

# OPTIMAL ADVANCE SCHEDULING

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# Morgan Stanley Children's Hospital

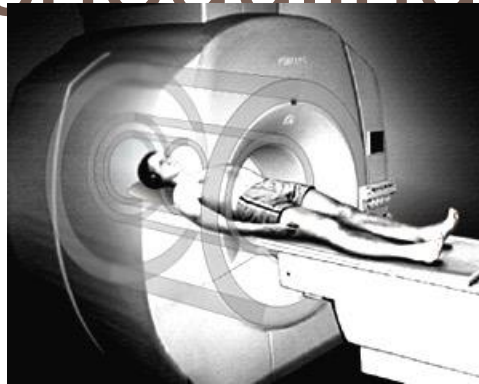


- Children's hospital of New York-Presbyterian
  - ▣ Academic hospital system of Columbia University and Cornell University
- *U.S. News & World Report*
  - ▣ "America's Best Children's Hospitals"

# Pediatric radiology department

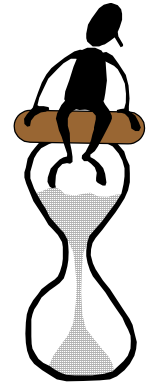
- Performs and interprets imaging studies
- Approximately 53,000 examinations per year.
  - 39,000 plain radiographs
  - 7,600 ultrasound studies
  - 2,300 fluoroscopy studies
  - 3,500 CT scans of the body excluding neuro-imaging
  - **525 body and musculoskeletal MRI scans**

# MRI Scheduling



# Service issues in MRI Imaging

- Multiple patient classes
  - ▣ urgent patients
    - prioritized to be served within 24h
  - ▣ large outpatient community
    - make advance appointments.
  
- Limited capacity and slow rate of service
  - ▣ 1 MRI machine
  - ▣ variable exam time (30min - 4h per exam)
  
- 8-10 week wait for an MRI scan for outpatients
  - ▣ frustration, diminished quality of care, patient attrition



# MRI Scheduling Problem

- Operational strategies
  - ▣ Long-term: increase capacity
  - ▣ Short-term: increase “rate of processing”
- This work is about how to dynamically manage the rate of processing
  - ▣ More overtime work or longer patient waiting?

# Model

- Days  $1, 2, 3 \dots T$ 
  - ▣  $T$  infinite (later consider finite  $T$ )
- Resources  $1, 2, 3 \dots R$ , each with finite daily capacity
  - ▣ Usage in excess of capacity incurs overtime costs
- New demand (urgent and regular) arises each day. Patients take earliest appointment offered.

# Model: 2-step scheduling

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- First step: **Patients are assigned to service days (focus of talk)**
- Second step: Appointments are sequenced and timed within a day



# Assigning patients to days

## 1: Allocation scheduling

- ▣ All patients waitlisted
- ▣ Manager chooses number of patients to serve each day
- ▣ More efficient and flexible system from perspective of manager
  - Can change rate of service with minimal notice
- ▣ Inconvenient for patients
- ▣ Sometimes used in public health systems



# Assigning patients to days

## 2: Advance scheduling (we will use this)

- ▣ Patients always receive advance appointments
- ▣ Convenient for patients
- ▣ Less flexible system
  - Service commitments made in advance cannot be changed
- ▣ **Predominant paradigm in all domains**



# Model: patient classes

- Urgent: served same day
  - Total amount of resource  $r$  consumed at time  $t$  is  $\epsilon_r^t$ ,  
,  
i.i.d. over time.
- Regular: make advance appointments
  - Each patient consumes an i.i.d. amount of capacity of type  $r$   $S_r(N)$
  - $N$  patients consume amount

# Model: costs

- A unit waiting cost  $W$  per day per regular patient
- Overtime cost  $u_r(.)$  for total resource used to treat all (urgent and regular) patients each day
  - Convex increasing in amount of resource  $r$  used per day
- Discount factor  $\gamma$

# Model: events on day $t$

- Beginning schedule is obser  $x^t = (x_1^t, \dots, x_{T-t+1}^t)$ 
  - $x_i^t$  - how many people are scheduled  $i-1$  days into the future, in period  $t+i-1$
  - $x_1^t$  - how many people are scheduled for today

# Model: events on day $t$

- New demand  $\delta^t$  for regular exam arises
- New demand is scheduled into  $\{t, \dots, T\}$
- Ending schedule  $z^t = (z_1^t, \dots, z_{T-t+1}^t)$ ,
- Urgent demand is observed

# Model: events on day $t$

- Waiting costs are incurred  $W|z^t|$
- $z_1^t$  regular patients and all urgent patients are served

- Overtime costs are incurred  $\sum_{r=1}^R \mathbf{E}[u_r^t (\epsilon_r^t + S_r(z_1^t))]$ .

