Optical parametric oscillator frequency tuning and control

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Received May 15, 1990; accepted August 7, 1990

The frequency-tuning and -control properties of monolithic doubly resonant optical parametric oscillators are analyzed for stable single-mode pump radiation. Single-axial-mode operation is observed on the idler and the signal for both pulsed and continuous pumping. Projections are made for tuning-parameter tolerances that are required for maintenance of stable single-frequency oscillation. Continuous frequency tuning is possible through the simultaneous adjustment of two or three parameters; thus the synthesis of specific frequencies within the broad tuning range of the doubly resonant optical parametric oscillator is permitted.

1. INTRODUCTION

An analysis of the frequency-tuning properties of doubly resonant optical parametric oscillators (DRO's), based on both experimental observations and theoretical modeling, is presented. Specific details in this presentation of frequency control and synthesis apply to monolithic DRO's constructed from $LiNbO_3$. Where possible, however, results are given with more general applicability. The purpose is achievement of a quantitative understanding of the conditions required for stable single-axial-mode parametric oscillation and the resulting frequency stability of the DRO output. Approaches to frequency synthesis and continuous frequency tuning that are based on the simultaneous adjustment of two or three tuning variables are described.

The potential of optical parametric oscillators (OPO's) for the generation of tunable coherent radiation was recognized more than twenty-five years ago.[1] The complex tuning properties of DRO's were also revealed in early demonstrations and analyses.[2–4] Optical parametric oscillation has been discussed in detail in a number of reviews,[5–7] and it is a subject treated in more general terms in a number of books that discuss nonlinear optics.[8] Improvements in the quality of nonlinear-optical materials and in the coherence of pump sources led to a number of advances in the performance of OPO's. Using recent experimental results obtained with stable single-mode pump sources and monolithic DRO's constructed from high-quality $LiNbO_3$ nonlinear-optical material, we are able to apply and to extend the earlier analyses.

Resonance of both the signal and the idler frequencies, double resonance, offers the advantage of a lower threshold for parametric oscillation than in single resonance. Double resonance also provides additional frequency selectivity in OPO operation. These desirable properties of double resonance, however, come with a considerable increase in the complexity of tuning and with more restrictive tolerances on pump stability and cavity stability. Diode-pumped solid-state lasers provide the required pump-frequency stability, and monolithic cavities provide the required mechanical stability in the OPO. Continuous tuning is difficult in DRO's, which typically tune with axial mode hops and cluster jumps over hundreds of axial modes. Nevertheless, with improved pump sources and nonlinear-optical materials coupled with multiple-parameter control, DRO's can in principle be operated stably and tuned continuously, thus widening their range of applications.

DRO's can provide highly coherent output, reproducing the statistical properties of the pump with little additional noise. This was shown theoretically by Graham and Haken[9] in a quantum-mechanical analysis of the DRO, and it was demonstrated in experimental measurements of the coherence properties of the DRO. The quantum-mechanical analysis showed that the diffusion of the sum of the signal and the idler wave phases follows the phase diffusion of the pump wave adiabatically. Although the phase difference of the signal and the idler may diffuse in an undamped manner, the statistical properties of a DRO are basically the same as those of an ideal laser. A result of these properties is the addition of only a small amount of phase noise in the output of the DRO above that present in the pump. This has been confirmed in coherence measurements of the output of a cw DRO.[10] For periods of ~1 min, the free-running DRO that was not servo locked oscillated on a single mode pair without a mode hop. That the DRO did not add significant excess linewidth over that present on the pump was demonstrated with the measurement of beating between the DRO output and an independent diode-laser-pumped solid-state laser during the periods between mode hops. The beat-note linewidth was 13 kHz, which was the expected value for the typically 10-kHz linewidths of the pump laser and the independent reference laser. Additional coherence measurements showed that the signal and the idler were phase anticorrelated when referenced to the pump laser. Also, the width of the signal–idler beat note with the DRO near but not at degeneracy was less than 1 kHz. The signal–idler beat note indicates the frequency fluctuations added to the DRO output in addition to those present on the pump.

The results of the classical stationary analysis presented here are consistent with the earlier analyses and measurements. The main purpose of this presentation is to explain the complex tuning properties of the DRO in order to permit fuller use of its remarkable coherence and spectral properties. The theoretical presentation of

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Section 2 begins in Subsection 2.A with a qualitative overview of DRO tuning. This overview is used to establish the extensive terminology required for the discussion. In Subsection 2.B the threshold condition for parametric oscillation is reviewed and recast in terms that are more easily adapted to tuning calculations. The theoretical basis of frequency selection is discussed in Subsection 2.C. Experimental tuning data are presented in Section 3. The degree to which our theoretical model describes the observed tuning justifies some confidence in its use for predictive calculations in Section 4. Results are summarized and discussed in Section 5. Finally, the properties of $MgO{:}LiNbO_3$ that are required for modeling the experimental data are reviewed in Appendix A.

2. THEORY

A. DRO Tuning Overview

A nonlinear-optical material pumped by intense optical radiation at frequency ω_p can provide gain at two lower frequencies, called the signal and the idler and related by the conservation-of-energy condition

$$\omega_p = \omega_s + \omega_i . \tag{1}$$

The parametric interaction is phase dependent, and proper phasing is required for energy to flow from the from the pump field to the signal and the idler fields. Phase-velocity matching ensures that the relative phases of the three waves do not change with propagation through the nonlinear material. Phase matching is described by the wave-vector mismatch, which for the case of collinear propagation can be expressed by the scalar relationship

$$\Delta k = k_p - k_s - k_i = (n_p\omega_p - n_s\omega_s - n_i\omega_i)/c, \tag{2}$$

where k_p, k_s, and k_i are the respective wave-vector magnitudes of the pump, the signal, and the idler waves, with corresponding indices of refraction given by n_p, n_s, and n_i, and c is the velocity of light. Useful parametric gain exists in the range of signal and idler frequencies for which $|\Delta k| \leq \pi/l$, where l is the length of the nonlinear material. The parametric gain is maximum near $\Delta k = 0$. Phase matching is often achieved by controlling the birefringence of a nonlinear crystal through temperature or angle of propagation.

An OPO requires feedback at either (or both) the signal or the idler frequencies. If there is feedback at only one frequency, the device is called a singly resonant oscillator. Doubly resonant oscillators have feedback at both the signal and the idler frequencies. Feedback can be provided by placing the nonlinear material in a cavity formed by two external mirrors, or, in the case of monolithic OPO's, highly reflecting coatings can be applied directly to the nonlinear material. Ring-cavity configurations offer the advantages of reduced feedback to the pump source and improved OPO conversion efficiency.[11] Figure 1 illustrates schematically several configurations for parametric oscillators. Both standing-wave and ring-cavity monolithic DRO's were used for the experimental observations described in this paper. The tuning properties were quite similar, and the same model of tuning properties could be used for both, because the return path length differed little from the gain path in the ring resonators.

Phase matching is the major factor in determining broad tuning properties of an OPO, although cavity resonances have the major effect on details of frequency tuning. The conditions $\omega_p = \omega_s + \omega_i$ and $\Delta k = 0$ define phase-matching curves. The most commonly shown OPO phase-matching curve is the parabolalike shape for type-I phase matching in a birefringent crystal, for which the signal and the idler waves have the same polarization and the pump wave has the orthogonal polarization. Figure 2(a) shows a near-degeneracy ($\omega_s \sim \omega_i$) section of the temperature-tuning curve for a $LiNbO_3$ noncritically phase-matched OPO. Propagation is along a crystal principal axis in noncritical phase matching, which reduces dependence on propagation direction and eliminates birefringent walk-off.

The spectral width of the parametric gain is also determined by phase matching. A typical spectral distribution for single-pass gain at a fixed temperature is shown in Fig. 2(b). Doubly resonant oscillation also entails simultaneous signal and idler resonance. Dispersion causes different cavity axial-mode frequency spacings for the two waves, and the simultaneous resonance condition thus occurs only at intervals in frequency. The regions of simultaneous resonance, called cluster frequencies, are indicated in Fig. 2(c). Early DRO's were observed to oscillate on a group or cluster of adjacent cavity axial modes. The wavelength of the cluster would at first shift continuously with tuning and then exhibit a discontinuous

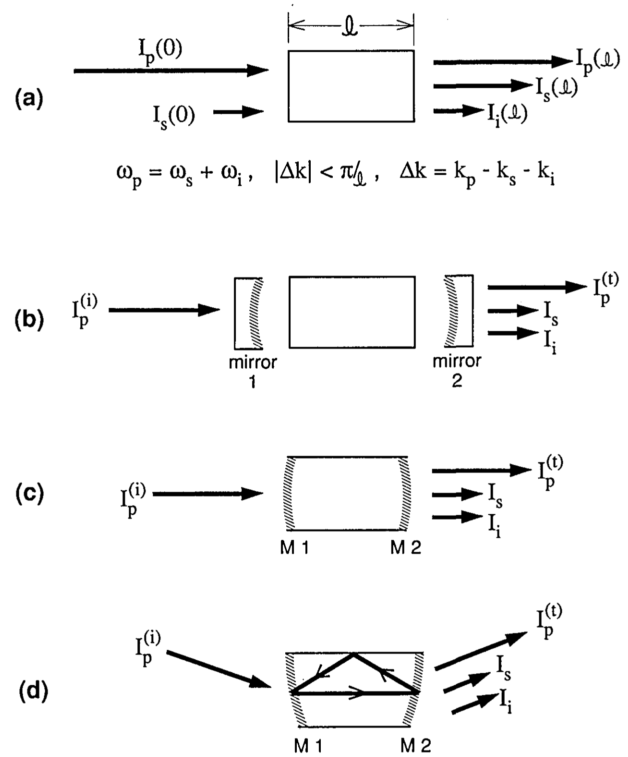

Fig. 1. (a) Schematic representation of optical parametric amplification. Optical parametric oscillators can be formed by the addition of mirrors that are separate from the nonlinear material, as shown in (b). Monolithic oscillators (c) and (d), with highly reflecting coatings (M's) applied directly to the nonlinear material, offer the advantages of low loss and rigidity that are important in stable, single-frequency DRO operation. Ring oscillators (d) offer the advantages of reduced feedback and improved conversion efficiency over standing-wave oscillators.

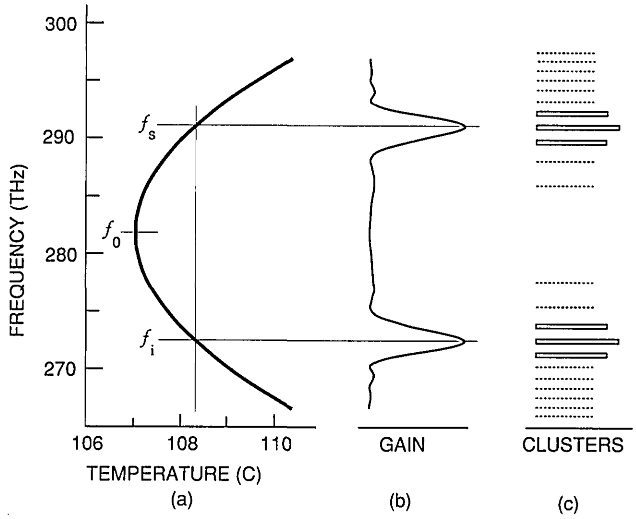

Fig. 2. (a) Typical OPO tuning curve near the degeneracy frequency $f_0 = f_p/2$, where f_p is the pump frequency. The signal and the idler frequencies are shown for a $LiNbO_3$ OPO as a function of the tuning parameter, in this case temperature. For a fixed value of the tuning parameter, single-pass parametric gain exists in bands that are centered on the phase-matching wavelengths, as shown in (b). DRO's have the added constraint that the signal and the idler cavity resonances must coincide in satisfying the condition $f_p = f_s + f_i$, which results in output at cluster frequencies (c). Only two or three clusters, represented by open horizontal bars, are located within the gain bandwidth. Usually one cluster, represented by the longest open bar, dominates.

jump to another cluster of modes. The curves of Fig. 2 are intended to illustrate some general properties of the frequency tuning of DRO's. The curves were calculated for the 12.5-mm-long monolithic $MgO{:}LiNbO_3$ oscillators, pumped at 564 THz (532 nm), that are discussed in this paper.

A useful device for understanding the requirement for simultaneous cavity resonances is a type of diagram used by Giordmaine and Miller[2]; the cavity resonances near the oscillating signal and idler frequencies are plotted as a function of the respective frequencies, as shown schematically in Fig. 3. The difference between signal and idler axial mode spacings, $\delta\omega_s$ and $\delta\omega_i$, respectively, is exaggerated in this figure for the purpose of illustration. One frequency, here the signal, increases from left to right. The other frequency scale, the idler, is determined by the first scale and the conservation-of-energy condition in such a way that a vertical line drawn through the diagram will give signal and idler frequencies that satisfy Eq. (1). If a signal–idler resonance pair lies on a vertical line, it satisfies the simultaneous resonance condition. If the temperature or the dc electric field applied to the crystal is changed, the position of the resonances will advance along the scales, one to the left and the other to the right, at slightly different rates because of dispersion, but the scales will not change. If pump frequency is changed, the frequencies of the cavity resonances will not change, but one of the frequency scales will be displaced with respect to the other, and the respective resonances will move with that scale.

Two types of discontinuous frequency shift are indicated in Fig. 3. One is an axial mode hop, and the other is a cluster jump. As a tuning variable is changed, better coincidence in satisfying the conservation-of-energy condition is attained on adjacent signal and idler axial modes. It then becomes advantageous for the oscillation frequencies to hop to the adjacent modes, to one higher in frequency and the other lower. This type of discontinuous frequency change is referred to as a mode hop. Other factors such as phase matching also affect the selection of the oscillation frequencies. As the tuning variable changes, phase matching also changes, and at some point it is advantageous for the oscillator to jump to the next cluster. This is illustrated in the schematic tuning curve of Fig. 4. The signal or the idler oscillation frequency progresses along a cluster curve in a series of mode hops until another cluster curve more closely approaches phase matching. At that point the larger discontinuous frequency change of a cluster jump takes the oscillation to the next cluster curve. Figures 3 and 4 are only schematic, with dispersion greatly exaggerated. Typically there several hundred axial modes between adjacent cluster frequencies.

Simultaneous resonance of signal–idler mode pairs occurs as a tuning parameter is continuously adjusted. In general, however, coincidence is not perfect, and oscillation of a particular mode pair depends on the degree of frequency matching and phase matching. The frequency mismatch $\Delta\omega$ of a signal–idler mode pair can be defined as the shift in frequency that is required of either the signal or the idler in order to bring the two resonances into coin-

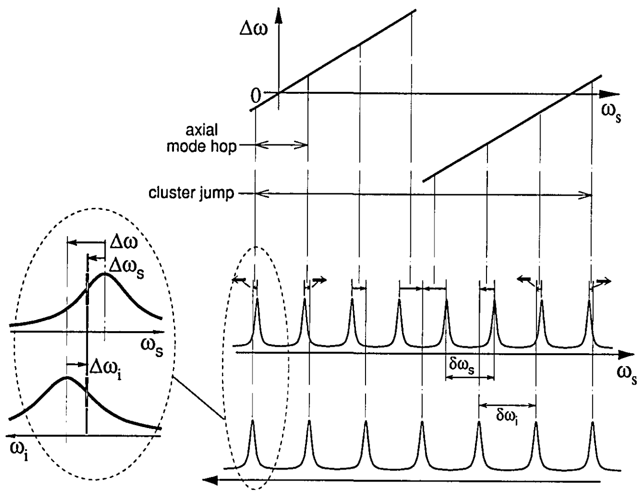

Fig. 3. Diagram[2] that shows the relationship between the DRO signal and idler resonance frequencies and the conservation-of-energy condition. Signal resonances are plotted as a function of signal frequency ω_s on an ordinary linear scale, with frequency increasing from left to right. The idler frequency scale is determined by that of the signal and the relationship $\omega_p = \omega_s + \omega_i$. In the display of idler resonances, therefore, frequency increases from right to left. A signal–idler pair that has both resonances centered on a vertical line are in coincidence, satisfying $\omega_p = \omega_s + \omega_i$. In general there will be some frequency mismatch $\Delta\omega$ for each mode pair. The frequency mismatch is the frequency shift that is required in order for either signal or idler resonance to produce coincidence. The detail on the left-hand side shows the frequency mismatch $\Delta\omega$ for a mode pair and its components $\Delta\omega_s$ and $\Delta\omega_i$, which are the respective frequency displacements from the centers of the signal and the idler cavity resonances to the frequencies most favorable for parametric oscillation. Dispersion is exaggerated in this schematic representation. There are typically hundreds of cavity axial modes between the cluster frequencies for which $\Delta\omega = 0$.

