OPTIMAL CERTIFICATION POLICY, ENTRY, AND INVESTMENT IN THE PRESENCE OF PUBLIC SIGNALS[*]

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ABSTRACT. We explore the optimal disclosure policy of a certification intermediary in an environment where (i) the seller's decision on entry and investment in product quality are endogenous and (ii) the buyers observe an additional public signal on quality. The intermediary mutes the seller's entry incentives but enhances investment incentives following entry, and the optimal policy maximizes rent extraction from the seller in the face of this trade-off. We identify conditions under which full, partial or no disclosure can be optimal. The intermediary's report becomes noisier as the public signal gets more precise, but if the public signal becomes too precise, the intermediary resorts to full disclosure. In the presence of an intermediary, a more precise public signal may also lead to lower social welfare.

1. INTRODUCTION

Certification intermediaries are a common feature in many markets where the consumers may not be able to readily assess the quality of the sellers' product. For example, credit rating agencies certify financial instruments, auditors certify the financial standings of organizations, numerous professional groups certify the qualifications and skills of their members, and a large numbers of agencies and laboratories offer certification service for product safety. It is also interesting to note that in such markets, the certification intermediaries are often not the only source of information as the consumer may have access to some information that is publicly available. For example, in the U.S., the investors in a publicly traded company not only consider the firm's credit rating but also its filings with the Securities and Exchange Commission as they are informative about the firm's overall financial strengths. Similarly, though the firm may get its product certified by an intermediary, the ratings given by some non-profit independent agency such as *Consumer Report* can be an additional source of imperfect public signal on product quality.

There is a vast literature on certification intermediaries that studies how their presence might improve trade efficiencies by alleviating the adverse selection problem in the marketplace through the provision of relevant information (Lizzeri, 1991). The reduction in

information asymmetry between the trading parties can also enhance the sellers' incentives to invest in quality and improves allocational efficiency (Albano and Lizzeri, 2001). But the presence of an intermediary also has redistributive consequences. Clearly, the seller's investment incentives get muted as the intermediary captures a share of the resulting gains. Moreover, the seller can be worse off in the presence of an intermediary if the intermediary can extract more surplus than what gets created due to the increased investment incentives. The extant literature has not fully explored the interplay between such redistributive and the efficiency effects as the extent of entry by sellers is typically assumed to be exogenously fixed.

In this paper we consider a setting where the level of entry by the sellers is endogenous. In such an environment, the presence of the intermediary gives rise to a novel trade-off: it enhances the seller's incentives to invest in quality following entry but mutes his incentive to enter at the first place. The goal of the paper is to explore the intermediary's optimal certification policy in the face of this trade-off. In the presence of a public signal on quality, we analyze how the informativeness of the intermediary's signal interacts with the precision of the public signal and draw out its welfare implications.

In order to explore the aforementioned trade-off between entry and investment incentives, we consider a model of certification intermediary with the following key features. The intermediary first commits to a policy that specifies a certification fee and a disclosure policy (we will elaborate on this shortly). Upon observing the certification policy, the seller decides whether to enter by incurring a fixed cost, and following entry, whether to invest to improve its product quality. The investment increases the likelihood of producing a high quality product and the cost of investment depends on the seller's private type. After the quality is realized, the seller decides whether to use the intermediary's service. The consumers cannot directly observe the quality but can obtain relevant information from two sources: a public signal whose precision is exogenously fixed, and the intermediary's signal (i.e., whether the seller has certified his product, and if so, what signal the intermediary has released). The consumers offer a price that is driven by their belief about the product quality given the available information.

As in Lizzeri (1999), we define a disclosure policy as probability distribution over a set of signals conditional on the quality of the seller's product. Such a specification accommodates the extreme cases of full and no disclosure as well as the more generic case of partial disclosure where the intermediary reveals a garbled information about the underlying product quality. Also, in order to highlight the key trade-off between the entry and investment incentives in the most transparent way, we restrict parameters such that entry is always efficient but investment may be inefficient in the absence of the intermediary.

We derive three key results. First, we characterize a "full-disclosure" benchmark (Proposition 2) where the intermediary is required to reveal the quality without any noise should the seller opt to use its service. In such a setting, the extent of efficiency in entry and investment critically depends on the size of the entry cost. In particular, we find that when the entry cost is below a cutoff, the entry is efficient. On the other hand the investment in quality, though improves compared to the scenario where there is no intermediary, remains inefficiently low compared to the first-best (i.e., when the product quality is perfectly observed by the buyers). In contrast, if the entry cost exceeds the cutoff, then both entry and investment remain inefficiently low.

The intuition is as follows. With mandated full disclosure, the intermediary's problem is similar to that of a monopolist—by charging a higher certification fee it can extract more rents from the seller (i.e., the buyer of its certification service) but also reduces the likelihood that the seller would purchase the certification service. Note that with full disclosure, only a seller with high quality product opts to certify. Thus, in order to ensure that the intermediary faces a robust demand for its service, it must protect both the entry and the investment incentives of the seller. In other words, the intermediary chooses the certification price so as to trade-off the gains from rent extraction with the losses from both the entry and the investment distortions. When the entry cost is low, the optimal certification fee does not affect the entry incentives and trades off rent extraction with distortions in investments only. But as the entry cost increases, the entry gets distorted and the intermediary needs to reduce its certification price to induce entry.

However, if the entry cost becomes sufficiently large, accommodating entry for all types of the seller becomes too costly. In response, the intermediary chooses a price such that entry is viable only if the seller also invests in quality (following entry), and all seller types with relatively high investment cost are foreclosed from the market. Hence, under full disclosure, when the entry costs are relatively low the intermediary leads to more efficient investment (compared to the case when there is no intermediary) and also maintains efficiency in entry. But, if the entry cost is relatively high, the presence of the intermediary leads to a trade-off as it enhances the efficiency in investment but distorts the efficiency in entry.

Next, we characterize the optimal disclosure policy for the intermediary and explore the interplay between the informativeness of the intermediary and the precision of the public signal (Propositions 4 and 5). If the public signal is sufficiently precise, the optimal policy calls for full disclosure irrespective of the cost of entry. Otherwise, the disclosure policy is more nuanced. If the cost of entry remains either too low or too high, full disclosure is still optimum. But for a moderate cost of entry the intermediary resorts to partial disclosure where, with some probability, the low quality product receives the same certification that the high quality product gets. Moreover, as the precision of the public signal increases, the intermediary's report becomes increasingly more noisy and eventually absolutely uninformative. But once the public signal becomes sufficiently precise, the intermediary reverts to full disclosure.

To see the argument, recall that under full disclosure the intermediary has exactly one instrument, namely the certification fee, to trade off the gains from rent extraction with the losses from diminished incentives for both entry and investment. For low entry cost the optimal fee does not distort entry, but as the cost increases the trade-off becomes more acute as the certification fees not only mutes investment incentives of the seller but also his incentives to enter the market at the first place. The intermediary must significantly lower its certification fee in order to incentivize entry of all types, and as a result, ends up leaving large amount rents to a high quality seller.

A noisy disclosure can attenuate this trade-off as it allows the intermediary to extract more rents from the seller while the resulting damage to entry incentives is partly restored through the garbling of information. By partly pooling the low and high quality seller, the intermediary can increase its payoff by ensuring efficient entry and inducing the seller use its service irrespective of the realized quality level. This is due to the fact that under partial disclosure, even a low quality seller expects to be pooled with the high quality and fetch a

better price from the consumers, whereas the consumers may believe the seller to be of low quality if he does not use the intermediary.

However, when the cost of entry is too high, even if the intermediary resorts to partial disclosure efficient entry would still call for a significant reduction of the certification fee. And at this point it may be more profitable for the intermediary to switch back to full disclosure and charge a higher fee in order to extract rents from the high quality sellers only. This implies that partial disclosure can be an useful strategy when the entry cost is intermediate and the trade-off between the rent extraction and seller's entry and investment incentives are most acute under full disclosure.