- Next period's starting schedule is

$$s(z^t) = (z_2^t, z_3^t, \dots, z_{T-L+1}^t)$$

# Problem formulation

- Minimize total discounted expected cost

$$V^t(\delta^t, x^t) = \min \left\{ W|z^t| + \sum_{r=1}^R \mathbf{E}[u_r^t(\epsilon_r^t + S_r(z_1^t))] + \gamma \mathbf{E}_{\delta^{t+1}} [V^{t+1}(\delta^{t+1}, \mathbf{s}(z^t))] \right\}$$

*s.t.*  $z^t \geq x^t$

$$|z^t| = \delta^t + |x^t|.$$

- Large state and decision space
- Feasible region has no nice structure





# Relation to the literature

# Same Day Scheduling

- How to sequence or time appointments within a day
  - ▣ Wang (1999), Rohleder and Klassen (2000), Denton and Gupta (2003), Robinson and Chen (2003), Klassen and Rohleder (2004), Green, Savin and Wang (2006), Kaandorp and Koole (2007), Hassin and Mendel (2008), Jouini and Benjaafar (2010), Erdogan and Denton (2011), Cayirli, Yang and Quek (2012), Luo, Kulkarni and Ziya (2012).

# Same Day Scheduling

- Controls patient's in-office wait and provider's idle time between appointments.
- Assumes number treated per day is known/fixed.
- Stage-2 decisions

# Multi-day Scheduling



- How to allocate patients to days
- Actively controls number of days that elapse until appointment

# Importance of Multi-day Scheduling

- Primary means of matching demand and capacity to cope with day-to-day variability
- Wait impacts no-shows and cancellations
- Wait can adversely affect outcome
- “Open Challenge” (Gupta and Denton 2008)

# Multi-day Scheduling

## □ Allocation scheduling

- Gerchak, Gupta and Henig (1996), Denton, Miller, Balasubramanian and Huschka (2010), Ayvaz and Huh (2010), Min and Yih (2010), and Huh, Liu and Truong (2012)
- More tractable but less used

## □ Advance scheduling

- Patrick, Puterman and Queyranne (2008): similar model but with  $N$  priority classes, one resource, and deterministic service times.
- Gocgun and Ghate (2012) develop an approximate dynamic programming method that uses Lagrangian relaxation
- Also Liu, Ziya and Kulkarni (2010), Feldman, Liu, Topaloglu and Ziya (2012)
- No known characterization of optimal policies.

# Multi-day Scheduling

- Why is advance scheduling less well understood?
  - ▣ “Most naturally modeled” as an infinite-horizon dynamic program
  - ▣ Curse of dimensionality

# Summary of Contributions

- For this 2-class model of advance scheduling
  - ▣ Exhibit reduction of the problem to a single dimension
  - ▣ Provide complete characterization of optimal policy
  - ▣ Provide easy method to compute optimal policy exactly



# Main result

There is an increasing and efficiently  
computable  $Z$ -function

such that in every period, given that there are  $n$   
patients in total, the optimal schedule is

$$z_1 = f(n),$$

$$z_2 = f(n - z_1),$$

$$z_3 = f(n - z_1 - z_2) \dots$$



# Solution methods

# Recall advance-scheduling formulation

Minimize total discounted expected cost

$$V^t(\delta^t, x^t) = \min \left\{ W|z^t| + \sum_{r=1}^R \mathbf{E}[u_r^t(\epsilon_r^t + S_r(z_1^t))] + \gamma \mathbf{E}_{\delta^{t+1}} [V^{t+1}(\delta^{t+1}, \mathbf{s}(z^t))] \right\}$$

*s.t.*  $z^t \geq x^t$

$$|z^t| = \delta^t + |x^t|.$$

# Allocation-scheduling counterpart

State is number of people remaining to be served after observation of demand.

Decision is how many people to serve today.

