Progress of Theoretical Physics, Vol. 35, No. 5, May 1966

# Optical Model Parameters for Composite Particles

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(Received January 24, 1966)

Approximate relations are given between the strengths and the shapes of real, imaginary and spin-orbit terms of the optical potentials for nucleons, deuterons, tritons and $He^3$-ions.

## § 1. Introduction

Recently, many interesting articles[1)-5)] treated the problem of relating the optical model parameters of a composite particle to the corresponding parameters of its constituent particles. One of these articles, by Gammel et al.,[2)] presented a method for calculating the optical model potential experienced by a deuteron passing through a nucleus in terms of its constituent nucleon potentials. This method neglects the distortion and the disintegration of the deuteron in the field of the target nucleus. The problem is reduced to a three-body problem; the two nucleons of the deuteron and the target nucleus. Each of the deuteron's nucleons interacts with the target nucleus via the optical potential. This approximation could lead to expressions suitable for numerical calculations for the optical parameters of the deuteron.

In the present paper we use the formalism of Gammel et al.[2)] to obtain some relations between the optical model parameters of composite projectiles and their constituent particles. Neglecting the $D$ state of the deuteron, we calculate in § 2 the parameters of its optical potential starting with nucleon optical potentials having radial dependence suitable for integrations. These forms have been checked[6),7)] and found to give the same scattering cross sections as that obtained by using the commonly used potentials.[8)] In § 3, we generalize the treatment to consider the case of three-body projectiles and derive expressions for the optical model potentials of tritons and $He^3$-ions.

The obtained relations between the optical model potentials of nucleons, deuterons, tritons and $He^3$-ions are in agreement with the predictions of the optical model analysis[9),12)] of elastic scattering and the distorted wave analysis of direct reactions.[13)-15)] The main interest in these relations is that they may serve to discriminate between different sets of optical potentials which give equally good fits to elastic scattering data. The latter problem is of particular importance for the study of direct nuclear reactions.

## § 2. The deuteron optical model potential

In this section we use the expressions obtained by Gammel et al.[2] to calculate the parameters of the deuteron optical potential. We neglect the $D$ state of the deuteron and describe the wave function of the deuteron by a Gaussian functions:

$$u = (2\beta/\pi)^{3/4} \exp(-\beta r^2), \quad \beta = 0.053\ \mathrm{fm}^{-2}.$$

The optical model potential of the deuteron is then expressed as[2]

$$V^d(R) + iW^d(R) + V_s^d(R)\boldsymbol{\sigma}_d \cdot \boldsymbol{L},$$

where $\boldsymbol{L} = -i\boldsymbol{R} \times \boldsymbol{\nabla}_R$, $\boldsymbol{\sigma}_n(p)$ is the Pauli matrix for the neutron (proton) with $\boldsymbol{s} = (\hbar/2)\boldsymbol{\sigma}$, $\boldsymbol{\sigma}_d = \boldsymbol{\sigma}_n + \boldsymbol{\sigma}_p$,

$$V^d + iW^d = [4(2\beta/\pi)^{1/2}R]^{-1} \exp(-8\beta R^2) \int_0^\infty r dr \exp(-8\beta r^2) \times \mathrm{sh}\, 16\beta r R [V^p(r) + V^n(r) + 2iW^N_{(r)}] \tag{2·1}$$

and

$$V_s^d = [4(2\beta/\pi)^{1/2}R]^{-1} \exp(-8\beta R^2) \frac{d}{dR} R^{-1} \int_0^\infty r dr \times \exp(-8\beta r^2)\, \mathrm{sh}\, 16\beta r R V_s^N(r). \tag{2·2}$$

The indices $p$, $n$, $N$ and $d$ refer to protons, neutrons nucleons and deuterons.