The argument for the optimality of partial disclosure is also useful in understanding how the intermediary responds to the changes in the precision of the public signal. The optimal partial disclosure policy stipulates a certain amount of spread between the expected prices that a high and a low quality seller gets from the customer. Recall that the price that the consumers offers for a product is tied to their posterior belief about the product's quality. Hence, given the precision of the public signal, the intermediary garbles its own report so as to attain the optimal spread in the consumer's belief.

But notice that as the intermediary's signal is (weakly) informative, its ability to influence the consumer's posterior belief is constrained by the precision of the public signal. The spread in the consumers' posterior beliefs that a low and a high quality seller expects to emerge following the intermediary's report is at least as large as the spread that would have been obtained when the consumers can access the public signal only. Until this constraint is binding, the intermediary adds more noise in its signals as the public signal becomes more precise. Thus, the informational content provided by the public signal and the certifier interacts as substitutes. But once the constraint becomes binding, it is optimal for the intermediary not to disclose any further information. Finally, as discussed earlier, when the public signal becomes very precise, the intermediary switches from partial disclosure to full disclosure; i.e., the intermediary's and public signal become complements.

Our third key result analyzes how the social welfare under the intermediary's optimal policy varies with the precision of the public signal. We show that in the presence of the intermediary, the precision of the public signal affects the social welfare only if the precision is at a moderate range. Also, the welfare under intermediary need not be monotone with the public signal's precision. And finally, the "value of the intermediary"—i.e., the change in social welfare due to the intermediary's presence—decreases in the precision of the public signal.

Recall that while the intermediary may improve the efficiency in investment, it can induce inefficiently low entry, particularly when the entry cost is relatively high and the intermediary resorts to full disclosure. The findings above stem from the fact that an increase in the precision of the public signal can lead to a regime change in the intermediary's disclosure policy where it switches from partial disclosure that induces efficient entry to a full disclosure policy where entry is restricted. The resulting welfare loss due to the entry inefficiency can outweigh the gains due to stronger investment incentives, and can lead to an overall drop in the social welfare. In contrast, as the public signal becomes more precise, welfare in the absence of the intermediary always increases as it provides more incentives to invest in quality. This implies that when the public signal is sufficiently precise, the intermediary's presence is detrimental to social welfare.

Literature review: There is by now a large literature on the role of certification inter-mediaries in markets plagued by asymmetric information (some early contributions include, among others, Biglaiser, 1993; Biglaiser and Friedman, 1994; Lizzeri, 1999; and Albano and Lizzeri, 2001; also see Dranove and Jin, 2010, for a survey). In relation to this literature, our paper is closest to Lizzeri (1999) and Albano and Lizzeri (2001). Lizzeri analyzes the role of certification intermediaries in a model of adverse selection, and shows that the optimal choice for the intermediaries often entails no disclosure or partial closure in the form of minimum quality certification. While Lizzeri assumes that the seller's product quality is exogenously fixed, Albano and Lizzeri (2001) extends Lizzeri's model to endogenize the quality. They analyze the issue of the optimal degree of information revelation and show that the presence of intermediary enhances efficiency by increasing the sellers' incentives to provide high qual-ity (though the full information first-best allocation remains infeasible).[1] But in contrast to our setup, these models abstract away from the question of the seller's entry incentives and the interplay between the public signal on quality and the intermediaries' disclosure policy (i.e., they assume that the seller is already in the market and the intermediary is the only source of information for the buyers).

A recent paper that explores how certification intermediaries may influence the sellers' entry decision is Harbaugh and Rasmusen (2018). They consider a pure adverse selection model without investment and show that "exact grading" is never optimal. As in our model, coarse grading—i.e., partial disclosure—is used to induce more participation by senders; in their model with a continuum of types, the middle types have more incentives to participate in the certification process under a pass-fail scheme than an exact grading scheme as they would like to distinguish themselves from the low types but pool with the high types. They assume a non-profit certifier whose objective is to enhance the amount of information available to the buyers while the seller incurs an exogenous certification cost. Our model, however, assumes a profit-maximizing certifier with an endogenous certification price and the cost of garbling information comes from reduced investment incentives rather than the loss of information per se. In a related work, Dubey and Geanakoplos (2010) consider the optimal grading scheme in inducing students' efforts, that is, they focus on the optimal disclosure policy for investments (but without considering participation constraints). They show that coarse grading (i.e., partial disclosure) often motivates students to work harder when the students care about their status (relative rank in the class).[2] While these papers present theoretical analyses of the intermediary's impact on the agents' behavior, Hui et al. (2018) provide an empirical analysis on the effects of certification policies on the evolution of markets. By using a change in the certification policy of eBay as a natural experiment, they investigate how the stringency of certification policy affects the types of entrants and seller behavior. However, the certification policy in their model is exogenous and the service is unilaterally provided by a platform owner (i.e., eBay) without any price. In contrast, our focus is on characterizing the optimal certification policy by a profit-maximizing certifier.

Our paper is also related to the literature on the optimal design of information structure (Ostrovsky and Schwarz, 2010; Kamenica and Gentzkow, 2011; Rayo and Segal, 2010). Kamenica and Gentzkow analyze the general Bayesian persuasion problem in which a sender

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[1]In a related paper, Belleflamme and Peitz (2014), assume that consumers observe the investment (but not the realization of the product quality) and show that the firm overinvest in quality compared to the full informatioin benchmark.

[2]Similar issues are also discussed by Costrell (1994) and Boleslavsky and Cotton (2015).

chooses the optimal information structure for a signal to be revealed to a receiver, and derive general conditions under which the ability to control the informational environment is beneficial to the sender.[3] As in the Bayesian persuasion literature, we also assume that the intermediary can precommit to a particular disclosure policy. However, in our setting the intermediary's problem cannot be modeled as a standard sender's problem because the distribution of the product quality (i.e., the underlying "state of the world" that the sender reveals information on) is endogenous to the sender's disclosure policy.

In a more recent paper, Boleslavsky and Kim (2018) develop a framework to analyze the optimal design of information structure in the presence of moral hazard. In particular, they consider a three-player Bayesian persuasion game which includes an agent, in addition to the sender and the receiver, whose private effort determines the distribution of an unobservable state. Our setup is similar to Boleslavsky and Kim in that the distribution of the "state of the world", i.e., the product quality is endogenously affected by the seller's entry and investment decisions. Nonetheless, our framework does not conform to the Boleslavsky and Kim's setup because we also have an element of adverse selection with heterogeneous agent types and additional constraint on the entry whereas their model focuses only on the investment margin with only one type of agent.[4]

Finally, it is worth noting that our analysis on the interplay between the informativeness of the intermediary and the precision of the public signal is related to the literature on the adverse consequences of transparency (if we interpret more precise public signal as more transparency in an agency relationship). Prat (2005), for instance, shows that an agent with career concerns may ignore his signal, to the detriment of the principal, and behave as a conformist when his action is observed (i.e., transparent). Levy (2007) considers the effect of transparency on committee decisions and identifies circumstances under which a secretive committee that uses a particular voting rule makes better decisions on average. Their analyses, however, are in completely different contexts and rely on career-based reputation effects. Also, these papers only consider a binary choice between no transparency and full transparency. In contrast, we consider a continuum of precision levels in the public signal, which enables us to conduct a comparative static analysis and derive the non-monotonicity result.

The remainder of the paper is organized as follows. In Section 2, we set up our basic model of adverse selection with investments and endogenous entry in the presence of intermediary. In Section 3, we characterize the market equilibrium in the absence of intermediary as a benchmark. Section 4 explores the role of intermediary in our set-up, but consider an intermediary who is constrained to fully disclose the quality. In Section 5, we present a general analysis of optimal disclosure policy in which the intermediary can choose both the certification price and the disclosure policy. We identify conditions under which the intermediary may adopt partial disclosure by garbling information. Section 6 discusses the implications of the optimal policy, its welfare consequences, and presents a conclusion. All proofs are given in the Appendix.

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[3]The Bayesian persuasion literature has been extended to many different directions to address the possibilities of multiple senders (Au and Keiichi, 2017; Gentzkow and Kamenica, 2017; Li and Norman, 2015), multiple receivers (Chan *et al.* 2017), and a privately informed receiver (Kolotilin *et al.* 2017), for instance. The framework has also been applied to a variety of contexts including voting (Alonso and Camara, 2016), monopoly pricing (Roesler and Szentes, 2017), price discrimination (Bergemann *et al.* 2015), and career concerns (Rodina, 2017).