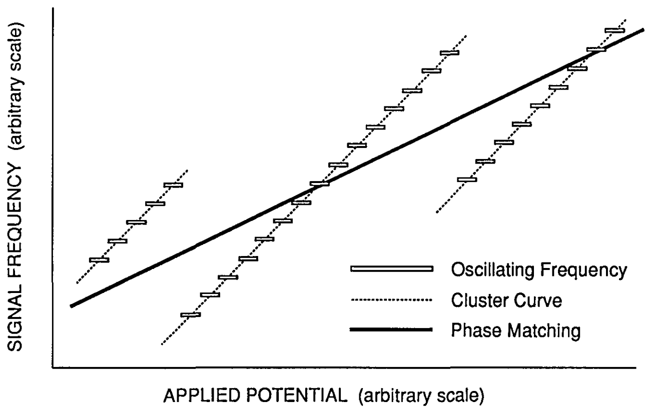

Fig. 4. Schematic representation of a detailed portion of an idealized tuning curve for a DRO. Oscillation progresses along cluster curves in discontinuous frequency changes, called axial mode hops, as a tuning variable is changed. A larger discontinuous frequency change, a cluster jump, occurs when better phase matching exists on an adjacent cluster curve.

cidence to satisfy Eq. (1). It is convenient to express the frequency mismatch as the sum of two components:

$$\Delta\omega = \Delta\omega_s + \Delta\omega_i\,. \tag{3}$$

Here $\Delta\omega_s$ is the frequency shift from the peak of the signal resonance to the signal frequency that is most favorable for oscillation for that mode pair. Correspondingly, $\Delta\omega_i$ is the frequency shift from the peak of the idler resonance to the idler frequency that is most favorable for oscillation, as illustrated in Fig. 3. The signal component is measured on the signal frequency scale, and the idler component is measured on the idler frequency scale. The directions of these scales are opposite. One increases from left to right, and the other is reversed, increasing from right to left. Therefore $\Delta\omega_s$ and $\Delta\omega_i$ appear in opposite directions in Fig. 3, even though they have the same sign. The frequency displacements of the signal and the idler from their respective resonance peaks are discussed in detail in Subsection 2.C. It is useful to consider the dependence of the OPO threshold on frequency mismatch and phase mismatch first.

B. DRO Threshold with Imperfect Signal-Idler Frequency Coincidence

Even an extremely small frequency mismatch can have significant effects on frequency selection and threshold of the DRO, particularly when cavity finesse is high. The threshold relationship obtained here is the same as that derived in the quantum-mechanical analysis by Graham and Haken[9] and is similar to but more detailed than the threshold equation given by Giordmaine and Miller.[2] The result given here is in terms of classical electromagnetic theory and is more easily applied to the tuning analysis that follows. This threshold relationship is limited to cavities with moderate-to-infinitesimal losses. The effect of phase and frequency mismatch on the thresholds of DRO's with arbitrary cavity losses was discussed by Falk.[12] Falk's results are used to estimate the conditions under which the threshold equation used here is appropriate.

The threshold for oscillation is obtained by setting the parametric gain equal to the cavity losses. The electric fields of the pump, the signal, and the idler waves are expressed in terms of complex amplitude and exponentials:

$$E_j(z,t) = \tfrac{1}{2}[E_j(z)\exp i(k_j z - \omega_j t) + \text{c.c.}],$$

where the subscript j indicates signal, idler, or pump, k is the wave-vector magnitude, ω is the angular frequency, z is the coordinate in the direction of propagation, and t is time. The coupled equations that describe parametric amplification of monochromatic plane waves traveling in the z direction are[7]

$$\frac{\mathrm{d}E_s}{\mathrm{d}z} = i\kappa_s E_p E_i^* \exp(i\Delta kz), \tag{4a}$$

$$\frac{\mathrm{d}E_i}{\mathrm{d}z} = i\kappa_i E_p E_s^* \exp(i\Delta kz), \tag{4b}$$

and

$$\frac{\mathrm{d}E_p}{\mathrm{d}z} = i\kappa_p E_s E_i \exp(-i\Delta kz), \tag{4c}$$

where mks units are used and $\kappa_s = \omega_s d_{\text{eff}}/(n_s c)$, $\kappa_i = \omega_i d_{\text{eff}}/(n_i c)$, and $\kappa_p = \omega_p d_{\text{eff}}/(n_p c)$, with n_s, n_i, and n_p the respective refractive indices for the signal, the idler, and the pump, c the velocity of light, and d_{eff} the effective nonlinear-optical coefficient. The solution used here is derived under the assumptions that at threshold pump depletion is insignificant and that the respective changes in signal and pump amplitudes, ΔE_s and ΔE_i, are small compared with the amplitudes. Hence E_s and E_i are treated as constants in calculating the changes, that is,

$$\Delta E_s = i\kappa_s E_p E_i^* l\ \text{sinc}(\Delta kl/2) \tag{5a}$$

and

$$\Delta E_i = i\kappa_i E_p E_s^* l\ \text{sinc}(\Delta kl/2). \tag{5b}$$

The length of the nonlinear crystal is again given by l, and the sinc function is defined by $\text{sinc}\,x = \sin(x)/x$.

For the low-loss DRO's considered here, Eqs. (5) are adequate for modeling the parametric gain. Other solutions to Eqs. (4) include general monochromatic plane-wave solutions[13] that permit both pump depletion and arbitrary changes in E_s and E_i and somewhat more restrictive solutions that involve no pump depletion but have arbitrarily large changes in E_s and E_i.[5,7] A solution of the latter type was used[12] for threshold analysis of DRO's with arbitrary strength of resonance.

The parametric gain must compensate both for a decrease in amplitude and for the phase change that is due to propagation in the cavity. The phasor diagram shown in Fig. 5 helps to illustrate this discussion. After a round-trip cavity transit the signal electric-field amplitude is reduced by a factor $(1 - a_s)$, and the phase is shifted by an angle ψ_s. Similarly, the idler amplitude is reduced by $(1 - a_i)$, and the phase is shifted by ψ_i. At threshold this change is balanced by the increments of the electric field ΔE_s and ΔE_i, added by the parametric interaction:

$$(1 - a_s)\exp(i\psi_s)E_s + \Delta E_s = E_s \tag{6a}$$

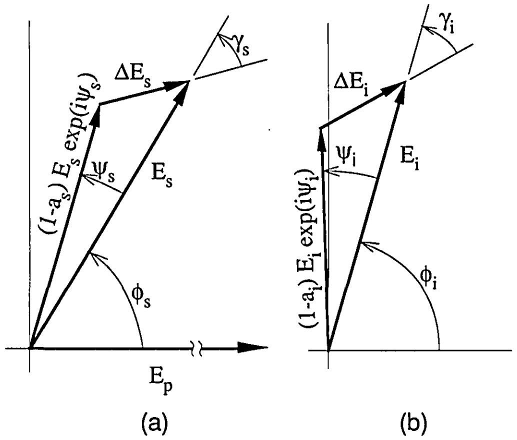

Fig. 5. Phasor diagrams schematically show amplitude losses a_s and a_i and phase shifts ψ_s and ψ_i after one round-trip cavity transit for the signal and the idler, respectively. At threshold the increments of electric-field amplitude added by optical parametric amplification, ΔE_s and ΔE_i, must restore the original fields.

and

$$(1 - a_i)\exp(i\psi_i)E_i + \Delta E_i = E_i . \tag{6b}$$

Choosing time so that the pump amplitude is real, $E_p = |E_p|$, expressing the signal and idler amplitudes as $E_s = |E_s|\exp(i\phi_s)$ and $E_i = |E_i|\exp(i\phi_i)$, and applying the conditions that a_s, a_i, ψ_s, and ψ_i are all small, we can write Eqs. (6) as

$$\Delta E_s = |E_s|(a_s^2 + \psi_s^2)^{1/2} \exp(i\phi_s - i\gamma_s) \tag{7a}$$

and

$$\Delta E_i = |E_i|(a_i^2 + \psi_i^2)^{1/2} \exp(i\phi_i - i\gamma_i), \tag{7b}$$

where $\gamma_s = \tan^{-1}(\psi_s/a_s)$ and $\gamma_i = \tan^{-1}(\psi_i/a_i)$.

Substituting Eqs. (7) into Eqs. (5) results in two equations for the complex arguments and two equations for the magnitudes. The relationships for the complex arguments,

$$\phi_s + \phi_i = \gamma_s + \pi/2 \tag{8a}$$

and

$$\phi_s + \phi_i = \gamma_i + \pi/2, \tag{8b}$$

immediately yield $\gamma_s = \gamma_i$ for the stationary solution, or

$$\frac{\psi_s}{a_s} = \frac{\psi_i}{a_i}. \tag{9}$$

Note that Eqs. (8) are consistent with the result that the sum of the signal and the idler phases is constant when referenced to the phase of the pump for stable single-mode-pair operation.[10]

The sum of the unpumped-cavity-round-trip phase shifts,

$$\psi = \psi_s + \psi_i, \tag{10a}$$

is useful for the purpose of comparison with the results of Ref. 12 and for conversion to frequency mismatch. When Eqs. (9) and (10a) are combined, the individual phase shifts can be expressed in terms of the sum by

$$\psi_s = \frac{a_s}{a_s + a_i}\psi \tag{10b}$$

and

$$\psi_i = \frac{a_i}{a_s + a_i}\psi. \tag{10c}$$

The threshold equation is obtained by taking the product of the two equations for the magnitudes that are obtained when Eqs. (7) are substituted into Eqs. (5), with the result that

$$[(a_s^2 + \psi_s^2)(a_i^2 + \psi_i^2)]^{1/2} = \kappa_s\kappa_i E_p^2 l^2 \,\mathrm{sinc}^2(\Delta k l/2) = \Gamma^2 l^2 \,\mathrm{sinc}^2(\Delta k l/2). \tag{11}$$

The quantity Γ^2 is the parametric gain for perfect phase matching, and it is proportional to the pump intensity.[7] With Eqs. (10b) and (10c), the threshold relationship given by Eq. (11) can be written in the form

$$\Gamma^2 l^2 = \frac{a_s a_i}{\mathrm{sinc}^2(\Delta k l/2)}\left[1 + \frac{\psi^2}{(a_s + a_i)^2}\right]. \tag{12}$$

Figure 6 shows the DRO threshold parameter $\Gamma^2 l^2$ as a function of the phase-shift sum ψ as given by Eq. (12) for two sets of cavity losses $a_s = a_i = 0.0087$ and $a_s = a_i = 0.0033$, corresponding to finesse values $\mathcal{F} = 360$ and $\mathcal{F} = 960$ of the DRO's used in the experimental measurements. The shape of the threshold curve does not change significantly, but the minimum value is translated toward zero as the losses decrease. The width of the threshold curves, defined as the region over which threshold is less than twice the minimum value, therefore decreases as cavity losses decrease.

In the application of Eq. (12) thresholds are expressed in terms of cavity finesse because it is easier to measure the ratio of resonance width to spacing than it is to measure losses directly. A comparison of the above analysis with

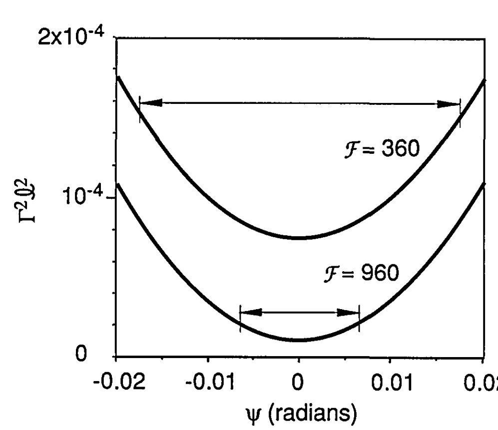

Fig. 6. Comparison of thresholds for DRO's with differing cavity finesses. Thresholds are calculated as a function of the sum of the cavity-round-trip phase shifts ψ with Eq. (12) for two DRO's with cavity finesse $\mathcal{F}_s = \mathcal{F}_i = 360$ and $\mathcal{F}_s = \mathcal{F}_i = 960$. The shape of the curve does not change, but the width, defined as the region over which threshold is less than twice its minimum value, decreases for higher finesse.

that of a parallel-plate interferometer[14] or of optical cavities in general[15] shows that the amplitude loss coefficients a_s and a_i are related to the cavity finesse at signal and idler frequencies $\mathcal{F}_s$ and $\mathcal{F}_i$, respectively, by

$$\mathcal{F}_s \approx \pi/a_s, \qquad \mathcal{F}_i \approx \pi/a_i. \tag{13}$$

It is also more convenient to use frequency mismatch than phase shift. The components of the frequency mismatch are related to the phase shifts by

$$\Delta\omega_s = \frac{\delta\omega_s\psi_s}{2\pi}, \qquad \Delta\omega_i = \frac{\delta\omega_i\psi_i}{2\pi}. \tag{14}$$

When Eqs. (3), (10), and (12)–(14) are combined, the threshold equation becomes

$$\Gamma^2 l^2 = \frac{\pi^2}{\mathcal{F}_i\mathcal{F}_s \operatorname{sinc}^2(\Delta k l/2)}\left[1 + \left(\frac{2\Delta\omega\mathcal{F}_i\mathcal{F}_s}{\mathcal{F}_i\delta\omega_s + \mathcal{F}_s\delta\omega_i}\right)^2\right]. \tag{15}$$

The threshold relationship given by Eq. (12) or (15) agrees with other threshold expressions under appropriate conditions. This result was obtained with a first-order plane-wave approximation for parametric gain. In the case of perfect phase matching, $\Delta k = 0$, and no frequency mismatch, $\Delta\omega = 0$, these equations reduce to $\Gamma^2 l^2 = a_s a_i$, which is the result obtained directly for this case.[7] Focusing and coupling to cavity modes[16] must be considered for quantitative threshold calculations. The plane-wave derivation of threshold is adequate for the analysis of tuning when it is necessary to know only the relative dependence of threshold on Δk and $\Delta\omega$.