We start with the calculation of the real part of the deuteron central potential. We express the corresponding nucleon potential as[3]

$$V^{p,n}(r) = -V_0^{p,n} \rho_N(r), \tag{2·3}$$

where $V_0^p(E) = V_0(E) + C(N-Z)/A + 0.4Z/A^{1/3}$, $V_0^n = V_0(E) - C(N-Z)/A$, $V(E) = 53\text{–}0.55E$ MeV and $C = 27$ MeV. In order to obtain an analytical expression for $V^d$, we take the form-factor

$$\rho_N(r) = \left[1 - \frac{1}{2}\exp(-R_0/a)\right]^{-1} \times \begin{cases} \left[1 - \frac{1}{2}\exp\{(r-R_0)/a\}\right]; & r < R_0 \\ \frac{1}{2}\exp\{(R_0 - r)/a\}; & r > R_0 \end{cases} \tag{2·3'}$$

with $R_0 = 1.25A^{1/3}$ fm and $a = 0.81$ fm. Glassgold and Kellogg[6] showed that this form-factor is equivalent to the Saxon-Woods form-factor for proton scattering at energies up to 30 MeV, when the depth and the radius of the well are kept unchanged and the diffuseness of the latter potential is 0.65 fm. Substituting this form-factor into Eq. (2·1) one finds

$$V^d(R) = -\left[1-\frac{1}{2}\exp(-R/a)\right]^{-1}\left[2V_0\left(\frac{E+\varepsilon}{2}\right)+0.4Z/A^{1/3}\right]$$
$$\times\left[\frac{1}{2}\{\Phi[\sqrt{8\beta}(R-R_0)]+\Phi[\sqrt{8\beta}(R_0+R)]\}\right.$$
$$-\frac{1}{4R}\exp\left(\frac{1}{32\beta a^2}\right)\left\{\left(R+\frac{1}{16\beta a}\right)\exp\left(\frac{R-R_0}{a}\right)\left(\Phi\left[\sqrt{8\beta}\left(R_0-R-\frac{1}{16\beta a}\right)\right]\right.\right.$$
$$\left.+\Phi\left[\sqrt{8\beta}\left(R+\frac{1}{16\beta a}\right)\right]\right)$$
$$+\left(R-\frac{1}{16\beta a}\right)\exp\left(-\frac{R_0+R}{a}\right)\left(\Phi\left[\sqrt{8\beta}\left(R_0+R-\frac{1}{16\beta a}\right)\right]-\Phi\left[\sqrt{8\beta}\left(R-\frac{1}{16\beta a}\right)\right]\right)$$
$$-\left(R-\frac{1}{16\beta a}\right)\exp\left(\frac{R_0-R}{a}\right)\left(1-\Phi\left[\sqrt{8\beta}\left(R_0-R+\frac{1}{16\beta a}\right)\right]\right)$$
$$\left.\left.-\left(R+\frac{1}{16\beta a}\right)\exp\left(\frac{R_0+R}{a}\right)\left(1-\Phi\left[\sqrt{8\beta}\left(R_0+R+\frac{1}{16\beta a}\right)\right]\right)\right\}\right], \qquad (2\cdot4)$$

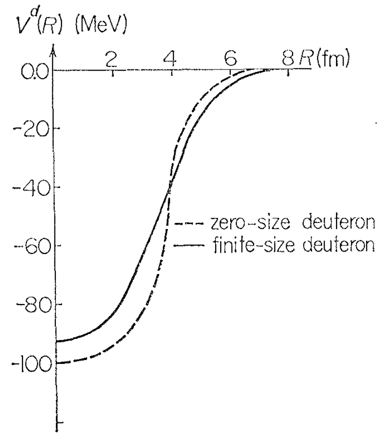


Fig. 1. The real part of the deuteron optical model potential for the nucleus with $Z=13$ and $A=27$.

