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Kort, P.M.

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...RSITY
...RSITEIT
BRABANT
POSTBOX 90153
5000 LE TILBURG
THE NETHERLANDS

[redacted]

DEPARTMENT OF ECONOMICS

RESEARCH MEMORANDUM

OPTIMAL ABATEMENT POLICIES WITHIN A STOCHASTIC DYNAMIC MODEL OF THE FIRM

Peter M. Kort

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Peter M. Kort

Economics Department

Tilburg University

P.O. Box 90153

5000 LE Tilburg

The Netherlands

Jel.No. 522

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Abstract: In this paper a stochastic dynamic model of the firm, originally developed by Bensoussan and Lesourne (1980), is used as a framework to study the optimality of abatement investments. Production causes pollution as an inevitable byproduct and in this model the latter is taxed by the government. The firm can diminish these tax payments through reducing its pollution output by carrying out abatement investments. It turns out that abatement investment is never optimal when the governmental environmental policy is weak, i.e. when the emissions tax rate is low, and that, when the emissions tax is sufficiently high, abatement investment can only be optimal when the expected marginal profitability of productive investment is too small to justify additional growth and the amount of cash availably high enough to guarantee a sufficiently strong liquidity position.

1. INTRODUCTION

Nowadays, the improvement of environmental quality has become one of the most important objectives in the industrialized world. From an economic point of view one could argue that a non-polluted environment has become a scarce commodity. Consequently, environmental use is an allocation problem (Siebert (1987)) and should be taken into consideration by economic theory. This seems to be the reason that more and more books are devoted to environmental economics (e.g. Baumol and Oates (1988), Wicke (1982)).

An important question in this respect is what kind of policy instruments the government, in its role as social planner, should choose to reduce the level of pollution. One class of instruments includes direct controls by setting limits to specific elements. These restrictions are called standards which in practice can take many forms: restrictions on pollution emissions, restrictions on pollution per unit of output or per unit of an input, restrictions on the output level, restrictions on the use of a polluting input, or mandated use of a particular pollution-control technology. In Helfand (1991) the effects on the firm's decisions of each of these standards are examined. According to Van der Ploeg and De Zeeuw (1991) the problem with standards is that they are difficult to enforce, that they are associated with high administrative costs, and that they lead to economic inefficiencies. Another possibility for the government to reduce pollution is to impose an emissions tax rate. In Van der Ploeg and De Zeeuw (1991) international aspects of such an emission charge are analyzed within a game theoretic framework, while in Kort, Van Loon and Luptacik (1991) the reaction of the firm on such a measure is studied in a deterministic dynamic setting.

In this paper we extend the work of Kort, Van Loon and Luptacik (1991) by introducing uncertainty into the analysis. To do so we incorporate activity analysis (see e.g. Takayama (1985)) into a stochastic dynamic model of the firm developed by Bensoussan and Lesourne (1980) (see also Kort (1989)). We consider two activities: the first one is productive but also generates pollution that is taxed by the government. The second one is non-productive but cleans pollution instead.
The paper is organized as follows. In Section 2 we present the stochastic dynamic model of the firm, while in Section 3 the candidate policies for optimality are inferred. In Section 4 the optimal behavior of the firm is presented in cases of a weak, a moderate and a severe governmental environmental policy. Section 5 concludes the paper and the Appendix contains some mathematical proofs.

2. MODEL FORMULATION

In this model the firm can invest in two different sorts of capital goods. One is productive but also causes pollution as an inevitable byproduct. The other one is non-productive but cleans pollution. We assume that pollution is homogeneous by nature:

$$E(K_1,K_2) = e_1K_1 - e_2K_2 \tag{1}$$

in which:

$E(K_1,K_2)$: amount of emissions being a function of K_1 and K_2

$K_1 = K_1(t)$: stock of productive capital goods at time t

$K_2 = K_2(t)$: stock of abatement capital goods at time t

e_1 : emission to capital ratio of the productive capital goods ($e_1 > 0$ and constant)

e_2 : abatement to capital ratio of the abatement capital goods ($e_2 > 0$ and constant)

We suppose that the amount of earnings that is left after paying emissions tax is stochastic and satisfies the following expression:[1)]

$$R(K_1,K_2)dt = \{S(K_1) - \tau(e_1K_1-e_2K_2)\}\{dt + \sigma\ dB\} \tag{2}$$

in which:

$B = B(t)$: standard Wiener process, where the increments $dB(t)$ are independent over time and normally distributed with mean zero and variance dt

$R(K_1,K_2)$: net earnings (i.e. earnings net from emissions tax) being a function of K_1 and K_2.

$S(K_1)$: expected earnings before taxes when the stock of productive capital goods equals K_1, $S(0) = 0$, $S'(K_1) > 0$, $S''(K_1) < 0$

τ : emissions tax rate ($\tau > 0$ and constant)

σ : a constant ($\sigma > 0$)

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1) The same expression can be found in Bensoussan and Lesourne (1980) except that the emissions tax is missing there.

The capital stocks are of the non-depreciating type and can be increased by investment. Furthermore, it is assumed that the firm starts out without having assigned any capital goods to the abatement activity yet. In this way we obtain the following state equations for productive and abatement capital stock:

$$dK_1 = I_1 dt, \ K_1(0) > 0 \tag{3}$$

$$dK_2 = I_2 dt, \ K_2(0) = 0 \tag{4}$$

in which:

$I_1 = I_1(t)$: rate of investment in productive capital goods at time t

$I_2 = I_2(t)$: rate of investment in abatement capital goods at time t

The firm is not able to borrow funds or issue new shares. Empirically (cf. Sinn (1987)) issues of new shares have turned out to be a marginal means of finance in postwar western economies. The assumption of no borrowing is more restrictive since in practice debt is an important means of finance. Admittedly, the reason for this assumption is mathematical convenience but we suspect that the inclusion of borrowing would not lead to dramatic changes of the results, qualitatively speaking. We return to this matter when we conclude the paper.

Besides investing in (non-)productive capital goods, the firm can also use its net earnings to pay out dividend or to increase the cash balance.

Further, we fix the value per unit of capital goods at one unit of money. Taking these into account we obtain the third state equation of the model:

$$dM = \{S(K_1) - \tau(e_1K_1 - e_2K_2) - I_1 - I_2 - D\}dt +$$

$$\sigma\{S(K_1) - \tau(e_1K_1 - e_2K_2)\}dB, \; M(0) > 0 \tag{5}$$

in which:

$D = D(t)$: rate of dividends at time t

$M = M(t)$: cash balance at time t

The firm behaves as if it maximizes the shareholders' value of the firm which can be expressed as the mathematical expectation of the discounted dividend stream over the planning period. Hence, the objective function becomes:

$$\text{maximize } E_0\left[\int_0^T D \exp(-it)dt\right] \tag{6}$$

in which:

T : horizon date

i : shareholders' time preference rate ($i > 0$ and constant)

Unlike most dynamic models, in the Bensoussan-Lesourne-framework the horizon date T is endogenously determined such, that it equals bankruptcy time. We assume that the firm is bankrupt as soon as the cash balance becomes negative, which is expressed in the following equation for the horizon date:

$$T = \inf\{t \mid M(t) \leq 0\} \tag{7}$$

Concerning this bankruptcy condition it is important to remark that capital goods cannot be sold to increase cash. This is because here investments are assumed to be irreversible (see e.g. Demers (1991), Pindyck (1988)), meaning that the firm cannot disinvest, so the expenditures are sunk costs. Irreversibility usually arises because capital is industry- or firm-specific, that is, it cannot be used in a different industry or by a different firm.
The assumption of irreversibility of investments and the nonnegativity of the dividend rate are captured by the following inequalities:

$$D \geq 0 \tag{8}$$

$$I_1 \geq 0 \tag{9}$$

$$I_2 \geq 0 \tag{10}$$

It is assumed that the firm does not spend more on investment and dividend than the expected net earnings:

$$D + I_1 + I_2 \leq S(K_1) - \tau(e_1K_1-e_2K_2) \tag{11}$$

This is a reasonable rule, since the firm should consider that possible profits arising from the positivity of the stochastic increment dB (cf. (2)) may be changed into losses one moment later.

