

Optical injection locking based amplification in phase coherent transfer of optical frequencies

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We demonstrate use of an optical injection phase locked loop (OIPLL) as a regenerative amplifier for optical frequency transfer applications. The optical injection locking (OIL) provides high gain within a narrow bandwidth (< 100 MHz), and is capable of preserving the fractional frequency stability of the incoming carrier to better than 10^{-18} at 1000 s. The OIPLL was tested in the field as a mid-span amplifier for the transfer of an ultra-stable optical carrier, stabilized to an optical frequency standard, over a 292 km-long installed dark fiber link. The transferred frequency at the remote end reached a fractional frequency instability of less than 1×10^{-19} at averaging time of 3200 s. © 2015 Optical Society of America

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The ultra-precise optical frequency standards have been improved progressively down to the frequency instability of a few parts in 10^{18} at 10000 s integration time [1]. For comparison of these frequency standards between National Metrology Institutes (NMI) (separated by hundreds of kilometers in Europe) or to distribute the precise frequency information, it is necessary to have means of its precise transfer/distribution. Unfortunately, the instability of current satellite based frequency transfer is only at the 10^{-16} level for a measurement time as long as an entire day. An alternative is using fibers of existing telecommunication networks [2,3]. To date, transfer of optical frequencies through fiber over distances of up to 1840 km has been demonstrated with a fractional frequency instability of 4×10^{-19} for an averaging time of 100 s [4]. Generally, there are two main challenges: the phase noise imparted on the optical carrier by thermal/acoustic perturbation of the fiber and the fiber loss. To detect the fiber induced phase noise, a portion of the light is sent back to the transmitter end typically through the same fiber. Subsequently, the environmentally-induced phase noise on the optical carrier at the user end is cancelled by measuring the round-trip noise and applying a corresponding correction at the transmitter end. Bi-directional erbium-doped fiber

amplifiers (EDFA) are employed to enable propagation in both directions for the optical carrier in the same fiber. However, the bi-directional nature of these amplifiers poses a limit on the maximum gain that can be used. Point reflections and Rayleigh back-scattering create unwanted optical feedback into the EDFAs (that for bi-directional operation [5,6] cannot employ any optical isolators), readily triggering lasing effects already at relatively low gains (typically 18-20 dB). As an alternative to the EDFA, fiber Brillouin [7,8] and Raman optical amplifiers [9] have been proposed and demonstrated. The advantages of the fiber Brillouin amplifier are a small signal bandwidth (typically <30 MHz) and quasi-unidirectional amplification. As in a typical frequency transfer scenario the signals are traveling in opposite directions and are frequency shifted (typically by tens of MHz), they can be amplified separately, avoiding amplification of Rayleigh back-scattered signal. However, due to the small gain bandwidth, the Brillouin pump laser carrier frequency needs to be locked to the signal carrier frequency. As for Raman fiber amplifiers, they do not suffer from parasitic lasing due to the distributed nature of the amplification, but require very high pump powers (up to 750 mW) to be launched into the fiber.

In this letter, we demonstrate the use of an optical injection phase locked loop (OIPLL) [10] as an alternative approach for phase coherent signal regeneration following on from our recent proposal [11]. A laser can be optically injection-locked (OIL) with an input optical signal provided that the frequency detuning ($\Delta\omega$) between the carrier of the free running laser and the injected signal is less than the OIL bandwidth. The OIL-laser then oscillates at the same frequency as the incoming signal. The amplitude of the incoming signal can be significantly weaker than that of the OIL-laser and thus the OIL-laser can be viewed as an amplifier of the input signal. The OIL bandwidth is proportional to the ratio (A_{inj}/A_{fr}), where A_{inj} is the field magnitude of the injection light to the OIL-laser and A_{fr} is the field magnitude of the free-running OIL-laser. Consequently, the OIL bandwidth can range from several GHz to below 100 MHz depending on A_{inj} [12]. Bandwidths below 100 MHz (as mentioned earlier - of interest in the amplification for precise frequency transfer) are obtained for injection ratios (defined as $(A_{inj}/A_{fr})^2$) below -40 dB, effectively providing > 40 dB of gain, making OIL an ideal candidate as an amplifier in this application. For such low injection ratios, the power emitted by the OIL-laser is almost identical to that of the free running laser and thus almost independent on the injected power [13].