[4]See Rosar (2017) for an analysis of a Bayesian persuasion game with voluntary participation.

2. Model

PLAYERS: We consider an environment with three types of players: a seller, a certification intermediary, and a set of identical buyers.

ACTIONS AND PAYOFFS: The seller decides whether to enter a market to sell his product to a set of identical buyers. The buyers' valuation of the seller's product is based on its quality. The product quality could be either high or low and generates a value $v \in \{0, 1\}$ for the buyer, where $v = 0$ if the quality is low and $v = 1$ if it is high.

If the seller decides to enter the market, he incurs an entry cost of k. Upon entry he can undertake an investment in quality in order to increase the likelihood of producing a high quality product. Let $I \in \{0, 1\}$ denote the seller's investment decision where $I = 1$ if he invests in quality and $I = 0$ otherwise. We have

$$(1) \qquad \Pr(v = 1 \mid I = 1) = \alpha > \frac{1}{2} = \Pr(v = 1 \mid I = 0).$$

The cost of this investment is determined by the seller's type $\theta \in [0, 1]$, which is assumed to be uniformly distributed on $[0, 1]$. For a seller of type θ, the cost of investment is c/θ for $\theta \neq 0$ (and $C > 1$ for $\theta = 0$).[5] The seller's type is his private information and known to him before he makes his entry decision.

The product quality is also privately observed by the seller, leading to an information asymmetry in the product market. But the buyers can obtain information on quality through two channels. First, the seller can hire a certification intermediary who can verify the quality and disclose additional information. The intermediary is assumed to be a monopolist in the market for certification services. At the beginning of the game, the intermediary commits to a certification price p and a disclosure policy $\mathcal{D}$ that specifies what it may disclose to the buyers, given the underlying quality of the product. However, the intermediary may not fully reveal the quality of the product and can potentially garble its report. In order to allow for such a noisy disclosure, we define a disclosure policy as a mapping $\mathcal{D} : \{0, 1\} \to \Delta X$ where X is a pre-specified signal space (and hence, a part of the policy). That is, the disclosure policy sends a signal $x \in X$ that is drawn according to a given probability distribution, conditional on the true quality of the product. We assume that the intermediary does not incur any cost to evaluate the product quality.

Second, in addition to the intermediary's signal, the buyers observe a public signal $z \in \{0, 1\}$ that provides noisy information about the product quality, where

$$\Pr(z = j \mid v = j) = \pi \in [1/2, 1), \text{ for } j = 0, 1.$$

The parameter π represents the precision of the public signal; when $\pi = \frac{1}{2}$ the pubic signal is completely uninformative.

The seller makes his entry decision after observing the intermediary's offer $(p, \mathcal{D})$. Also, the seller decides on whether to hire the intermediary after the quality realization but before

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[5]For simplicity we normalize the seller's probability of producing a high quality product without any investment to be $\frac{1}{2}$. Also, if $\Pr(v = 1 \mid I = 0) = \alpha_0 < \frac{1}{2} < \alpha_1$, we may not have a unique equilibrium even in the absence of the intermediary, and the characterization of the optimal policy becomes analytically intractable.

the public signal is realized. Observing the available signals on the quality, the buyers simultaneously bid for the product and the product is sold at the highest bid. All players are assumed to be risk neutral. This implies that the product is sold at the expected value of the product given the information available to the consumers.

TIMING: The following time line summarizes the game.

- *Stage 1:* The intermediary commits to his certification policy $(p, \mathcal{D})$.
- *Stage 2:* The seller observes his type θ and the intermediary's policy and decides whether to enter. (If there is no entry, the game ends.)
- *Stage 3:* If the seller enters, he decides whether to investment on quality, and observing his product quality, decides on whether to hire the certification intermediary.
- *Stage 4:* The intermediary reveals its signal x on the product quality.
- *Stage 5:* The public signal z on quality is revealed. Observing x (if available) and z, the buyers bid for the product and the product is sold at the highest bid.

STRATEGIES AND EQUILIBRIUM CONCEPT: The strategies of the players are as follows: the intermediary's strategy is to choose a certification policy $(p, \mathcal{D})$. The seller's strategy has three components: (i) entry and (ii) investment decisions given his type and the intermediary's policy, and (iii) decision on hiring the intermediary given his product quality and the intermediary's policy. Finally, the buyers' strategy is to choose a bid given the available information (i.e., the public signal, the intermediary's certification policy, whether the intermediary was hired or not, and if hired, the intermediary's report). We use (*pure strategy*) perfect Bayesian Equilibrium as the solution concept. The optimal disclosure policy is defined as the $(p, \mathcal{D})$ pair that induces the highest feasible equilibrium payoff for the intermediary.

Below, we maintain the following parametric restrictions to streamline our analysis. Let $\Delta := \alpha - \frac{1}{2}$ denote the social value of investment in quality.

Assumption 1. (i) $\sqrt{c\alpha} < \Delta$; (ii) $\frac{c}{2\Delta} < k < \frac{1}{2}$.

Observe that Assumption 1 (i) implies $c < \Delta$, i.e., at least for some types of the seller, it is socially efficient to invest in quality. We maintain a stronger assumption (than simply requiring $c < \Delta$) so as to rule out certain corner solutions that clutter our analysis without offering any important economic insight. Assumption 1 (ii) plays a similar role by stipulating a moderate range of entry cost where the interplay of entry and investment decisions is non-trivial and leads to a rich set of equilibrium characteristics.

3. BENCHMARKS: FIRST-BEST AND MARKET EQUILIBRIUM IN ABSENCE OF INTERMEDIARY

In this section, we consider two benchmark cases that inform our subsequent analysis of the intermediary's disclosure policy.

First, consider the social first-best—the entry and investment rule that maximizes the aggregate surplus of the seller and the buyers. Note that the surplus created by a seller of type θ with investment $I \in \{0, 1\}$ is $\Pr(v = 1 \mid I) - Ic/\theta - k$. Hence, the first-best requires

all types of the seller to enter (recall from (1) and Assumption 1 (ii), $\Pr(v = 1 \mid 0) - k > 0$) and all types $\theta \geq \theta_I^{FB}$ to invest, where:

$$\alpha - \frac{c}{\theta_I^{FB}} = \frac{1}{2}, \ i.e., \ \theta_I^{FB} = \frac{c}{\Delta}.$$

As a second benchmark, consider the investment and entry incentives of the seller in absence of the intermediary where the buyers' only source of information is the public signal z. Let $\Theta_E \subseteq [0,1]$ and $\Theta_I \subseteq \Theta_E$ be the set of the seller's types that enter the market and invest in quality, respectively. Since the buyers competitively bid for the product, the equilibrium is defined as follows: (i) The seller obtains a price of $\mathbb{E}(v \mid z, \Theta_E, \Theta_I)$, i.e., the expected valuation of the product given the realized public signal and set of seller types that enter and invest. (ii) Given the seller's expected price (prior to the realization of the public signal), only the types in Θ_E enters, and among the entrants, only the types in Θ_I invests.

Denote

$$v_i(\Theta_E, \Theta_I) = \sum_{z \in \{0,1\}} \mathbb{E}(v \mid z; \Theta_E, \Theta_I) \Pr(z \mid I = i),$$

which is the expected price the seller receives when his investment decision is I and the buyers believe that the set of types that enter is Θ_E and the set of types that invest is Θ_I. Now, for type θ, the expected payoff from entry and investment decision I is:

$$V(I; \theta, \Theta_E, \Theta_I) = \begin{cases} v_1(\Theta_E, \Theta_I) - k - \frac{c}{\theta} & if \ I = 1 \\ v_0(\Theta_E, \Theta_I) - k & if \ I = 0 \end{cases}.$$

The following proposition characterizes the equilibrium.

Proposition 1. *(**Equilibrium without intermediary**) If there is no intermediary in the market, the equilibrium has the following features:*

(i) Entry is always efficient—all types of the seller enter (i.e., $\Theta_E = [0,1]$).