Of course it makes no sense that the amount of emissions becomes negative, so we have to impose the following inequality:

$$E = e_1K_1 - e_2K_2 \geq 0 \tag{12}$$

To summarize: the model contains three state variables K_1, K_2 and M, three control variables I_1, I_2, and D, and can be expressed as follows:

$$\underset{I_1,I_2,D}{\text{maximize}}\ E_0\left[\int_0^T D \exp(-it)dt\right] \tag{13}$$

s.t.

$$dK_1 = I_1dt,\ K_1(0) > 0 \tag{14}$$

$$dK_2 = I_2dt,\ K_2(0) = 0 \tag{15}$$

$$dM = \{S(K_1) - \tau(e_1K_1-e_2K_2) - I_1 - I_2 - D\}dt +$$

$$\sigma\{S(K_1) - \tau(e_1K_1-e_2K_2)\}dB,\ M(0) > 0 \tag{16}$$

$$e_1K_1 - e_2K_2 \geq 0 \tag{17}$$

$$D \geq 0 \tag{18}$$

$$I_1 \geq 0 \tag{19}$$

$$I_2 \geq 0 \tag{20}$$

$$S(K_1) - \tau(e_1K_1-e_2K_2) - I_1 - I_2 - D \geq 0 \tag{21}$$

Finally, as an additional assumption we impose that at the start of the planning period the firm's productive capital stock is that low (notice that $S''(K_1) < 0$) that the expected increase in net earnings due to an extra unit of K_1 (i.e. $S'(K_1) - \tau e_1$) exceeds the emissions tax decrease due to an extra unit of K_2 (i.e. τe_2) as well as the shareholders' time preference rate:

$$S'(K_1(0)) - \tau e_1 > \max(\tau e_2, i) \tag{22}$$

3. CANDIDATE POLICIES FOR OPTIMALITY

To solve the model we use dynamic programming. Then the first step is to define a value function:

$$V(M(t),K_1(t),K_2(t)) = \max_{\substack{I_1,I_2,D\geq 0 \\ D+I_1+I_2\leq S(K_1)-\tau E}} E_t\left[\int_t^T D\exp(-i(s-t))ds\right] \tag{23}$$

in which:

$V(M,K_1,K_2)$: value of the firm being a function of M, K_1 and K_2

V equals the expected discounted dividend stream during a time interval that begins at an arbitrary instant $t \in [0,T]$ and ends at the horizon date T. Because this horizon date depends completely on the value of M (cf. (7)), we can conclude that V depends only on M, K_1 and K_2, and not explicitly on T.

Throughout the rest of the paper we assume that all partial derivatives of first and second order exist. Now, we may derive the Hamilton-Jacobi-Bellman equation in case the state constraint $E = e_1K_1 - e_2K_2 \geq 0$ is not binding:

$$iV = \max_{\substack{I_1, I_2, D \geq 0 \\ D+I_1+I_2 \leq S(K_1)-\tau E}} \left[D + V_M\{S(K_1)-\tau E-I_1-I_2-D\} + V_{K_1}I_1 + V_{K_2}I_2\right] +$$

$$\frac{1}{2}\sigma^2\{S(K_1) - \tau E\}^2 V_{MM} \tag{24}$$

The term between brackets can be maximized by comparing the values of V_M, V_{K_1} and V_{K_2} to each other and to 1. If V_M, V_{K_1} and V_{K_2} are less than 1, then the term between brackets reaches its maximum by putting D as high as possible which can be done by equating I_1 and I_2 to zero and D to $S(K_1) - \tau E$. If V_M is greater than the maximum of 1, V_{K_1} and V_{K_2}, then the term is maximized by making $S(K_1) - \tau E - I_1 - I_2 - D$ as high as possible, which implies that I_1, I_2 and D must be zero. Further, it is optimal to equate I_1 to $S(K_1) - \tau E$ and D and I_2 to zero if V_{K_1} is greater than V_M, V_{K_2} and 1, and, similarly, if V_{K_2} has the largest value then it is optimal to equate I_2 to $S(K_1) - \tau E$ and D and I_1 to zero. In this way (24) can be rewritten as:

$$iV = \{S(K_1) - \tau E\} \max\{1, V_M, V_{K_1}, V_{K_2}\} + \frac{1}{2}\sigma^2\{S(K_1) - \tau E\}^2 V_{MM} \quad (25)$$

to which we adjoin the boundary condition:

$$V(0, K_1, K_2) = 0 \quad (26)$$

Depending on the relative size of 1, V_M, V_{K_1} and V_{K_2}, the policies maximizing the right-hand side of (25) differ. Four policies have to be considered, which can be economically interpreted since:

1 = the marginal profitability of an additional dollar used to increase dividend

V_M = the marginal increase of the value of the firm due to one extra dollar kept in cash

V_{K_1} = the marginal increase of the value of the firm due to an additional productive investment of one dollar

V_{K_2} = the marginal increase of the value of the firm due to an additional abatement investment of one dollar.

The four policies can then be summarized by the following:

Cash policy: $dM = \{S(K_1) - \tau E\}(dt + \sigma dB)$, $D = I_1 = I_2 = 0$

optimal if:

$$V_M \geq \max(1, V_{K_1}, V_{K_2}) \quad (27)$$

Due to (27) we can conclude that for this policy it is marginally better:

- to increase cash than to pay out dividend;
- to increase cash than to invest in productive capital goods;
- to increase cash than to invest in abatement capital goods.

Dividend policy: $dM = \sigma\{S(K_1) - \tau E\}dB$, $D = S(K_1) - \tau E$, $I_1 = I_2 = 0$ optimal if:

$$1 \geq \max(V_M, V_{K_1}, V_{K_2}) \tag{28}$$

For this policy it is marginally better:

- to pay out dividend than to increase cash;
- to pay out dividend than to invest in productive capital goods;
- to pay out dividend than to invest in abatement capital goods.

Investment policy: $dM = \sigma\{S(K_1) - \tau E\}dB$, $I_1 = S(K_1) - \tau E$, $D = I_2 = 0$ optimal if:

$$V_{K_1} \geq \max(1, V_M, V_{K_2}) \tag{29}$$

For this policy it is marginally better:

- to invest in productive capital goods than to increase cash;
- to invest in productive capital goods than to pay out dividend;
- to invest in productive capital goods than to invest in abatement capital goods.