Consequently, the OIL can erase the amplitude fluctuations of the injected signal as a further benefit. It is worth mentioning that an additional reduction in the relative intensity noise (RIN) of the OIL-laser as compared to the free-running laser is achieved through suppression of the amplified spontaneous emission (ASE) at laser modes that are below the threshold [14]. Bi-directional amplification can be easily implemented using a 4-port cyclic optical circulator and two OIL-lasers. Tab. 1 summarizes the performance of the above-discussed technologies, showing the potential advantages of the OIL-based amplifier.

Table 1. Comparison of various amplification technologies for phase coherent optical frequency transfer.

	Small bandwidth	Gain (dB)	Optical pump required	Amplitude-noise reduction
Bi-directional EDFA [5,6]	No	< 20	Yes, 100s of mW	No
Brillouin [7]	Yes 10 MHz	> 50	Yes > 30 mW	No
Raman [9]	No	22	Yes, 750 mW	No
OIL-based	Yes <100 MHz	40-50	No	Yes
Ideal	Yes	> 40	No	Yes

Within the OIL bandwidth the OIL-laser is frequency locked to the input signal, but the relative phase between the OIL-laser output and the input signal at the front facet of the OIL-laser is given by (at point (A) in Fig. 1)

$$\Delta\phi_L = \sin^{-1} \left\{ -\frac{\Delta\omega}{\kappa\sqrt{1+\alpha^2}} \frac{A_o}{A_{inj}} \right\} - \tan^{-1} \alpha \quad (1)$$

where κ is the coupling rate, α is the OIL-laser linewidth enhancement factor, A_o is the field magnitude of OIL-laser, and g is the gain coefficient [13]. For $\alpha \gg 1$, $\Delta\phi_L$ ranges from $-\pi/2$ to 0. For typical quantum-well laser diodes α is between 2 to 5. To counteract variations of $\Delta\phi_L$ (e.g. due to a drift in the OIL-laser current that changes $\Delta\omega$), we employ a slow feedback loop, forming an OIPLL, Fig. 1. The bandwidth of the feedback loop (<1 kHz) was limited by the low-pass filter used in the laser driver to reduce the driver noise. The input signal was dithered at f_m (1 GHz), generating two 1-GHz side-tones. Subsequently, the signal was injected into the OIL-laser via a 3-port optical circulator. The OIL-laser was an isolator-free discrete-mode semiconductor laser which has a linewidth of 100 kHz and an output power of 5 dBm (Eblana Photonics). For an injection power (at point (A), Fig. 1) of -37 dBm, corresponding to an injection ratio of -42 dB in our experiment, we measured the OIL bandwidth to be 80 MHz. Consequently, while $\Delta\phi_L$ of the OIL-laser is given by Equation (1), the 1-GHz side-tones were always well outside the OIL bandwidth with their phase not influenced by the OIL process. This allowed $\Delta\phi_L$ to be determined by measuring the phase of the 1-GHz dither tone at the OIL-laser output. This is achieved by extracting a portion of the OIL-laser output (using a 50/50 coupler in our experiment), and comparing the detected 1-GHz beat signal with the 1-GHz reference signal by means of an analog RF-mixer. The electric field in front of the photodiode (PD) is given by:

$$E_{PD} = A_o \exp\{i(\omega_0 t - k_0 L + \Delta\phi_L)\} + A_R \exp\{i(\omega_0 t - k_0 L + \varphi)\} + A_S \exp\{i(\omega_{+S} t - k_{+S} L)\} + A_S \exp\{i(\omega_{-S} t - k_{-S} L)\} \quad (2)$$