(ii) Investment may be inefficient. There exists a threshold $\bar{c} > 0$ such that if $c > \bar{c}$ no type invests. Otherwise, there exists a unique cutoff $\theta_I^{NI} \ (> \theta_I^{FB})$ such that all types $\theta \geq \theta_I^{NI}$ invest. Moreover, θ_I^{NI} is decreasing in π.

The proposition above has a simple intuition. Note that even if the seller does not invest, he earns at least $\frac{1}{2} - k > 0$ by entering the market irrespective of his own type ($v_0(\Theta_E, \Theta_I) \geq \frac{1}{2}$ for all Θ_E and Θ_I). Hence, all types enter. Moreover, as the seller's payoff from investment is monotonically increasing in his type, the investment decision must follow a cutoff strategy: unless the cost of investment is too high, all types above a threshold invest.

However, there is under-investment compared to the first-best because the seller cannot receive a fair return from his investment due to asymmetric information on the product quality. But the investment inefficiency can be mitigated via the availability of a public signal, and the extent of such inefficiency decreases with the signal's precision. The more precise is the public signal the higher is the return from investment, and the stronger investment

incentive moves the type threshold closer to the first-best. In the limit, if the public signal is perfectly informative (i.e., $\pi = 1$), there is no information asymmetry and we restore the first-best (i.e., $\theta_I^{NI} = \theta_I^{FB}$). In contrast, if the public signal is completely uninformative (i.e., $\pi = \frac{1}{2}$), there is no incentive to invest for the seller regardless of his type (i.e., $\theta_I^{NI} = 1$).

4. Intermediary with Full Disclosure

Given the benchmark analyses above, we now explore the role of intermediary in our model. We first consider a simple case where the intermediary, *if hired*, is obligated to fully disclose the quality. That is, under (mandatory) full disclosure, the intermediary can only set the price $p \geq 0$ for its certification service. The analysis of the full disclosure case clearly illustrates the trade-offs that the intermediary faces while choosing its certification fee, and it is also instructive in understanding when and how a partial disclosure policy may be optimal.

Let $\mu(x, z) = \Pr(v = 1 \mid x, z)$ be the buyers' (posterior) belief on quality given the intermediary's report x and the public signal z. Note that under full disclosure, $x \in \{0, 1, \emptyset\}$ where, with a slight abuse of notation, we denote $x = \emptyset$ when the seller does not use the intermediary. (Clearly, as the seller decides on hiring the intermediary after observing his product quality, the buyers would update their beliefs even when the seller decides not to hire the intermediary.) In what follows, we analyze the optimal PBE for the intermediary, i.e., we derive the equilibrium certification price p and the associated belief μ that maximize the intermediary's profit given that the beliefs are consistent and the seller's entry and investment decisions are sequentially rational (conditional on his type).

Lemma 1. *In the optimal PBE under full disclosure, only the high quality seller hires the intermediary.*

The argument is straightforward (hence, we omit the formal proof): Notice that in any PBE with $p > 0$, it cannot be the case that the seller hires the intermediary irrespective of his product quality. Under full disclosure, the low quality seller is sure to get a price of 0 from the buyers if he hires the intermediary, and he can save on the certification fee by not hiring the intermediary at the first place. Likewise, in the optimal PBE, it cannot be the case that the seller never certifies irrespective of his product quality. If such an equilibrium exists, the intermediary makes 0 whereas (at the time of the certification stage) the high quality seller can only expect from the buyers a bid that is strictly less than 1 (as the public signal is noisy). Any such equilibrium, even if it exists, gives lower payoff to the intermediary compared to an equilibrium where the intermediary charges a sufficiently low price $\varepsilon > 0$ (with full disclosure), and $\mu(\emptyset, z) = 0$ for all $z \in \{0, 1\}$ (i.e., the buyers take a "skeptical posture" à la Milgrom and Roberts (1986) and believe that the product is of low quality if the seller does not hire the intermediary). For such a price, the high quality seller would find it profitable to use the intermediary, and the intermediary would earn a strictly positive payoff.

In light of the above lemma, we focus on the fully separating PBE where only the high quality seller hires the intermediary. If such a PBE exists, notice three salient features of such an equilibrium: First, the public signal does not affect beliefs. Second, for any price of certification, p, a seller of type θ enters the market only if

(2)
$$\max\left\{\alpha\left(1-p\right)-\frac{c}{\theta}-k,\ \frac{1}{2}\left(1-p\right)-k\right\}\geq 0.$$

Hence, in any equilibrium, we must have:

$$p\leq\bar{p}:=1-\frac{k+c}{\alpha},$$

as otherwise even the most efficient type $(\theta=1)$ would choose to stay out. Third, all types of the seller enter if $\frac{1}{2}\left(1-p\right)-k>0$, i.e.,

$$p<p_E:=1-2k.$$

Note that for $p<p_E$, the seller gets a strictly positive (expected) payoff by entering the market even if he does not invest following entry. Also, for any $p\in[p_E,\bar{p})$ entry is profitable only if the seller also invests in quality ($p_E<\bar{p}$ by Assumption 1 (ii)).

Based on the above observations, we can now characterize the optimal certification price for the intermediary. In any PBE the set of seller's types that enter (Θ_E) and invest (Θ_I) are pinned down by their respective type cutoffs, θ_E and θ_I (say), where $\Theta_E=[\theta_E,1]$ and $\Theta_I=[\theta_I,1]$. (The argument is the same as in the benchmark cases discussed above, and follows from the fact that the cost of investment is decreasing in the seller's type).

Now, the intermediary can adopt one of two possible types of pricing strategies: (i) charge $p\leq p_E$ and induce full entry, i.e., $\Theta_E=[0,1]$, or (ii) charge $p\in(p_E,\ \bar{p}]$ such that low types stays out and all types that enter also invests in quality, i.e., $\Theta_I=\Theta_E$. Thus, in the most profitable PBE for the intermediary, he charges a price that solves the following problem:

$$\max_p\ \Pi\left(p\right)\ :=p\Pr\left(v=1\mid\theta_E,\theta_I\right)=p\left[\tfrac{1}{2}(\theta_I-\theta_E)+\alpha\left(1-\theta_I\right)]\right]$$
$$=\begin{cases}\Pi_E(p):=p\left(\tfrac{1}{2}\theta_I+\alpha\left(1-\theta_I\right)\right)&\text{if }p\leq p_E\\\Pi_I(p):=p\alpha\left(1-\theta_I\right)&\text{if }p\in[p_E,\bar{p})\end{cases},$$

where θ_E and θ_I are the marginal types of the seller for whom it is profitable to enter and profitable to invest (respectively), given the intermediary's price. (Notice that $\theta_I-\theta_E$ is the mass of entrant types who do not invest whereas $1-\theta_I$ is the mass of types who invest following entry.) The lemma below states the solution to the above problem.

Lemma 2. *There exist three entry cost cutoffs $k_E<k_I<k^*$ such that the intermediary's payoff at the most profitable PBE is:*

$$\Pi^*=\begin{cases}\Pi_E^*&\text{if }k<k^*\\\Pi_I^*&\text{otherwise}\end{cases},$$

where

(3)
$$\Pi_E^*=\max_{p\leq p_E}\ \Pi_E(p)=\begin{cases}\overline{\Pi}_E:=\alpha+c-2\sqrt{c\alpha}&\text{if }k<k_E\\\underline{\Pi}_E:=\alpha\left(1-2k\right)\left(1-\frac{c}{2k\alpha}\right)&\text{otherwise}\end{cases},$$

and

$$(4) \qquad \Pi_I^* = \max_{p > p_E} \Pi_I(p) = \begin{cases} \overline{\Pi}_I := \alpha + c - k - 2\sqrt{c(\alpha - k)} & \text{if } k \geq k_I \\ \underline{\Pi}_I := \alpha(1 - 2k)\left(1 - \frac{c}{2k\Delta}\right) & \text{otherwise} \end{cases}.$$

The intuition for the above result is as follows. By raising the certification price the intermediary can extract more rents from the seller but also reduces the likelihood that the seller would hire the intermediary at the first place. The latter effect on the intermediary's demand stems from the fact that the certification price distorts both the investment and the entry incentives of the seller. Recall that under full disclosure only the high-quality seller hires the intermediary, and the seller's product quality is more likely to be high if he invests in quality. But the larger is the certification price the weaker is the seller's incentive to invest as the intermediary keeps a larger share of the trade surplus. And if there is not enough surplus left for the seller to cover his entry cost, he may not enter the market.