Abatement policy: $dM = \sigma\{S(K_1) - \tau E\}dB$, $I_2 = S(K_1) - \tau E$, $D = I_1 = 0$ optimal if:

$$V_{K_2} \geq \max(1, V_M, V_{K_1}) \tag{30}$$

For this policy it is marginally better:

- to invest in abatement capital goods than to increase cash;
- to invest in abatement capital goods than to pay out dividend;
- to invest in abatement capital goods than to invest in productive capital goods.

As already stated in the beginning of this section the value of the firm V only depends on M, K_1 and K_2 and not on t. Then the same holds, of course, for the partical derviatives V_M, V_{K_1} and V_{K_2}. Now, we can conclude from the conditions (27) through (30) that it completely depends on the values of the state variables M, K_1 and K_2 which of the four policies is optimal for the firm to carry out. Hence, in what follows we try to establish for which values of M, K_1 and K_2 which policy is optimal. To so so, we have to divide the M-K_1-K_2 space in four regions, each of them belonging to one of the four candidates for an optimal policy, which are collections of those values of M, K_1 and K_2 for which the corresponding policy is optimal. In this way we get the following regions: investment-region, cash-region, dividend-region and abatement-region. In what follows these regions will be denoted by I_1-region, M-region, D-region and I_2-region, respectively.

4. SOLUTION

We study this problem for different scenarios. Each scenario is characterized by a different set of values for: shareholders' time preference rate, the parameter σ (cf. (2)), abatement to capital rate and emissions tax rate. In this paper we will consider only those scenarios under which the solutions with the most realistic properties arise. Mathematically this means that we take the following two inequations as starting-point for the remaining analysis (in the sequel we explain in footnotes what happens when (31) and (32) do not hold):

$$1/i - \sigma/\sqrt{2i} > 0 \tag{31}$$

$$\{S'(0) - \tau e_1\}(1/i - \sigma/\sqrt{2i} - \rho) > 1 \tag{32}$$

in which:

ρ : a constant which satisfies:

$$\exp\{(r_1-r_2)\rho\} = \{1 - r_2(1/i - \sigma/\sqrt{2i})\}/\{1 - r_1(1/i - \sigma/\sqrt{2i})\} \tag{33}$$

in which:

$$r_1 = (-1 + \sqrt{1+2\sigma^2 i})/\sigma^2 \tag{34}$$

$$r_2 = (-1 - \sqrt{1+2\sigma^2 i})/\sigma^2 \tag{35}$$

Now we are ready to formulate our first proposition.

Proposition 1

Under condition (31) only the M-region can include the K_1-K_2 plane where $M = 0$.[2)]

Proof.

Follows the same steps as the proof of Proposition 1 in Kort (1989, Ch. 4) and is therefore omitted.

4.1. Optimal firm behavior under a weak environmental policy of the government

Here we analyze a scenario for which the following condition holds:

$$\tau e_2 < i \tag{36}$$

Hence, we assume for the moment that the emissions tax rate is relatively low.

One extra unit of abatement investment (which costs one dollar) decreases the amount of emissions by e_2 implying that the emissions tax to be paid

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2) In case (31) does not hold it can be proved (cf. Kort (1989)) that a Dividend policy is better than a Cash policy. Economically this could be interpreted by saying that now the earnings level is so uncertain, i.e. σ is relatively high when the sign of inequality (31) is reversed, that the firm still has a fair chance of going bankrupt when it carries out a Cash policy. Therefore, shareholders prefer to obtain dividends immediately before bankruptcy actually occurs. This statement can also be explained by observing that the shareholders' time preference rate is relatively high, implying that immediate obtainment of dividends is attractive, when the sign of inequality (31) is reversed.

by the firm decreases with the amount τe_2. Hence, the marginal value of abatement investment equals τe_2. One extra dollar dividend pay out can be used by the shareholders to invest outside the firm, generating a return of i. As the firm maximizes the shareholders' value of the firm, we can conclude that the firm always assigns a higher value to dividend pay out than to abatement investments when condition (36) holds. Therefore, the Dividend policy will always dominate the Abatement policy and, consequently, the firm will never carry out an Abatement policy when the governmental environmental policy is weak.

The above implies that there will be no abatement investments, so the stock of abatement capital remains zero throughout the whole planning period (cf. (4)). Therefore, we can restrict ourselves to the problem of dividing the M-K_1 plane into an I_1-region, a M-region and a D-region.

Except that their contents and proofs are slightly adjusted for the presence of activity analysis and emissions tax, the following propositions and their proofs also hold for the original Bensoussan-Lesourne model without these extensions. Therefore, we only present these propositions and for their proofs refer to Kort (1989), in which the Bensoussan-Lesourne model is extensively treated.

Proposition 2

The boundary between the M-region and the D-region can be expressed as $M = \rho\{S(K_1) - \tau(e_1K_1-e_2K_2)\}$, where ρ is given by (33).

Proposition 3

The boundary between the I_1-region and the D-region increases in the M-K_1 plane (K_2 being constant) and lies below a horizontal asymptote which is situated on the level K_1^*, determined by $S'(K_1^*) - \tau e_1 = i$. The D-region lies at the left-hand side of this boundary.

At the intersection point $(\bar{M},\bar{K}_1)$ of the boundary between the I_1-region and the D-region and the boundary between the M-region and the D-region it must hold that $\{S'(\bar{K}_1) - \tau e_1\}(1/i - \sigma/\sqrt{2i} - \rho) = 1$.[3)]

Proposition 4

In the M-K_1 plane the boundary between the I_1-region and the M-region starts at the origin and ends at the intersection point $(\bar{M},\bar{K}_1)$ of the boundaries between the M-region and the D-region and between the I_1-region and the D-region.

Notice, that we already concluded that K_2 will always be zero under condition (36) so that in this subsection we consider the above propositions for $K_2 = 0$. Now, the contents of the Propositions 1-4 can be transformed into the solution as depicted in Figure 1.

[place Figure 1 about here]

This solution has the same configuration as one of the solutions of the Bensoussan-Lesourne model without activity analysis and emissions tax (see Kort (1989), Figure 4.1)). Resemblances are that the firm increases cash if the cash balance is low, it invests if the stock of capital goods is

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3) Notice that if the sign of inequality (32) is reversed, then the intersection point $(\bar{M},\bar{K}_1)$ does not exist. This implies that it becomes optimal to pay out dividend for very low values of K_1. In reality such a policy can only be observed in exceptional situations (cf. Kort (1989), p. 87).

low and it pays out dividends if both the cash balance and the stock of capital goods are high enough.

A first difference is that in this model the horizontal asymptote of the boundary between the I_1-region and the D-region is situated on the level K_1^*, which satisfies $S'(K_1^*) - \tau e_1 = i$ (cf. Proposition 3), while in the Bensoussan-Lesourne model this asymptote corresponds to K^*, which satisfies $S'(K^*) = i$. For both situations the economic interpretation is similar: the firm never carries out an Investment policy if the expected marginal earnings (after emissions tax) fall below the return that the shareholders can obtain outside the firm, which is equal to i. This actually happens at the moment that K is above the asymptote (to understand this remember that $S'' < 0$). Furthermore, we can conclude that in the present model the asymptote corresponds to a lower level of the stock of capital goods, implying a smaller investment-region. Hence, investing becomes less attractive to the firm which can be declared by the fact that now investment is less profitable due to the emissions taxation costs.

