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OPTIMAL BUFFER STOCKS AND PRECAUTIONARY SAVINGS WITH DISAPPOINTMENT AVERSION

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ABSTRACT

Developing countries use various risk reduction schemes, ranging from active management of buffer stocks and international reserves to commodity stabilization funds. The purpose of this paper is to reexamine the design of these schemes in a generalized expected utility maximization model where agents are disappointment averse. We derive first the generalized risk premium, showing that disappointment aversion increases the conventional risk premium by a term proportional to the standard deviation times the degree of disappointment aversion. Next, we show that disappointment aversion modifies the characteristics of precautionary saving. The concavity of the marginal utility continues to determine precautionary saving, but its effect is of a *second order* magnitude (proportional to the variance) compared to the *first order* effect (proportional to the standard deviation) induced by disappointment aversion. Hence, higher volatility increases the precautionary saving of a disappointment averse agent. This result applies even if the income process approaches a random walk. Finally, we reexamine the optimal size of buffer stocks, showing that disappointment aversion increases its size by a *first order* magnitude. A buffer stock that is rather small when agents are maximizing the conventional expected utility is rather large when agents are disappointment averse.

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1. Introduction and summary

Most developing countries are managing actively various stabilization funds of one sort or anther. These funds range from international reserves managed by central banks, to Commodity Stabilization Funds (CSF henceforth) and buffer stocks[1,2]. The impact of volatility on the demand for these funds has been the focus of significant theoretical research. Various studies identified rules for maximizing expected utility agents who manage stabilization funds and buffer stocks.[3] A frequent conclusion is that the gains from the optimal management of stabilization funds and buffer stocks are rather small. For example, Newbery and Stiglitz (1981, page 420) concluded that "even for large risk and low storage costs the average buffer stock is small, and that the buffer rule which would be optimal if costs are ignored often looks relatively unattractive compared with alternatives once the costs are included." Similar results have been obtained by Deaton (1991) who studied precautionary savings in a model where agents are liquidity constrained.[4]

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1 Chile, Colombia, Oman and Papua new Guinea have had stabilization funds for some time. Many oil exporting countries established some type of a fund with the additional oil income stemming from the period of the gulf war [e.g., Ecuador, Mexico, Nigeria and Venezuela]. For further details see Arrau and Claessens (1993).

2 Various studies found a well defined demand for international reserves, a demand whose functional form is similar among countries choosing different exchange rate regimes [for example, see Frenkel (1974) and Edwards (1983)].

3 See Newbery and Stiglitz (1981), Deaton (1991), Basch and Engle (1993), Hausmann, Powell and Rigobon (1993).

4 Deaton's contribution evaluated the saving behavior of impatient, liquidity constrained agents. He showed that if the agent's income is stationary, i.i.d. distributed over time, the equilibrium path is characterized by a low average level of assets, which accomplishes significant smoothing. If the income is characterized by a positively auto correlated process, the role of assets

The practice of various developing countries suggests that most manage a sizable stock of international reserves, and some manage large CSFs. An example of a stabilization fund that is frequently praised as a success story is the Chilean Copper Stabilization Fund. Applying Deaton's framework, Arrau and Claessens (1993) found that the optimal size of the Chilean fund is very small (about US$ 70 million, roughly 20% of copper exports).[5] In fact, the size of the *actual* fund is much larger [more than US$ 1.8 billion that year].[6] The possible merits of a stabilization fund are of obvious concern to countries which specialize in exporting few primary products. In these circumstances terms of trade volatility translate into volatility of fiscal revenue.[7]

The purpose of this paper is to show that a generalized-expected utility maximization framework may lead to a much larger optimal size of stabilization funds and precautionary savings than the size predicted if agents are maximizing expected utility. We show that disappointment

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as buffer stock will decline the higher the auto correlation is, and will practically disappear when the income process approaches a random walk.

5 However, their results show that even their small fund could have had a considerable effect on expenditure volatility (reducing the standard deviation of expenditure by about 40%).

6 The Chilean CSF operated since 1987 to dampen fluctuations in fiscal expenditure arising from fluctuation in copper prices [copper accounts for about 30 percent of Chile's export]. It defines a reference price for copper linked directly to a six years moving average of the spot prices. If the realized copper price exceeds (falls short) of the reference price plus (minus) a threshold, the extra revenue is deposited (withdrawn) from the fund. This fund proved useful during turbulent periods such as the "poisoned grapes" period where Chilean exports to the US were severely restricted and exporters were compensated using US$ 363 million drawn on the Chilean CSF account. Further discussion on rules for shock management in Latin America see Hausmann and Gaivn (1995).

7 For example, estimates of the fiscal impact of the oil shocks in Venezuela in the 1970's were of the order of 11% of the GDP, and of about 5% of the GDP in Chile in the 1980's.

aversion increases precautionary saving by a first order magnitude, and this result applies even if income follows a random walk process. Specifically, we investigate the impact of volatility on the optimal buffer stock and on precautionary saving in a disappointment aversion utility framework, as postulated by Gul (1991). His approach provides an axiomatic extension of the expected utility approach to account for the Allias paradox. This is done by modeling an agent who maximizes a weighted sum of utility, where the weights deviate from the probabilities so as to reflect disappointment aversion. Gul's specification is a convenient framework as it encompasses the expected utility approach as a special case, allowing one to analyze the impact of disappointment aversion in comparison to the case where agents are maximizing a conventional expected utility.