where A_R and φ are the field magnitudes of the reflected carrier light and its relative phase, A_S is the field magnitude of the side-tones, ω_0 is the angular frequency of the carrier, $\omega_{\pm S}$ is the angular frequency of side-tones, L is the propagation length of the optical fiber from the OIL-laser to the PD, and k is the wavenumber. Then, the RF-beat signal (P_{RF} , at 1-GHz) detected by PD can be represented as:

$$P_{RF} = A_o A_S \{ \cos(\Delta\omega t + \psi + \Delta\phi_L) + \cos(\Delta\omega t + \psi - \Delta\phi_L) \} + A_R A_S \{ \cos(\Delta\omega t + \psi + \varphi) + \cos(\Delta\omega t + \psi - \varphi) \} \quad (3)$$

where $\Delta\omega = \omega_0 - \omega_{-S} = \omega_{+S} - \omega_0$ and $\psi = k_{-S} L - k_0 L = k_0 L - k_{+S} L$. The baseband error signal (S) at the output of the mixer (at point (B), Fig. 1) is given by

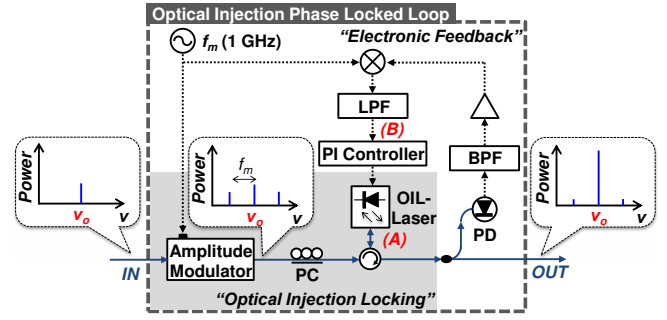


Fig. 1. Regenerative amplification based on optical injection phase locked loop (PC: Polarization Controller, PD: Photodiode, LPF: Lowpass Filter, BPF: Bandpass Filter).

$S = 2K_m A_S A_{Ref} \cos(\psi - \theta) \{A_o \cos(\Delta\phi_L) + A_R \cos(\varphi)\} \quad (4)$ where, A_{Ref} the magnitude of the 1-GHz reference signal, K_m is the conversion factor of the RF mixer, and θ is the phase of the 1-GHz reference signal. As α for our OIL-laser is 5, the error signal voltage varies nearly linearly with the frequency detuning. Although there is an offset ($A_R \cos(\varphi)$) due to the reflection of the injected light, we neglect it as $A_R \ll A_o$. The filtered error signal was fed into a proportional-integral (PI) controller.

The total optical loss of the OIPLL was 12 dB (amplitude modulator: 7 dB, circulator: 2 dB, and optical coupler: 3 dB). The output optical power of the OIPLL was approximately 1 dBm.

The OIPLL was tuned and characterized using the set-up shown in Fig. 2. We characterized the short-term performance by measuring the single sideband (SSB) phase noise using an analog phase detector and the long-term performance by measuring the Allan deviation by using a Pi-Type zero-dead time counter with a gate time of 1 s. The input signal was a free-running continuous wave (CW) laser emitting at 1542.1 nm with <5 kHz linewidth (RIO Orion from Rio Redfern Integrated Optics). The RIO laser output was split into two arms, one used for the OIL and the other as the reference. Two polarization controllers (PC) were used: one to align the polarization in front of the amplitude modulator used inside the OIPLL, the other to align the polarization of the reference arm relative to the polarization of the OIPLL. The injection power into the OIL-laser (P_{in}) was controlled using a variable attenuator. In the reference arm, the optical frequency was shifted by +35 MHz with an acousto-optic modulator (AOM). The frequency-shifted reference signal was recombined with the output of the OIPLL and the beat signal was detected with a photodiode (PD). The detected signal was sent to an analog mixer together with the 35 MHz RF reference signal. For the phase noise measurement, it was necessary to keep these two signals in quadrature, which was done by controlling the phase of the reference arm with a slow feedback loop (<10 Hz) using a piezoelectric-fiber stretcher (PZT). For the long-term stability measurement, the PZT feedback loop was disabled and the signal from the PD was band-pass filtered (27.5 MHz – 50 MHz) before being analyzed using a digital frequency counter.