There is also agreement with the central result of Falk's analysis[12] in the limit of high cavity finesse, that is, $a_s \ll 1$ and $a_i \ll 1$. Rewritten in the notation used here, Falk's Eq. (9) becomes

$$\frac{\Gamma^2 l^2}{2} = \frac{B \sin\left[2\tan^{-1}\left(\frac{\sin\psi}{C + \cos\psi}\right) - \psi\right]}{\sin\left[\tan^{-1}\left(\frac{\sin\psi}{C + \cos\psi}\right)\right]\operatorname{sinc}^2\left(\frac{\Delta k l}{2}\right)} - \frac{1}{\operatorname{sinc}^2\left(\frac{\Delta k l}{2}\right)}, \tag{16}$$

where, defining $R_s = 1 - a_s$ and $R_i = 1 - a_i$,

$$B = \frac{R_i(1 - R_s^2)}{R_i^2 - R_s^2}, \qquad C = \frac{R_s}{R_i}\frac{1 - R_i^2}{1 - R_s^2}.$$

Equation (16) is a more accurate approximation of the DRO threshold applicable for arbitrary cavity loss. However, it is unwieldy and must be evaluated as a limit when $\psi = 0$ or when $a_s = a_i$. Evaluation of Eq. (12) yields threshold values that differ from those obtained from Eq. (16) by approximately the fraction $(a_s a_i)^{1/2}$. There is a fortuitous partial compensation for this disparity in the approximation for finesse given in Eqs. (13). For $\mathcal{F}_s = \mathcal{F}_i = 5$, the difference between Eqs. (15) and (16) is less than 13% at the frequency mismatch for which threshold is twice its minimum value, and the difference decreases with decreasing frequency mismatch. For $\mathcal{F}_s = \mathcal{F}_i = 10$, the difference is 4% at twice minimum threshold, and the agreement again improves as minimum threshold is approached.

Equation (12) or (15) could be used directly to determine the mode pair with the lowest threshold for oscillation. It is more convenient, however, to restrict the possible mode pairs on which oscillation may take place to a small number, based on frequency mismatch and wave-vector mismatch considerations. This is done in Subsection 2.C, where it is shown that there are three mode pairs in the phase-matching bandwidth for which the frequency mismatch is a minimum. Which of these three mode pairs has the lowest threshold depends on the respective values of Δk, $\Delta\omega$, and the cavity finesse.

C. Frequency Selection in the DRO

The selection of signal and idler frequencies in a cw DRO operating on a single mode pair is determined by two conditions: the conservation of energy stated in Eq. (1) and the minimum threshold for oscillation. An approximation for the threshold condition was given in Eq. (15). The conservation-of-energy condition becomes implicit in the analysis of the condition of minimum threshold. In this analysis it is convenient to follow the approach used by Boyd and Ashkin[3] and to define the signal and the idler axial mode numbers

$$m_s = 2ln_s/\lambda_s = ln_s\omega_s/(\pi c) \tag{17a}$$

and

$$m_i = 2ln_i/\lambda_i = ln_i\omega_i/(\pi c), \tag{17b}$$

which are continuous variables that take on integer values at cavity resonances. The free-space wavelengths of the signal and the idler are given by λ_s and λ_i, respectively, and $2ln_s$ and $2ln_i$ are the respective optical lengths for a round-trip cavity transit. The free spectral ranges or mode spacings of the signal–idler resonances, $\delta\omega_s$ and $\delta\omega_i$, respectively, are the frequency changes that change the mode numbers m_s and m_i by one; that is,[4]

$$\frac{\partial m_s}{\partial\omega_s} = \frac{l}{\pi c}\left(n_s + \omega_s\frac{\partial n_s}{\partial\omega_s}\right) = \delta\omega_s^{-1} \tag{18a}$$

and

$$\frac{\partial m_i}{\partial\omega_i} = \frac{l}{\pi c}\left(n_i + \omega_i\frac{\partial n_i}{\partial\omega_i}\right) = \delta\omega_i^{-1}. \tag{18b}$$

The sum of the mode numbers

$$m = m_s + m_i, \tag{19}$$

which is also a continuous variable, is useful for the description of cluster effects. A signal frequency and an idler frequency that satisfy Eq. (1) and for which m is an integer compose a cluster frequency pair. In general, cavity resonances are not located precisely at the cluster frequencies. Only the sum $m_s + m_i$ must be an integer at the cluster frequencies; the individual mode numbers in general differ from integers by amounts equal in magnitude but opposite in sign. The cavity resonance pairs that most closely satisfy the conservation-of-energy condition and therefore that are most favorable for oscillation are also the resonances for which m is most nearly an integer. Equivalently, oscillation frequencies of a DRO are displaced from a frequency pair at which m is an integer by no more than one half of the respective axial mode spacings, whereas there are typically hundreds of modes between adjacent signal or idler cluster frequencies at which m is an integer.

Two further quantities that are useful for the description of the mode hops and cluster jumps of DRO tuning, Δm and Δm_s, are obtained by subtracting the integer nearest the mode number from the mode number:

$$\Delta m = m - \mathrm{ROUND}(m) \tag{20a}$$

and

$$\Delta m_s = m_s - \mathrm{ROUND}(m_s). \tag{20b}$$

These quantities are used in the calculation of oscillation frequencies and tuning-variable tolerances.

At optimum operating conditions, the quantities Δk, Δm, and Δm_s will all be zero, indicating perfect phase matching and simultaneous cavity resonances at the desired signal and idler frequencies. Adjustment of three independent parameters is necessary in order to reach this condition. The discussion presented here is given in general terms with quantities Δk, m, and m_s and in specific terms of the tuning or control parameters that are used in the experimental observations. The experimental observations use temperature T and applied potential V as adjustable parameters to control the output signal frequency ω_s. Pump frequency ω_p is used as the required third adjustable parameter for the calculations. Simple Taylor expansions for Δk, m, and m_s were found to be adequate for modeling the observed frequency tuning:

$$\begin{aligned}\Delta k = {} & \left(\frac{\partial \Delta k}{\partial \omega_s}\right)_{\omega_p}(\omega_s - \omega_{s,0}) + \frac{1}{2}\left(\frac{\partial^2 \Delta k}{\partial {\omega_s}^2}\right)_{\omega_p}(\omega_s - \omega_{s,0})^2 \\ & + \left(\frac{\partial \Delta k}{\partial \omega_p}\right)_{\omega_s}(\omega_p - \omega_{p,0}) + \frac{\partial \Delta k}{\partial T}(T - T_0) \\ & + \frac{\partial \Delta k}{\partial V}V + \Delta k_0,\end{aligned} \tag{21}$$

$$\begin{aligned}m = {} & \left(\frac{\partial m}{\partial \omega_s}\right)_{\omega_p}(\omega_s - \omega_{s,0}) + \frac{1}{2}\left(\frac{\partial^2 m}{\partial {\omega_s}^2}\right)_{\omega_p}(\omega_s - \omega_{s,0})^2 \\ & + \left(\frac{\partial m}{\partial \omega_p}\right)_{\omega_s}(\omega_p - \omega_{p,0}) + \frac{\partial m}{\partial t}(T - T_0) + \frac{\partial m}{\partial V}V + m_0,\end{aligned} \tag{22}$$

and

$$\begin{aligned}m_s = {} & \frac{\partial m_s}{\partial \omega_s}(\omega_s - \omega_{s,0}) + \frac{1}{2}\frac{\partial^2 m_s}{\partial {\omega_s}^2}(\omega_s - \omega_{s,0})^2 \\ & + \frac{\partial m_s}{\partial T}(T - T_0) + \frac{\partial m_s}{\partial V}V + m_{s,0}.\end{aligned} \tag{23}$$

A second-order derivative is used for signal frequency because the first-order derivatives $(\partial \Delta k/\partial \omega_s)_{\omega_p}$ and $(\partial \Delta m/\partial \omega_s)_{\omega_p}$ become zero at degeneracy, and dispersion of $\partial m_s/\partial \omega_s$ is essential to the analysis.

The notation of a partial derivative in parentheses with a parameter subscript to the right-hand parenthesis indicates that the parameter of the subscript is held constant for the differentiation. The conservation-of-energy condition is introduced through this device. Consider a function that is dependent on the signal, the idler, and the pump frequencies $f = f(\omega_s, \omega_i, \omega_p)$, and require that the conservation-of-energy condition $\omega_s + \omega_i = \omega_p$ hold. Differentiation with respect to ω_s, with ω_p held constant, requires that as ω_s is increased ω_i must decrease, or

$$\left(\frac{\partial f}{\partial \omega_s}\right)_{\omega_p} = \frac{\partial f}{\partial \omega_s} - \frac{\partial f}{\partial \omega_i},$$

and, similarly,

$$\left(\frac{\partial f}{\partial \omega_p}\right)_{\omega_s} = \frac{\partial f}{\partial \omega_p} + \frac{\partial f}{\partial \omega_i}.$$

The derivatives used in Eqs. (21)–(23) are expanded in Table 1. The differentiation is straightforward and can be verified by inspection of Eqs. (2), (17), and (19).

1. Phase-Matching Curve and Cluster Curves

A number of equations considered below are identical except for the exchange of the tuning variables. An economy of notation is possible through the use of a general tuning parameter ζ, which is T, V, or ω_p in the specific example or, more generally, is any single parameter used to tune a DRO. A phase-matching curve as used here gives the signal frequency for which $\Delta k = 0$ as a function of the tuning parameter ζ and is denoted by $\omega_{s,\mathrm{PM}}(\zeta)$. This curve is obtained from Eq. (21) or from a similar

Table 1. Derivatives Used to Calculate Tuning of a Monolithic DRO

$$\frac{\partial^2 m_s}{\partial {\omega_s}^2} = \frac{l}{\pi c}\left(2\frac{\partial n_s}{\partial \omega_s} + \omega_s \frac{\partial {n_s}^2}{\partial {\omega_s}^2}\right)$$

$$\frac{\partial m_s}{\partial T} = \frac{l\omega_s}{\pi c}\frac{\partial n_s}{\partial T} + \frac{\omega_s n_s}{\pi c}\frac{\partial l}{\partial T}$$

$$\frac{\partial m_s}{\partial V} = \frac{\omega_s}{\pi c}\left(n_s \frac{\partial l}{\partial V} + l\frac{\partial n_s}{\partial V}\right)$$

$$\left(\frac{\partial m}{\partial \omega_s}\right)_{\omega_p} = \frac{l}{\pi c}\left(n_s - n_i + \omega_s \frac{\partial n_s}{\partial \omega_s} - \omega_i \frac{\partial n_i}{\partial \omega_i}\right)$$

$$\left(\frac{\partial^2 m}{\partial {\omega_s}^2}\right)_{\omega_p} = \frac{l}{\pi c}\left[2\left(\frac{\partial n_s}{\partial \omega_s} + \frac{\partial n_i}{\partial \omega_i}\right) + \omega_s \frac{\partial^2 n_s}{\partial {\omega_s}^2} + \omega_i \frac{\partial^2 n_i}{\partial {\omega_i}^2}\right]$$

$$\left(\frac{\partial m}{\partial \omega_p}\right)_{\omega_s} = \frac{l}{\pi c}\left(n_i + \omega_i \frac{\partial n_i}{\partial \omega_i}\right)$$

$$\frac{\partial m}{\partial T} = \frac{l}{\pi c}\left(\omega_s \frac{\partial n_s}{\partial T} + \omega_i \frac{\partial n_i}{\partial T}\right) + \frac{1}{\pi c}(\omega_s n_s + \omega_i n_i)\frac{\partial l}{\partial T}$$

$$\frac{\partial m}{\partial V} = \frac{n_s \omega_s + n_i \omega_i}{\pi c}\frac{\partial l}{\partial V} + \frac{l}{\pi c}\left(\omega_s \frac{\partial n_s}{\partial V} + \omega_i \frac{\partial n_i}{\partial V}\right)$$

$$\left(\frac{\partial \Delta k}{\partial \omega_s}\right)_{\omega_p} = \left(n_i - n_s + \omega_i \frac{\partial n_i}{\partial \omega_i} - \omega_s \frac{\partial n_s}{\partial \omega_s}\right)\Big/ c = -\frac{\pi}{l}\left(\frac{\partial m}{\partial \omega_s}\right)_{\omega_p}$$

$$\left(\frac{\partial^2 \Delta k}{\partial {\omega_s}^2}\right)_{\omega_p} = -\frac{\pi}{l}\left(\frac{\partial^2 m}{\partial {\omega_s}^2}\right)_{\omega_p}$$

$$\left(\frac{\partial \Delta k}{\partial \omega_p}\right)_{\omega_s} = \left(n_p - n_i + \omega_p \frac{\partial n_p}{\partial \omega_p} - \omega_i \frac{\partial \omega_i}{\partial \omega_i}\right)$$

$$\frac{\partial \Delta k}{\partial T} = \frac{\omega_p}{c}\frac{\partial n_p}{\partial T} - \frac{\omega_s}{c}\frac{\partial n_s}{\partial T} - \frac{\omega_i}{c}\frac{\partial n_i}{\partial T}$$

$$\frac{\partial \Delta k}{\partial V} = \frac{\omega_p}{c}\frac{\partial n_p}{\partial V} - \frac{\omega_s}{c}\frac{\partial n_s}{\partial V} - \frac{\omega_i}{c}\frac{\partial n_i}{\partial V}$$

equation by setting the other adjustable parameters to fixed values and setting $\Delta k = 0$. The cluster curves $\omega_{s,\mathrm{Cl}}(\zeta)$ give the signal cluster frequencies as a function of the tuning parameter ζ. The cluster curves can be obtained from Eq. (22) or from a similar equation by setting m to integer values and again setting the other adjustable parameters to fixed values. For type-I phase matching, the phase-matching curve is a parabolalike curve, as shown in Fig. 1, and the cluster curves are a family of parabolalike curves. The oscillating frequencies are closest to the cluster curves near the points where the cluster curves intersect the phase-matching curve.

The signal frequency separation of adjacent clusters Ω_s can be obtained from the second-order approximation

$$\Delta m(\text{cluster}) = \pm 1 = \left(\frac{\partial m}{\partial \omega_s}\right)_{\omega_p} \Omega_{S\pm} + \frac{1}{2}\left(\frac{\partial^2 m}{\partial {\omega_s}^2}\right)_{\omega_p} {\Omega_{S\pm}}^2. \quad (24)$$

Away from degeneracy the first-order term dominates, and Eq. (24) is approximated by

$$\Omega_{S\pm} \approx \pm\left(\frac{\partial m}{\partial \omega_s}\right)_{\omega_p}^{-1} = \pm\frac{\delta\omega_i \delta\omega_s}{\delta\omega_i - \delta\omega_s}, \quad (25)$$

in agreement with Ref. 4. Phase-matching limitations result in a gain bandwidth with half-maximum values at the frequencies for which $\Delta k = \pm 0.886\pi/l \approx \pm\pi/l$ and a corresponding signal frequency full width at half-maximum of

$$\Delta\omega_s(\text{Gain FWHM}) \approx \left|\frac{2\pi}{l}\left(\frac{\partial \Delta k}{\partial \omega_s}\right)_{\omega_p}^{-1}\right|. \quad (26)$$

In the specific case of the monolithic DRO, for which derivatives are given in Table 1, $\pi/l(\partial m/\partial\omega_s)_{\omega_p} = -(\partial\Delta k/\partial\omega_s)_{\omega_p}$ and $\Delta\omega_s(\text{Gain FWHM}) \approx 2\Omega_S$. Since the frequency separation of the clusters is approximately one half of the parametric gain bandwidth, there are two or three clusters within the gain bandwidth. This is true for any DRO in which the nonlinear crystal is the only dispersive component and the crystal is traversed twice in each round-trip cavity transit but has parametric gain in only one direction.