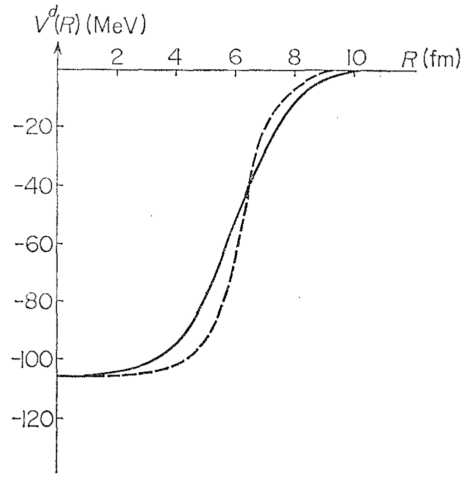


Fig. 2. The real part of the deuteron optical potential for the nucleus with $Z=53$ and $A=125$.

where $\varepsilon$ is the binding energy of the deuteron, $\Phi$ is the error function. Now using the expression $(2\cdot4)$ we calculated the real part of the deuteron optical potential for nuclei with $Z=13$, $A=27$ and $Z=53$, $A=125$. The results are given in Figs. 1 and 2 and are compared with the results corresponding to the case when the deuteron radius is equated to zero $(u^2(r)=\delta(\boldsymbol{r}))$. In the latter case the geometrical parameters of the deuteron potential take the same values as the corresponding nucleon parameters, while the depth of the deuteron potential is equal to the sum of depths of the proton and the neutron potentials. We see that the effect of the finite dimensions of the deuteron is to decrease the strength and the radius and to increase the diffuseness of the real part of the optical potential. We see also that the effect is stronger for the lighter nucleus where the radius of the deuteron is not small compared with the nuclear radius. We expect that these effects are increased if we take into account the distortion of the internal state of the deuteron, since Redondo et al.[16] have shown that the radius of the deuteron is increased when it enters the nuclear matter. We note that the numerical analysis of deuteron scattering[9),10)] and deuteron stripping reactions[13)] shows that the best fits are obtained by using deuteron optical potential with depth a little less than the sum of the proton and neutron potentials depths, with radius smaller and diffuseness larger than the corresponding values for the nucleon potentials.

Now we consider the imaginary part of the deuteron optical potentials. A formula similar to Eq. $(2\cdot4)$ is obtained in the case of a volume nucleon absorption. If the imaginary part of the nucleon optical potential is a surface potential and has a Gaussian shape

$$W^N(r) = -W_0^N \exp\{-(R_I-r)^2/b_N^2\}, \tag{2·5}$$

then, using Eq. $(2\cdot1)$, we obtain the following expression for the absorptive deuteron potential:

$$\begin{aligned} W^d(R) = &-\{2\beta b_N^2/(8\beta b_N^2+1)\}^{3/2} W_0^N (\beta R)^{-1} \\ &\times\left[\left(8\beta R+\frac{R_I}{b_N^2}\right)\exp\left\{-\frac{8\beta(R-R_I)^2}{8\beta b_N^2+1}\right\}\left\{1+\Phi\left(\frac{R_I+8\beta b_N^2 R}{b_N\sqrt{8\beta b_N^2+1}}\right)\right\}\right. \\ &\left.+\left(8\beta R-\frac{R_I}{b_N^2}\right)\exp\left\{-\frac{8\beta(R+R_I)^2}{8\beta b_N^2+1}\right\}\left\{1+\Phi\left(\frac{R_I-8\beta b_N^2 R}{b_N\sqrt{8\beta b_N^2+1}}\right)\right\}\right]. \end{aligned} \tag{2·6}$$

Equation $(2\cdot6)$ can be approximated as

$$W^d(R) \approx -W_0^d \exp\{-(R_I-R)^2/b_d^2\}, \tag{2·7}$$

where $W_0^d = 2(1+1/8\beta b_N^2)^{-1/2} W_0^N$ and $b_d = (1+1/8\beta b_N^2)^{1/2} b_N$. It is interesting to note that the quantities $W_0^d$ and $b_d$ are dependent on the size-parameter of the deuteron wave function, whereas their product is independent of this parameter

$$W_0^d\, b_d = 2W_0^N\, b_N. \tag{2·8}$$

Hence, we expect the relation $(2\cdot8)$ to be independent of the shape of the deuteron wave function. Therefore this relation should be used for testing the validity of the model and distinguishing between optical potential having different imaginary parts.