The interest in this approach stems from several observations. First, there is a significant body of controlled experiments that supports the need to model behavior using the generalized expected utility framework.[8] Second, the results of the generalized-expected utility approach are in contrast with the predictions of the neoclassical approach, where the impact of volatility on precautionary saving is found to be ambiguous, and typically of a second order magnitude [i.e., proportional to the variance].[9]

After a brief overview of Gul's framework, we derive the risk premium for a disappointment averse agent. We point out that while volatility induces second order effects on the risk premium if agents are maximizing expected utility, volatility induces first order effects on the risk premium when agents are disappointment averse. This outcome has a direct bearing on all the

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8 See Epstein (1992), Harless and Camerer (1994) and Hey and Orme (1994) for a useful assessment and further references. In concluding their paper Harless and Camerer (1992) pointed out that "The pairwise-choice studies suggest that violations of expected utility are robust enough that modeling of aggregate economic behavior based on alternatives to expected utility is well worth exploring" (page 1286).

9 Further discussion on volatility in a non-expected utility framework can be found in Segal and Spivak (1990) and Epstein (1992).

analytical results characterizing demands for assets as a function of the risk premium. We illustrate this point by extending a model advanced by Newbery and Stiglitz (1981) who characterized the optimal buffer stock for a risk averse agent. Our analysis extends this model for the case where agents are disappointment averse, showing that a stabilization fund that is rather small when agents are maximizing the conventional expected utility, turns out to be rather large when agents are disappointment averse.

The issue of the optimal CSF has many other aspects that are ignored by our paper. For example, in weak political systems a large CSF may be unfeasible due to rent seeking activities -- any liquid funds may be expropriated by pressure groups maximizing their narrow agenda.[10] On the other hand, significant adjustment costs to changes in government consumption would increase the optimal size of the CSF.[11] Hence, the issue of the usefulness of CSF hinges upon various considerations that were not dealt with in this paper. The purpose of this paper is not to claim that CSF are always welfare enhancing. Instead, we illustrate that results that hinge upon the size of the risk premium in an expected utility maximization model should be reexamined in light of the generalized expected utility maximization literature. Various predictions of previous frameworks may be modified substantially, even before adding factors like adjustment costs and political economy considerations.

We close the paper with a reexamination of precautionary savings. As was shown by Leland (1968) and Sandmo (1970), higher volatility of future income has an ambiguous effect on precautionary saving, an effect the sign of which hinges on the concavity of the marginal utility. We point out that their results are significantly modified when agents are disappointment averse. While the concavity of the marginal utility has the same effect as in the above studies, its magnitude is of a *second order*, whereas disappointment aversion has a *first order* effect on precautionary saving. For a disappointment averse agent, higher volatility will increase precautionary saving in

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10 See Tornell and Lane (1994) and Aizenman and Powell (1995).

11 See Basch and Engle (1993) and Hausmann, Powell and Rigobon (1993).

proportion to the standard deviation. Hence, for a disappointment averse agent higher volatility increases the demand for saving independently from the concavity of marginal utility. This result applies even if the income process follows a random walk. The rationale is that with a random walk process the future income is distributed symmetrically around the present income. In these circumstances a disappointment averse agent will have a much greater incentive to hold assets as buffer stock compared to a disappointment neutral agent. The first treats the future asymmetrically, assigning a greater weight to bad states of nature. A greater coefficient of disappointment aversion increases the gap between the weights attached to the bad versus the good states, increasing thereby the gains from the buffer stock. Hence, even if the income process approaches a random walk, a disappointment averse agent still has a powerful incentive to save. In contrast, a disappointment neutral agent treats the future states of nature symmetrically. The only reason to save stems from the curvature of the marginal utility, which generates only a second order magnitude of saving.

Section 2 reviews the concept of disappointment aversion agents. Section 3 applies it to characterize an optimal buffer stock for a disappointment averse agent. Section 4 evaluates the impact of disappointment aversion on precautionary saving, and section 5 closes with concluding remarks.

2. Disappointment aversion utility

The preferences of a disappointment aversion agent may be summarized by [u(x), β], where u is a conventional utility function describing the utility of consuming x , [u' > 0, u" < 0], and β ≥ 0 is a number that measures the degree of disappointment aversion.[12] The disappointment adverse expected utility is defined implicitly, by describing its key features. In the absence of risk, the agent's utility level is simply u(x). Suppose that our consumer faces risky income $\{x_s\}$ in *n* states of nature, s = 1, ...n . Let us denote by $V(\beta;\{x_s\})$ the expected utility of a disappointment averse agent (whose disappointment aversion rate is β) . Let μ denote the certain income that yields the same utility level as the risky income: $V(\beta;\{x_s\})= u(\mu)$.[13] Our consumer is revealing disappointment aversion if he/she attaches extra disutility to circumstances where the realized income is below μ . A convenient way to define $V(\beta;\{x_s\})$ is

$$(1)\ V(\beta;\{x_s\}) = E(u(x)) - \beta E[u(\mu) - u(x) \mid \mu > x]$$

where *E* is the expectation operator, $E[u(\mu) - u(x) \mid \mu > x]$ is the expected value of $u(\mu) - u(x)$, conditional on the realized consumption being below the certainty equivalent consumption. The term $E[u(\mu) - u(x) \mid \mu > x]$ measures the average "disappointment," being defined by the expected difference between the certainty equivalence utility and the actual utility u in states of nature where the realized income is below the certainty equivalence income. The disappointment averse expected utility equals the conventional expected utility, adjusted downwards by a measure of disappointment aversion (β) times the "expected disappointment."

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12 Gul (1991) considered the more general case, where β ≥ -1 and u may be both convex and concave. We focus on the case where β ≥ 0 because we restrict our attention to u" < 0.

13 I.e., the consumer is indifferent between the prospect of a safe income μ and risky income $\{x_s\}$ in n states of nature (s = 1, ...n).