Same cost structure.

$$\underline{V}^t(\bar{w}^t) = \min_{\bar{w}^t \geq q^t \geq 0} \left\{ W\bar{w}^t + \sum_{r=1}^R \mathbf{E}_{\delta^{t+1}} [u_r^t(\epsilon_r^t + S_r(q^t))] + \gamma \mathbf{E} [\underline{V}^{t+1}(\bar{w}^t - q^t + \delta^{t+1})] \right\}.$$

# Allocation-scheduling properties

- Value function is convex and increasing in size of waitlist
- Value function is submodular
- It is optimal to serve  $q^*(n)$  patients in this period when the waitlist size is  $n$ 
  - $q^*(n)$  is *allocation function*
  - $q^*(n)$  is increasing in  $n$
  - $q^*(n+1) \leq q^*(n) + 1$

# Allocation-scheduling properties

- First proved by Gerchak, Gupta and Henig (1996)
- Generalized here to
  - ▣ Multiple resources
  - ▣ Non-stationary demands and capacities (will use later)

# Advance-scheduling properties

- No properties of the optimal solution are known
- What can we do?
- How to deal with the rigid constraints of advance scheduling?

# Bridging model

Let's drop the commitment constraints from advance scheduling (i.e. prior commitments do not have to be upheld)

This model doesn't make physical sense.

But perfectly valid theoretically.

$$\underline{V}^t(\delta^t, x^t) = \min \left\{ W|z^t| + \sum_{r=1}^R \mathbf{E}[u_r^t(\epsilon + S_r(z_1))] + \gamma \mathbf{E}_{\delta^{t+1}} [\underline{V}^{t+1}(\delta^{t+1}, \mathbf{s}(z^t))] \right\}$$

s.t.  $|z^t| = \delta^t + |x^t|.$

*Non-committal advance scheduling*



# Non-committal advance scheduling

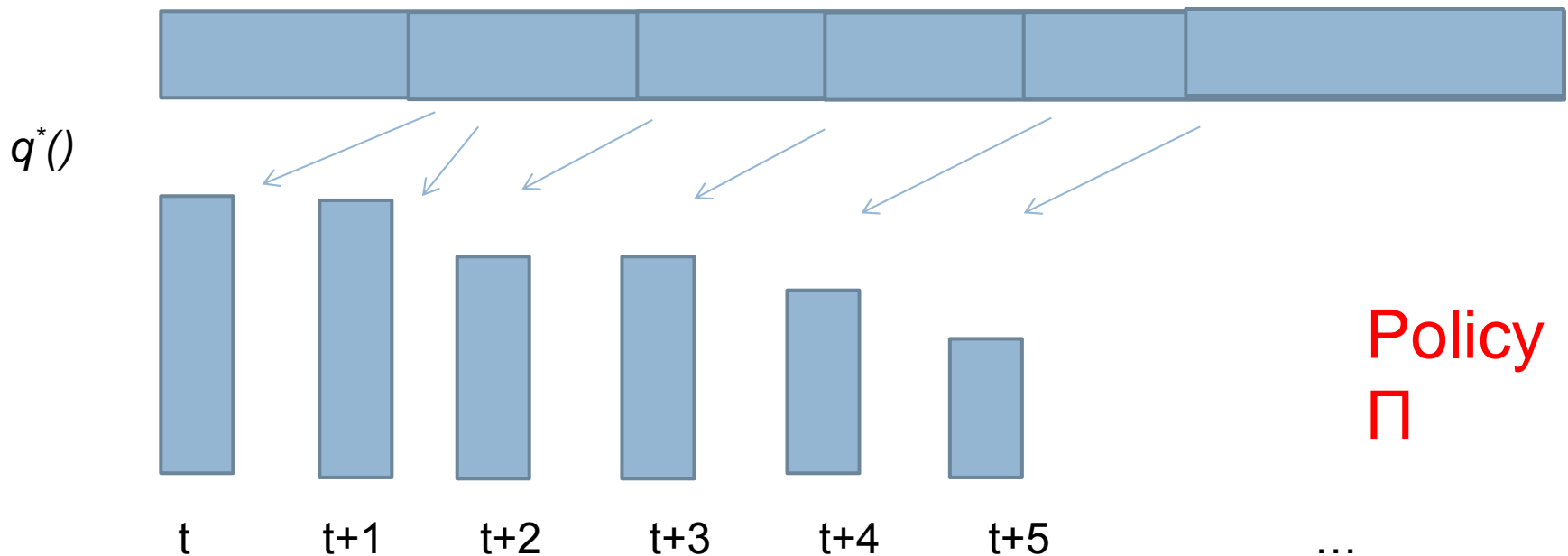
- Must be as efficient as allocation-scheduling

$$\begin{aligned} & \text{Cost ( allocation scheduling )} \\ = & \text{Cost ( non-committal advance scheduling )} \\ \leq & \text{Cost ( advance scheduling )} \end{aligned}$$

- Optimal solution must be “the same as” allocation-scheduling solution.