The imaginary part of the nucleon optical potential is sometimes taken as[8)]

$$W^N(r) = 4W_0^N a_N \frac{d}{dr}\rho_N(r), \tag{2·9}$$

where $\rho_N(r)$ is given by Eq. $(2\cdot3)$. Rosen et al.[7)] found that this potential is equivalent to the potential (2·5) when $b_N \approx 2.5 a_N$, as might be expected from a comparison of the second moments of the two forms about $r=R_0$. For the choice (2·9), the relation (2·8) may be rewritten as

$$W_0^d a_d = 2W_0^N a_N, \qquad (2\cdot10)$$

where $W_0^N = 11$ MeV and $a_N = 0.47$ fm. We calculate the product $W_0^d a_d$ for different sets of parameters obtained by the Pereys[7] from the analysis of 11.8–, 15– and 21.6–MeV deuteron scattering data. The results of the calculations are given in Table I. It can be seen that these results agree with the predictions of the model under investigations. The fluctuations of the value of the product is expected as these fluctuations exist for the imaginary part of the nucleon optical potential.[7),8)]

Table I. The product $W_0^d a_d$ in MeV. fm of different sets of optical model parameters calculated by the Pereys.

| set | $E$=11.8 MeV | $E$=15 MeV | $E$=21.6 MeV |
|---|---|---|---|
| A | 10.58±1.85 | 8.78±0.54 | 9.18±0.95 |
| B | 12.75±2.81 | 10.87±1.00 | 11.62±1.59 |
| C | 12.03±1.27 | 10.55±1.45 | 9.62±0.40 |
| D | 14.75±1.69 | 13.95±1.66 | 12.99±1.40 |

Now we consider the spin-orbit term in the deuteron optical potential. It is usually assumed that the spin-orbit term of the nucleon potential has a Thomas form

$$V_s^N(r) = \lambda_\pi^2 V_{s0}^N r^{-1} d\rho_N(r)/dr \qquad (2\cdot11)$$

where $\lambda_\pi$ is the pion compton wave length, $\rho_N(r)$ is given by Eq. (2·3′) and $V_{s0}^N$ is about 8 MeV. Substituting this into Eq. (2·2), one obtains the following expression for the deuteron spin-orbit potential:

$$V_s^d(R) = \frac{1}{2}\lambda_\pi^2 V_{s0}^d R^{-1}\frac{d}{dR} 4R^{-1}(2\beta/\pi)^{1/2}\exp(-8\beta R^2)$$
$$\times\int_0^\infty r dr \exp(-8\beta r^2)\,\mathrm{sh}(16\beta r R)\,\rho_N(r). \qquad (2\cdot12)$$

Comparing this with Eqs. (2·1) and (2·3), one finds

$$V_s^d(R) = \frac{1}{2}\lambda_\pi^2 V_{s0}^d R^{-1}\frac{d}{dR}\frac{V^d(R)}{V_0^n + V_0^p}. \qquad (2\cdot13)$$

Thus the obtained deuteron spin-orbit potential has also a Thomas form. Actually if $V^d(R) = -V_0^d \rho_d(R)$, then

$$V_s^d(R) = \lambda_\pi^2 V_{s0}^d R^{-1}\frac{d}{dR}\rho_d(R), \qquad (2\cdot14)$$

where

$$V_{s0}^{d} = V_{s0}^{N} V_0^{d} / 2(V_0^{p} + V_0^{n}) \approx \frac{1}{2} V_{s0}^{N}. \tag{2·15}$$

The result (2·14) agrees with the investigations by Bassel et al.,[10] who found that the optimum fit of deuteron scattering on $Ca^{40}$ is obtained by adding a spin-orbit term of the type (2·13) with strength $V_{s0}^{d} = 4.74$ MeV to the deuteron optical potential.