A second difference is that here the cash-dividend boundary equals $M = \rho\{S(K_1) - \tau e_1 K_1\}$ (cf. Proposition 2 for $K_2 = 0$) and in the Bensoussan-Lesourne model this boundary is expressed by $M = \rho S(K)$. This implies that in the present model a Dividend policy is more popular to the firm than a Cash policy compared to the Bensoussan-Lesourne model. The reason is that in this model the cash situation is negatively influenced by the presence of emissions tax, implying a greater risk for the firm to go bankrupt (cf. (7)). Therefore, the shareholders want to obtain dividend as soon as possible. They are less interested in increasing the cash balance first because of the increased risk of the firm going bankrupt before the dividend payout starts.

The overall conclusion is that the presence of emissions tax negatively influences the economic prospects of the firm and therefore the shareholders are less interested in building up the firm first, i.e. increasing the stock of capital goods and the cash balance, before demanding any dividends. So, an immediate obtainment of dividends becomes more popular implying an increased D-region. The latter is confirmed by the fact that, compared to the Bensoussan-Lesourne model, here the central intersection point $(\bar{M},\bar{K}_1)$ has lower values for the stocks of cash and capital goods. Because now $\bar{M}$ and $\bar{K}_1$ are fixed by the equalities $\bar{M} = \rho\{S(\bar{K}_1) - \tau e_1\bar{K}_1\}$ and $\{S'(\bar{K}_1) - \tau e_1\}(1/i - \sigma/\sqrt{2i} - \rho) = 1$ (cf. Proposition (3)), while in the Bensoussan-Lesourne model $(\bar{K},\bar{M})$ is determined by $\bar{M} = \rho S(\bar{K})$ and $S'(\bar{K})(1/i - \sigma/\sqrt{2i} - \rho) = 1$.

4.2. Optimal firm behavior under a moderate environmental policy of the government

In this subsection it is assumed that the parameters satisfy the following inequalities:

$$\tau e_2 > i \tag{37}$$

$$\tau e_2(1/i - \sigma/\sqrt{2i} - \rho) < 1 \tag{38}$$

Comparing (36) and (37) we conclude that here governmental environmental policy will have a stronger influence than in the solution of the previous subsection. Because of (37) the Abatement policy is no longer dominated by the Dividend policy implying that now we must try to find out where the

I_2-region is situated in the M-K_1-K_2 space. Let us concentrate first on the boundary between the I_1-region and the I_2-region. This boundary consists of a collection of points in the M-K_1-K_2 space, which have in common that investments in productive capital goods and in abatement capital goods have the same (expected) profitability. The marginal expected net earnings of productive investment equal $S'(K_1) - \tau e_1$, while τe_2 is the marginal tax decrease of abatement investment. Hence, on the boundary between the I_1-region and the I_2-region K_1 must be constant, say $\hat{K}_1$, such that it satisfies:

$$S'(\hat{K}_1) - \tau e_1 = \tau e_2 \tag{39}$$

The properties of the boundary between the I_2-region and the D-region are formulated in the following proposition:

Proposition 5

The boundary between the I_2-region and the D-region has the following properties:

1. $(\bar{K}_1, \bar{M})$ (cf. Proposition 3) is not situated on the I_2/D-boundary.
2. (39) is a necessary condition for the I_2/D-boundary to intersect the I_1/D-boundary.
3. The I_2/D-boundary increases in the K_1-M plane for K_2 fixed.
4. For the I_2/D-boundary it holds that: $K_1 \to \infty \Leftrightarrow M \to \infty$.
5. The I_2-region is situated at the right-hand side of this boundary.

Proof

See Appendix.

In Figure 2 the solution is depicted in case that $K_2 = 0$. The correctness of this solution can be verified by checking that the Propositions 1-5 are satisfied and that the I_1/I_2-boundary is given by (39).

[place Figure 2 about here]

Compared to Figure 1, the big difference is that Figure 2 contains a region where it is optimal to carry out an Abatement policy. Apparently such a policy can only be optimal if productive capital stock is that large that the marginal emissions tax decrease resulting from an abatement investment is higher than the expected earnings due to marginal productive investment. Furthermore the stock of cash must be high enough to guarantee a sufficiently strong liquidity position. Remarkable is that the I_2-region is situated at the right of the D-region.

Notice that, due to (37), we could think that, opposite to the situation in Subsection 3.1, the Abatement policy dominates the Dividend policy. But we have to remember that the objective of the firm is to maximize the expected discounted dividend stream. Hence it could be dangerous to postpone paying out dividend by, for instance, carrying out abatement investments first, because in the mean time bankruptcy could occur and then the shareholders get nothing. Therefore carrying out an Abatement policy only turns out to be optimal when the bankruptcy risk is limited, i.e. when the stock of cash is high.

Figure 2 actually holds at time-point zero, because the firm starts out with a stock of abatement capital being zero. Also it holds that the productive capital stock is less than $\hat{K}_1$ (cf. (22), (39)) at the start of the planning period. The I_2-region is situated above $\hat{K}_1$ implying that K_2 can only increase at the moment that K_1 becomes at least as high as $\hat{K}_1$. In the following proposition it is stated in what way the configuration in the K_1-M plane changes when K_2 increases.

Proposition 6

When K_2 increases the configuration of the regions in the K_1-M plane changes in the following way compared to Figure 2:

1. the M/D-boundary shifts to the right;
2. $(\bar{M},\bar{K})$ shifts to the right;
3. the I_1/D-boundary shifts to the right;
4. the I_2/D-boundary shifts to the right;

Proof

See Appendix.

The results stated in Proposition 6 can be explained as follows: due to an increase in the stock of abatement capital emissions are reduced resulting in a reduced payment of emissions tax by the firm. Therefore the firm is more able to prevent bankruptcy which explains that the Cash policy becomes more popular when the stock of abatement capital increases. This causes an expansion of the M-region and through this swelling the other regions are pushed to the right.

Carrying out abatement investments only make sense as long as there is something to abate, i.e. as long as the amount of emissions is positive. When this amount becomes zero the state constraint (12) becomes binding and a further increase of abatement capital stock is not possible.