Fig. 3 (a) shows the measured phase noise and integrated phase noise (10 Hz – 17.5 MHz) with various injection ratios. The interferometer noise floor was measured by removing the circulator and the OIL-laser. At frequencies above the OIPLL electronic feedback bandwidth (<1 kHz), the phase noise increases as the injection ratio (injected power) decreases. This is because variations of $\Delta\phi_L$ (as a function of $\Delta\omega$) increase as A_{inj} decreases as follows from Equation (1) [15]. The noise bump around 750 Hz is due to the OIPLL electronic feedback.

The measured Allan deviation (ADEV) and modified ADEV (Mod ADEV) are shown in Fig. 3 (b). Whilst the frequency stability and phase noise (Fig. 3a) showed degradation when the injection power is reduced (higher injection ratio), the frequency stability was always

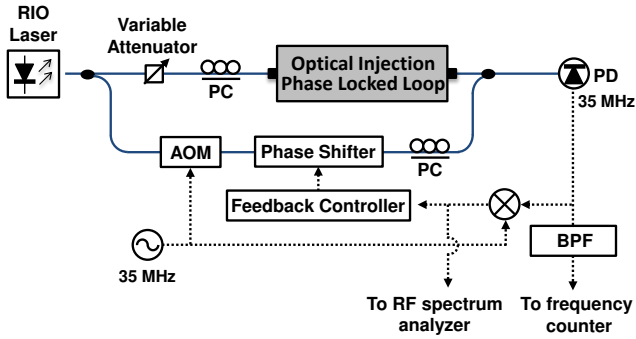


Fig. 2. Experimental set-up to characterize the optical injection phase locked loop (PC: Polarization Controller, PD: Photodiode, AOM: Acousto-Optic Modulator, BPF: Bandpass Filter).

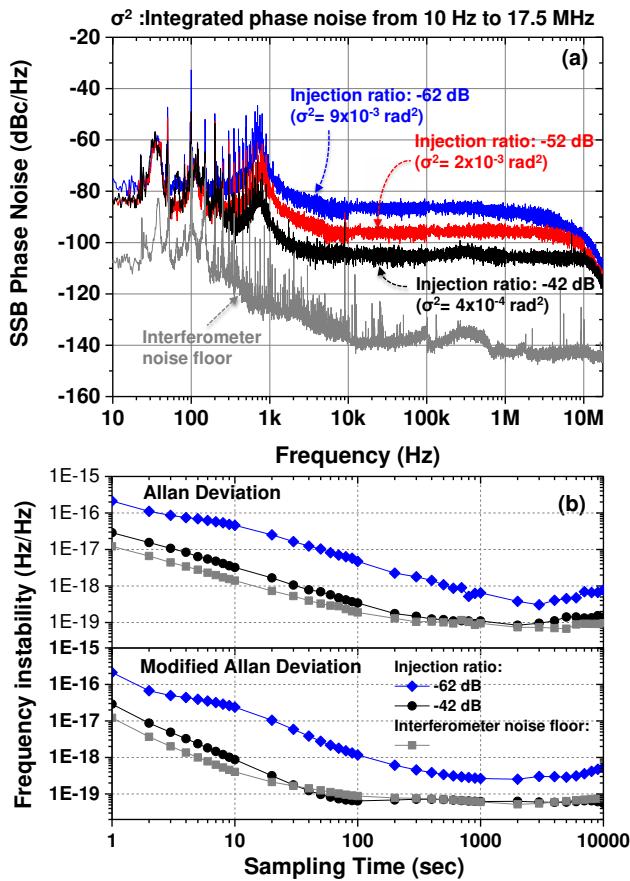


Fig. 3. (a) Measured single sideband (SSB) phase noise for three injection ratios (σ^2 is integrated phase noise from 10 Hz to 17.5 MHz), and (b) measured frequency instability of the optical injection phase locked loop (ADEV: Allan Deviation, Mod ADEV: Modified Allan Deviation).

better than 3×10^{-16} at 1 s and improved to better than 10^{-18} at 1000 s, making this technique suitable for state-of-the-art frequency metrology applications. For averaging time < 100 s, the Mod ADEV showed a $\tau^{-3/2}$ behavior suggesting that white phase noise is the dominant noise source at this time scale. At > 100 s, however, the Mod ADEV exhibits a reduced slope, most likely limited by environmentally-induced instability such as temperature changes in the measurement interferometer.