## § 3. The optical model potentials for tritons and $He^3$-projectiles

Let us now proceed to calculate the optical model parameters for the three-nucleon projectiles (tritons and $He^3$-ions). The potential of the triton ($He^3$) may be expressed as

$$V^{t(h)}(\boldsymbol{R}, \boldsymbol{s}_1, \boldsymbol{s}_2, \boldsymbol{s}_3) = \int d\boldsymbol{r} d\boldsymbol{\rho} u^*(\boldsymbol{r}, \boldsymbol{\rho}) [V^{p}(\boldsymbol{r}_1, \boldsymbol{s}_1) + V^{n}(\boldsymbol{r}_2, \boldsymbol{s}_2) + V^{n(p)}(\boldsymbol{r}_3, \boldsymbol{s}_3)] u(\boldsymbol{r}, \boldsymbol{\rho}), \tag{3·1}$$

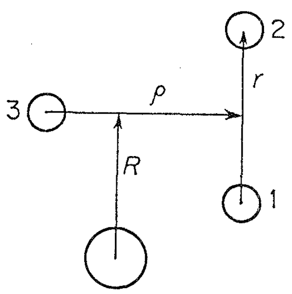


Fig. 3. A Schematic representation for the elastic scattering of three-nucleon projectile.

where $u(\boldsymbol{r}, \boldsymbol{\rho})$ is the internal wave function of the triton ($He^3$) and the variables involved are shown in Fig. 3. The superfixed $t$ and $h$ refer to the tritons and $He^3$ ions respectively. Now we take the nucleon optical potential as

$$V^{p,n}(\boldsymbol{r}, \boldsymbol{s}) = V^{p,n}(r) + iW^{N}(r) + V_s^{N}(r)\boldsymbol{l}\cdot\boldsymbol{\sigma}. \tag{3·2}$$

Substituting this potential into Eq. (3·1) and taking into account the symmetry of the internal wave function of the triton ($He^3$) with respect to the exchange of the spatial coordinates of its constituent nucleons, one finds

$$V^{t(h)} = V^{t(h)}(R) + iW^{th}(R) + V_s^{th}(R)\boldsymbol{L}\cdot\boldsymbol{\sigma}_{t(h)}, \tag{3·3}$$

where

$$\boldsymbol{\sigma}_{t(h)} = \boldsymbol{\sigma}_p + \boldsymbol{\sigma}_n + \boldsymbol{\sigma}_{n(p)},$$

$$V^{t(h)}(R) = \int d\boldsymbol{r} d\boldsymbol{\rho} u^2(\boldsymbol{r}, \boldsymbol{\rho}) [V^{p}(|\boldsymbol{R} + \frac{2}{3}\boldsymbol{\rho}|) + V^{n}(|\boldsymbol{R} + \frac{2}{3}\boldsymbol{\rho}|) + V^{n(p)}(|\boldsymbol{R} + \frac{2}{3}\boldsymbol{\rho}|)], \tag{3·4}$$

$$W^{th}(R) = 3\int d\boldsymbol{r} d\boldsymbol{\rho} u^2(\boldsymbol{r}, \boldsymbol{\rho}) W^{N}(|\boldsymbol{R} + \frac{2}{3}\boldsymbol{\rho}|), \tag{3·5}$$

and

$$V_s^{th}(R) = \frac{1}{3}\int d\boldsymbol{r}\, d\boldsymbol{\rho} u^2(\boldsymbol{r},\boldsymbol{\rho})\, V_s^N(|\boldsymbol{R}+\frac{2}{3}\boldsymbol{\rho}|)\frac{\boldsymbol{R}\cdot(\boldsymbol{R}+(2/3)\boldsymbol{\rho})}{R^2}. \tag{3·6}$$