# Optimal schedule for non-committal advance scheduling

- Use allocation function  $q^*(\cdot)$  from allocation scheduling to construct a non-committal advance schedule

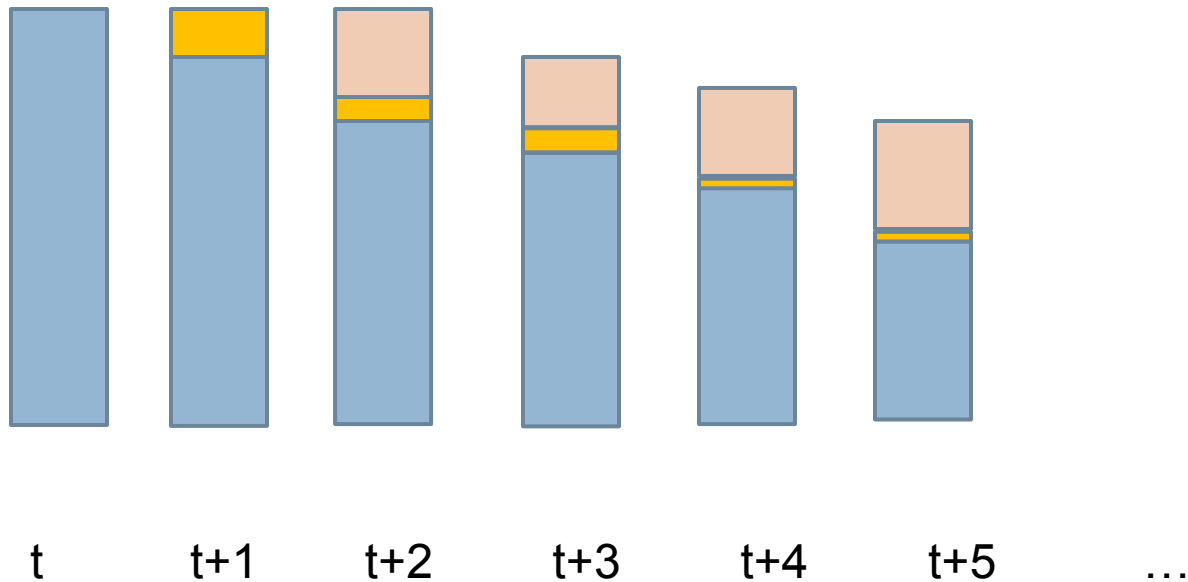


# Non-committal advance scheduling

- Properties of constructed policy  $\Pi$ 
  - $\Pi$  creates a valid schedule
    - Assigns a day to each of  $\delta^t + |x^t|$  patients
  - $\Pi$  is optimal for non-committal advance scheduling
  - $\Pi$  has the *successive refinability* property

# Non-committal advance scheduling

- *Successive refinability* of schedule



# Proof of successive refinability

$$\delta^{t+1}$$



$$q^* (\text{yellow box} + z_2^t + z_3^t + \dots )$$

$$z_1^{t+1} = \dots$$

$$x_1^{t+1}$$

$$q^* (z_2^t + z_3^t + \dots )$$

$$z_2^t$$



t+1

$$q^* (\text{yellow box} + z_3^t + z_4^t + \dots )$$

$$z_2^{t+1} = \dots$$

$$x_2^{t+1}$$

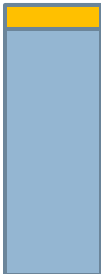
$$q^* (z_3^t + z_4^t + \dots )$$

$$z_3^t$$



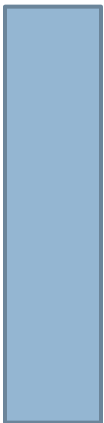
t+2

...