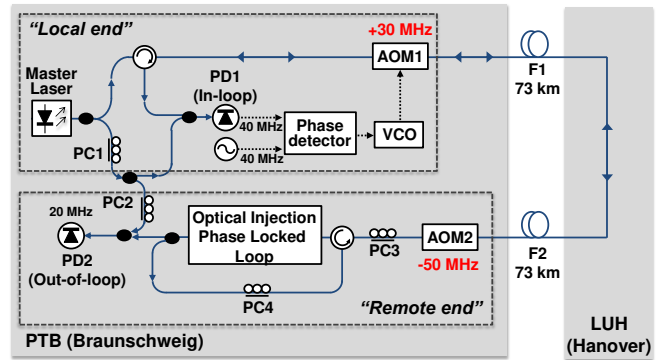


Fig. 4. Schematic diagram of 146 km fiber link with optical injection phase locked loop (AOM: Acousto-optic Modulator, VCO: Voltage Controlled Oscillator, PC: Polarization Controller, PD: Photodiode, F: Fiber)

Following the OIPLL characterization, which was done at the University of Southampton, we tested the OIL-based amplifier in transferring a precise frequency over an installed dark fiber link as seen in Fig. 4. The link connects the Physikalisch-Technische Bundesanstalt (PTB) in Braunschweig, Germany and the Leibniz Universität Hannover (LUH) in Hannover, Germany. The dark fiber link consists of two fibers, each 73 km long. At LUH both fibers were connected forming a fiber link with a total length of 146 km. This allowed us to have 'local' and 'remote' end in the laboratory at PTB. The transmitted signal was derived from a CW laser (RIO module), phase-locked to an ultralow-expansion (ULE)-cavity stabilized, ultrastable reference laser having sub-Hz linewidth ('master laser') [5]. The frequency of this master laser is close to 194.4 THz, corresponding to the ITU-channel 44.

At the remote end, the signal, attenuated due to propagation through the optical fiber (loss of 43 dB), was amplified using our OIPLL and sent back in the opposite direction via the same optical fiber, Fig. 4. The returned signal from the 'remote end' was compared with the local signal thus generating an error signal for phase-stabilization of the fiber link, Fig. 4. A PC (PC3, Fig. 4) prior to the circulator was used to align the polarization of the incident light to the amplitude modulator used in the OIPLL. Another PC (PC4, Fig. 4) was used to keep the state of polarization (SOP) of the back-reflected signal perpendicular to that of the transferred signal. AOM2 was used in order to be able to distinguish the return signal (toward the local end) from stray reflections from the link. After a full round-trip of 292 km at the local end a beat note of 40 MHz, between the return light and the reference light, was compared with a signal at the same frequency from an RF reference by means of a phase detector. The error signal was fed to the voltage-controlled oscillator (VCO) to control AOM1 for fiber noise compensation. All the measurements that follow were obtained with the fiber link stabilization turned on. The injection power at the input of the OIPLL was -43 dBm. The loss of the dark fiber link to the remote end was 43 dB. Additionally, the loss due to circulator, AOM2, and optical connectors was 5 dB in total. The power of the injected optical carrier into the fiber link from the local end was 5 dBm. Since the optical power re-injected into the fiber link after regeneration was 1 dBm, the net gain provided by our OIPLL was 44 dB. As mentioned earlier, because of the additional loss of the optical components inside the OIPLL of 12 dB, the actual OIL injection ratio was -56 dB. First, we measured the SSB phase noise, Fig 5(a). The bandwidth of the loop was estimated to be about 350 Hz by $1/4\tau$, where τ is the time delay introduced by the fiber link [5]. Inside the loop bandwidth, the phase noise is kept below -10 dBc/Hz thanks to the link stabilization.