2. Oscillation Frequencies

The oscillation frequencies are determined by phase matching, the center frequencies of the signal and the idler cavity resonances, the frequency mismatch of the resonances, the finesse values of the resonances, and the axial mode spacings. To model experimental observations, we calculate the frequencies of the parametric oscillation by the following procedure. First the signal frequency for phase matching $\omega_{s,\mathrm{PM}}$ is found for specified tuning parameters with the condition $\Delta k = 0$. Next the signal cluster frequency $\omega_{s,\mathrm{Cl}}$ that is closest to $\omega_{s,\mathrm{PM}}$ is found with the condition $\Delta m = 0$. If the DRO cavity has only moderate or low finesse and if the precise oscillating frequency and mode hops are not of concern, these two steps are all that are required. The extra resolution of frequency tuning can be obtained by using the value of Δm_s at the cluster frequency $\omega_{s,\mathrm{Cl}}$. This value is called $\Delta m_{s,\mathrm{Cl}}$ to indicate that it is calculated at the cluster frequency $\omega_{s,\mathrm{Cl}}$ for the specified tuning conditions.

The procedure for determining the fine details of tuning is illustrated in Fig. 7. The signal cluster frequency $\omega_{s,\mathrm{Cl}}$, obtained by the steps described above, is a reference point from which to start. The center of the nearest signal resonance is displaced from $\omega_{s,\mathrm{Cl}}$ by frequency $-\Delta m_{s,\mathrm{Cl}}\delta\omega_s$, where $\delta\omega_s$ is the signal axial mode frequency separation. The center of the nearest idler resonance is displaced by $-\Delta m_{i,\mathrm{Cl}}\delta\omega_i$ from the complementary idler cluster frequency $\omega_{i,\mathrm{Cl}} = \omega_p - \omega_{s,\mathrm{Cl}}$, where $\delta\omega_i$ is the idler mode frequency separation and ω_p is the pump frequency. Since $\omega_{s,\mathrm{Cl}}$ is a cluster frequency with $\Delta m = 0$, then $\Delta m_{i,\mathrm{Cl}} = -\Delta m_{s,\mathrm{Cl}}$, which permits the frequency mismatch of the signal–idler mode pair to be expressed as

$$\Delta\omega = \Delta m_{s,\mathrm{Cl}}(\delta\omega_s - \delta\omega_i). \quad (27)$$

Recall that the frequency mismatch is the shift in frequency of either the signal or the idler resonance that is necessary in order to bring the resonance pair into coincidence to satisfy the conservation-of-energy condition.

The displacement of the signal oscillation frequency $\omega_{s,\mathrm{Osc}}$ from the center of the signal cavity resonance is $\Delta\omega_s$, and the displacement of the idler oscillation frequency $\omega_{i,\mathrm{Osc}}$ from the center of the idler cavity resonance is $\Delta\omega_i$. From the above definitions and conditions imposed on round-trip cavity phase shifts for a stationary solution given in Eqs. (3), (10), (13), and (14), it follows that

$$\Delta\omega_s = \Delta\omega\delta\omega_s \mathcal{F}_i/(\delta\omega_s \mathcal{F}_i + \delta\omega_i \mathcal{F}_s) \quad (28a)$$

and

$$\Delta\omega_i = \Delta\omega\delta\omega_i \mathcal{F}_s/(\delta\omega_s \mathcal{F}_i + \delta\omega_i \mathcal{F}_s). \quad (28b)$$

Finally, the signal oscillation frequency is given by the sum of the cluster frequency plus the frequency separation of the signal cavity resonance from the cluster frequency plus the frequency shift from the center of the

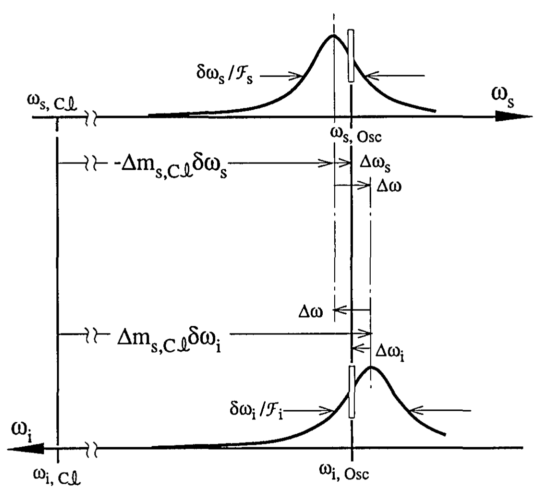

Fig. 7. Signal–idler resonance diagram similar to Fig. 3, expanded in detail to show the relationships between quantities. The signal and the idler cavity resonances on which oscillation occurs are displaced from the respective cluster frequencies $\omega_{s,\mathrm{Cl}}$ and $\omega_{i,\mathrm{Cl}}$ for the general case of nonzero frequency mismatch. The DRO oscillating frequencies $\omega_{s,\mathrm{Osc}}$ and $\omega_{i,\mathrm{Osc}}$ divide the frequency mismatch $\Delta\omega$ into the components $\Delta\omega_s$ and $\Delta\omega_i$.

signal cavity resonance, that is,

$$\omega_{s,\mathrm{Osc}} = \omega_{s,\mathrm{Cl}} - \Delta m_{s,\mathrm{Cl}}\delta\omega_s + \Delta\omega_s$$
$$= \omega_{s,\mathrm{Cl}} - \Delta m_{s,\mathrm{Cl}} \frac{\delta\omega_s \delta\omega_i (\mathcal{F}_s + \mathcal{F}_i)}{\delta\omega_s \mathcal{F}_i + \delta\omega_i \mathcal{F}_s}. \quad (29)$$

Equations (27) and (28a) are used to obtain the second step of Eq. (29).

The rate of frequency change associated with a general tuning parameter ζ could be obtained by direct differentiation of Eq. (29) or more simply by considering the tuning rates of the cavity resonances. The tuning rate for the frequency of the signal mode is $-\delta\omega_s(\partial m_s/\partial\zeta)$, and that for the frequency of the idler is $-\delta\omega_i(\partial m_i/\partial\zeta)$. The tuning rates of the cavity resonances are combined to yield the rate of change of the frequency mismatch,

$$\frac{\partial\Delta\omega}{\partial\zeta} = \delta\omega_s \frac{\partial m_s}{\partial\zeta} + \delta\omega_i \frac{\partial m_i}{\partial\zeta} = (\delta\omega_s - \delta\omega_i)\frac{\partial m_s}{\partial\zeta} + \delta\omega_i \frac{\partial m}{\partial\zeta}.$$

The tuning rate of the signal oscillation frequency is the sum of the tuning rate of the signal resonance plus the fraction $\delta\omega_s\mathcal{F}_i/(\delta\omega_s\mathcal{F}_i + \delta\omega_i\mathcal{F}_s)$ of $\partial\Delta\omega/\partial\zeta$, that is,

$$\frac{\partial\omega_{s,\mathrm{Osc}}}{\partial\zeta} = \frac{\delta\omega_s\delta\omega_i}{\delta\omega_s\mathcal{F}_i + \delta\omega_i\mathcal{F}_s}\left(\mathcal{F}_i\frac{\partial m}{\partial\zeta} - (\mathcal{F}_s + \mathcal{F}_i)\frac{\partial m_s}{\partial\zeta}\right). \quad (30)$$

This tuning is limited to a small range by mode hops or increased cavity losses as the oscillation is pulled off the peaks of the cavity resonances. On a broader scale, tuning progresses along a cluster curve in a series of mode hops. If finesse is high, it is possible that the oscillation jumps back and forth between adjacent cluster curves and also hops from one mode pair to the next along each of the cluster curves. The analysis of cluster jumps requires that the mode-hop structures on the two or three cluster curves that are closest to phase matching be compared in order to determine which cluster curve provides conditions most favorable for oscillation.

The tuning rates in the regions between mode hops and cluster jumps, which are described by Eq. (30), are strongly dependent on the relative values of finesse of the signal and the idler cavity resonances. Some caution, however, is required in the use of Eq. (30). For example, the calculation of $\partial\omega_{s,\mathrm{Osc}}/\partial V$ and $\partial\omega_{s,\mathrm{Osc}}/\partial T$ for DRO's with nearly equal signal and idler finesses involves the small difference of two quantities. In such situations it is important that the terms on the right-hand side of Eq. (30) be evaluated accurately for the specified operating conditions.

3. Tuning Limits and Mode Hops

Mode hops are periodic along the cluster curves, occurring every time $m_{s,\mathrm{Cl}}$ changes by one. Recall that $m_{s,\mathrm{Cl}}$ is the value of m_s on the cluster curve for which $\Delta m = 0$. The change of the tuning parameter $\Delta\zeta_{\mathrm{Hop\,spacing}}$ that corresponds to a mode hop is a quantity that is easily measured experimentally. Since a mode hop corresponds to a change of one in $m_{s,\mathrm{Cl}}$, it follows that the tuning parameter change that corresponds to the mode-hop spacing is

$$\Delta\zeta_{\mathrm{Hop\,spacing}} = \left|\left(\frac{\partial m_{s,\mathrm{Cl}}}{\partial\zeta}\right)^{-1}\right|. \quad (31)$$

In the evaluation of $\partial m_{s,\mathrm{Cl}}/\partial\zeta$ it is helpful to use the derivative

$$\frac{\partial\omega_{s,\mathrm{Cl}}}{\partial\zeta} = -\frac{\partial m}{\partial\zeta}\left(\frac{\partial m}{\partial\omega_s}\right)^{-1}_{\omega_p}$$
$$= -\frac{\delta\omega_s\delta\omega_i}{\delta\omega_i - \delta\omega_s}\frac{\partial m}{\partial\zeta}. \quad (32)$$

The first step simply states that the cluster frequency $\omega_{s,\mathrm{Cl}}$ must change with the tuning parameter ζ in such a way that m does not change, and the second step is accomplished by using Eqs. (18) and (19). It is possible to expand the derivative $\partial m_{s,\mathrm{Cl}}/\partial\zeta$ by using first the chain rule of differentiation and then Eqs. (18) and (32) to obtain

$$\frac{\partial m_{s,\mathrm{Cl}}}{\partial\zeta} = \frac{\partial m_s}{\partial\zeta} + \frac{\partial m_s}{\partial\omega_s}\frac{\partial\omega_{s,\mathrm{Cl}}}{\partial\zeta} = \frac{\partial m_s}{\partial\zeta} - \frac{\delta\omega_i}{\delta\omega_i - \delta\omega_s}\frac{\partial m}{\partial\zeta}. \quad (33)$$

Another useful parameter is the maximum frequency shift from the cluster curve that can be achieved without a mode hop $(\omega_{s,\mathrm{Hop}} - \omega_{s,\mathrm{Cl}})$. This maximum is obtained directly when the extreme values of $\Delta m_{s,\mathrm{Cl}} = \pm 1/2$ are inserted into Eq. (29), yielding

$$\omega_{s,\mathrm{Hop}} - \omega_{s,\mathrm{Cl}} = \pm\frac{1}{2}\frac{\delta\omega_i\delta\omega_s(\mathcal{F}_i + \mathcal{F}_s)}{\delta\omega_i\mathcal{F}_s + \delta\omega_s\mathcal{F}_i}. \quad (34)$$

Cavity finesse can also limit the single-parameter tuning range. It follows from Eq. (15) that threshold is double its minimum value when the frequency mismatch $\Delta\omega$ reaches the value

$$\Delta\omega = \pm\frac{\mathcal{F}_i\delta\omega_s + \mathcal{F}_s\delta\omega_i}{2\mathcal{F}_i\mathcal{F}_s}.$$

The corresponding value of $\Delta m_{s,\mathrm{Cl}}$ is obtained from Eq. (27). On substitution into Eq. (29) the maximum displacement of signal oscillation frequency from the cluster frequency allowed by cavity finesse is found to be

$$\omega_{s,\mathrm{Fin}} - \omega_{s,\mathrm{Cl}} = \pm\frac{\mathcal{F}_s + \mathcal{F}_i}{2\mathcal{F}_s\mathcal{F}_i}\frac{\delta\omega_s\delta\omega_i}{\delta\omega_i - \delta\omega_s}. \quad (35)$$

If finesse is large the frequency displacement allowed by Eq. (35) becomes significantly smaller than the frequency displacement required for a mode hop described by Eq. (34). In this case parametric oscillation on the cluster curve closest to phase matching ceases in a region near the mode hop. It is then possible for the parametric oscillation frequencies to jump to an adjacent cluster curve that is still within the phase-matching gain bandwidth if a favorable coincidence of signal and idler resonances exists on that cluster curve.

Plotting the mode-hop frequency limits $\omega_{s,\mathrm{Hop}}$ and the finesse frequency limits $\omega_{s,\mathrm{Fin}}$ in addition to the cluster curve provides additional information concerning the fine detail of tuning. On a broader scale, it is informative to display curves defining the phase-matching gain bandwidth along with the phase-matching curve. It may also be useful to display more than one cluster curve near the phase-matching curve.

An attempt was made to keep the results of this section general. For application to the specific case of a mono-

Table 2. Derivatives used to Model Tuning of MgO:LiNbO$_3$ Monolithic DRO's^a

$$\frac{\partial m_s}{\partial \omega_s} = 3.05 \times 10^{-11}\ (\text{rad/sec})^{-1}$$
$$\left(\frac{\partial m}{\partial \omega_s}\right)_{\omega_p} = 0$$
$$\left(\frac{\partial \Delta k}{\partial \omega_s}\right)_{\omega_p} = 0$$
$$\frac{\partial^2 m_s}{\partial {\omega_s}^2} = 1.10 \times 10^{-27}\ (\text{rad/sec})^{-2}$$
$$\left(\frac{\partial^2 m}{\partial {\omega_s}^2}\right)_{\omega_p} = 2.20 \times 10^{-27}\ (\text{rad/sec})^{-2}$$
$$\left(\frac{\partial^2 \Delta k}{\partial {\omega_s}^2}\right)_{\omega_p} = -5.53 \times 10^{-25}\ \frac{\text{rad/m}}{(\text{rad/sec})^2}$$
$$\frac{\partial m_s}{\partial \omega_p} \equiv 0$$
$$\left(\frac{\partial m}{\partial \omega_p}\right)_{\omega_s} = 3.05 \times 10^{-11}\ (\text{rad/sec})^{-1}$$
$$\left(\frac{\partial \Delta k}{\partial \omega_p}\right)_{\omega_s} = 5.44 \times 10^{-10}\ \frac{\text{rad/m}}{\text{rad/sec}}$$
$$\frac{\partial m_s}{\partial T} = 1.02°\text{C}^{-1}$$
$$\frac{\partial m}{\partial T} = 2.03°\text{C}^{-1}$$
$$\frac{\partial \Delta k}{\partial T} = 749\ \frac{\text{rad/m}}{°\text{C}}$$
$$\frac{\partial m_s}{\partial V} = \frac{-1.81 \times 10^{-6}\ \text{m/V}}{t}$$
$$\frac{\partial m}{\partial V} = \frac{-3.63 \times 10^{-6}\ \text{m/V}}{t}$$
$$\frac{\partial \Delta k}{\partial V} = \frac{3.61 \times 10^{-4}}{t}\ \frac{\text{rad/m}}{\text{V/m}}$$

^{a}The parameter t is the effective thickness of the crystal in meters; l = 0.0125 m, $\omega_{p,0} = 3.54070 \times 10^{15}$ rad/sec, $\omega_{s,0} = 1.77035 \times 10^{15}$ rad/sec, $T_0 = 107.04°$C, $V_0 = 0$.

lithic DRO tuned by temperature, applied potential, and pump frequency, the appropriate variables and derivatives from Table 1 are directly substituted for the terms involving the general tuning parameter ζ. Evaluation of the derivatives for the case of monolithic DRO's made from MgO:LiNbO$_3$ with propagation in the x direction and with an electric field applied in the y direction is discussed in Appendix A. Temperature-dependent dispersion, thermal expansion, the electro-optic effect, and the piezoelectric effect of the nonlinear-optical material are used in the evaluation. Results of this evaluation for experimental conditions described in Section 3 are given in Table 2.