We restrict our attention first to the simplest example -- of two states of nature. Suppose that the agent will receive income x_i in state i (i = 1, 2), where $x_1 > x_2$, with probabilities (α, 1- α), respectively. Applying (1), the disappointment averse expected utility is defined by:

$$(2) \qquad V(\beta) = \alpha u(x_1) + (1-\alpha)u(x_2) - \beta(1-\alpha)[V(\beta) - u(x_2)],$$

where for notation simplicity we henceforth use $V(\beta)$ for $V(\beta;\{x_s\})$. Thus,

$$(2') \qquad V(\beta) = \frac{\alpha}{1+(1-\alpha)\beta}u(x_1) + \frac{(1-\alpha)(1+\beta)}{1+(1-\alpha)\beta}u(x_2)$$

Note that for $\beta = 0$, V is identical to the conventional expected utility.[14] We turn now to identify the risk premium in the presence of disappointment aversion. Suppose that a disappointment averse agent described above faces income $[Y+\varepsilon, Y-\varepsilon]$, and let $\alpha = 0.5$. Define the risk premium τ by:

$$u(Y-\tau) = \frac{0.5}{1+0.5\beta}u(Y+\varepsilon) + \frac{0.5(1+\beta)}{1+0.5\beta}u(Y-\varepsilon)$$

Applying a second order Taylor approximation leads to

$$(3) \qquad \frac{\tau}{Y} \approx \frac{0.5\beta}{1+0.5\beta}\sigma_y + 0.5R[\sigma_y]^2.$$

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14 An alternative way of writing the disappointment averse expected utility is $V(\beta) = \alpha[1 - \frac{(1-\alpha)\beta}{1+(1-\alpha)\beta}]u(x_1) + (1-\alpha)[1 + \frac{\alpha\beta}{1+(1-\alpha)\beta}]u(x_2)$. If the agent is disappointment averse ($\beta > 0$), he attaches extra weight to "bad" states where he would be disappointed (relative to the probability weight used in the conventional utility), and attaches a lighter weight to "good" states.

where R, σ_y are the coefficient of relative risk aversion and the coefficient of variation of income, respectively: $R = -Y\frac{u''}{u'}$, $\sigma_y = \frac{\varepsilon}{Y}$. Note that the risk premium increases with the degree of disappointment aversion times the coefficient of variation. Furthermore, if $\beta > 0$ then the disappointment aversion may dominate the determination of the risk premium, as the relative risk aversion R is playing only a secondary role [I.e., the impact of R is proportional to the variance, whereas the impact of β is proportional to the standard deviation]. Hence, the addition of disappointment aversion may modify substantially all the results that hinge on calculations involving risk premium. To better appreciate this observation, we turn now to examine how disappointment aversion affects the demand for stabilization funds.

3. Optimal buffer stock and disappointment aversion

There exists a large literature dealing with optimal buffer stock and CSF rules, with varying degrees of complexity of assumptions regarding preferences, budget constraints, and stochastic processes. To illustrate our point we focus on one of the simpler example advanced by Newbery and Stiglitz (1981, pp. 415-420) [NS henceforth]. This allows us to exemplify the implications of disappointment aversion on buffer stocks and international reserves in a rather tractable way. Similar points can be made in the context of more complex models.

Consider the case where there are equally likely two states of the world: high and low real output (h):

$$(4) \qquad h = 1 \pm \sigma, \sigma \geq 0$$

The consumer's utility function is

$$(5) \qquad u(c) = \begin{cases} \dfrac{c^{1-R}}{1-R}, & R \neq 1, R \geq 0 \\ \\ \log c, & R = 1 \end{cases}$$

For simplicity of exposition, we ignore discounting and consider a simple storage rule: the goal is to stabilize consumption around the mean real income (*E(h)* = 1). In good years agents put into storage σ, as long as the storage capacity has not been reached. In bad years agents take away from the storage σ, as long as the storage is not empty.[15] Suppose that the capacity of the storage is K, and to simplify assume that K is an integer, and define a 'storage unit' as σ units of output. Hence, there are K+1 possible levels of stock [0, 1, ... K units of storage]. Let π_V be the probability that the stock contains exactly *V* units. NS showed that in these circumstances[16]

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15 This rule can be showed to be optimal if the storage cost is zero.

16 This follows from the fact that if the stock at time t is V, it was either V - 1 or V +1 at time t - 1 with equal probabilities (unless V = 0 or V = K). Hence, $\pi_0 = 0.5[\pi_1 + \pi_0]$,

(6) $$\pi_V = \frac{1}{K+1}$$

Hence, there is a uniform distribution of amounts of storage, and consumption is stabilized at level 1 in *K/[K+1]* of the time. It is $1 + \sigma$ in 0.5/[K+1] of the time in cases where the storage is full and there is a good state of nature, and $1 - \sigma$ in 0.5/[K+1] of the time in cases where the storage is empty and there is a bad state of nature. Hence, the expected utility with buffer stock (in the absence of discounting) is

(7) $$V_b = \frac{K}{K+1}u(1) + \frac{u(1-\sigma)+u(1+\sigma)}{2(K+1)}.$$

where index b indicates an active buffer stock. In the absence of a buffer stock, the expected utility is

(8) $$V_n(0) = \frac{u(1-\sigma)+u(1+\sigma)}{2}.$$

where index n indicates no active buffer stock. Applying a second order approximation the expected benefit from the buffer stock is proportional to the conventional risk premium:

(9) $$B = \frac{V_b(0) - V_n(0)}{u'(1)} \approx \frac{R\sigma^2 K}{2(K+1)}.$$

Let the storage cost per period per unit of storage (inclusive of capital cost) be s, and let ξ be the cost of maintaining a storage capacity of a unit. The total expected cost of running the buffer stock (per period) is

(10) $$[0.5s + \xi]K\sigma$$

$\pi_V = 0.5[\pi_{V+1} + \pi_{V-1}]$ for $1 \le V < K-1$, and $\pi_K = 0.5[\pi_K + \pi_{K-1}]$. Solving these K+1 equations we infer (6).