Now if

$$V^{n,p}(r) = -V_0^{n,p}\rho_N(r),\ \ W^N = -W_0^N q_N(r)\ \text{and}\ V_s^N = \lambda_\pi^2 V_{s0}^N \frac{1}{r}\frac{d}{dr}\rho_N(r)$$

and if the internal wave function of the triton ($He^3$) is taken as[17] $(2\sqrt{3}\gamma/\pi)^{3/2}\times \exp(-2\gamma\rho^2 - 3/2\cdot\gamma r^2)$ where $\gamma = 0.16\,\text{fm}^{-2}$, then changing the variables of integration $\boldsymbol{\rho}$ by $(3/2)(\boldsymbol{x}-\boldsymbol{R})$ integrating over $\boldsymbol{r}$ and $\Omega_x$, Eqs. (3·4)–(3·6) become

$$V^{t(h)}(R) = -6\sqrt{\gamma/\pi}(V_0^p + V_0^n + V_0^{n(p)})\,[1-\frac{1}{2}\exp(-R_0/a)]^{-1}R^{-1}$$

$$\times\exp(-9\gamma R^2)\int_0^\infty x\,dx\exp(-9\gamma x^2)\,\mathrm{sh}(18x\gamma R)\,\rho_N(x) \tag{3·7}$$

$$= -V_0^{t(h)}\rho_t(R),$$

$$W^{th}(R) = -18\sqrt{\gamma/\pi}W_0^N R^{-1}\exp(-9\gamma R^2)\int_0^\infty x\,dx\exp(-9\gamma x^2)\mathrm{sh}(18x\gamma R)\rho_N(x) \tag{3·8}$$

$$= -W_0^{th} q_t(R),$$

$$V_s^{th}(R) = \frac{1}{3}V_{s0}^N\lambda_\pi^2\frac{1}{R}\frac{d}{dR}\frac{6\sqrt{\gamma/\pi}}{R}\int_0^\infty x\,dx\exp\{-9\gamma(x^2+R^2)\}\,\mathrm{sh}(18\gamma xR)\,\rho_N(x)$$

$$= \frac{1}{3}V_{s0}^N\frac{V_0^{t(h)}}{V_0^p+V_0^n+V_0^{n(p)}}\lambda_\pi^2\frac{1}{R}\frac{d}{dR}\rho_t(R). \tag{3·9}$$

Now we compare these expressions with the corresponding ones in § 2. First we consider the real part of the triton ($He^3$) optical potential. We note that the integrals in Eq. (3·7) is similar to Eq. (2·4) if one puts in the latter $\beta = (9/8)\gamma$. Thus one may write the following expression for the real part of the triton ($He^3$) optical potential:

$$V^{t(h)}(R) = -[1-\frac{1}{2}\exp(-R_0/a)]^{-1}[V_0^n + V_0^p + V_0^{n(p)}]$$

$$\times\Bigg[\frac{1}{2}\Big\{\Phi[3\sqrt{\gamma}(R_0-R)] + \Phi[3\sqrt{\gamma}(R_0+R)]\Big\}$$

$$+\frac{1}{4R}\exp\left(\frac{1}{36\gamma a^2}\right)\left\{\left(R+\frac{1}{18\gamma a}\right)\exp\left(\frac{R-R_0}{a}\right)\left\{\Phi\left[3\sqrt{\gamma}\left(R_0-R-\frac{1}{18\gamma a}\right)\right]\right.\right.$$

$$\left.\left.+\Phi\left[3\sqrt{\gamma}\left(R+\frac{1}{18\gamma a}\right)\right]\right\}\right.$$