3. EXPERIMENTAL OBSERVATIONS AND MODELING

A. Experimental Conditions

Two monolithic DRO's, which were described previously,[17,18] were used in the experimental observations. One DRO had lower finesse and had to be pulse-pumped in order to achieve the higher threshold power needed for parametric oscillation. The higher-finesse DRO operated above threshold with the available continuous pumping. The pump source was a diode-laser-pumped nonplanar ring oscillator,[19,20] constructed of neodymium-doped yttrium aluminum garnet (Nd:YAG) with the 1064-nm laser output converted to 532 nm by externally resonant second-harmonic generation.[21] Approximately 30 mW of cw pump radiation was generated. The laser operated in a single longitudinal and a single transverse mode. For cw operation of the laser, fundamental frequency stability was typically 10 kHz over short periods of time.[22,23] This value was doubled at the second harmonic. Higher peak power at similar average power was obtained by driving the laser into relaxation oscillations by 10% amplitude modulation of the diode-laser output at 320 kHz. Good frequency stability and high optical quality of the pump radiation, such as that achieved with the diode-laser-pumped solid-state laser, are important for obtaining stable DRO performance.

Both monolithic DRO's were operated with a ring-resonator configuration. They were constructed from 5% magnesium-oxide-doped lithium niobate (MgO:LiNbO$_3$).[24,25] Each of these monolithic resonators was 12.5 mm long, with the crystal x axis in the long direction. A ring path in these resonators was formed by reflections from two multilayer dielectric coated surfaces with 10-mm radii of curvature and a totally internally reflecting surface. The centers of curvature of the spherical surfaces were on a line parallel to and 180 μm inside the flat totally internally reflecting surface. A drawing of the monolithic DRO's is shown in Fig. 8. The 532-nm pump beam was mode matched for collinear propagation on the segment of the ring path parallel to the crystal x axis. The pump beam with extraordinary polarization did not follow the closed path of the signal and the idler waves with ordinary polarization because of bireflection.

Metal coatings for electric-field tuning were applied to the crystal surfaces perpendicular to the y axis. The thickness of the crystals between the electrodes was 2.2 mm. The finesse of both DRO's at 1.064 μm was measured with the Nd:YAG laser output directly, without second-harmonic generation. Electric-field tuning was used to scan the resonators through a free spectral range, and transmission through the resonators gave a measure of resonance width relative to the mode spacing. One DRO had a finesse of 360, and the other had finesse of 960. The lower-finesse device had an experimentally observed threshold for cw parametric oscillation of 35 mW, and the higher-finesse DRO had a threshold of 12 mW. The pump source could produce approximately 30 mW of

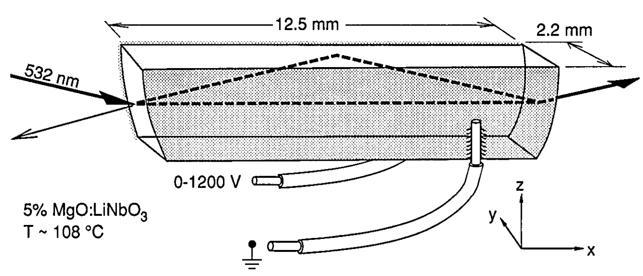

Fig. 8. DRO geometry used for experimental observations.

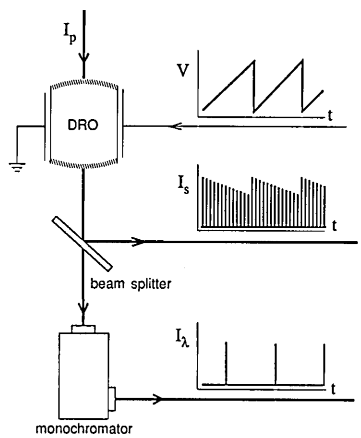

Fig. 9. Schematic representation of the setup used for DRO tuning measurements.

cw radiation at 532 nm. The higher-threshold OPO was pumped by 532-nm second harmonic, which consisted of 400-nsec pulses with 230-mW peak power at a 320-kHz repetition rate.

The output of the DRO's was tuned by temperature and electric field. Noncritical phase matching in $MgO:LiNbO_3$ was achieved for degeneracy at 107°C, and as temperature was increased the signal and the idler wavelengths separated from the 1.064-μm degeneracy point. For the tuning studies the potential applied to the crystal was repetitively ramped at fixed temperature. Output wavelength measurements were repeated at incrementally changed temperatures. An *f*/10, 1-m grating monochromator with a 600-line/mm grating was used for wavelength measurement. The DRO output directed into the monochromator consisted of a series of pulses; these pulses resulted either from the pulse pumping or from the mode hops produced by the ramped voltage with continuous pumping. The radiation transmitted by the monochromator usually consisted of a few pulses in a narrow spectral band that could be correlated with the potential applied to the DRO electrodes. A schematic representation of the experimental setup is shown in Fig. 9.

B. Cluster Tuning

The tuning of the high-finesse DRO involved spectral jumps back and forth between cluster curves as well as mode hops along the cluster curves. This behavior is illustrated in Fig. 10, where DRO output is displayed for a small voltage range at a constant temperature. In this figure output is resolved on three separate cluster curves. The monochromator slits were opened to provide a 5-nm transmission width, one sufficient to resolve the individual clusters while transmitting a number of mode hops. The central cluster curve with signal wavelength near 1043 nm dominated. Two other cluster curves fit within the phase-matching gain bandwidth, and output on these curves was observed near 1053 and 1037 nm. Competition with the central cluster curve, which depletes the pump wave, is evident in the two cluster curves to either side.

Three adjustable parameters were used to fit Eqs. (21) and (22) to the observations. A temperature-offset correction is used to fit the calculated phase-matching curve. There are inaccuracies in both the absolute measurement of temperature and in the temperature dependence of the dispersion relationships that make this necessary. The temperature adjustment was accomplished by shifting the data a fraction of 1°C but could have equally well been done by changing the parameter Δk_0 in Eq. (21). Another fitting parameter is required because the optical lengths of the DRO's are not precisely known. This fit is accomplished by changing the value of m_0 in Eq. (22) and has the effect of adjusting the placement of the cluster curves. The thickness of the DRO crystals is also used as a fitting parameter. The electrodes do not completely cover the surfaces, and fringing effects are not considered. Instead, it is assumed that there is a uniform electric field in the y direction given by $E_y = V/t$, where V is the applied potential and t is an effective thickness. Adjusting the thickness has the effect of changing the slopes of the cluster curves and voltage-tuned phase-matching curve. It is interesting to note that the piezoelectric effect, in addition to the electro-optic effect, is needed to model the observed tuning. When only the electro-optic effect is used, the calculated voltage-tuned cluster curves are parallel to the voltage-tuned phase-matching curve. A fourth fitting parameter not used here is the constant $m_{s,0}$ in Eq. (23). Adjustment of $m_{s,0}$ would allow for different phase shifts at the mirror surfaces for the signal and the idler and the possibility of different cavity lengths.[3] Adjustment of $m_{s,0}$ would change the calculated position of the mode hops.

The observed and fitted tuning curves for the DRO's are shown in Figs. 11 and 12. The theory is most easily expressed in terms of frequency. Frequency therefore is used as the primary ordinate scale in these graphs, and wavelength is included as a secondary scale for reference.

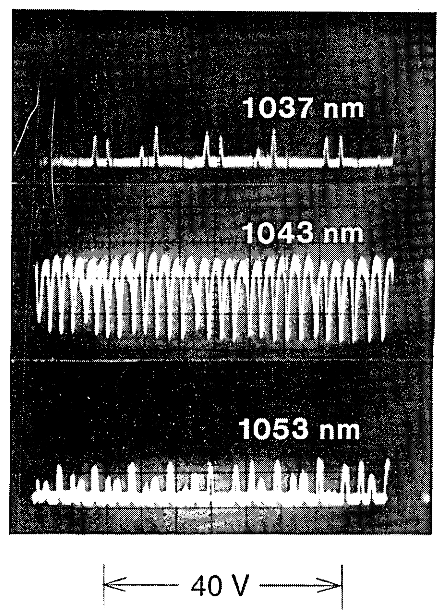

Fig. 10. Oscillograms of cw-pumped DRO output, showing simultaneous output on three cluster curves. The signal displayed is that produced by a photodiode placed after a monochromator with slits adjusted for a 5-nm bandpass. Each of the oscillograms corresponds to the same portion of the ramped voltage applied to the DRO. The change in applied potential is indicated. The oscillograms differ only in the wavelength setting of the monochromator, indicated for the individual traces. The output on the central cluster dominates and is so strong that the oscilloscope trace does not return to the baseline.

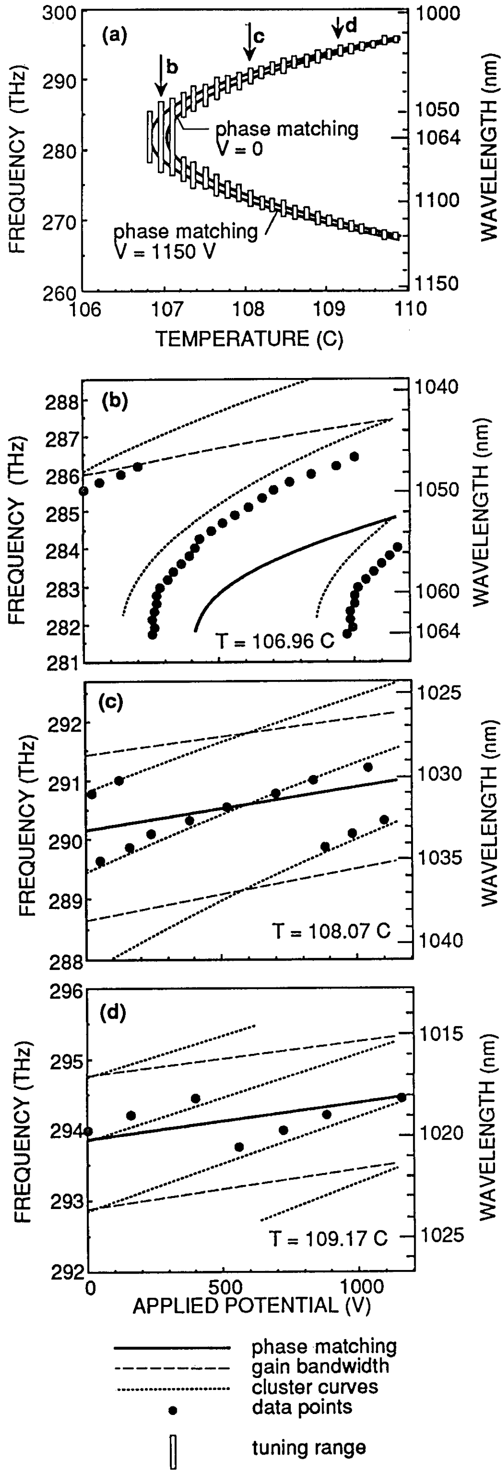

Fig. 11. Observed and calculated tuning for the pulsed-pumped DRO with finesse of 360. The open vertical bars in (a) show the extent of tuning observed as applied potential was ramped from 0 to 1150 V at a constant temperature. The solid curves behind the vertical bars are calculated phase-matching curves for the extreme voltages. Voltage tuning for three temperatures is shown in (b)–(d), where the bold central curves are the calculated phase-matching curves and the dashed curves indicate the limits of the phase-matching bandwidth. The dotted curves are calculated cluster curves, and the filled circles are observed operating points of the DRO. This DRO, which has only moderate finesse, exhibits few jumps between cluster curves as the voltage is ramped. The data are measurements of the applied potential for a limited sampling of output frequencies and do not represent individual mode hops.

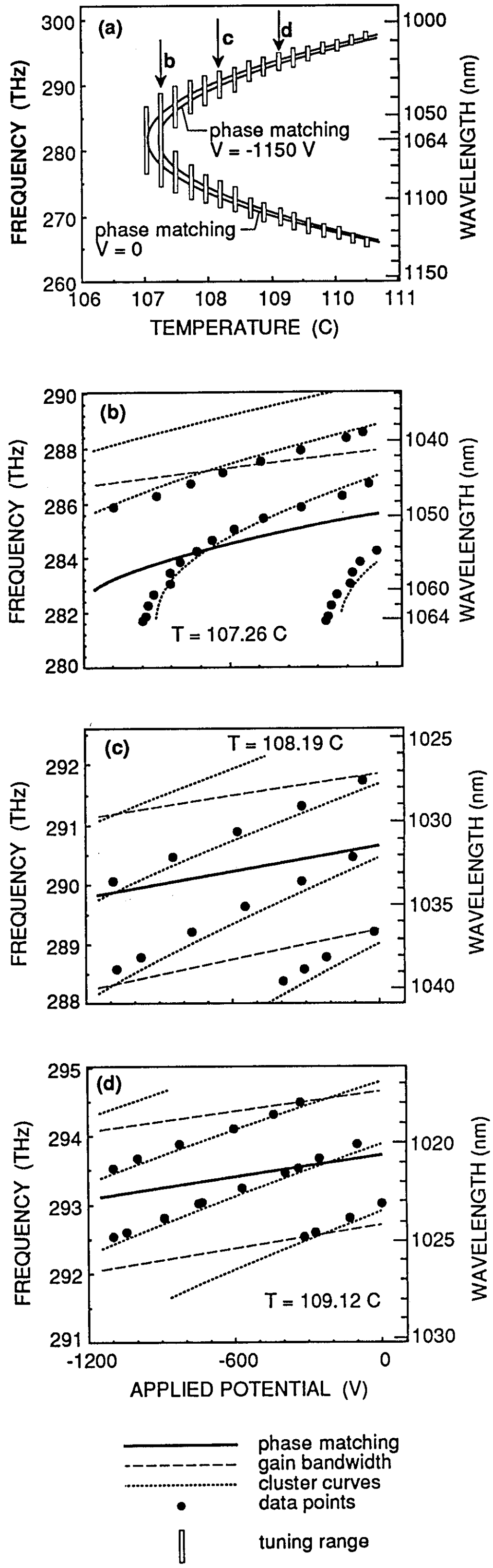

Fig. 12. Observed and calculated tuning for the cw-pumped DRO with finesse of 960. As in Fig. 11, the open bars in (a) indicate the range of tuning as voltage was ramped, in this case between −1150 V and 0, and the solid curves behind the vertical bars are the calculated phase-matching curves for the two extreme voltages. Voltage tuning is shown for three temperatures in (b)–(d). This DRO, which has higher finesse, exhibits a number of frequency jumps between three cluster curves as voltage is tuned.

The temperature-tuned phase-matching curves are shown in Figs. 11(a) and 12(a). At each temperature setting a range of output wavelengths is obtained by voltage tuning. In most cases the observed tuning ranges cover the space between and extend slightly beyond the calculated temperature-dependent tuning curves for the extreme voltages. Voltage was ramped from 0 to 1150 V, and when the crystal was reversed, from 0 to −1150 V. Figures 11(b)–11(d) and 12(b)–12(d) show the voltage tuning at selected fixed temperatures. The data are the voltages at which output was observed at selected frequencies. Calculated phase-matching curves, gain–bandwidth curves, and cluster curves are shown for comparison. In some instances the data are located in lines parallel to the calculated cluster curves but not on the cluster curves. This most likely is caused by inaccuracy in temperature measurement, and coincidence could be obtained by choosing a different temperature calibration for each setting. In practice DRO tuning may provide an accurate measurement of its temperature. The DRO sensitivity to temperature will become more apparent below when the details of tuning are discussed.