The optimal buffer stock capacity K^* is obtained by maximizing the expected net cost of operating the buffer stock.

$$(11) \quad \underset{K}{Max}\left\{B - [0.5s + \xi]K\sigma\right\}$$

Leading to

$$(12) \quad K^* \approx \sqrt{\frac{R\sigma}{s + 2\xi}} - 1$$

For example, if R=1, $\sigma = 0.5$, $s + 2\xi = 0.1$, the optimal storage is K* = 1, the net benefit from the storage is 3.75%, and income is stabilized in half of the time (these numbers were used by NS).

These results have lead NS to conclude that "even for large risk and low storage costs the average buffer stock is small, and that the buffer rule which would be optimal if costs are ignored often looks relatively unattractive compared with alternatives once the costs are included."[17]

We turn now to a recalculation of the optimal storage for the case where agents are disappointment averse. Specifically , suppose that our agent is characterized by the utility specified in (1). This does not change the operation of the buffer stock, as characterized by (6), but it modifies the evaluation of the expected benefit. The disappointment averse expected utility in the absence of stabilization is:

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17 One can apply NS methodology to get similar welfare inferences using Deaton's results. For example, if the income process is i.i.d. with a standard deviation of 10%, the optimal application of precautionary saving by an impatient, liquidity constrained agent reduces the consumption standard deviation to 5% for a consumer whose coefficient of risk aversion is R = 2 [see Deaton (1991, page 1234)]. The resultant consumption smoothing can be translated into an income gain of about 0.75% [i.e., the consumer is willing to sacrifice up to .75% of his average income for the option to smooth consumption optimally].

$$(13) \qquad V_n(\beta) = \frac{0.5}{1+0.5\beta}\left[u(1+\sigma)+(1+\beta)u(1-\sigma)\right].$$

Applying (1) the disappointment aversion expected utility (with active stabilization) is

$$(14) \qquad V_b(\beta) = \frac{K}{K+1}u(1) + \frac{u(1-\sigma)+u(1+\sigma)}{2(K+1)} - \beta\frac{0.5}{K+1}\left\{V_b(\beta)-u(1-\sigma)\right\}$$

Alternatively,

$$(15) \qquad V_b(\beta) = \frac{Ku(1)+0.5u(1+\sigma)+(1+\beta)0.5u(1-\sigma)}{K+1+0.5\beta}$$

Applying a second order Taylor expression we obtain that

$$(16) \qquad B(\beta) = \frac{V_b(\beta)-V_n(\beta)}{u'(1)} \approx \frac{K}{K+1+0.5\beta}\left[0.5R\sigma^2 + \frac{0.5\beta}{1+0.5\beta}\sigma\right].$$

A comparison of (3) and (16) reveals that the benefit from the buffer stock is proportional to the modified risk premium, which in turn depends linearly on the degree of disappointment aversion times the standard deviation of the shock. The optimal buffer stock is obtained by:

$$(17) \qquad \underset{K}{Max}\left\{B-\left[0.5s+\xi\right]K\sigma\right\}$$

Leading to

$$(18) \qquad K^* \approx \sqrt{\frac{(1+0.5\beta)R\sigma+\beta}{s+2\xi}} - (1+0.5\beta).$$

Figure 1 (A) summarizes the dependency of the buffer stock (K) and the average buffer stock (Kσ) capacity on the disappointment aversion. The NS results are obtained for β = 0 (the parameters in the simulation are the ones that NS used in their base case). Note that disappointment aversion has first order effects on the buffer stock. For example, if $\beta = 2$, then the optimal storage capacity is K = 4 [keeping in mind the integer restriction]. In this case we would observe that the average buffer stock is a two years output, and that consumption is stabilized in 4/5 the time (whereas it is stabilized only in half of the time if $\beta = 0$, as was the case considered by NS). Figure 1 (B) plots the gain from the buffer stock, as a fraction of the annual output, revealing a gain of 15% of the GDP if $\beta = 2$ (and only 3.75% if $\beta = 0$). Both results reveal that disappointment aversion induces first order effects, changing significantly the predictions of the model.

In general, it is both the degree of disappointment aversion (measured by β) and the curvature of u (measured by R) that determine the ultimate use and the gains from buffer stocks. While the two aversions interact, the disappointment aversion has a robust role that is independent from the role of risk aversion. To better appreciate this point, we focus in Figure 2 on the case where the coefficient of risk aversion is rather small (R = 0.25). Note that in the absence of disappointment aversion, the use of buffer stocks and their welfare implications are minimal. This changes drastically with disappointment aversion, which induces significant welfare gains from using a sizable buffer stock. Thus, from the two aversions, it is the disappointment aversion that has the more decisive role in explaining the demand for buffer stocks.

The logic of our discussion is applicable beyond the buffer stock issue. It suggests that welfare calculations and demand for assets that depend on risk premiums should be adjusted if the agent is disappointment averse. The needed adjustment is proportional to the disappointment aversion times the standard deviation. This adjustment may change significantly previous results, as its impact is of a first order magnitude [compared to a second order magnitude of the conventional risk premium]. We close our discussion with another illustration of this point, identifying the impact of volatility on precautionary saving.

4. Disappointment aversion and precautionary saving

Consider the problem of an agent who lives two periods. His first and second period budget constraints are

$$\text{(19)} \qquad \begin{aligned} c_1 &= x_1 - S \\ c_2 &= x_2 + S(1+r) \end{aligned}$$

where c_i, x_i denote the consumption and endowment in period i (i = 1, 2), S is the saving, and r is the real interest rate. The agent determines saving by maximizing an intertemporal time additive utility:

$$\text{(20)} \qquad U = u(c_1) + \frac{V_2(\beta)}{1+\rho}$$

where $V_2(\beta)$ is the disappointment averse second period utility, defined by equations (1)-(2). Suppose that the second period income rate may be either high or low, with equal probabilities:

$$\text{(21)} \qquad x_2 = \begin{cases} \bar{x} + \sigma & \textit{prob.}\ 0.5 \\ \bar{x} - \sigma & \textit{prob.}\ 0.5 \end{cases}$$

for notation convinces we assume that the real interest rate equals the rate of time preferences. Applying (2') the generalized expected utility is:

$$\text{(22)} \qquad U = u(x_1 - S) + 0.5\frac{u[\bar{x} + S(1+\rho) + \sigma] + (1+\beta)u[\bar{x} + S(1+\rho) - \sigma]}{(1+\rho)(1+0.5\beta)}$$