Suppose the firm is in the I_2-region and K_2 reaches its upperbound, resulting from emissions becoming zero, for the first time. Due to the facts that the firm starts out with a productive capital stock being less than $\hat{K}_1$ (cf. (22), (39)), that above $\hat{K}_1$ the firm never carries out productive investments, when (12) is not binding, and below $\hat{K}_1$ there is no abatement-region (cf. Figure 2), the stock of productive capital then exactly equals $\hat{K}_1$. It is not possible to perform an Abatement policy so the firm has to choose between the other three policies. In the I_2-region it holds that $M > \rho\{S(K_1) - \tau(e_1K_1-e_2K_2)\}$, implying that it is always better to pay out dividend than to increase cash (cf. Proposition 2). Therefore, at the moment that emissions become zero in the I_2-region the firm will make a choice between investing in productive capital stock and paying out dividend. If K_1 equals $\hat{K}_1$ the firm will without doubt choose for productive investment, because marginal expected net earnings of productive investment $(S'(\hat{K}) - \tau e_1)$ then equal marginal earnings of abatement investment (τe_2, cf. (39)) and the latter should be the best policy if emissions were not zero. As a result of productive investment K_1 increases, implying that the amount of emissions turn positive which means that abatement investments are allowed again. Investment in abatement capital stops when emissions again become zero. Then for the second time the firm must choose between productive investment and dividend pay out. Given the fact that the D/I_1-boundary increases in the K_1-M plane where the D-region is situated at the left and the I_1-region at the right of this boundary (cf. Proposition 3), the firm prefers to invest as long as the stock of cash is sufficiently large. This alternating investment/abatement behavior can go on for some time and in this way we obtain a mixed Investment/Abatement policy. Notice that if the stock of cash remains to be sufficiently large the stock of productive capital goods will ultimately reach K_1^{**} that satisfies:

$$\frac{e_2}{e_1+e_2} S'(K_1^{**}) = i \tag{40}$$

K_1^{**} is determined such that the expected extra profit arising from the application of an additional capital good exactly equals the shareholders' time preference rate. Namely, from this capital good $e_2/(e_1+e_2)$ is assigned to activity 1, implying an expected return of $S'(K_1^{**})e_2/(e_1+e_2)$, and $e_1/(e_1+e_2)$ to activity 2 (this division is the result of solving two equations with two unknowns: taking into consideration one additional capital good implies that $\Delta K_1+\Delta K_2 = 1$ and having no pollution results in $e_1\Delta K_1 = e_2\Delta K_2$). This equality between expected marginal return to capital goods and shareholders' time preference rate implies that at this stage the firm preferes paying out dividend instead of productive investments. Hence, the stock of productive capital remains constant at the level K_1^{**} and therefore emissions remain zero, so further investment in abatement capital makes no sense. This implies that the only policy left is the Dividend policy. The above story is depicted in Figure 3.

[place Figure 3 about here]

If the assumption "M sufficiently large" is dropped we must study Figure 3 in connection with Figure 2. From Figure 2 we obtain that for $K_1 < \bar{K}_1$ the I_1-policy is replaced by the M-policy if the stock of cash becomes sufficiently low, and for $K > \bar{K}_1$ the policies depicted in Figure 3 are replaced by the D-policy for intermediate values of M and by the M-policy for low values of M.

4.3. Optimal firm behavior under a strong environmental policy of the government

Here, the firm's optimal policy is studied under the following parameter constraint:

$$\tau e_2(1/i - \sigma/\sqrt{2i} - \rho) > 1 \qquad (41)$$

Comparing (38) and (41) we conclude that in this subsection the marginal decrease of emissions tax due to abatement investment has become larger relative to the shareholders' time preference rate and the parameter σ (this is true because ρ is a function of i and σ (cf. (33)-(35)) and by tedious calculations (see Kort (1988), Appendix 2) it can be shown that both the signs of the derivatives from ρ to i and σ are not clearly positive or negative).

Now, we formulate the following proposition:

Proposition 7

The M/D-boundary does not exist if condition (40) holds.

Proof

See Appendix.

Proposition 8

If M goes to infinity, K_1 and K_2 are finite and the state constraint (12) is not binding, it cannot be optimal for the firm to pay out dividend.

Proof

See Appendix.

Like in the solution of the previous subsection also here the I_1/I_2-boundary will be situated at the level $K_1 = \hat{K}_1$, where $\hat{K}_1$ is given by (39). Below $\hat{K}_1$ marginal expected earnings of productive capital is greater than the marginal tax decrease of abatement capital and above $\hat{K}_1$ the reverse is true. From Proposition 7 we obtain that the D-region is not directly bounded to the M-region. Therefore we can conclude that in the K_1-M plane the M-region is directly connected to the I_1-region if $K_1 < \hat{K}_1$ and for $K_1 > \hat{K}_1$ the M/I_2-boundary exists.
The contents of Propositions 7 and 8 do suggest that there is no D-region at all. This can be economically explained by the fact that, as stated before, expression (41) implies that the shareholders' time preference rate and the parameter σ have rather low values compared to the emissions tax decrease caused by a marginal investment in abatement capital. A low value of the shareholders' time preference rate means that there is no rush for immediate dividend pay out, so there is time to carry out productive and abatement investment first. A low value of σ implies that earnings are rather certain which means that bankruptcy risk is not so high. Hence, unlike in the solution of Subsection 3.2, here there is no need to be afraid that postponing dividend pay out is dangerous because of the fact that the occurrence of bankruptcy then implies that the shareholders

get nothing. Therefore, as long as emissions are positive, we expect that the Dividend policy is completely dominated by the Abatement policy. Notice that also here it holds that $\tau e_2 > i$.

The above story is depicted in Figure 4. Unfortunately we have to state that we were not able to prove mathematically that there is no D-region at all. The Propositions 7 and 8 are still satisfied if in Figure 4 a D-region occurs somewhere between the M-region and M being infinite. But, due to the fact that we cannot think of an economic reason for such a D-region, we skipped it in Figure 4.

[place Figure 4 about here]

Of course, the solution can also be drawn in the K_1-K_2 plane. For M sufficiently large also here Figure 3 applies with the only difference that under condition (41) $\bar{K}_1$ (cf. Proposition 3) is greater than $\hat{K}_1$. Of course if the assumption "M sufficiently large" is dropped Figure 3 has to be read in connection with Figure 4. So, the policies depicted in Figure 3 can be interrupted at any time by the need to raise the stock of cash through an M-policy if M decreases such that (K_1,M) goes to the left of the I_1/M-boundary or the I_2/M-boundary.

5. CONCLUSIONS

The aim of this paper is to study the optimality of abatement investments within an uncertain environment. To reach this aim we extended a stochastic model of Bensoussan and Lesourne (1980) by incorporating activity analysis. We considered two activities: the first one is productive but also causes pollution that is taxed by the government. The second one is non-productive but abates pollution so that tax payments are reduced.

The problem was modelled as a dynamic model of the firm. Models belonging to this category attempt to describe a firm, in broad terms and over its entire lifetime, with respect to basic characteristics such as its objective, production technology and financial structure. Taking primarily a normative point of view, the theory attempts to derive optimal time paths for key decision variables of the firm such as investment in productive capital, employment and dividend policy. One of the main purposes of designing and analyzing dynamic models of the firm is to develop a theoretical background for managerial policies. An extensive survey of the area of "The Dynamics of the Firm" is provided by Jørgensen (1991).

It turned out that investments in the abatement activity can be optimal when the following conditions are satisfied:

- the firm has a strong liquidity position;
- the reduction in emissions tax due to an additional abatement investment must be greater or equal than the shareholders' time preference rate and the expected marginal net earnings of productive investment.