Figures 11 and 12 appear to be similar in a cursory examination; however, one aspect of tuning's dependence on finesse is illustrated. The lower-finesse DRO, with the tuning shown in Fig. 11, usually oscillates on the single cluster curve nearest the phase-matching curve. Sometimes the oscillation jumps back and forth between two cluster curves when they are nearly equally distant from phase matching. The output of the higher-finesse DRO with the tuning shown in Fig. 12 jumps between two or three cluster curves. This is in agreement with theoretical predictions that show the tuning limit imposed on the $\mathcal{F} = 960$ DRO by the resonance widths; that is, the finesse limit of tuning is reached before the mode-hop limit of tuning for most conditions encountered.

The cluster curves are also dependent on temperature. The data displayed in Fig. 12 are interpreted to show temperature tuning at constant voltage in Fig. 13. Here the data points are either interpolated from measurements of cluster tuning with voltages both higher and lower than the selected voltage or extrapolated from measurements of the cluster curves that nearly reach the selected voltage. The calculated phase matching, gain bandwidth, and cluster curves are again in reasonable agreement with observation.

C. Axial Mode-Hop Tuning

There is good agreement between observation and the calculated voltage change required to produce a mode hop. Observations similar to those illustrated in Fig. 10 were performed under various conditions. The results are shown in Fig. 14, in which $\Delta V_{\text{Hop spacing}}$ is displayed as a function of detuning from degeneracy. The calculated line is obtained from Eqs. (31) and (33). The tuning parameter is voltage, and it is necessary to substitute V for ζ in the equations and further to substitute the appropriate values from Table 2, and to evaluate the derivative $(\partial m/\partial \omega_s)_{\omega_p}$ as a function of signal frequency. The approximations given in Section 4 by relations (37) and (38) also work well in the evaluation of Eq. (31).

Calculations of axial-mode-hop tuning along cluster curves were performed for conditions that would approximate those used to produce Fig. 10. The same fitting parameters were used as in Figs. 12 and 13. An operating temperature and center voltage were chosen to give three cluster curves centered on phase matching at the observed operating frequencies. This was done by manipulation and solution of Eqs. (21) and (22). Calculated phase-matching, gain-bandwidth, and cluster curves in this region are shown in Fig. 15(a). Calculated tuning along the three cluster curves with the detail of mode hops is shown in Figs. 15(b)–15(d). These tuning curves were obtained by using Eq. (22) to calculate the cluster signal frequency $\omega_{s,\text{Cl}}$, Eq. (23) to calculate the signal mode number $m_{s,\text{Cl}}$ at the cluster frequency, and Eq. (29) to calculate the signal frequency of the oscillation. The mode-hop frequency limits given by Eq. (34) and the finesse frequency limits given by Eq. (35) are also shown.

When the oscillating signal frequency differs from the cluster signal frequency by more than the mode-hop limit, it is advantageous for the oscillation to shift to another signal–idler resonance pair. When the oscillating frequency excursion from the cluster frequency reaches the finesse limit, the threshold for parametric oscillation is double the value that it had when the oscillation frequencies coincided with the cluster frequencies, and the threshold increases for greater excursions of the oscillation frequency from the cluster frequency. Figure 15 illustrates how cluster jumps can be interspersed with the mode hops of a single cluster curve. For the calculation presented in Fig. 15, the finesse limit of tuning is reached before the mode-hop limit is reached on the central cluster curve shown in Fig. 15(c). Parametric oscillation on the central cluster curve usually dominates, because phase matching is best there. When the finesse limit of the frequency excursion from the cluster curve is reached, however, the parametric oscillation on the central cluster curve decays, and it is possible to have oscillation build up on an adjacent cluster curve before oscillation can build up on the next mode pair of the central cluster curve.

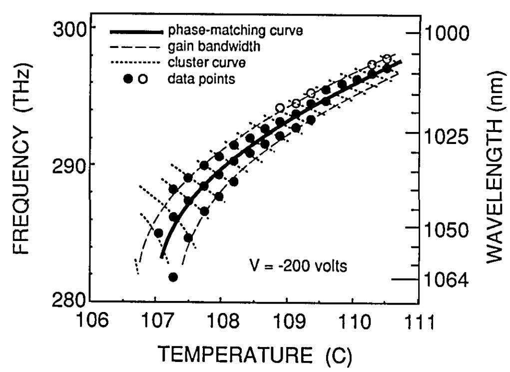

Fig. 13. Observed and calculated tuning for the cw-pumped DRO as a function of temperature. The same tuning data that were used in Fig. 12 are used here. A fixed voltage of −200 V was chosen. For the cases in which oscillation on a cluster curve was observed at voltages both higher and lower than this voltage, frequencies were obtained by interpolation and are represented by filled circles. For the cases in which cluster tuning came near but did not reach this voltage, frequencies were obtained by extrapolation and are represented by open circles. The dotted curves are portions of the calculated temperature-dependent cluster curves. The calculated phase-matching curve is the central bold curve, and the dashed curves show the approximate gain-bandwidth limits.

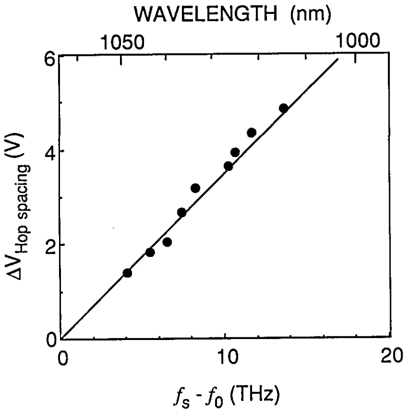

Fig. 14. Mode-hop spacing in an applied potential as a function of detuning from degeneracy. The dots are data points, and the solid line is calculated from theory.

Notice that the mode-hop spacing that is measured by the change in the tuning variable is different for the three adjacent cluster curves of Fig. 15(b)–15(d). Also, the relative positions of the finesse and mode-hop limits change with detuning from degeneracy.

These comparisons show quantitative agreement between observations and the tuning theory of Section 2. The tuning theory describes the cluster tuning of the DRO as well as the effects of cavity finesse on the cluster structure in the occurrence of cluster jumps. Also, the theory is able to predict the observed spacing in the tuning variable of axial mode hops on a microscopic scale of tuning. This is done with temperature-dependent dispersion, thermal expansion, and electro-optical and piezoelectric effects. Only three fitting parameters are used: a temperature calibration, which entailed the translation of a temperature scale by a fraction of a degree Celsius, an adjustment of cavity length l of less than one wavelength, and the use of a effective crystal thickness that compensated for the nonuniformity of the electric field inside the crystal. With this agreement it is reasonable to consider applying the theory to analysis of the DRO for optical frequency synthesis. Specifically, the analysis addresses conditions that are necessary to reproduce the coherence of a frequency-stable pump with a small degree of tunability at any frequency in the tuning range of the DRO.

4. FREQUENCY SYNTHESIS AND TUNING-VARIABLE TOLERANCES

Parameter tolerances and continuous-frequency tuning are topics that can be addressed with the theory presented above. Knowledge of tolerances is important for stable DRO operation and for tuning to oscillation at specific frequencies. Continuous-frequency tuning is of interest in many applications. Fixed-frequency operation with resolution finer than a mode hop may be required, or perhaps truly continuous frequency coverage may be necessary. The DRO output frequencies lie within the widths of cavity resonances. The extent of continuous tuning depends on several factors, including frequency shifts of the cavity modes, the spectral range over which the conservation-of-energy condition can be satisfied while oscillation is maintained within a selected mode pair, and the spectral range

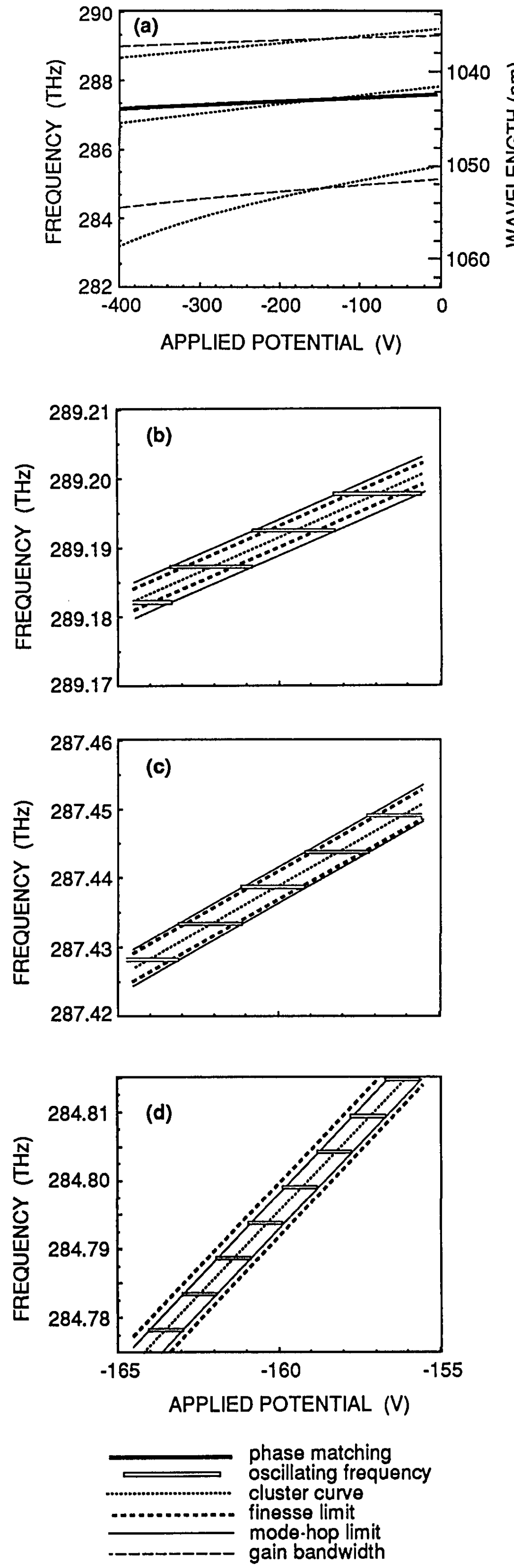

Fig. 15. Detailed display of calculated DRO tuning as a function of applied potential for conditions that would produce output similar to that shown in Fig. 10. All calculations are for a fixed temperature of 107.540°C. Detailed calculations of tuning for three cluster curves are shown in (b)–(d). Here the DRO output frequency is indicated by the open horizontal bars. A finesse of 960 is used. The slope of the continuous portions of the detailed tuning curves (b)–(d) is dependent on the relative values of finesse at the signal and the idler frequencies, but in all cases this slope is much less than the slope of the cluster curves.

over which higher net parametric gain is not available on another mode pair. Multiple-parameter tuning, in which two or more parameters are synchronously changed, is required for continuous tuning over the full free spectral range of the oscillator. Single-parameter tuning will provide frequency coverage over small regions that are separated by the discrete mode hops.

It is easiest to think of tolerances for situations in which only a single parameter is permitted to change. In practice there are advantages in dealing with parameter tolerances in pairs. For example, voltage and temperature adjustments could be used to maintain stable oscillation at a fixed frequency. It may not be possible to control temperature to the precision that is required if voltage is fixed, but the lack of adequate temperature control could be offset by voltage control. Feedback techniques could be used to adjust voltage in order to maintain stable oscillation on a signal–idler mode pair even in the presence of temperature fluctuations that by themselves would cause mode hops. The change in voltage required for stable operation could be used as an error signal that would in turn be used to return the temperature to the desired value.

Simultaneous adjustment of three parameters could also be used to tune the output frequency of the DRO. As an example, consider a pump frequency that is ramped in some specified way. The conditions required for stable operation on a single signal–idler mode pair could be provided by feedback control of the potential applied for electro-optic and piezoelectric tuning. The tolerance required for phase matching would be much less stringent than that required for stable operation on a single signal–idler mode pair; adequate phase matching could be maintained by temperature control, based on a functional relationship that is dependent on the pump frequency and voltage required for stable operation. With two-parameter tuning frequency matching could be maintained, but it would not be possible to maintain optimum phase matching.

A. Tuning-Variable Tolerances

The parameter tolerances for stable operation are determined by the more restrictive of two conditions. Mode hops are avoided by operation within a range of adjustment over which higher gain does not develop on another signal–idler mode pair. The range of adjustment over which oscillation can be maintained on a mode pair may be limited to a smaller value by the resonance width or equivalently by the DRO finesse. These tolerances are closely related to the mode-hop spacing and spectral limits of tuning that were discussed above, and they can be obtained from detailed tuning curves such as those shown in Fig. 15(b)–15(d) for voltage tuning. Detailed tuning curves for temperature tuning and pump-frequency tuning are shown in Figs. 16 and 17, respectively.

The conditions for the calculations displayed in Figs. 16 and 17 are the same as those used to produce Fig. 15(c). These conditions are $\mathcal{F}_s = \mathcal{F}_i = 960$ for a $MgO:LiNbO_3$ DRO of length $l = 1.25$ cm, pumped at 564 THz (532 nm) with signal frequency near 287 THz (1043 nm). Fitting parameters used in these calculations, such as an effective thickness $t = 0.277$ cm and a length adjustment corresponding to a change in m of 0.42, are the same as those required to fit the experimental data in Figs. 12 and 13. These characteristics are carried forward to other calculations for the purpose of providing a specific example for comparison.

The detailed tuning curves of Figs. 15–17 are similar in many respects. The mode-hop and finesse limits of frequency displacement from the cluster curve are independent of the tuning parameter. In each of the figures the slope of the continuous portion of tuning between mode hops is much smaller than the slope of the cluster curve. The slopes of the continuous portions of tuning are dependent on the relative finesse of the signal and the idler resonances. The case of equal finesse is shown in the calculated tuning curves. The illustrated curves show that voltage must be held within a tolerance of approximately 1 V, temperature within 0.0006°C, and pump frequency within 7 MHz for stable operation in this example.

Analytic approximations for the parameter tolerances for stable operation are not limited to a specific example. The range over which a parameter can be changed without causing a mode hop is obtained from the condition $\Delta m_{s,\mathrm{Cl}} = \pm 1/2$. Recall that $m_{s,\mathrm{Cl}}$ is the value m_s for a point on the associated cluster curve, and the cluster curve is

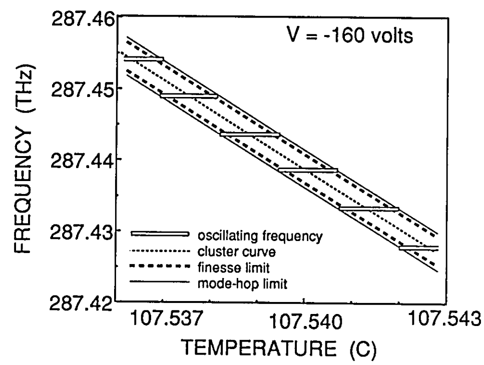

Fig. 16. Calculated detailed tuning as a function of temperature. For this calculation temperature is adjusted while other parameters are held constant at values that correspond to a point near the center of Fig. 15(c). Here, also, the slope of the continuous portions of the tuning curve are dependent on the relative values of signal and idler finesse, and this slope is small compared with the slope of the cluster curve.

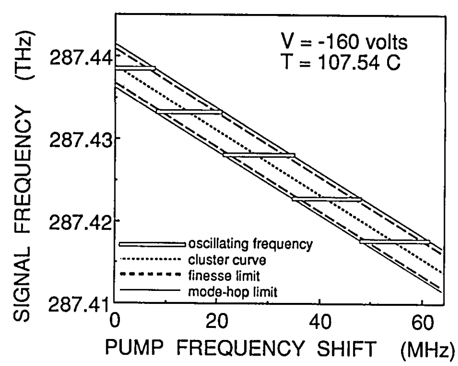

Fig. 17. Calculated detailed tuning as a function of pump frequency. For this calculation pump frequency is adjusted while other parameters are held constant at values that correspond to a point near the center of Fig. 15(c). For pump-frequency tuning with equal signal and idler cavity finesse, the slope of the continuous portions of the tuning curve is ~0.5. Because of the scale necessary to display the much greater slope of the cluster curve, the continuous portions of the tuning curve appear to be horizontal.