The first order condition characterizing saving is

$$(23) \qquad u'(x_1 - S) = 0.5\frac{u'[\bar{x} + S(1+\rho) + \sigma] + (1+\beta)u'[\bar{x} + S(1+\rho) - \sigma]}{1 + 0.5\beta}$$

We identify the impact of volatility by a second order expansion of (23) around $\sigma = 0$. It can be shown that

$$(24) \qquad dS = \frac{\frac{0.5\beta}{1+0.5\beta}\sigma + 0.5\left[\frac{u'''[\bar{x} + S(1+\rho)]}{u'[\bar{x} + S(1+\rho) + \sigma]}\right]\sigma^2}{A_1 + (1+r)A_2}$$

where A_i is the coefficient of absolute risk aversion at time i [i.e., $A_i = -\frac{u_i''}{u_i'} > 0$].

In the absence of disappointment aversion, (24) states the results advanced by Leland (1968) and Sandmo (1970) -- the impact of volatility on precautionary saving is determined by the concavity of the marginal utility of consumption, and its sign is ambiguous for a general utility. If the agent is disappointment averse, however, the concavity of the marginal utility is playing only a secondary role relative to the role of the disappointment aversion (as the first term is proportional to the standard deviation, and the second to the variance). Thus, volatility tends to induce a first order positive effect on precautionary saving independently of the concavity pattern of the marginal utility. Figure 3 plots a simulation of precautionary saving for the case where R = 1.5. Note that disappointment aversion magnifies precautionary saving, which is of secondary importance in the absence of disappointment aversion. Figure 4 illustrates this point for a quadratic utility function. While the precautionary demand for a quadratic utility is zero in the absence of disappointment aversion, it is of a first order magnitude in the presence of disappointment aversion.

We close our discussion with a reexamination of one implication of Deaton (1991), who considered an impatient, liquidity constrained agent. Deaton's contribution showed that the role of assets as a buffer stock practically disappears when the income process approaches a random walk. A way to gain insight regarding the impact of disappointment aversion on a liquidity constrained agent is to specialize the model of this section for the case where the income follows an AR(1) process. Suppose that

$$(25) \quad x_1 = \bar{x} + v_1, \qquad x_2 = \begin{cases} \bar{x} + \phi v_1 + \sigma & \textit{prob.}\ 0.5 \\ \bar{x} + \phi v_1 - \sigma & \textit{prob.}\ 0.5 \end{cases}$$

where v_1 is the income shock affecting period 1, ϕ is the autocorrelation of income shocks, and σ is the second period income shock. The case where $\phi = 0$ corresponds to i.i.d. disturbances, and the case where $\phi = 1$ corresponds to a random walk process. Suppose that u is a CRRA utility, as specified in (5). As in Deaton, the consumer is impatient -- $r < \rho$. Figure 5 plots the dependency of the saving rate on the autocorrelation, for various coefficients of disappointment aversion. While the simulation allows borrowing, for a liquidity constrained consumer the realized saving rate is bounded below by zero. The simulation considers the case where the income shocks in both periods are ±10% of the initial income, $\bar{x}$, and is drawn for $R = 2$, $r = 0.02$, $\rho = 0.05$, $\sigma = 0.1$. Figure 5A (5B) corresponds to the case of an adverse (favorable) income shock in period 1. Notice that for a disappointment neutral agent (whose $\beta = 0$) saving is close to zero when the income process approaches a random walk, as is predicted by Deaton's analysis.[18] The saving rate is positive and significant, however, if the agent is disappointment averse. The rationale is that with a random walk process the future income is distributed symmetrically around the present

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18 The saving rate of a liquidity constrained consumer whose $\beta = 0$ is zero if $v = 0.1$, and 0.2% if $v = -0.1$. The saving rate of a liquidity constrained consumer whose $\beta = 10$ is 3.2% if $v = 0.1$, and 4.1% if $v = -0.1$, respectively.

income. In these circumstances a disappointment averse agent will have a much greater incentive to hold assets as buffer stock compared to a disappointment neutral agent. The first treats the future asymmetrically, assigning a greater weight to bad states of nature. A greater coefficient of disappointment aversion increases the gap between the weights attached to the bad versus the good states, increasing thereby the gains from the buffer stock. Hence, even if the income process approaches a random walk, a disappointment averse agent still has a powerful incentive to save. In contrast, a disappointment neutral agent treats the future states of nature symmetrically. The only reason to save stems from the curvature of the marginal utility, which generates only a second order magnitude of saving.

Deaton's analysis characterized the dynamic evolution of saving for a liquidity constrained consumer whose horizon is unbounded. Our example considered a much simpler set-up, where the consumer horizon is only two periods. Thus, one should be cautious in interpreting the implications of our results for a consumer whose horizon is long. Nevertheless, several conclusions emerge. Disappointment aversion modifies Deaton's results in two ways. First, the welfare gains from precautionary savings are much higher when the agent is disappointment averse [alternatively, the losses from not following the optimal saving path are much greater]. Second, as disappointment aversion increases the cost of volatility, a disappointment averse agent will exhibit a significantly greater demand for precautionary saving.