Of course we have to keep in mind that the above conclusions only hold under the assumptions of the model. For instance, in our quest to obtain analytical results we left financing possibilities like borrowing and issuing new shares aside by assuming that the firm can finance its investments only by retained earnings. However, as stated in Section 2, issuing new shares is not an important means of finance in real life. Concerning borrowing possibilities we refer to Bensoussan and Lesourne

(1981) in which, like in the present paper, the self-financing model of Bensoussan and Lesourne (1980) was used as a departure-point. But, while we incorporated activity analysis to gain insights on environmental policies, they incorporated debt to study the implications of borrowing. They concluded that borrowing may be optimal for three reasons: to increase permanently the size of the firm's equipment, to accelerate the growth of the firm and/or to avoid bankruptcy by improving the cash situation. Comparing these results with our conditions for optimality of abatement investments, we may suspect that these conditions will not be affected, qualitatively speaking, when the model is extended to allow for borrowing.

APPENDIX

Proof of Proposition 5

Due to (25) and (28) we obtain that in the D-region it holds that:

$$iV = S(K_1) - \tau(e_1K_1 - e_2K_2) + \sigma^2\{S(K_1) - \tau(e_1K_1 - e_2K_2)\}^2V_{MM} \tag{A.1}$$

Solving this differential equation leads to:

$$V = \{S(K_1) - \tau(e_1K_1 - e_2K_2)\}/i +$$

$$c_1(K_1,K_2)\ \exp[M\sqrt{2i}/\sigma\{S(K_1) - \tau(e_1K_1 - e_2K_2)\}] +$$

$$c_2(K_1,K_2)\ \exp[-M\sqrt{2i}/\sigma\{S(K_1) - \tau(e_1K_1 - e_2K_2)\}] \tag{A.2}$$

in which:

$c_1(K_1,K_2)$ and $c_2(K_1,K_2)$ are arbitrary functions.

Since from an economic point of view it is unlikely that the value of the firm increases or decreases exponentially when M increases, while keeping K_1 and K_2 constant, we put $c_1(K_1,K_2)$ equal to zero.
We now turn to the five properties:

Proof of property 1

From Proposition 3 we obtain that $\bar{K}_1$ satisfies:

$$S'(\bar{K}_1) - \tau e_1 = 1/(1/i - \sigma/\sqrt{2i} - \rho) \tag{A.3}$$

From (38) we get:

$$\tau e_2 < 1/(1/i - \sigma/\sqrt{2i} - \rho) \tag{A.4}$$

Now, from (A.3) and (A.4) we can conclude that in $(\bar{K}_1,\bar{M})$ the expected marginal profitability of productive investment $(S'(K_1) - \tau e_1)$ exceeds the profitability of one dollar abatement investment (τe_2). Hence carrying out abatement investments will never be optimal in the direct neighbourhood of $(\bar{K}_1,\bar{M})$, which leads to the conclusion that the I_2/D-boundary does not intersect this point.

Hence the I_2-region is not situated near $(\bar{K}_1,\bar{M})$ which implies that in the direct neighbourhood of this point the solution is the same as in Figure 1. Thus we have a M/D-boundary given by (cf. Proposition 2):

$$M = \rho\{S(K_1) - \tau(e_1K_1 - e_2K_2)\} \tag{A.5}$$

Due to the fact that on the M/D-boundary it also holds that $V_M = 1$ (because (27) and (28) must both be satisfied) we can obtain from (A.2) and the fact that we already put $c_1(K_1,K_2)$ equal to zero (cf. Bensoussan and Lesourne (1980) p. 265):

$$c_2(K_1,K_2) = \sigma\{S(K_1) - \tau(e_1K_1 - e_2K_2)\}\exp(\rho \frac{\sqrt{2i}}{\sigma})/\sqrt{2i} \tag{A.6}$$

If we substitute $c_1(K_1,K_2) = 0$ and (A.6) into (A.2) we obtain for the D-region:

$$V = \{S(K_1) - \tau(e_1K_1 - e_2K_2)\}$$

$$(1/i - \sigma \exp[(\rho - M/\{S(K_1) - \tau(e_1K_1 - e_2K_2)\})\sqrt{2i}/\sigma]/\sqrt{2i}) \tag{A.7}$$

Proof of property 2

From (A.7) we obtain:

$$V_{K_1} = \{S'(K_1) - \tau e_1\}\left[\frac{1}{i} - \left\{\frac{\sigma}{\sqrt{2i}} + \frac{M}{S(K_1)-\tau(e_1K_1-e_2K_2)}\right\}\right.$$

$$\left.\exp\left[\left[\rho - \frac{M}{S(K_1)-\tau(e_1K_1-e_2K_2)}\right]\frac{\sqrt{2i}}{\sigma}\right]\right] \tag{A.8}$$

$$V_{K_2} = \tau e_2\left[\frac{1}{i} - \left\{\frac{\sigma}{\sqrt{2i}} + \frac{M}{S(K_1)-\tau(e_1K_1-e_2K_2)}\right\} \exp\left[\left[\rho - \frac{M}{S(K_1)-\tau(e_1K_1-e_2K_2)}\right]\frac{\sqrt{2i}}{\sigma}\right]\right] \tag{A.9}$$

At the intersection point of the I_2/D-boundary and the I_1/D-boundary it must hold that $V_{K_1} = V_{K_2} = 1$. Now we can immediately conclude from (A.8) and (A.9) that property 2 is valid.

Proof of property 3

From (A.9) we get:

$$V_{K_2M} = \tau e_2 M\sqrt{2i}/\left[\sigma\{S(K_1) - \tau(e_1K_1-e_2K_2)\}^2\right]$$

$$\exp\left[\left[\rho - \frac{M}{S(K_1)-\tau(e_1K_1-e_2K_2)}\right]\frac{\sqrt{2i}}{\sigma}\right] > 0 \qquad (A.10)$$

$$V_{K_2K_1} = -\frac{\sqrt{2i}}{\sigma}\,\frac{\{S'(K_1)-\tau e_1\}\tau e_2M^2}{[S(K_1)-\tau(e_1K_1-e_2K_2)]^3}$$

$$\exp\left[\left[\rho - \frac{M}{S(K_1)-\tau(e_1K_1-e_2K_2)}\right]\frac{\sqrt{2i}}{\sigma}\right] < 0 \qquad (A.11)$$

From (A.10) we derive that V_{K_2} increases if M increases. On the whole I_2/D-boundary it must hold that $V_{K_2} = 1$, so if M increases and we keep K_2 constant we have to find a K_1 which cancels the increase in V_{K_1} due to M. From (A.11) we can conclude that V_{K_2} decreases if K_1 increases. Hence, on the boundary a higher level of M corresponds to a higher level of K_1, given that K_2 is kept constant, and therefore the boundary is an increasing function in the M-K_1 plane.

Proof of property 4

We study two cases:

1. $M \to \infty$, K_1 being finite:

now, we obtain from (36) and (A.9):

$$V_{K_2} = \tau e_2/i > 1 \tag{A.12}$$

Hence, because on the I_2/D-boundary it holds that $V_{K_2} = 1$, we conclude from (A.12) that if M goes to infinity K_1 cannot have a finite value on the I_2/D-boundary.

2. $K_1 \to \infty$, M being finite:
now, (36) and (A.9) lead to:

$$V_{K_2} = \tau e_2\left[\frac{1}{i} - \frac{\sigma}{\sqrt{2i}} \exp\left[\rho \frac{\sqrt{2i}}{\sigma}\right]\right] > 1 \tag{A.13}$$

So, if K_1 goes to infinity on the I_2/D-boundary then M cannot have a finite value.