Table 3. Calculated Single-Parameter Continuous Tuning Rates and Parameter Tolerances for Stable Operation of the Finesse = 960 DRO Pumped at 563.6 THz with Signal Frequency 287.44 THz

Tuning Parameter ζ	$\frac{\partial f_{s,\mathrm{Osc}}}{\partial \zeta}$	$\Delta\zeta_{\mathrm{Hop\ tolerance}}$	$\Delta\zeta_{\mathrm{Fin\ tolerance}}$
V (Voltage)	72 kHz/V	±0.98 V	±0.80 V
T (Temperature)	−124 MHz/°C	±0.00064°C	±0.00051°C
f_p (Pump Frequency)	0.499 Hz/Hz	±6.7 MHz	±5.4 MHz

a line that gives the values of signal frequency ω_s and a tuning parameter ζ for which $\Delta m = 0$, with the other parameters held at fixed values. It follows that the tuning-parameter tolerance is

$$\Delta\zeta_{\mathrm{Hop\ tolerance}} = \pm\frac{1}{2}\left(\frac{\partial m_{s,\mathrm{Cl}}}{\partial \zeta}\right)^{-1}. \tag{36}$$

The derivative in the above equation can be evaluated with Eq. (33). An approximation of that equation for the case of type-I phase matching is given by

$$\frac{\partial m_{s,\mathrm{Cl}}}{\partial \zeta} \approx -\frac{\delta\omega_0}{\delta\omega_i - \delta\omega_s}\frac{\partial m}{\partial \zeta}, \tag{37}$$

where $\delta\omega_0$ is the mode spacing at degeneracy ($\omega_s = \omega_i = \omega_0 = \omega_p/2$). The difference between the idler and the signal mode spacing can also be expanded about degeneracy by using Eqs. (18) to obtain

$$\delta\omega_i - \delta\omega_s \approx 2\delta{\omega_0}^2\frac{\partial^2 m_s}{\partial {\omega_s}^2}(\omega_s - \omega_0). \tag{38}$$

Equation (36) and relations (37) and (38) can be combined to give the desired approximation for the mode-hop parameter tolerance, namely,

$$\Delta\zeta_{\mathrm{Hop\ tolerance}} \approx \pm\delta\omega_0\frac{\partial^2 m_s}{\partial {\omega_s}^2}(\omega_s - \omega_0)\Big/\frac{\partial m}{\partial \zeta}. \tag{39}$$

The adjustable parameter tolerance related to cavity finesse can be obtained from the cavity-round-trip phase-shift sum for which the pumping threshold is twice its minimum value. From Eqs. (12) and (13) this phase-shift sum is

$$\psi_{\mathrm{Fin}} = \pm\pi\left(\frac{1}{\mathcal{F}_i} + \frac{1}{\mathcal{F}_s}\right). \tag{40}$$

Since $\psi = 2\pi\Delta m$, the parameter tolerance determined by cavity finesse is

$$\Delta\zeta_{\mathrm{Fin\ tolerance}} \approx \pm\frac{\frac{1}{2}(1/\mathcal{F}_i + 1/\mathcal{F}_s)}{\partial m/\partial \zeta}. \tag{41}$$

The parameter tolerance related to mode hops, given by relation (39), is zero at degeneracy and increases linearly with detuning from degeneracy. In practice, however, operation precisely at degeneracy was stable for tens of minutes with no adjustments to the DRO.[10] The parameter tolerance related to cavity finesse, given by relation (41), remains approximately constant independent of detuning from degeneracy as long as finesse remains constant. Calculated tuning-variable tolerances are given in Table 3 for the $MgO{:}LiNbO_3$ DRO for the conditions used to generate Figs. 15(c), 16, and 17.

B. Single-Parameter Tuning

The cavity resonances associated with the signal wave and the idler wave can have significantly different finesse. The oscillating frequencies of the DRO will align more closely with the higher-finesse cavity resonance than with the complementary resonance with lower finesse and greater width. If the frequencies of the cavity resonances change, the oscillation will follow the higher-finesse resonance more closely, to the extent possible without a mode hop or a cluster jump. If the pump frequency changes, the frequency of the wave oscillating on the higher-finesse cavity resonance will remain more nearly constant than that of the wave oscillating on the lower-finesse resonance. In this sense the higher-finesse resonance pulls the frequencies of oscillation more strongly.

It has been noted by Smith[4] that the continuous tuning of a DRO is relatively insensitive to tuning-parameter changes that change the optical length of the DRO resonator, but the signal frequency and the idler frequency both display approximately one half of the change that occurs in pump frequency. Single-parameter continuous tuning is described by Eq. (30), which can be rewritten with $m = m_s + m_i$ as

$$\frac{\partial\omega_{s,\mathrm{Osc}}}{\partial\zeta} = \frac{\delta\omega_s\delta\omega_i}{\delta\omega_i\mathcal{F}_s + \delta\omega_s\mathcal{F}_i}\left(\mathcal{F}_i\frac{\partial m_s}{\partial\zeta} - \mathcal{F}_s\frac{\partial m_i}{\partial\zeta}\right).$$

In the mathematical development used here, ω_s and ω_p are used as independent variables, with ω_i determined by Eq. (1). Choosing pump frequency ω_p as the variable parameter ζ requires the substitutions

$$\frac{\partial m_i}{\partial\zeta} \to \left(\frac{\partial m_i}{\partial\omega_p}\right)_{\omega_s} = \frac{\partial m_i}{\partial\omega_i}$$

and

$$\frac{\partial m_s}{\partial\zeta} \to \left(\frac{\partial m_s}{\partial\omega_p}\right)_{\omega_s} = 0.$$

The signal and the idler mode spacings, $\delta\omega_s$ and $\delta\omega_i$, will differ by only a small amount, and if $\mathcal{F}_s$ and $\mathcal{F}_i$ are nearly equal, the tuning rate is $\partial\omega_{s,\mathrm{Osc}}/\partial\omega_p \approx 1/2$. More generally, for differing values of finesse, the tuning rate is in the range $0 < \partial\omega_{s,\mathrm{Osc}}/\partial\omega_p < 1$. Even though approximately half the pump frequency tuning will be reflected in signal tuning, only a relatively small spectral range will be covered before a mode hop is encountered. Calculated single-parameter tuning rates for the special case of $\mathcal{F}_s = \mathcal{F}_i$, corresponding to the DRO's described in Section 3, are given in Table 3. The partial derivatives needed in this calculation were evaluated for $\omega_{s,\mathrm{Osc}} = 287.44$ THz (1043 nm) and $T_0 = 107.51$°C instead of being taken from Table 2. As explained in Subsection 2.C.2, this procedure is required for the evaluation of $\partial f_{s,\mathrm{Osc}}/\partial V$ and $\partial f_{s,\mathrm{Osc}}/\partial T$, which involve the small differences in two quantities, but has little effect on the other values in the table.

Table 4. Calculated Values for Two-Parameter Tuning of a Monolithic MgO:LiNbO$_3$ DRO at f_s = 287.44 THz or λ_s = 1043 nm

Adjustable Parameters	Fixed Parameter	$\frac{df_s}{d\zeta_1}$	$\frac{d\zeta_2}{d\zeta_1}$	$\frac{d\Delta k}{d\zeta_1}$
$\zeta_1 = f_p$, $\zeta_2 = V$	T = 107.54°C	0.510	1.47×10^{-7} V/Hz	2.26×10^{-8} (rad/m)/Hz
$\zeta_1 = f_p$, $\zeta_2 = T$	V = −160 V	0.511	(-9.4×10^{-11})°C/Hz	-6.7×10^{-8} (rad/m)/Hz
$\zeta_1 = V$, $\zeta_2 = T$	f_p = 563.6 THz	−7.9 kHz/V	(6.4×10^{-4})°C/V	0.613 (rad/m)/V

C. Multiple-Parameter Tuning

It is possible to extend the continuous-tuning range by synchronously adjusting two or three parameters. Adjusting two parameters simultaneously permits the conditions $\Delta m = 0$ and $\Delta m_s = 0$ to be maintained, but Δk will change. Adjusting three parameters simultaneously permits tuned parametric oscillation while Δm, Δm_s, and Δk all remain equal to zero for a specified mode pair, and tuning is limited only by the extent that the parameters can be changed.

Generalized tuning parameters with adjustable ζ_1 and ζ_2 and fixed ζ_3 are used for the discussion of two-parameter tuning. For the specific case treated here, any permutation of voltage, temperature, and pump frequency can be used for these three parameters. The conditions $\Delta m = 0$ and $\Delta m_s = 0$ determine a relationship between ζ_1 and ζ_2, and use of this relationship permits ω_s and Δk to be expressed as functions of ζ_1. These relationships can be obtained by first differentiating Eqs. (21)–(23) with respect to ζ_1, yielding

$$\frac{dm}{d\zeta_1} = 0 = \left(\frac{\partial m}{\partial \omega_s}\right)_{\omega_p} \frac{d\omega_s}{d\zeta_1} + \frac{\partial m}{\partial \zeta_1} + \frac{\partial m}{\partial \zeta_2}\frac{d\zeta_2}{d\zeta_1}, \tag{42a}$$

$$\frac{dm_s}{d\zeta_1} = 0 = \frac{\partial m_s}{\partial \omega_s}\frac{d\omega_s}{d\zeta_1} + \frac{\partial m_s}{\partial \zeta_1} + \frac{\partial m_s}{\partial \zeta_2}\frac{d\zeta_2}{d\zeta_1}, \tag{42b}$$

and

$$\frac{d\Delta k}{d\zeta_1} = \left(\frac{\partial \Delta k}{\partial \omega_s}\right)_{\omega_p} \frac{d\omega_s}{d\zeta_1} + \frac{\partial \Delta k}{\partial \zeta_1} + \frac{\partial \Delta k}{\partial \zeta_2}\frac{d\zeta_2}{d\zeta_1}. \tag{42c}$$

Only a small spectral region is being considered, and it is unnecessary to consider second partial derivatives with respect to signal frequency. The first partial derivatives, however, must be evaluated for the operating conditions that are being considered. A specific case for which this consideration is important is frequency tuning for voltage and temperature adjustment, which again involves a small difference of terms. Equations (42a) and (42b) are solved for $d\omega_s/d\zeta_1$ and $d\zeta_2/d\zeta_1$. These values are substituted into Eq. (42c) to yield a value for $d\Delta k/d\zeta_1$. Results are calculated for three sets of tuning parameters and are listed in Table 4. The conditions used for these calculations are the same as those used for Figs. 15–17 and Table 3; the partial derivatives in Eqs. (42) again were evaluated for $\omega_{s,\mathrm{Osc}}$ = 287.44 THz (1043 nm) and T_0 = 107.51°C.

Two of the examples given in Table 4 are briefly noted. The voltage–temperature tuning mentioned above is of interest for operation at a fixed frequency. The rate of change of the output frequency with applied potential when the voltage and the temperature are changed simultaneously in order to maintain $\Delta m = 0$ and $\Delta m_s = 0$ is calculated to be −7.9 kHz/V. The magnitude of this tuning rate is approximately 10 times smaller than the single-parameter voltage tuning rate given in Table 3 and significantly smaller than the 3.5-MHz/V tuning rate of the cavity resonance. The reduced sensitivity is important in stable-frequency operation of the DRO. Simultaneous pump-frequency and voltage tuning is useful for continuous coverage of the spectral region between the mode hops of single-parameter tuning. Calculated tuning curves for this case are shown in Fig. 18. The same conditions used for Figs. 15–17 apply again to Fig. 18. Tuning is taken to the limits of $\Delta k = \pm\pi/l$ in the figure.

Two methods of three-parameter tuning are presented. First a method is described for achieving oscillation at a specified frequency while satisfying the conditions Δk = 0, $\Delta m = 0$, and $\Delta m_s = 0$. The description is in mathematical terms but is analogous to what could be done experimentally. The first step in this method is adjustment of the temperature in order to achieve phase matching for the specified frequency. This is just a matter of changing the temperature to the value determined by Eq. (21). Next, the pump frequency and the temperature are adjusted simultaneously in order to maintain the Δk = 0 phase-matching condition and to satisfy the condition $\Delta m = 0$. Numerically this is done by setting $\Delta k = 0$ and $\Delta m = 0$ in Eqs. (21) and (22) and solving for T and ω_p with V and ω_s held constant. Next, three parameters are adjusted simultaneously by solving Eqs. (21)–(23) for T, ω_p, and V with ω_s again held constant and Δk, Δm, and Δm_s set to zero.

In practice this mathematical procedure would be analogous to setting temperature to a value calculated for phase matching and observing the location of the cluster curve nearest the phase-matched signal frequency. Next, the temperature and the pump frequency are adjusted

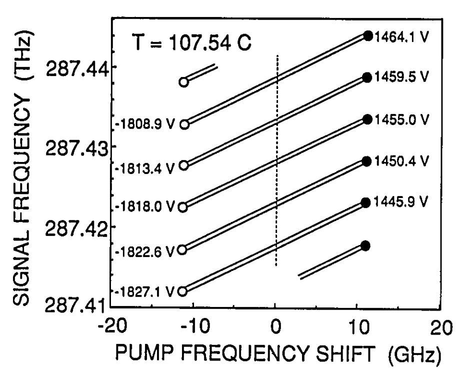

Fig. 18. Calculated tuning for varying the voltage and the pump frequency simultaneously so as to maintain $\Delta m = 0$ and Δm_s = 0. The dashed line is the cluster curve of Fig. 17. Tuning limits taken are the points at which $\Delta k = \pm\pi/l$ in the 1.25-cm-long crystal.

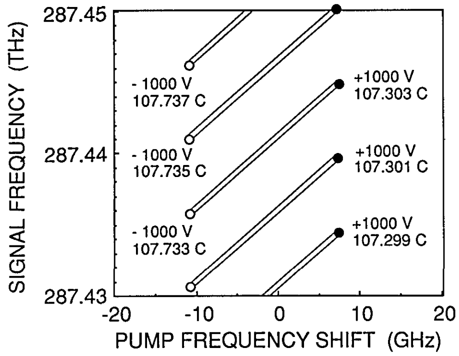

Fig. 19. Calculated tuning for varying the voltage, the pump frequency, and the temperature simultaneously so as to maintain $\Delta m = 0$, $\Delta m_s = 0$, and $\Delta k = 0$.

simultaneously in order to move that cluster curve to intersect the desired frequency at phase matching. At this point oscillation is on the resonance nearest the specified frequency. Finally the temperature, the pump frequency, and the voltage are adjusted simultaneously in order to bring the cavity resonance to the desired frequency while coincidence of the signal and the idler modes and phase matching is maintained.