As seen in Fig. 5(b), the out-of-loop ADEV (at the remote end) was 4×10^{-16} for 1s averaging time, which is almost an order of magnitude better than the value obtained with an EDFA [5]. Further, ADEV and

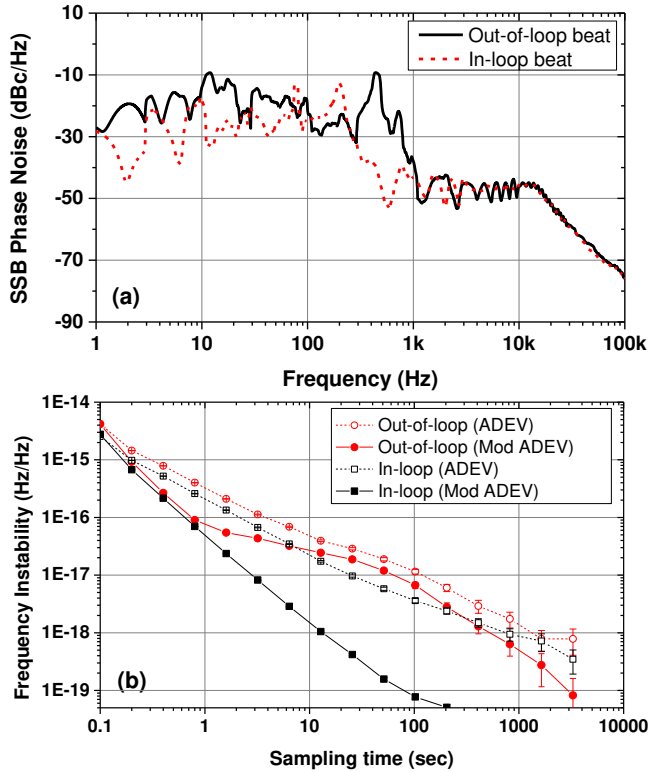


Fig. 5. Measured (a) single sideband (SSB) phase noise and (b) frequency instability of 146 km fiber link with the optical injection phase locked loop (ADEV: Allan Deviation, Mod ADEV: Modified Allan Deviation).

Mod ADEV (Fig. 5(b)) at 1 s averaging time was comparable to that in Fig. 3(b) for a similar injection ratio (-62 dB), suggesting that the noise added by the stabilized fiber link has a negligible effect in comparison with the noise added by the OIPLL. For an averaging time shorter than 1 s, the Mod ADEV follows a $\tau^{-3/2}$ slope, suggesting the phase noise is white. Although there was a small bump from 1 to 100 sec, Mod ADEV reached 1×10^{-19} at 3200 s. The bump is typically caused by noise pickup in the fibers in the set-up in which the signal propagates only uni-directionally (and thus is not noise-cancelled). This could be improved, as we did not use any particular measures to thermally shield them. When comparing the performance at longer averaging times (e.g., 1000 s), the EDFA-assisted system gives an ADEV of 3×10^{-17} [5] as compared to our OIPLL that gives an ADEV of 1.5×10^{-18} .

In conclusion, we demonstrated an optical injection phase locked loop as a regenerative amplifier for phase coherent optical frequency transfer. As compared to a system using bi-directional EDFAs it can provide higher gain, and a narrower gain bandwidth. Further, it does not require a pump to be propagated through the fiber link (which is the case for both Brillouin and Raman amplifiers). In our proof-of-principle experiment we used the OIPLL as a mid-stage amplifier in a bi-directionally operated 292-km long installed dark fiber link (total loss of 86 dB) in which the frequency of a stabilized laser (1542 nm) was transferred with a fractional frequency instability (Mod ADEV) of $< 1 \times 10^{-16}$ at 1 s, reaching 10^{-19} (Mod ADEV) at 3200 s averaging time. Another unique property of the OIPLL is its inherent reduction of amplitude noise. In future, we plan to implement automatic polarization control to further improve the robustness of the OIPLL for continuous operation.

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