Consider the certification price p^* (say) that the intermediary would have charged if the seller were already on the market. In such a setting, the optimal price trades off the gains from the rent extraction with the losses that emanates only due to weakened investment incentives of the seller. Notice that for sufficiently low entry cost $(k < k_E)$, $p^* < p_E$. So, even in our setting the intermediary would charge p^* as such a price would not affect entry, and all types of the seller would be on the market.

But as the entry cost increases beyond the threshold k_E, we have $p^* > p_E$, and there would be inefficiently low entry at the price p^*—the seller may stay out if his type is sufficiently low (i.e., cost of investment is sufficiently high). As long as the entry cost is moderately low (i.e., when $k_E < k < k^*$) the intermediary is better off by lowering its price to p_E. By charging a lower certification price, the intermediary forgoes a part of the rent that it could have extracted from the seller. But such a loss is more than offset by the gains from increased likelihood of having a high-quality seller as the reduction in the certification price restores entry efficiency and also strengthens the seller's investment incentives.

However, when the entry cost is significantly large, accommodation of entry would considerably hurt the intermediary as it would require a large reduction in the certification price. As a result, when the entry cost crosses a threshold (k^*) the intermediary finds it optimal to restrict entry by raising his certification price such that all entrants types have incentive to invest in quality.

In light of the above lemma, we can characterize how the intermediary's optimal pricing policy (under full disclosure) affects the entry and investment efficiencies in our setting.

Proposition 2. (*Efficiency implications of intermediary under full disclosure*) *If $k < k^*$, there is efficient entry, i.e., all types enter, but investment remains inefficient as only the types $\theta \geq \theta_I^f$ invest, where*

$$\theta_I^f := \min\left\{\frac{\sqrt{c\alpha}}{\Delta}, \frac{c}{2k\Delta}\right\}.$$

But for $k > k^*$, both entry and investment remain inefficient as only the types $\theta \geq \theta_I^i$ enter and invest where

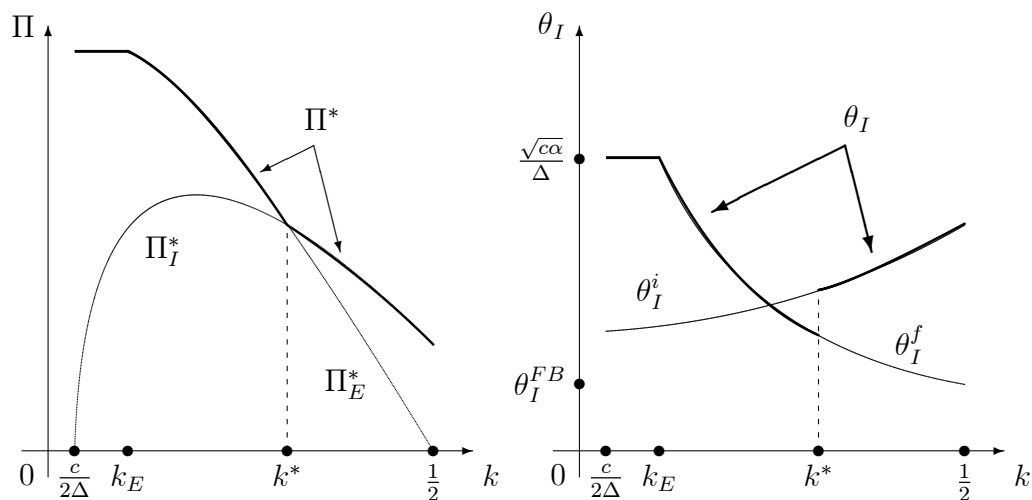$$\theta_I^i := \sqrt{\frac{c}{\alpha - k}}.$$

All other types stay out.

Figure 1. The intermediary's payoff (Π^*) and associated cutoff type
for investment (θ_I) as a function of entry cost k.

Figure 1 depicts the intermediary's equilibrium payoff as a function of entry cost k and the corresponding cutoff types for investment (as given by Lemma 2 and Proposition 2). Observe that θ_I^f is (weakly) decreasing whereas θ_I^i is increasing in the entry cost. The intuition is simple and, again, can be seen from Lemma 2: When $k < k_E$, entry is efficient and the intermediary's optimal price is independent of k—it is the same price that he would have charged if the seller were already on the market. So, the seller's share of the trade surplus is also independent of the entry cost, and, consequently, the seller's type cutoff for investment (θ_I^f) remains constant. But when $k_E < k < k^*$, the intermediary accommodates entry by setting $p = p_E = 1 - 2k$. Thus, higher k calls for lower certification price and since the seller gets to keep a larger share of the trade surplus, his incentive to invest increases. As a result θ_I^f is decreasing in k. In contrast, for $k > k^*$, entry is profitable only if the seller invests. So with larger entry cost only the more efficient types enter; i.e., θ_I^i is increasing in k.

To conclude our analysis of the full disclosure case, we compare the entry and investment incentives with and without the intermediary (as given by Proposition 1 and 2 above).[6]

Proposition 3. *(Comparing full- and no-disclosure)* *In the presence of the intermediary (under full disclosure) entry is inefficient iff $k \geq k^*$. The efficiency in investment is affected as follows:*

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[6]We omit the proof of Proposition 3 as it readily follows from the facts that θ_I^f is decreasing and θ_I^i is increasing in k (as given in Proposition 2), and θ_I^{NI} is decreasing in π (as given in Proposition 1).

(i) If $c \geq \bar{c}$ (i.e., when the intermediary is absent, all types enter but none invests), the intermediary always improves efficiency in investment.

(ii) If $c < \bar{c}$ (i.e., when the intermediary is absent, all types enter and all types $\theta > \theta_I^{NI}$ invest), for all k, there exists a cutoff π_k such that the intermediary improves investment incentives iff $\pi < \pi_k$. The threshold π_k is increasing in k for $k < k^*$ but decreasing otherwise.

5. Optimal Disclosure Policy

In this section, we turn to the question of optimal disclosure policy where the intermediary may choose both his certification price as well as his disclosure policy. In particular, we allow for partial disclosure where the intermediary may (partially) pool the low and high quality sellers by using a stochastic disclosure rule.

To simplify the analysis, we adopt the direct revelation approach. As the buyers bid competitively for the product, the equilibrium bid is simply the expected quality of the product given the intermediary's disclosure policy, the set of types Θ_E and Θ_I who enter and invest, and the realized signals (x, z). For a given disclosure policy, let t_0 and t_1 be the ex-ante expected bids (or "transfers" from the buyers) for a low and high quality seller (respectively) when he uses the intermediary but before he learns the realization of the signals (x, z). That is,

$$t_i = \mathbb{E}_{(x,z)|v=i}\mathbb{E}_v\left[v \mid x, \ z, \ \Theta_E, \ \Theta_I\right], \quad i \in \{0, 1\}.$$

With this direct revelation approach, the intermediary's certification policy—certification price p and disclosure policy $\mathcal{D}$—can be represented by the triplet (p, t_0, t_1). Under a strictly partial disclosure policy we have $0 < t_0 < t_1 < 1$ whereas under full disclosure, we have $t_0 = 0$ and $t_1 = 1$.

Unfortunately, the direct revelation approach to solving the intermediary's optimal policy still lacks algebraic tractability. The complication primarily stems from the fact that both entry and the hiring of intermediary are discrete choices of the seller, and the intermediary's payoff need not be continuous in his certification policy (p, t_0, t_1). To circumvent this problem, we proceed as follows:

First, we note that if a partial disclosure policy is optimal then we must have "full market coverage" where all types (θ) of the seller enter and the seller always hires the intermediary irrespective of his product quality. Next, we characterize the optimal partial disclosure policy under "full market coverage"—i.e., we impose the constraint that the policy must induce all seller types to enter and certify (regardless of the quality). Finally, we derive the optimal policy by comparing the intermediary's maximal payoff under the constrained partial disclosure policy (as defined above) and its full disclosure counterpart. We begin our analysis with the following lemma.