$$+\left(R-\frac{1}{18\gamma a}\right)\exp\left(-\frac{R_0+R}{a}\right)\left\{\Phi\left[3\sqrt{\gamma}\left(R-R_0-\frac{1}{18\gamma a}\right)\right]-\Phi\left[3\sqrt{\gamma}\left(R-\frac{1}{18\gamma a}\right)\right]\right\}$$
$$-\left(R-\frac{1}{18\gamma a}\right)\exp\left(\frac{R_0-R}{a}\right)\left\{1-\Phi\left[3\sqrt{\gamma}\left(R_0-R+\frac{1}{18\gamma a}\right)\right]\right\}$$
$$-\left(R+\frac{1}{18\gamma a}\right)\exp\left(\frac{R_0+R}{a}\right)\left\{1-\Phi\left[3\sqrt{\gamma}\left(R_0+R+\frac{1}{18\gamma a}\right)\right]\right\}\Bigg], \qquad (3\cdot 10)$$

where $\rho_N(r)$ is given by Eq. $(2\cdot 3')$. For not very small $R_0$, it is easy to show that the depth of the central real triton ($He^3$) optical potential almostly equals to the sum of the depths of the potentials for its constituent three nucleons. Equation $(3\cdot 10)$ was used to evaluate the real parts of the optical potentials for triton and $He^3$ interaction with nuclei with $A=27$ and $Z=13$ with the nucleon optical model potentials used in § 2. The resultant potential is given in Fig. 4,

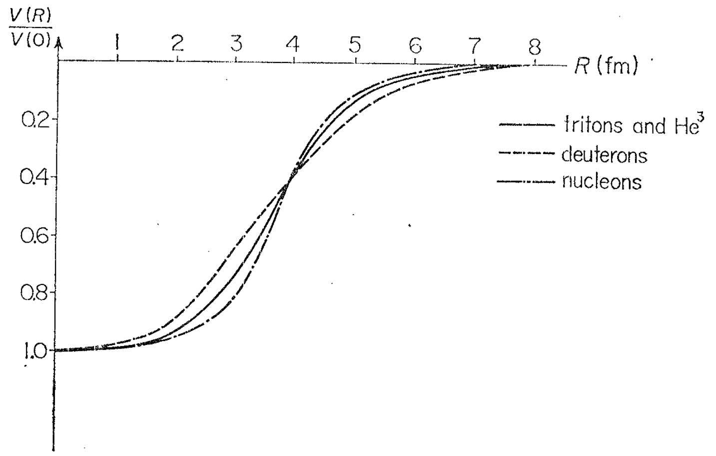


Fig. 4. Comparison between the real parts of the optical potential for nucleons, deuterons, tritons and $He^3$-ions scattered by a target nucleus with $A=27$.

and compared with the potentials obtained for the nucleon and the deuteron. It is interesting to note that the radius of these potentials are smaller than the corresponding potentials of the nucleon potentials but are larger than the radii for the corresponding deuteron potentials. This may be due to the fact that the triton and the $He^3$ nucleus are a more strongly bound system than the deuteron and that as we concluded in § 2, the decrease of the radius of the deuteron optical potential is a direct result of the finite dimensions of the deuteron.

Now comparing the integrals of Eq. $(3\cdot 8)$ and the corresponding integral of Eq. $(2\cdot 4)$ we find

$$W^{th}(R)=-\left[9\gamma b_N^2/(9\gamma b_N^2+1)\right]^{3/2}\frac{W_0^N}{6\gamma R}$$

$$\times\Big[\Big(9\gamma R+\frac{R_I}{b_N^2}\Big)\exp\Big\{-\frac{9\gamma(R-R_I)^2}{9\gamma b_N^2+1}\Big\}\Big\{1+\Phi\Big(\frac{R_I+9\gamma b_N^2R}{b_N\sqrt{9\gamma b_N^2+1}}\Big)\Big\}$$