$$q^* (z_1^t + z_2^t + \dots )$$

$$z_1^t$$



t

# Extension to advance scheduling

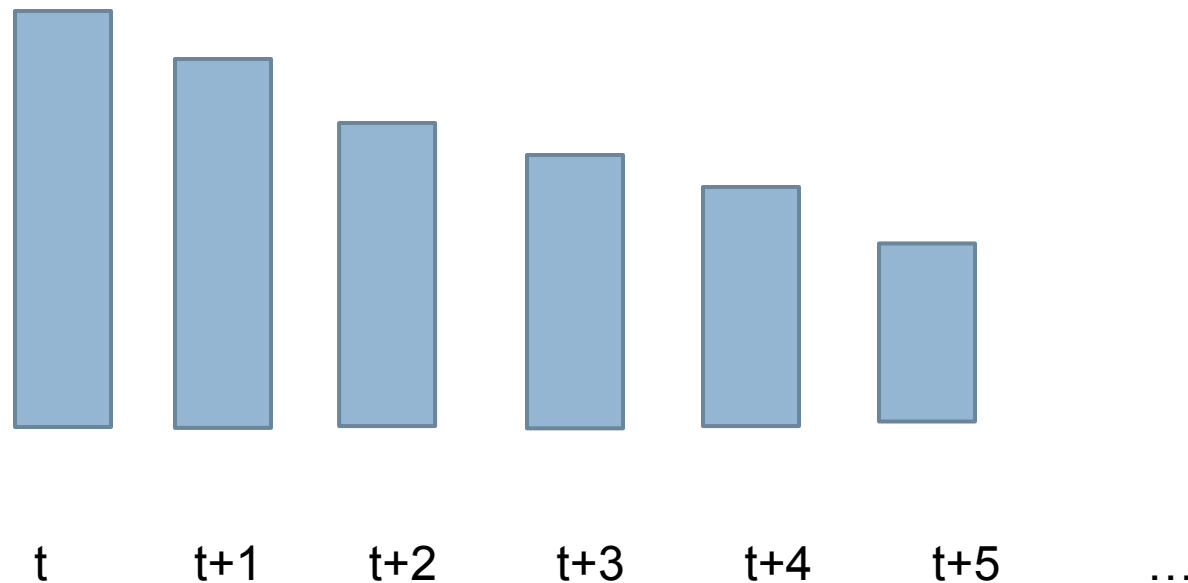
- Successive refinability ensures that optimal policy  $\Pi$  for non-committal advance scheduling is **feasible** for advance scheduling
- Commitment constraints satisfied automatically by schedules constructed under  $\Pi$ 
  - Assuming starting schedule in period 1 is 0

# Extension to advance scheduling

- Policy  $\Pi$  is **optimal** for advance scheduling
  - Assuming starting schedule in period 1 is 0

# Extension to advance scheduling

- Components of the optimal schedule are decreasing in time
  - work should be **front-loaded**





# Extension to advance scheduling

- The proof technique allows us to extend results from allocation scheduling (easy) to advance scheduling (hard)
- Successive refinability is the key property exploited
- Under what conditions does successive refinability hold?