Lemma 3. *If a partial disclosure policy is optimal, it must entail "full market coverage": in equilibrium, (i) all types (θ) of the seller enter the market, and (ii) the sellers for both product qualities use the intermediary.*

To see the intuition behind the above result, consider the latter part first. If the intermediary plans to sell its service only to the high quality seller, it is best to maximize the difference in prices that a low and a high quality seller would receive from the buyer. Clearly, this is attained under full disclosure as it removes all information asymmetries regarding the product quality. Thus, a partial disclosure policy can be optimal only when the intermediary intends to induce both the high and low quality sellers to certify their products.

A similar logic applies to the first part of the lemma. In any equilibrium, if it is the case that only some types of the seller enter, it also must be the case that those types who enter also invest. (Since the entry cost is the same for all types, if its profitable for some type θ to enter even though it would not invest following entry, then entry must be profitable for all types of the seller.) But if the intermediary only intends to induce entry of those types who would invest as well, it can be argued that it is (weakly) optimal to adopt the full disclosure policy. Thus, we can conclude that partial disclosure is adopted by the intermediary only when it intends to engage in full market coverage.

In light of Lemma 3, we can now formulate the intermediary's optimal partial disclosure policy. We maintain the off-equilibrium belief $\mu(\emptyset, z) = 0$ as it is the most favorable to the the intermediary, i.e., if the seller chooses not to use the intermediary, the buyers offer a price of 0. Thus, for all types of the seller to enter and certify regardless of the realized product quality, the certification price p should satisfy the following two conditions:

(IR_E) $$\frac{1}{2}(t_0 + t_1) - k - p \geq 0,$$

and

(IR_C) $$t_0 - p \geq 0.$$

The first condition above (IR_E) is the individual rationality condition for entry and states that it is profitable for all types (θ) of the seller to enter the market even if he decides not to invest in quality. Otherwise, a seller with low θ will not enter the market as his cost of investment may be prohibitively high; consequently, we would not have full market coverage. The second condition (IR_C) states that the seller has incentives to certify even when his product quality is low. Obviously, under full disclosure we have $t_0 = 0$, and (IR_C) cannot be satisfied with any positive certification price. Hence, under full market coverage, the intermediary needs to garble information if it were to receive a higher certification price. So, for any disclosure policy (t_0, t_1) with full market coverage, the intermediary's payoff is:

$$\Pi(p) = p = \min\left\{t_0, \frac{1}{2}(t_0 + t_1) - k\right\}.$$

As before, we can argue that the seller's investment decision follows a cutoff strategy as the cost of investment is monotonically decreasing in the seller's type. Let $v(\theta_I)$ be the "prior" expected value of the product—i.e., the probability that the seller's product is of high quality (so $v = 1$) given that all types enter and all types above θ_I invest, but without any information on the signals (x, z). That is,

$$v\left(\theta_I\right) = \frac{1}{2}\theta_I + \alpha\left(1 - \theta_I\right) = \frac{1}{2} + \Delta\left(1 - \theta_I\right).$$

Any disclosure policy (t_0, t_1) must satisfy the following four conditions: First, a Bayes rationality (BR) condition that requires the expected posterior mean quality must be equal to the prior mean quality:

$$(BR) \qquad\qquad v\left(\theta_I\right)t_1 + \left(1 - v\left(\theta_I\right)\right)t_0 = v\left(\theta_I\right).$$

Second, as the intermediary's signal is (weakly) informative, we must have the following bounds:

$$(L_1) \qquad\qquad t_1 \geq \underline{t}_1\left(\theta_I\right) := \mathbb{E}_{z|v=1}\mathbb{E}_v\left[v \mid z, \Theta_E = [0,1], \; \Theta_I = [\theta_I, 1]\right].$$

and

$$(U_0) \qquad\qquad t_0 \leq \bar{t}_0\left(\theta_I\right) := \mathbb{E}_{z|v=0}\mathbb{E}_v\left[v \mid z, \Theta_E = [0,1], \; \Theta_I = [\theta_I, 1]\right].$$

Finally, as discussed earlier in the case of full disclosure, if it is incentive compatible for all types $\theta \geq \theta_I$ to invest, we must have:

$$(IC) \qquad\qquad \theta_I = \min\left\{\frac{c}{\left(t_1 - t_0\right)\Delta}, 1\right\}.$$

Hence, the intermediary's optimal partial disclosure policy solves the following program:

$$\mathcal{P} : \begin{cases} \max_{t_0, t_1, \theta_I \in [0,1]} & \min\left\{\frac{1}{2}\left(t_0 + t_1\right) - k, t_0\right\} \\[2mm] \quad s.t.\ (BR),\ (L_1),\ (U_0),\ \text{and}\ (IC). \end{cases}$$

In order to solve the above program we first consider a relaxed problem that ignores the boundary conditions (L_1) and (U_0). If the optimal cutoff for the investment type, say $\hat{\theta}_I$, in the relaxed problem satisfies the boundary conditions, then $\hat{\theta}_I$ is also the solution to the original program $\mathcal{P}$. Otherwise, we need to adjust the investment type cutoff in order to account for the boundary conditions. As shown in the proof of Lemma 4, lowering the investment cutoff level θ_I (i.e., inducing investment by more types) relaxes both boundary conditions. Moreover, both of these conditions become strictly non-binding as θ_I becomes sufficiently small, as stronger incentives for investment calls for more precise information revelation; i.e., a larger spread between t_1 and t_0. Therefore, there is a positive cutoff level $\bar{\theta}_I$ such that $t_1 \geq \underline{t}_1\left(\theta_I\right)$ and $t_0 \leq \bar{t}_0\left(\theta_I\right)$ if and only if $\theta_I \leq \bar{\theta}_I$. When the boundary conditions are not satisfied, the solution to the original problem thus requires a reduction of the cutoff level from $\hat{\theta}_I$ to $\bar{\theta}_I$.

Lemma 4. (Optimal partial disclosure policy under full market coverage) *The solution to $\mathcal{P}$, $(\tilde{t}_0, \tilde{t}_1, \tilde{\theta}_I)$, is characterized as follows:*

(i) *The optimal cutoff above which all types of the seller invest in quality is given by $\tilde{\theta}_I = \min\{\hat{\theta}_I(k), \bar{\theta}_I(\pi)\}$, where $\hat{\theta}_I$ is independent of π but decreasing in k, and $\bar{\theta}_I$ is independent of k but decreasing in π.*

(ii) *There exist two thresholds, $\underline{\pi}$ and $\bar{\pi}$, where $0 < \underline{\pi} < \bar{\pi} < 1$, such that for any given k, $\tilde{\theta}_I = \hat{\theta}_I(k)$ if $\pi < \underline{\pi}$, and $\tilde{\theta}_I = \bar{\theta}_I(\pi)$ if $\pi > \bar{\pi}$. Otherwise, i.e., if $\pi \in [\underline{\pi}, \bar{\pi}]$, there exists a threshold $k_\pi \in [\sqrt{\frac{c}{4\alpha}}, \sqrt{\frac{c}{4\Delta}}]$ such that $\tilde{\theta}_I = \bar{\theta}_I(\pi)$ if $k < k_\pi$ and $\tilde{\theta}_I = \hat{\theta}_I(\pi)$ otherwise. Moreover, k_π is increasing in π.*

(iii) *The seller's prices $\tilde{t}_0$ and $\tilde{t}_1$ solve (BR) and (IC) evaluated at $\tilde{\theta}_I$.*

As the public signal becomes more precise, the boundary conditions (L_1) and (U_0) become tighter and there is less room for the intermediary to garble information. When the conditions start binding, the optimal partial disclosure policy implements $\tilde{\theta}_I = \bar{\theta}_I(\pi)$. Note that when the boundary conditions hold with equality (i.e, when they are binding), the intermediary's (garbled) signal is pure noise as it does not contain any information on the product quality. This implies that the investment threshold in this case is the same as the one under no intermediary, that is, $\bar{\theta}_I(\pi) = \theta_I^{NI}(\pi)$.