From the analysis of the above two cases we derive the validity of property 4.

Proof of property 5

From (A.10) we obtain that in the D-region V_{K_2} increases if M increases. For a Dividend policy to be optimal it must hold that $V_{K_2} < 1$. And because at the I_2/D-boundary it holds that $V_{K_2} = 1$ we conclude that property 5 is valid.

Proof of Proposition 6

Proof of property 1

Follows directly from the expression of this boundary (cf. Proposition 2)

Proof of property 2

Follows directly from the expression of this point (cf. Proposition 3).

Proof of property 3

At this boundary it holds that $V_{K_1} = 1$. From (A.8) we obtain:

$$V_{K_1M} = \frac{\sqrt{2i}}{\sigma} \frac{\{S'(K_1)-\tau e_1\}M}{\{S(K_1)-\tau(e_1K_1-e_2K_2)\}^2} \exp\left[\left[\rho - \frac{M}{S(K_1)-\tau(e_1K_1-e_2K_2)}\right]\frac{\sqrt{2i}}{\sigma}\right] > 0 \qquad (A.14)$$

From (A.11) we already know that $V_{K_1K_2} < 0$. Hence, when K_2 increases V_{K_1} decreases. To keep V_{K_1} equal to 1 M must be increased.

Proof of property 4

At this boundary it holds that $V_{K_2} = 1$. From (A.9) we obtain:

$$V_{K_1K_2} = -\frac{\sqrt{2i}}{\sigma} \frac{(\tau e_1)^2M^2}{\sigma[S(K_1)-\tau(e_1K_1-e_2K_2)]^3} \exp\left[\left[\rho - \frac{M}{S(K_1)-\tau(e_1K_1-e_2K_2)}\right]\frac{\sqrt{2i}}{\sigma}\right] < 0 \qquad (A.15)$$

From (A.10) we obtain that $V_{K_2M} > 0$. Hence when keeping K_1 constant and K_2 increases also M must increase to keep V_{K_2} equal to 1.

Proof of Proposition 7

If the M/D-boundary exists it is given by (cf. Proposition 2);

$$M = \rho\{S(K_1) - \tau(e_1K_1-e_2K_2)\} \tag{A.16}$$

Then in the D-region the value of the firm is given by (A.7) and from this expression we obtain:

$$V_{K_2} = \tau e_2\left[\frac{1}{i} - \left[\frac{\sigma}{\sqrt{2i}} + \frac{M}{S(K_1)-\tau(e_1K_1-e_2K_2)}\right]\right.$$

$$\left.\exp\left[\left[\rho - \frac{M}{S(K_1)-\tau(e_1K_1-e_2K_2)}\right]\frac{\sqrt{2i}}{\sigma}\right]\right] \tag{A.17}$$

After substitution of (A.16) into (A.17) we obtain the value of V_{K_2} on the M/D-boundary:

$$V_{K_2} = \tau e_2\left\{\frac{1}{i} - \frac{\sigma}{\sqrt{2i}} - \rho\right\} \tag{A.18}$$

Now, from (40) and (A.18) we obtain that on the M/D-boundary it holds that $V_{K_2} > 1$, which implies that here a Dividend policy cannot be optimal (cf. (28)). hence under condition (40) the M/D-boundary does not exist.

Proof of Proposition 8

Due to (A.2) and the fact that we have put $c_1(K_1,K_2)$ equal to zero (see below (A.2)) we have the following expression for the value of the firm in the D-region.

$$V = \{S(K_1) - \tau(e_1K_1-e_2K_2)\}/i + c_2(K_1,K_2)$$

$$\exp\left[\frac{-M\sqrt{2i}}{\sigma\{S(K_1)-\tau(e_1K_1-e_2K_2)\}}\right] \tag{A.19}$$

From this equation we derive:

$$V_{K_2} = \frac{\tau e_2}{i} + \left[\frac{\partial c_2}{\partial K_2} + \frac{\tau e_2\sqrt{2i}M\ c_2(K_1,K_2)}{\sigma\{S(K_1)-\tau(e_1K_1-e_2K_2)\}^2}\right]$$

$$\exp\left[\frac{-M\sqrt{2i}}{\sigma\{S(K_1)-\tau(e_1K_1-e_2K_2)\}}\right] \tag{A.20}$$

Due to (A.20) we can conclude that $V_{K_2} > 1$ if M goes to infinity and, due to (28), this implies that a Dividend policy can not be optimal.

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Figure captions

Figure 1. The optimal solution for the scenario where it holds that $\tau e_2 < i$.

Figure 2. The optimal solution in case that $K_2 = 0$ for the scenario where it holds that $\tau e_2 > i$ and $\tau e_2(1/i - \sigma/\sqrt{2i} - \rho) < 1$.

Figure 3. The optimal solution in case that M is sufficiently large for the scenario where it holds that $\tau e_2 > i$ and $\tau e_2(1/i - \sigma/\sqrt{2i} - \rho) < 1$.

Figure 4. The optimal solution for a fixed value of K_2 which is such that the amount of emissions is positive, and for parameter values that satisfy $\tau e_2(1/i - \sigma/\sqrt{2i} - \rho) > 1$.