5. Concluding remarks

In concluding the paper it is useful to put it in the context of recent literature. Disappointment aversion, while convenient, is only one of the many extensions of expected utility. Some of the other extensions have been already applied in Macroeconomics. For example, Schmeidler (1989) and Gilboa (1987) modeled a non additive subjective probability framework to account for rational decision making under Knightian uncertainty.[19] Other generalizations of expected utility include recursive utility, where the timing of risk resolution matters, along the lines of Kreps and Porteus (1978).[20]

The debate regarding the merits of generalized expected utility is not over, and various studies question the usefulness of going beyond the expected utility paradigm. For example, Hey and Orme (1994) close their discussion on various generalized expected utility approaches stating "...we are tempted to conclude by saying that our study indicates that behavior can be reasonably well modeled (to what might be termed a 'reasonable approximation') as 'expected utility plus noise' " (page 1322). Our paper points out that this "noise" has first order effects, and consequently the choice of the framework has important bearings on practical policy questions.

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19 This approach was applied by Epstein and Wang (1994) and Dow and Werlang (1992), who showed that excess volatility of the type reported by Shiller is consistent with equilibrium outcomes under Knightian Uncertainty, and by Aizenman (1995) who showed that uncertainty may inhibit growth and may lead to first order costs.

20 Versions of this approach were applied by Epstin and Zin (1991) and Bufman and Leiderman (1990) to explain macro consumption and asset price data.

References

Aizenman, J., 1995, Investment In New Activities and the Welfare Cost of Uncertainty, forthcoming, *Journal of Development Economics*.

Aizenman J. and A. Powell, 1995, The political economy of saving behavior and the role of the international financial institutions, presented at the CEPR workshop on Political Economy, Sovereign Debt and the IMF, Cambridge, UK.

Arrau, P. and S. Claessens, 1992, "Commodity Stabilization Funds," World Bank Policy Research Working Paper, WPS 835.

Basch, M. and E. Engel, 1993, 'Temporary shocks and stabilization mechanisms : the Chilean case' in Engel and Meller, 1993.

Bufman, G. and Leiderman, 1990, Consumption and assets returns under non-expected utility, *Economic Letters*, 34, pp. 231-62.

Deaton, A., 1991, Saving with liquidity constraints, *Econometrica*, 59, 5, 1221-1248.

Dixit, A. K. and R. S. Pindyck, 1994, *Investment Under Uncertainty* (MIT Press).

Dow, J. and Werlang Sergio Ribeiro de Costa, 1992, Uncertainty aversion, risk aversion, and the optimal choice of portfolio, *Econometrica* , Vol. 62, 197-204.

Edwards, S., 1983, The demand for international reserves and exchange rate adjustments: the case of LDCs, 1964-1972, *Economica*, Vol. 50, 269-280.

Engel, E. and P. Meller eds., 1993, "External shocks and stabilization mechanisms," Johns Hopkins Press Washington DC.

Epstein, L. G., 1992, Behavior under risk: recent developments in theory and applications, Chapter 1 in *Advances in economic theory: Sixth World Congress.*, Volume 1, Laffont-Jean-Jacques, ed. (Cambridge University Press), 1-63.

Epstein, L. G. and T. Wang., 1994, Intertemporal asset pricing under Knightian uncertainty, *Econometrica*, Vol. 62, 283 - 322.

Epstein, L.G. and S. E. Zin, 1991, "Substitution, risk aversion and the temporal behavior of consumption and assets returns: an empirical analysis," *Journal of political economy,* 99, 263-86.

Frenkel, J., 1974, The demand for international reserves by developed and less developed countries, *Economica*, Vol. 41, 14-24.

Gilboa, Itzhak, 1987, Expected utility theory with purely subjective non-additive probabilities, *Journal of Mathematical Economics* Vol. 16, 65-88.

Gul, F. 1991, A theory of disappointment aversion, *Econometrica*, Vol. 59, pp. 667-686.

Harless D. W. and C. Camerer, 1994, The predictive utility of generalized expected utility, *Econometrica*, Vol. 62, 1251-1290.

Hausmann, R. and M. Gavin, 1995, Macroeconomic Volatility in Latin America, manuscript, Inter-America Development Bank.

Hausmann, R.; A. Powell and R. Rigob6n. 1992, An optimal spending rule facing oil income uncertainty (Venezuela), in *External Shocks and Stabilization Mechanisms,* (E. Engel and P. Meller, eds.), IDB, Washington.

Hey, J. D. and C. Orme. 1994, The predictive utility of generalized expected utility theories, *Econometrica*, Vol. 62, pp. 1251-1289.

Kreps, D. M. and E. L. Porteus, 1978, Temporal resolution of uncertainty and dynamic choice theory," *Econometrica*, 46, 185-200.

Leland, H., E, 1968, Saving and Uncertainty: the Precautionary Demand for Saving, *Quarterly Journal of Economics*, pp. 465-473.

Newbery, D. and J. Stiglitz. (1981) *The Theory of Commodity Price Stabilization, A Study of the Economics of Risk,* Oxford, Claredon Press, 1981.

Sandmo, A., 1970, The effect of uncertainty on saving decisions, *Review of Economic Studies*, pp. 353-360.

Schmeidler, D., 1989, Subjective probability and expected utility without additivity, *Econometrica*, 57, 571-587.

Segal, U. and A. Spivak, 1990, First-Order versus Second-order risk aversion, *Journal of Economic Theory*, Vol. 51, 111-25.

Tornell A. and P. Lane, 1994, When good news is bad news: a non-representative model of the current account and fiscal policy, manuscript, Harvard University.