We can now characterize the intermediary's optimal policy by comparing his payoff under full disclosure and partial disclosure (with full market coverage).

Proposition 4. (*Optimal disclosure policy*) *There exists a threshold π^{FD} ($\pi^{FD} > \bar{\pi}$, where $\bar{\pi}$ is as defined in Lemma 4) such that:*

(i) *If $\pi > \pi^{FD}$, full disclosure is optimal for all k.*

(ii) *Otherwise, there exists an interval $(k_1(\pi), k_2(\pi))$, where $k_1(\pi) \leq k^* \leq k_2(\pi)$, such that partial disclosure is strictly optimal if and only if $k \in (k_1(\pi), k_2(\pi))$. Moreover, $(k_1(\pi), k_2(\pi)) = (\sqrt{\frac{c}{4\Delta}}, \frac{1}{2})$ for $\pi < \bar{\pi}$, and the interval shrinks with π for $\pi \in [\bar{\pi}, \pi^{FD}]$.*

Proposition 4 illustrates how the optimal disclosure policy varies with the entry cost and the precision of the public signal. When the public signal is relatively imprecise, partial disclosure is optimal only if the entry cost is sufficiently large. Otherwise, partial disclosure becomes optimal only for a moderate range of entry cost—if the cost is too high or too low, the intermediary resorts to full disclosure. Moreover, this range of entry cost (i.e., where partial disclosure is optimal) gets smaller as the public signal becomes more informative; when the public signal is sufficiently precise (i.e., $\pi > \pi^{FD}$) full disclosure is optimal regardless of the cost of entry.

To see the intuition for the result, recall the trade-off with certification fee under full disclosure. A larger fee extracts more rents from the seller who uses the intermediary but reduces the likelihood that the seller would use the intermediary at the first place. The reduction in the demand for the intermediary's service stems from the fact that when the intermediary extracts a larger share of the trade surplus, both the investment and the entry incentives of the seller become muted.

But this trade-off could be softened with partial disclosure. By partially pooling the low quality seller with the high quality one, the intermediary can ensure that an entrant gets a relatively high price from the buyer even if he ends up with a low quality product. Thus, such

a policy can induce all types of the seller to enter and use the intermediary irrespective of the product quality while allowing the intermediary to charge a moderately large certification fee. (Though investment incentives are dampened under partial disclosure it does not affect the intermediary's payoff under partial disclosure as the seller uses the intermediary irrespective of his product quality.) The resulting profit dominates its counterpart under full disclosure when the entry cost is sufficiently large. Recall that when the entry cost is high, under full disclosure the intermediary either offers a deep discount in order to induce entry for all types or forecloses the market for the types with relatively high cost of investment, considerably dampening the intermediary's payoff.
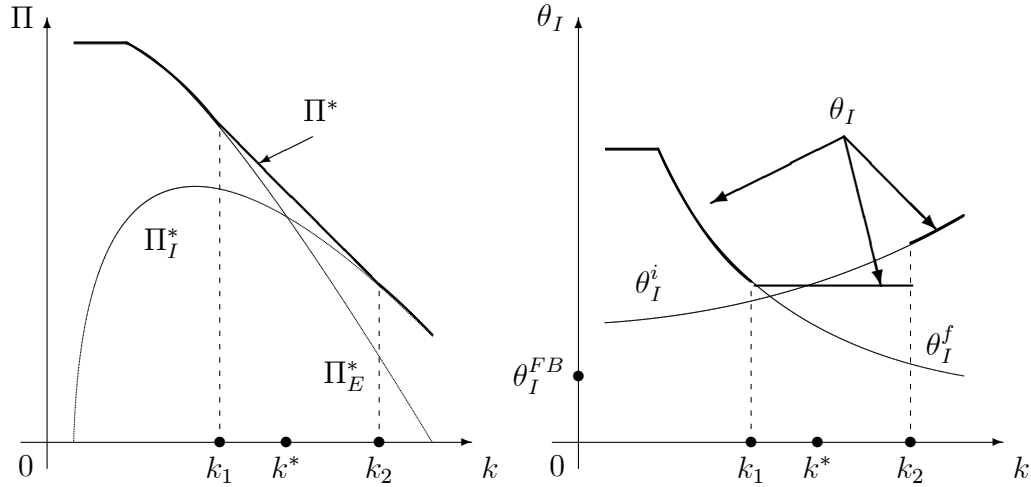

Figure 2. Intermediary's payoff (Π^*) and associated cutoff type for investment (θ_I) under optimal disclosure policy (partial disclosure is optimal for $k \in (k_1, k_2)$).

But when the public signal gets more precise the boundary conditions (L_1) and (U_0) start to bind—we have $t_0 = \bar{t}_0$ and $t_1 = \underline{t}_1$—and there is little room for the intermediary to garble information. So, even if the intermediary sends a completely uninformative signal, it must lower its certification fee considerably to meet the (IR_E) and (IR_C) constraints— i.e., to induce the seller to enter and certify his product.[7] Thus, the intermediary's payoff drops, and such a partial disclosure policy (i.e., "no disclosure") cannot be optimal unless the entry cost is at an intermediate level. Recall that when the entry cost is relatively low, the certification fee under full disclosure remains relatively large yielding a higher payoff. And for a sufficiently large entry cost, accommodating entry for all types would call for a significant price concession even under partial disclosure (i.e., (IR_E) gets tighter) and full disclosure with restricted entry becomes more profitable.

Finally, as the public signal gets more precise, $\bar{t}_0$ becomes smaller; i.e., (IR_C) gets tighter reducing the intermediary's payoff. As a result, partial disclosure policy (with full market coverage) becomes less attractive whereas the profit from full disclosure remains unaltered.

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[7]Notice that even if the intermediary does not provide any information whatsoever it still has a demand for its service. This is due to the fact that a seller who does not use the intermediary is believed to be a low-quality seller.

Hence, the range of entry cost for which partial disclosure is optimal shrinks; and full disclosure becomes optimal irrespective of the cost of entry when the public signal is sufficiently precise.

Figure 2 depicts a generic example of the intermediary's payoff and the associated investment cutoff as a function of the entry cost k under the optimal disclosure policy when $\pi < \pi^{FD}$. The intermediary resorts to partial disclosure for $k \in (k_1, k_2)$ but adopts a full disclosure policy otherwise. The equilibrium investment cutoff is weakly decreasing when $k < k_2$, and is constant over the range of k where partial disclosure is optimal. Also, for any $k < k_2$, entry is always efficient. However, for $k \geq k_2$ there is a discontinuous jump in the cutoff as the intermediary switches from partial disclosure with full entry to full disclosure with restricted entry.

6. Discussion and conclusion

6.1. **Implementation of the Optimal Disclosure Policy.**
The optimal disclosure policy, as given by Proposition 4, lends itself to a simple implementation rule. As shown in the following proposition the intermediary only needs to use a binary signal structure: the high quality always gets a "good" signal whereas the low quality may get a "good" signal some of the time and a "bad" signal otherwise.

Proposition 5. *Without loss of generality, we can restrict attention to disclosure policies where $X = \{x_0, x_1\}$, $\Pr(x_1 \mid v = 1) = 1$ and $\Pr(x_1 \mid v = 0) = \rho$. If the optimal policy is full disclosure, $\rho = 0$. If the optimal policy is partial disclosure with seller's expected bids $(\tilde{t}_0, \tilde{t}_1)$ and investment type cutoff $\tilde{\theta}_I$, then $\rho \in (0, 1]$ solves*

$$(PD) \quad \tilde{t}_1 = \frac{\pi^2 v(\tilde{\theta}_I)}{\pi v(\tilde{\theta}_I) + \rho(1 - \pi)\left(1 - v(\tilde{\theta}_I)\right)} + \frac{(1 - \pi)^2 v(\tilde{\theta}_I)}{(1 - \pi) v\left(\tilde{\theta}_I\right) + \rho \pi \left(1 - v(\tilde{\theta}_I)\right)} =: \tau(\rho, \pi).$$

Proposition 5 is also useful in uncovering how the informativeness of the intermediary's report interacts with the precision of the public signal. Fix any $\pi < \bar{\pi}$ and any $k > \sqrt{\frac{c}{4\Delta}}$ (i.e., partial disclosure is strictly optimal under such parameters). Recall from Lemma 4 that for any $\pi < \bar{\pi}$, the boundary conditions (L_1) and (U_0) are slack and $\tilde{\theta}_I = \hat{\theta}_I(k)$ for any k; therefore, $\tilde{t}_i$s remain constant. In other words, when the public signal gets more precise, as long as $\pi < \bar{\pi}$, the intermediary continues to filter information so that the expected bids for both low and high quality sellers remain unaltered. As the right-hand side of (PD) (i.e., $\tau(\rho, \pi)$) is increasing in π and decreasing in ρ, this implies that ρ must increase with π as long as $\pi < \bar{\pi}$.