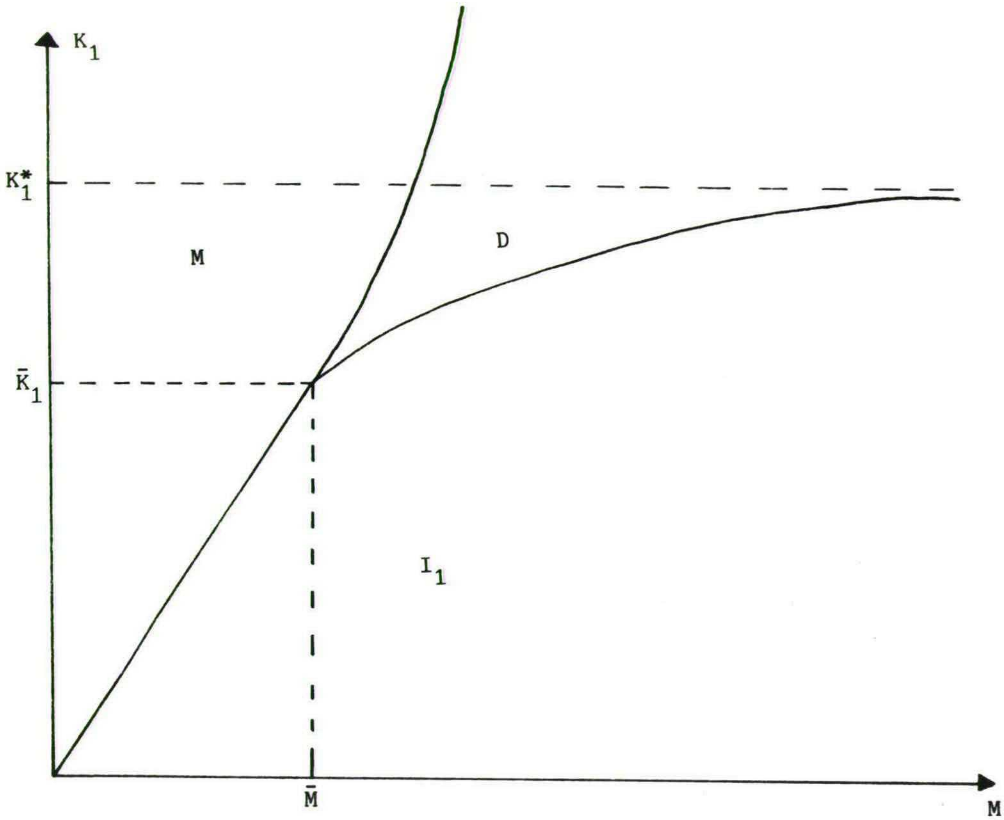

Figure 1

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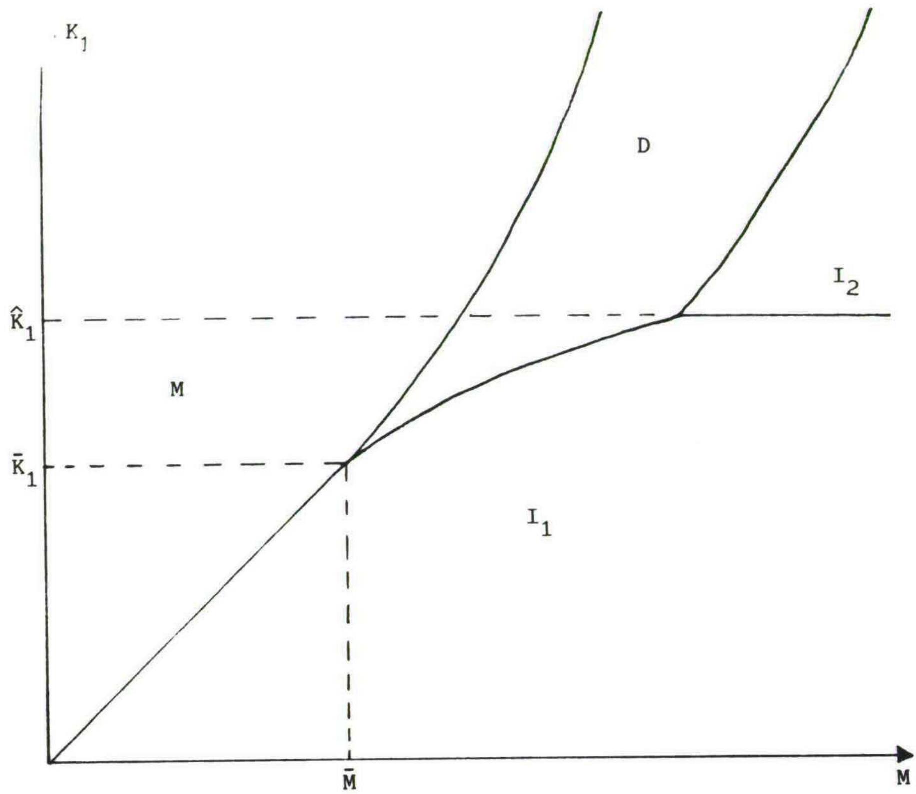

Figure 2

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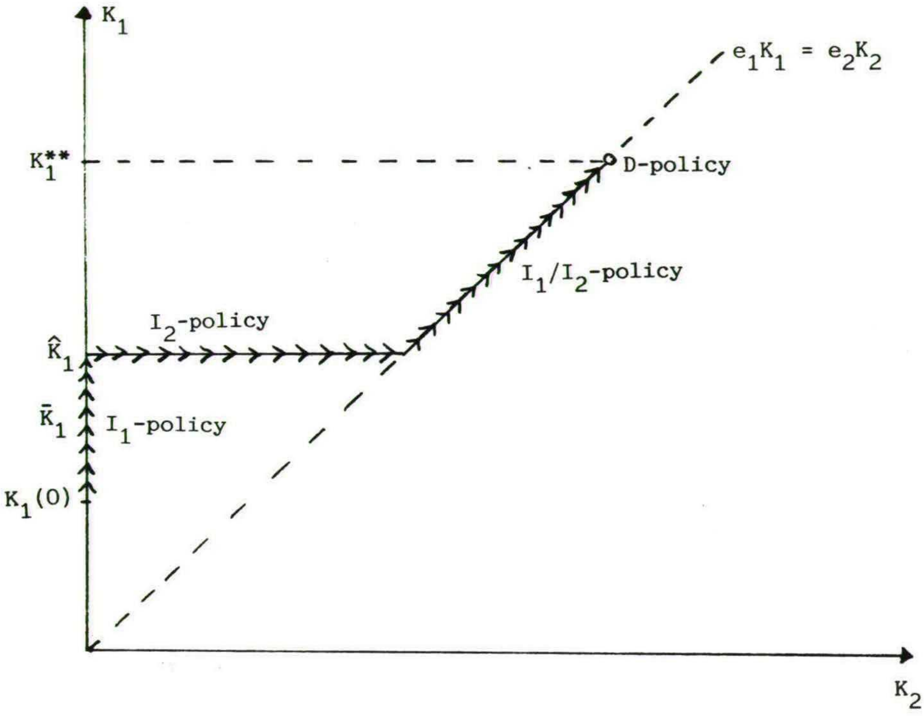

Figure 3

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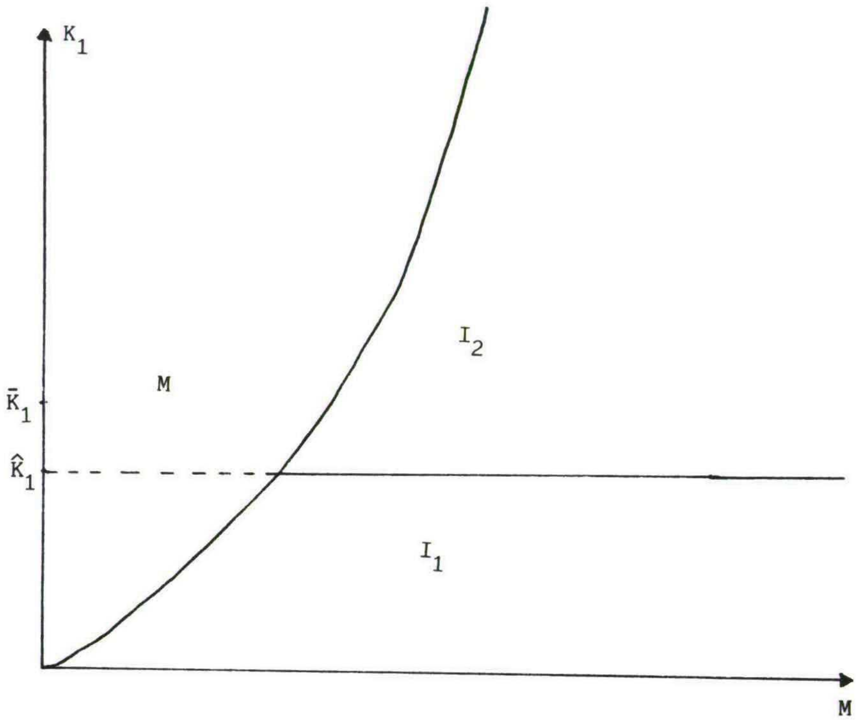

Figure 4

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