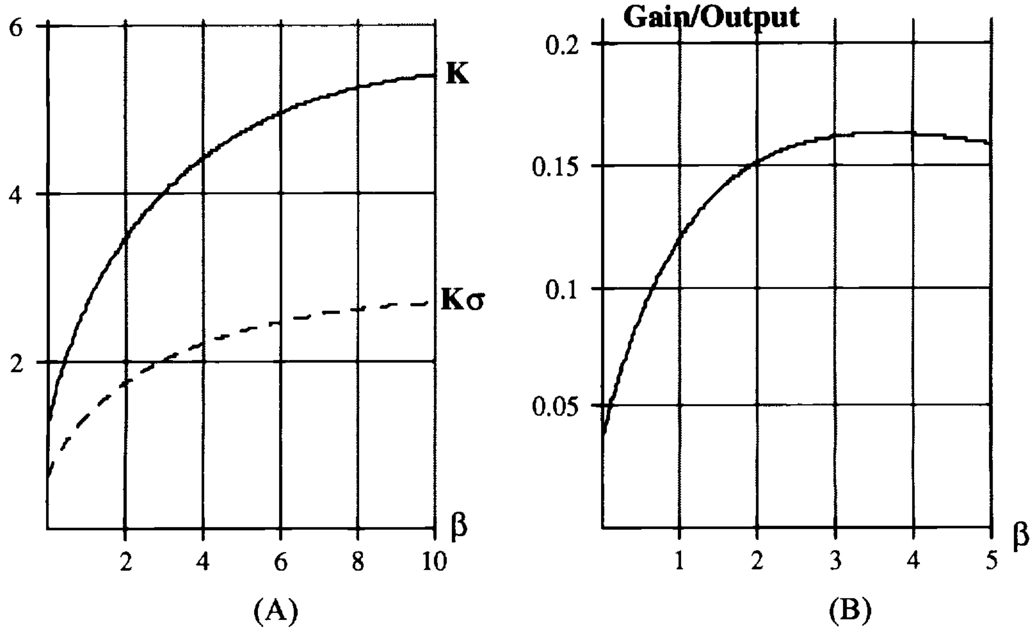


Figure 1
Disappointment aversion buffer stocks (drawn for $\sigma = 0.5$, $R = 1$, and $s + 2\xi = 0.1$)

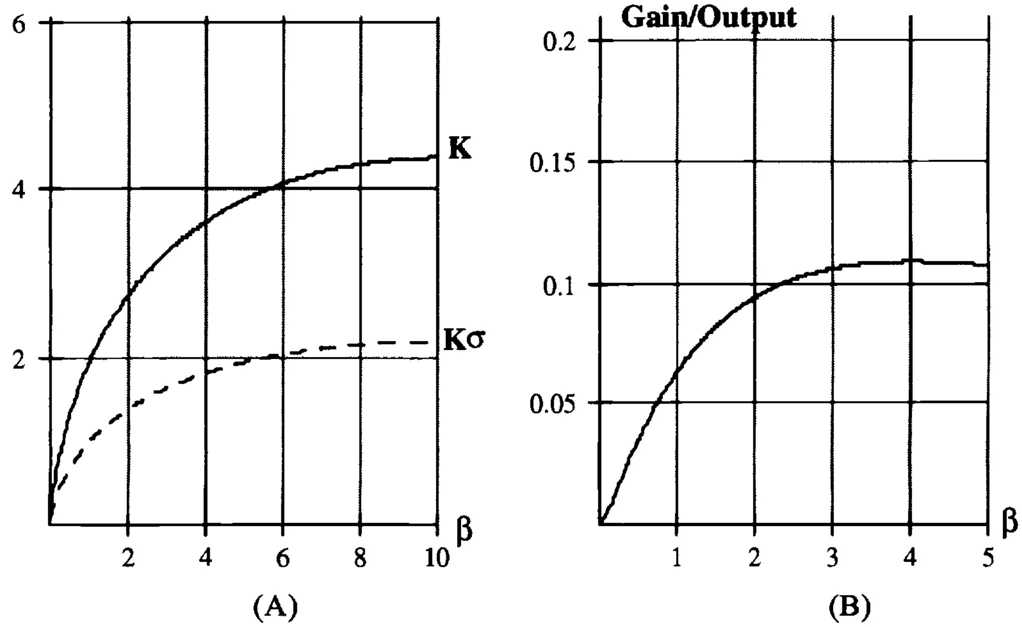


Figure 2
Disappointment aversion buffer stocks (drawn for $\sigma = 0.5$, $R = 0.25$, and $s + 2\xi = 0.1$)

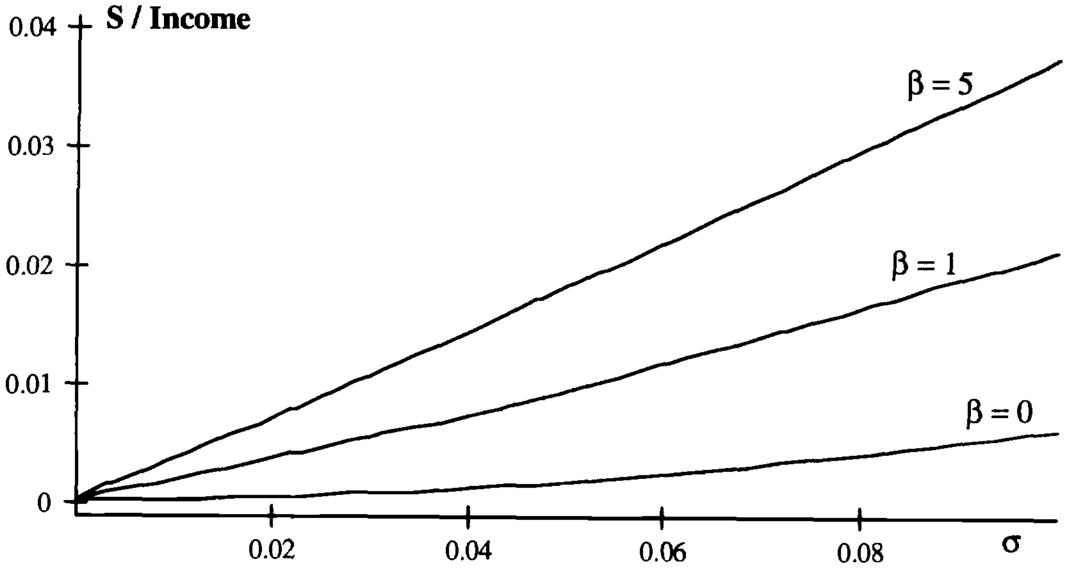


Figure 3
Disappointment aversion and the precautionary saving (plotted for ρ = 0.05, R = 1.5)