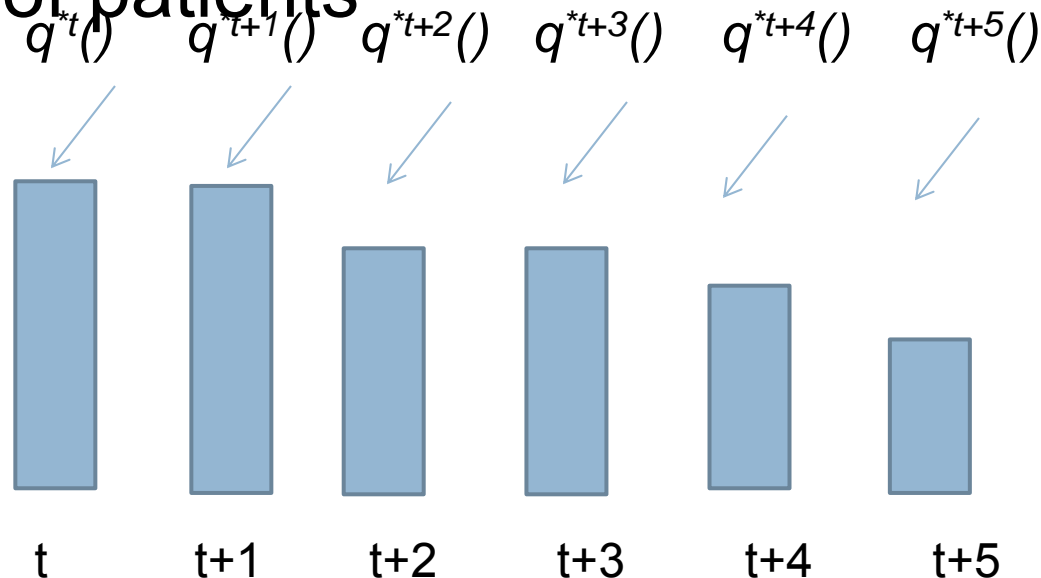
# Generalizing the model

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- Multiple resources and non-stationarity are important phenomena in healthcare scheduling not currently captured by most analytical models.

# Non-stationary demand and capacities

- The allocation function  $q^{*t}()$  from the allocation-scheduling counterpart becomes time-dependent
- Use a time-dependent allocation to exhaust the “list” of patients



# Starting from an arbitrary schedule

$x^1$

- We just showed that the problem with the 0 initial schedule is optimally solvable when capacity is allowed to be random and non-stationary
- In each period  $t$ , reduce capacity by the (random) usage of the  $x^1_t$  patients
  - ▣ Overtime cost functions  $u^t_r()$  change
  - ▣ Allocation functions  $q^{*t}()$  change

# Starting from an arbitrary schedule

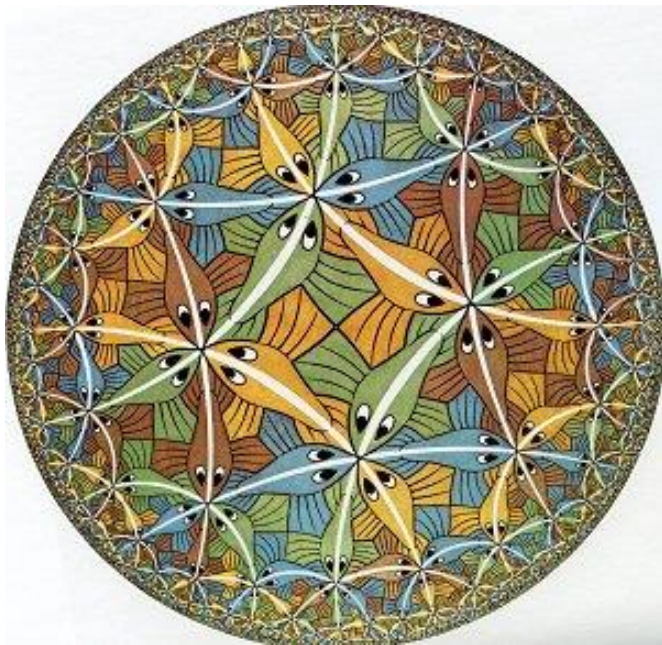
$x^1$

- Solve the problem as if starting with 0 patients
- Provides optimal solution for arbitrary start