$$+\Big(9\gamma R-\frac{R_I}{b_N^2}\Big)\exp\Big\{-\frac{9\gamma(R+R_I)^2}{9\gamma b_N^2+1}\Big\}\Big\{1+\Phi\Big(\frac{R_I-9\gamma b_N^2R}{b_N\sqrt{9\gamma b_N^2+1}}\Big)\Big\}\Big], \tag{3·11}$$

where $\rho_N(r)$ was taken as $\exp\{-(r-R_I)^2/b_N^2\}$. This potential may be approximated as

$$W^{th}(R)\approx -W_0^{th}\exp\{-(R-R_I)^2/b_{th}^2\},$$

where

$$W_0^{th}=(1+1/9\gamma b_N^2)^{-1/2}3W_0^N,$$
$$b_{th}=(1+1/9\gamma b_N^2)^{1/2}b_N.$$

Thus we find that

$$W_0^{th}b_{th}=3W_0^N b_N. \tag{3·12}$$

Now let us consider the spin-orbit term of the triton optical potential. Since Eq. (3·9) does not depend on the parameters of the triton ($He^3$) wave function we may conclude that this equation is independent of the choice of the wave function of the triton ($He^3$). Noting that the strength of the real part of triton ($He^3$) optical potential (3·10) is almost equal to the sum of the strengths of the potentials of the three nucleons, then we may write

$$V_{s0}^{th}\approx\frac{1}{3}V_{s0}^N\approx 3\text{ MeV},$$

i.e. the strength of the spin-orbit term in the triton ($He^3$) optical potential is a third of the strength of the corresponding term in the nucleon optical potential.

## § 4. Discussion

The approximate methods suggested by Watanabe and others[1)-3)] were used to obtain some analytical relations between the optical potential of a composite projectile and those potentials of its constituent particles. The results of the present investigations can be summerised as follows:

1. The strength of the real part of the central optical potential of a composite particle is approximately equal to the sum of the potential strengths of the constituent particles. A reduction in the strength of this potential for the composite projectile occurs when the spatial dimension of the projectile is comparable with the dimension of the target nucleus. This result is in agreement with the findings of the distorted wave analysis of deuteron stripping reactions,[13)] $(t, p)$ reactions[14)] and $(He^3, \alpha)$ reactions.[15)]
2. The radius of the real part of the optical potential for the composite projectile

is smaller than the radius of the corresponding potential for the nucleon, while the diffuseness of the former potential is larger. This is also a result of the finite dimension of the projectile. This conclusion is in agreement with the findings of the distorted wave Born approximation treatment of deuteron stripping reactions[13] where best fits were obtained when the real radius of the deuteron potential is smaller than the real radius of the proton potential. One would also expect for the optical potential of composite projectile that the real radius parameter $r_0(R_0 = r_0 A^{1/3})$ increases while the real diffuseness decreases with the atomic mass number, if the corresponding nucleon parameters are independent of the mass number. This latter conclusion contradicts the examination of best fit parameters obtained by the Pereys[9] for the elastic 11.8 MeV deuterons by heavy elements.

3. If the absorptive nucleon optical potential is a volume potential, then the corresponding potential for a composite projectile will also be a volume potential. In this case the conclusions 1 and 2 apply to the absorptive potential. Similarly, if the nucleon absorptive potential is a surface one, then the potential for the composite particle will be a surface potential. The strength of this potential increases with the mass of the projectile. The ratio between the product of the strength and width of the absorptive potential, and between the corresponding product for the nucleon potential is equal to the ratio of the masses of these particles.
4. If the spin-orbit term in the nucleon optical potential has a Thomas form, then this form will also remain for the composite projectile. In this case, the strength of the spin-orbit potential of the deuteron will be a half, and of the triton will be a third of the nucleon spin-orbit potential. Optical model analysis by Bassel et al.[10] for deuteron scattering show similar behaviour for the spin-orbit optical model potential.

## Acknowledgements

The authors are thankful to O. Zohni for useful discussions and to Abdel-Salam for carrying the numerical calculations.

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