without the feedback. The laser is biased at the threshold and is modulated by a relatively strong current of  $-2$  dBm at  $f_l$ . The spectrum shows a deteriorated noise performance due to the stochastic noise of gain switching and the enhancement by the relaxation resonance. By contrast, the frequency-locking state has a better noise performance due to the effect of the feedback loop.

To summarize, a technique of generating a precise microwave frequency comb is demonstrated using a semiconductor laser under both electrical feedback and modulation. By applying the modulation at a comb component, the component is injection-locked to attain a narrow linewidth. Locking of all the other components follows.

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