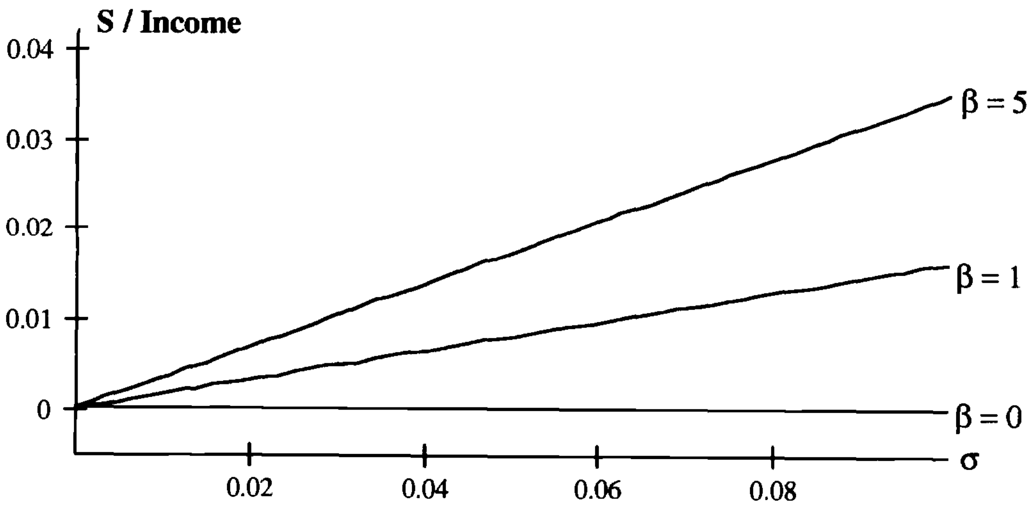
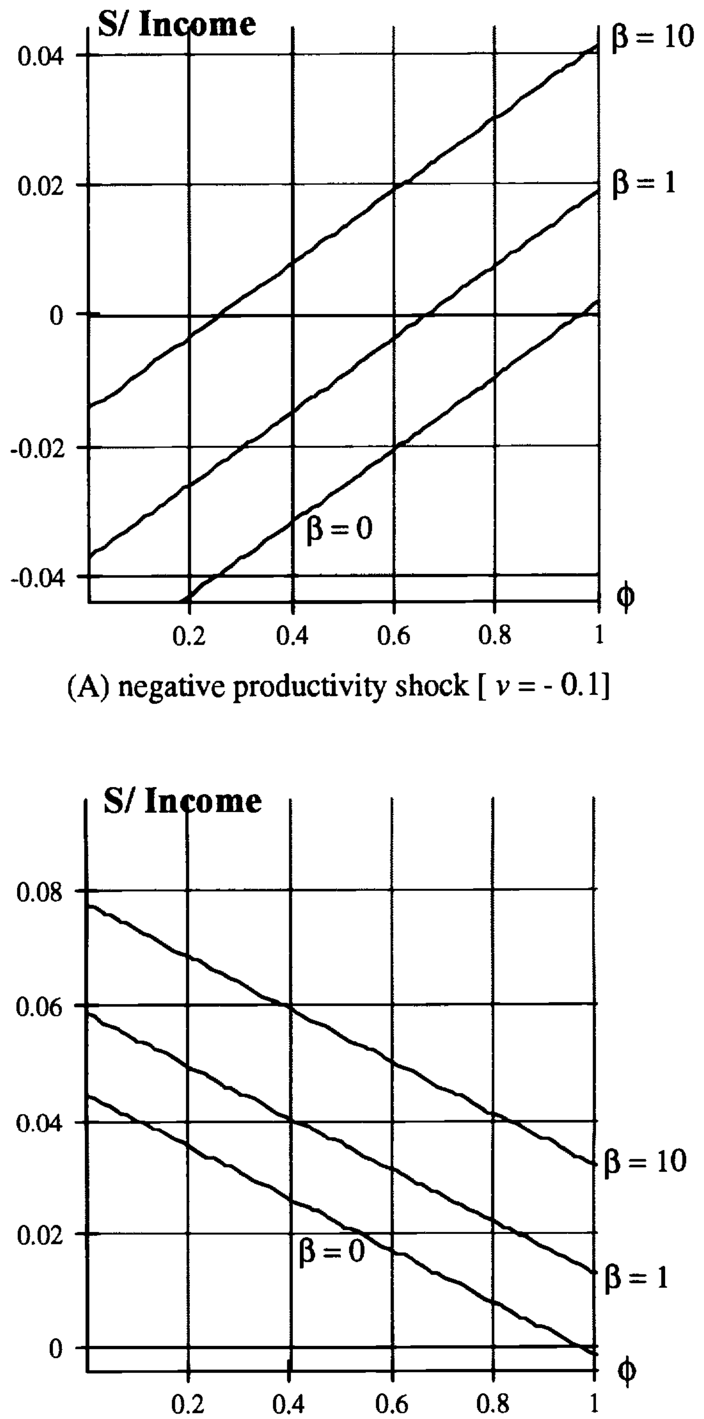


Figure 4
Precautionary saving, disappointment aversion and quadratic utility
(drawn for $u'(x) = 10 - 1x$).



(A) negative productivity shock [$\nu = -0.1$]

(B) positive productivity shock [$\nu = 0.1$]

Figure 5
Precautionary saving, disappointment aversion and serially correlated income

(drawn for $R = 2$, $r = 0.02$, $\rho = 0.05$, $\sigma = 0.1$).