The second method of three-parameter tuning concerns a situation in which oscillation is achieved with optimum phase matching and coincidence of the modes in satisfying the conservation-of-energy condition. Continuous output frequency tuning is possible over a limited range while optimum DRO operating conditions are maintained. One parameter can be changed arbitrarily, but the other parameters must be changed in a prescribed manner. The prescription for this change is again obtained by differentiating Eqs. (21)–(23) and this time setting all total derivatives equal to zero. The pump frequency ω_p is chosen as the independent parameter, and the differentiation yields

$$\frac{\mathrm{d}\Delta k}{\mathrm{d}\omega_p} = 0 = \left(\frac{\partial \Delta k}{\partial \omega_s}\right)_{\omega_p} \frac{\mathrm{d}\omega_s}{\mathrm{d}\omega_p} + \left(\frac{\partial \Delta k}{\partial \omega_p}\right)_{\omega_s} + \frac{\partial \Delta k}{\partial T}\frac{\mathrm{d}T}{\mathrm{d}\omega_p} + \frac{\partial \Delta k}{\partial V}\frac{\mathrm{d}V}{\mathrm{d}\omega_p}, \tag{43a}$$

$$\frac{\mathrm{d}m}{\mathrm{d}\omega_p} = 0 = \left(\frac{\partial m}{\partial \omega_s}\right)_{\omega_p} \frac{\mathrm{d}\omega_s}{\mathrm{d}\omega_p} + \left(\frac{\partial m}{\partial \omega_p}\right)_{\omega_s} + \frac{\partial m}{\partial T}\frac{\mathrm{d}T}{\mathrm{d}\omega_p} + \frac{\partial m}{\partial V}\frac{\mathrm{d}V}{\mathrm{d}\omega_p}, \tag{43b}$$

and

$$\frac{\mathrm{d}m_s}{\mathrm{d}\omega_p} = 0 = \frac{\partial m_s}{\partial \omega_s}\frac{\mathrm{d}\omega_s}{\mathrm{d}\omega_p} + \frac{\partial m_s}{\partial \omega_p} + \frac{\partial m_s}{\partial T}\frac{\mathrm{d}T}{\mathrm{d}\omega_p} + \frac{\partial m_s}{\partial V}\frac{\mathrm{d}V}{\mathrm{d}\omega_p}. \tag{43c}$$

Again, only the first partial derivatives, evaluated in the region of consideration, are required. All the partial derivatives are determined by material characteristics and the DRO configuration, resulting in three linear equations with three unknowns, which are solved by the usual methods. Continuous-frequency coverage can be obtained with an incremental series of continuous-frequency sweeps. A calculation of tuning in this manner is shown in Fig. 19. The extent of tuning for the individual sections will be limited by the range of parameter adjustment. For example, there may be a maximum voltage that can be applied, or the extent of pump frequency tuning may be limited. A limit of ±1000 V was used in Fig. 19.

The calculations of two- and three-parameter tuning show that a DRO can be tuned to any frequency in its operating range[3] with reasonable adjustment of the tuning parameters. Continuous tuning is possible over spectral ranges of approximately the extent of a free spectral range. Complete coverage of larger spectral regions has to be done by scanning a series of smaller regions. The control of individual parameters, particularly temperature, requires difficult tolerances. The control problem can be shifted to another, more easily controlled parameter, such as voltage with multiple-parameter control of the DRO. The degree of correction required on the second parameter can then be used as an error signal for control of the first parameter. Fortunately the oscillating frequencies of the monolithic DRO, exclusive of mode hops, are relatively insensitive to voltage and temperature changes. If mode hops and cluster jumps are avoided, the frequency change of the DRO is approximately one half the frequency change of the pump.

5. SUMMARY

The theory that is used to model tuning of the DRO is verified at many stages. The first-order threshold approximation agrees well with more general calculations in the limit of low cavity loss. The theory accurately models observed cluster curves for two monolithic $MgO:LiNbO_3$ DRO's. The modeling includes temperature-dependent dispersion, thermal expansion, and the electro-optic and piezoelectric effects in the nonlinear material. The effect of DRO cavity finesse on the fine details of tuning gives a reasonable explanation of observed cluster jumps. Further substantiation of the model in the fine details of tuning is provided by the accurate prediction of the axial-mode-hop rate for tuning-parameter change.

An understanding of DRO tuning is important for controlled stable operation. Continuous tuning rates were calculated for single- and multiple-parameter adjustment. Tolerances for stable operation were estimated. The results of these calculations will be useful for DRO design optimization. Multiple-parameter tuning, including pump-frequency adjustment, will be necessary for reaching any arbitrary frequency in the OPO operating range. With appropriate control the DRO will be able to produce stable outputs with a frequency stability as good as that available in the pump source.

The DRO should find application in the generation of stable fixed-frequency radiation. Incremental tuning in controlled mode hops or cluster jumps will have applications in spectroscopy and differential absorption lidar (light detection and ranging). Slow, high-resolution tuning will be possible over limited frequency ranges for spectroscopic applications.

The theory presented here could easily be extended to DRO configurations other than monolithic devices. Other degrees of freedom, such as direct length control in a discrete-component DRO, would provide greater versatility in operation. Independent control of signal and idler cavity lengths would be useful in providing greatly extended ranges of continuous tuning. The development

Table 5. Comparison of Measured and Calculated Values for Parametric Fluorescence in MgO:LiNbO$_3$

	514.5-nm Pump Wavelength		488-nm Pump Wavelength	
Experimental Temperature Setting (°C)	Obs. Fluorescence Wavelength (nm)	Calc. Temperature (°C)	Obs. Fluorescence Wavelength (nm)	Calc. Temperature (°C)
148	777.1	146.70	673.3	143.17
198	741.2	196.21	653.1	194.93
248	710.8	245.26	634.5	245.18
298	684.5	293.38	617.3	294.12
318	674.8	312.52		
348	660.8	341.50	601.3	341.86
398	639.2	389.31	585.7	390.38
448	619.2	436.60		

of stable DRO operation is now possible through the combination of improved nonlinear-optical materials and frequency-stable laser development, such as in diode-pumped solid-state lasers. Optical parametric oscillators again appear to be on the threshold of reaching a potential that was first understood twenty-five years ago.

APPENDIX A. MATERIAL PROPERTIES OF MgO:LiNbO$_3$ RELATED TO DRO TUNING

A. Temperature-Dependent Dispersion

Edwards and Lawrence[26] developed temperature-dependent dispersion equations for congruently grown LiNbO$_3$, based on data reported by Nelson and Mikulyak[27] and Smith *et al.*[28] They use dispersion equations of the form

$$n^2 = A_1 + \frac{A_2 + B_1 F}{\lambda^2 - (A_3 - B_2 F)^2} + B_3 F - A_4 \lambda^2, \quad \text{(A1)}$$

where

$$F = (T - T_0)(T + T_0 + 546), \quad \text{(A2)}$$

λ is wavelength in micrometers, and T is temperature in degrees Celsius. Coefficients for congruent LiNbO$_3$ are as follows:

	Ordinary	Extraordinary
A_1	4.9048	4.5820
A_2	0.11775	0.09921
A_3	0.21802	0.21090
A_4	0.027153	0.021940
B_1	2.2314×10^{-8}	5.2716×10^{-8}
B_2	-2.9671×10^{-8}	-4.9143×10^{-8}
B_3	2.1429×10^{-8}	2.2971×10^{-7}
T_0	24.5	24.5

The material used in this work is not congruent LiNbO$_3$; rather, it is 5%MgO:LiNbO$_3$. There are few refractometric data available this material. To obtain an approximate set of equations for 5%MgO:LiNbO$_3$, the extraordinary index was adjusted by changing A_{1E} from 4.5820 to 4.55207. This has the effect of increasing the calculated noncritical phase-matching temperature for 1064–532-nm second-harmonic generation from −19.4 to 107.04°C. The measured value for the MgO-doped material is 107°C.[29] This modification to the congruent dispersion equations accurately reproduces the observed tuning curve for a singly resonant OPO that was tuned between 0.85 and 1.48 μm by varying temperature between 122 and 190°C.[30] This modification also predicts parametric florescence pumped at 514.5 and 488 nm when the crystal is tuned between 100 and 450°C (Table 5).

B. Electro-Optic Effect

Electro-optical, piezoelectric, and thermal expansion characterizations of LiNbO$_3$ are reviewed by Räuber.[31] A somewhat more extensive tabulation of electro-optical coefficient measurements is given by Yariv and Yeh.[32] Their treatment of the electro-optical effect is followed here. The index ellipsoid in a principal coordinate system is given by

$$\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1. \quad \text{(A3)}$$

When an electric field is applied, the electro-optical effect is described by the modified index ellipsoid

$$\left(\frac{1}{n_x^2} + r_{1k}E_k\right)x^2 + \left(\frac{1}{n_y^2} + r_{2k}E_k\right)y^2 + \left(\frac{1}{n_z^2} + r_{3k}E_k\right)z^2 + 2r_{4k}E_k yz + 2r_{5k}E_k xz + 2r_{6k}E_k xy = 1, \quad \text{(A4)}$$

where

$$r_{ik}E_k = \sum_{k=1}^{3} r_{ik}E_k, \qquad i = 1, \ldots 6.$$

For point group $3m$, to which LiNbO$_3$ belongs, the following relationships apply: $n_x = n_y = n_o$, $n_z = n_e$, and

$$(r_{ik}) = \begin{bmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{bmatrix}. \quad \text{(A5)}$$

There are only four independent electro-optical coefficients. We consider only application of an electric field along the y axis ($E = E_y$), which further simplifies the index ellipsoid to

$$\left(\frac{1}{n_o^2} - r_{22}E_y\right)x^2 + \left(\frac{1}{n_o^2} + r_{22}E_y\right)y^2 + \frac{z^2}{n_e^2} + 2r_{51}E_y yz = 1. \quad \text{(A6)}$$

The presence of the y–z cross term shows that the electro-optical effect results in a slight rotation of the principal axes. This is a small effect that accounts for less than 1% of the refractive-index change even at the highest fields that are considered here; therefore this rotation is ignored. For propagation in the x direction we have

$$n_y \approx \left(\frac{1}{n_o^2} + r_{22}E_y\right)^{-1/2} \approx n_o - \frac{n_o^3 r_{22} E_y}{2} \qquad \text{(A7)}$$

and

$$n_z \approx n_e .$$

Yariv and Yeh[32] list three values for r_{22} of $LiNbO_3$, measured with a low-frequency applied electric field for various optical wavelengths:

$$r_{22}(633\ \text{nm}) = 6.8 \times 10^{-12}\ \text{m/V},$$

$$r_{22}(1.15\ \mu\text{m}) = 5.4 \times 10^{-12}\ \text{m/V},$$

$$r_{22}(3.39\ \mu\text{m}) = 3.1 \times 10^{-12}\ \text{m/V}.$$

These values suggest that we use $r_{22} = 5.5 \times 10^{-12}$ m/V near the wavelength 1.06 μm, the wavelength region at which our 5%MgO:$LiNbO_3$ DRO was operated. The value 6.8×10^{-12} m/V is from a measurement reported in 1967.[33] Note that this measurement was made even before the growth of congruent $LiNbO_3$ was reported.[34,35] We are working with an still slightly different material 5%MgO:$LiNbO_3$,[24,25] and caution is required in applying these values.

C. Thermal Expansion

Thermal expansion measurements to second order in temperature for $LiNbO_3$ are reported by Kim and Smith.[36] They express the fractional change of length with the quadratic function

$$\frac{\Delta l}{l} = \alpha(T - T_R) + \beta(T - T_R)^2, \qquad \text{(A8)}$$

where l is length, Δl is change in length, T is temperature in degrees Celsius and $T_R = 25°C$ is a reference temperature. The DRO length is measured along the crystal x axis. One set of coefficients,

$$\alpha_{11} = (1.54 \times 10^{-5})°\text{C}^{-1}$$

and

$$\beta_{11} = (5.3 \times 10^{-9})°\text{C}^{-2},$$

applies to expansion in the x–y plane, and a second set applies to expansion in the z direction. The spread in the measurements reported by Kim and Smith suggests an accuracy of ~10% in the two coefficients.

D. Piezoelectric Effect

The direct piezoelectric effect describes the electric polarization P that results when a stress T is applied to a material by the relationship

$$P_i = d_{ijk} T_{jk}, \qquad \text{(A9)}$$

where d_{ijk} are the piezoelectric moduli. The converse piezoelectric effect describes the strain S that results when an electric field is applied to a piezoelectric material by the relationship[37]

$$S_{jk} = d_{ijk} E_i . \qquad \text{(A10)}$$

Summation of the repeated indices is implied in both of the above equations, and the moduli d_{ijk} are the same in both equations. The elongation of the DRO cavity is given by S_{11}, the x component of the change of a vector that lies in the x direction. For these measurements an electric field is applied in the y direction, E_y, and the modulus d_{211} is required for the calculation of strain. The symmetry of the stress tensor permits the use of a contracted subscript notation in which the modulus d_{211} is expressed as d_{21}. The $3m$ symmetry of the lithium niobate crystal reduces the number of independent moduli to four. The reduced matrix of piezoelectric moduli for the point group $3m$ is

$$(d_{jm}) = \begin{pmatrix} 0 & 0 & 0 & 0 & d_{15} & -2d_{22} \\ -d_{22} & d_{22} & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{pmatrix}. \qquad \text{(A11)}$$

From the relationships among the moduli, it follows that strain is given by

$$S_{11} = d_{211}E_2 = d_{21}E_2 = -d_{22}E_y . \qquad \text{(A12)}$$

The value of the piezoelectric modulus reported by Smith and Welsh,[38] $d_{22} = (2.08 \times 10^{-11})$ C/N, is used. They identify the $LiNbO_3$, which they used as commercially grown, with a Curie point of 1165°C. Sound-propagation measurements were used to determine the piezoelectric moduli.

The derivatives used in Eqs. (21)–(23) are expanded in Eqs. (18) and Table 1. Evaluation for the experimental conditions described in Section 3 with the material properties described above is given in Table 2. Derivatives involving the electro-optic and the piezoelectric effects are given with respect to the voltage applied to the electrodes on the crystal surfaces perpendicular to the y axis. An effective crystal thickness between the electrodes, t, is used, and the derivatives are given by

$$\frac{\partial \Delta k}{\partial V} = (\omega_s n_s^3 + \omega_i n_i^3)\frac{r_{22}}{2ct}, \qquad \text{(A13)}$$

$$\frac{\partial m}{\partial V} = -\frac{l}{\pi c t}\left[(n_s\omega_s + n_i\omega_i)d_{22} + (\omega_s n_s^3 + \omega_i n_i^3)\frac{r_{22}}{2}\right], \qquad \text{(A14)}$$

and

$$\frac{\partial m_s}{\partial V} = -\frac{l}{\pi c t}\left(n_s\omega_s d_{22} + \omega_s n_s^3 \frac{r_{22}}{2}\right). \qquad \text{(A15)}$$

ACKNOWLEDGMENTS

This work was supported by the Office of Naval Research through contracts N00014-88-K-0576 and N00014-88-K-0701 and by NASA through grant NAGW-1760. C. D. Nabors and W. J. Kozlovsky are grateful to the Fannie and John Hertz Foundation for support of their graduate studies. We express our appreciation to Crystal Technology of Palo Alto, California, for supplying the

$MgO:LiNbO_3$ material. Our appreciation is also extended to many individuals who helped in a number of ways. Martin Fejer and Eric Gustafson provided many helpful discussions and suggestions in the course of this work. Dieter Jundt performed the parametric fluorescence measurements of phase matching for $MgO:LiNbO_3$. Stephen Yang operated the apparatus for the measurements shown in Fig. 10. The help of Janet Okagaki in formating and preparing the figures and the optical fabrication of the monolithic DRO's by Joe Vrhel are especially appreciated.

Note added in proof: The tuning properties of doubly resonant oscillation in waveguide OPO's have been observed by Suche and Sohler.[39] There are many similarities between the tuning properties described here and those reported for the waveguide cavities. Piskarskas *et al.*[40] have reported cluster effects in the output of DRO's synchronously pumped by cw mode-locked laser output. Wong[41] has proposed the use of DRO's for optical frequency synthesis. The increased photoconductivity and the trapping of charges in $MgO:LiNbO_3$ may pose some problems in the use of dc electric fields with that material.[25]

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