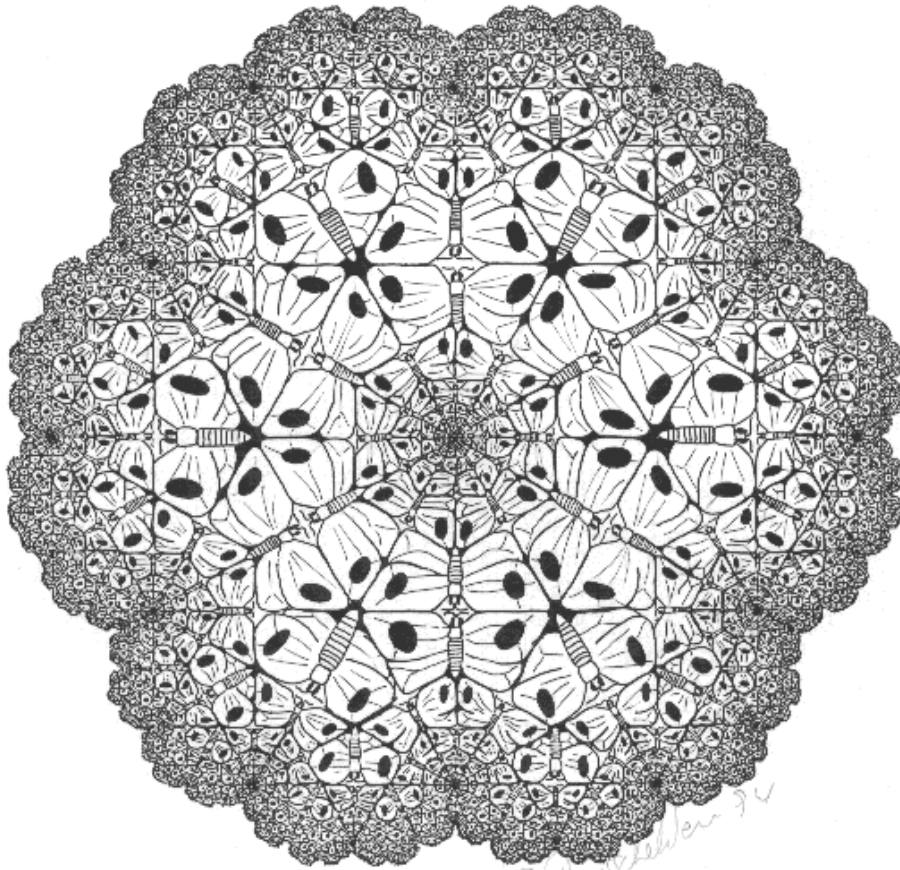
# Summary



- Intimate connection between allocation and advance scheduling
- Advance scheduling is as efficient in this case
- There is no cost to making advance commitments

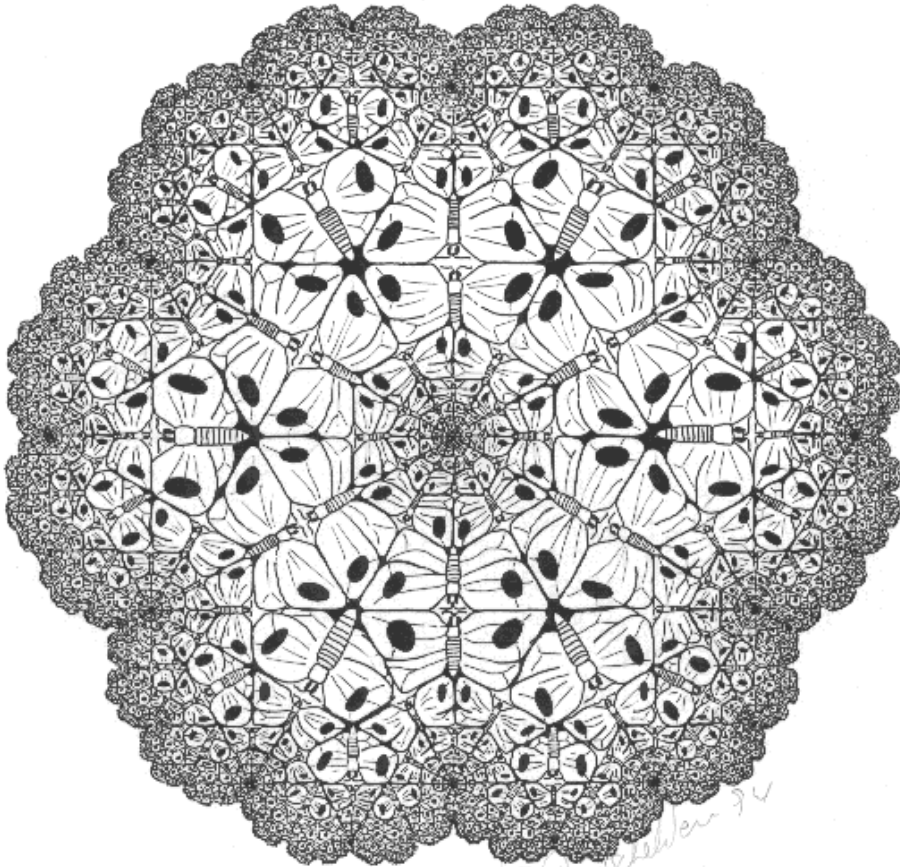


# Summary



- Introduced the property of **successive refinability**

# Future direction



- Investigate necessary and sufficient conditions for **successive refinability** in more complex settings
- Potential gateway to analysis of more general models of advance scheduling