In contrast, when the public signal is sufficiently precise, i.e., if $\pi \geq \bar{\pi}$, the boundary conditions bind; so for any k where partial disclosure is still optimal, we must have $\tilde{t}_1 = \underline{t}_1$ and $\tilde{\theta}_I = \bar{\theta}_I(\pi)$. But notice that

$$\underline{t}_1 = \tau(1, \pi).$$

So, the optimal partial disclosure policy sets $\rho = 1$ for all such π, i.e., "no disclosure" becomes optimal as the intermediary sends the same signal irrespective of the product quality.

Moreover, as π increases further, the optimal policy eventually switches from partial (or "no disclosure") to full disclosure. Thus, we have the following corollary.

Corollary 1. *For any given $k > \sqrt{\frac{c}{4\Delta}}$ (i.e., a value of k such that partial disclosure is strictly optimal if $\pi < \bar{\pi}$) there exists a threshold of π (that depends on k), π_k^{FD}, such that ρ is strictly increasing in π for $\pi < \bar{\pi}$, $\rho = 1$ for $\pi \in \left[\bar{\pi}, \pi_k^{FD}\right]$ and $\rho = 0$ if $\pi > \pi_k^{FD}$.*

The key implication of the above finding is that as the public signal gets more precise, the intermediary signal becomes less informative; but if the public signal becomes too precise, partial disclosure ceases to be optimal and the intermediary's signal becomes perfectly informative (as the intermediary resorts to full disclosure). In other words, when the public signal is relatively noisy, in the optimal policy the informativeness of the intermediary's signal behaves as a substitute to that of the public signal. However, if the public signal becomes sufficiently precise, they become complements as the intermediary opts for full disclosure.

6.2. **Welfare Implications.** How does the presence of the intermediary affect the social welfare in terms of aggregate surplus? Also, how does the aggregate surplus vary as the public signal gets more precise? Notice that for any given type cutoff for entry (θ_E) and investment (θ_I) the aggregate surplus is given as:

$$W\left(\theta_E, \theta_I\right) = \left(\theta_I - \theta_E\right)\left(\frac{1}{2} - k\right) + \int_{\theta_I}^{1} \left(\alpha - \frac{c}{\theta} - k\right) d\theta.$$

Recall that (from Proposition 1) in the absence of the intermediary, all types enter (i.e., $\theta_E = 0$) and all types above θ_I^{NI} invest if and only if the cost of investment c is not too large (i.e., $c < \bar{c} := \left(2\pi - 1\right)^2 \Delta$). Thus, the aggregate surplus in the absence of the intermediary is given as:

$$(5) \qquad W_{NI} = \begin{cases} W\left(0, 1\right) & if \ \pi < \frac{1}{2} + \sqrt{\frac{c}{4\Delta}} =: \pi^{NI} \\ W(0, \theta_I^{NI}) & otherwise \end{cases} .$$

In the presence of the intermediary, the type thresholds for entry and investment depend both on the entry cost (k) and the precision of the public signal (π). Using Proposition 4, the aggregate surplus in equilibrium can be obtained as follows:

$$(6) \qquad \begin{aligned} & if \ \left(k_1\left(\pi\right), k_2\left(\pi\right)\right) \neq \emptyset, \\ & \qquad W_I = \begin{cases} W(0, \theta_I^f) & if \ k \leq k_1\left(\pi\right) \\ W\left(\theta_I^i, \theta_I^i\right) & if \ k \geq k_2\left(\pi\right) \\ W(0, \tilde{\theta}_I) & otherwise \end{cases}, \\ & and \ if \ \left(k_1\left(\pi\right), k_2\left(\pi\right)\right) = \emptyset, \\ & \qquad W_I = \begin{cases} W(0, \theta_I^f) & if \ k \leq k^* \\ W\left(\theta_I^i, \theta_I^i\right) & if \ k \geq k^* \end{cases}. \end{aligned}$$

where θ_I^f and θ_I^i (type thresholds for investment under full disclosure with and without full entry, respectively) are as given in Proposition 2, and k^* is as given in Lemma 2. The following proposition describes the behavior of W_I with respect to π.

Proposition 6. *The following holds:*

(i) $\partial W_I / \partial \pi \neq 0$ *only if* $\pi \in \left[\overline{\pi}, \pi^{FD}\right]$.

(ii) *Suppose that* π *increases from* π' *to* π'' *where* $\pi' < \pi^{FD}$ *and* $\pi'' > \overline{\pi}$. *(a) If* $\pi'' < \pi^{FD}$, *i.e.,* $(k_1(\pi''), k_2(\pi'')) \neq \emptyset$, W_I *(strictly) increases if* $k \in [k_1(\pi'), k_2(\pi'')]$, *decreases if* $k \in [k_2(\pi''), k_2(\pi')]$, *and remains constant if* $k < k_1(\pi')$ *or* $k > k_2(\pi')$. *(b) If* $\pi'' > \pi^{FD}$, *i.e., full disclosure is optimal for all* k *at* π'' *as* $(k_1(\pi''), k_2(\pi'')) = \emptyset$, W_I *(strictly) increases if* $k \in [k_1(\pi'), k^*]$, *but decreases otherwise.*

(iii) $W_I - W_{NI}$ *is (weakly) decreasing in* π.

The above result shows that the welfare under intermediary is non-monotonic in the precision of the public signal. In particular, W_I increases in π only if the entry cost lies in a moderate range. However, the "value of the intermediary", i.e., the gains in the aggregate welfare when the intermediary is present vis-a-vis when it is not $(W_I - W_{NI})$, always decreases when the public signal gets more precise.

To see the intuition, notice that a change in π affects the social welfare, both with and without the intermediary's presence. In the absence of the intermediary, a more precise public signal elicits more investment (θ_I^{NI} is decreasing in π) and increases welfare. The optimal disclosure policy of the intermediary, on the other hand, varies with π and affect both entry and investment.

First, consider the impact of π on W_I. As π increases, say from π' to π'', the range of k over which partial disclosure is optimal—i.e., $(k_1(\pi), k_2(\pi))$—shrinks (Proposition 4), from $[k_1(\pi'), k_2(\pi')]$ to $[k_1(\pi''), k_2(\pi'')]$. So, when $(k_1(\pi''), k_2(\pi'')) \neq \emptyset$ we have $k_1(\pi') < k_1(\pi'') < k^* < k_2(\pi'') < k_2(\pi')$.

Now, for relatively low or high values of k (i.e., for $k < k_1(\pi')$ or $k > k_2(\pi')$), the disclosure policy stays the same and there is no change in welfare. But for moderately low values of k (i.e., for $k \in [k_1(\pi'), k_2(\pi'')]$) welfare increases as more types invest in quality. The investment increases due to two reasons: (i) for $k \in [k_1(\pi'), k_1(\pi''))$, the policy switches from partial to full disclosure leading to stronger incentive for investment (without distorting entry). (ii) And for $k \in (k_1(\pi''), k_2(\pi''))$, while the intermediary continues to adopt partial disclosure, investment incentives are still stronger as the buyers have more information (notice that the public signal is now more informative, and the intermediary's signal must (weakly) augment the information already available from the public signal). Finally, for moderately high values of k (i.e., $k \in (k_2(\pi''), k_2(\pi'))$), the policy switches from partial to full disclosure leading to inefficiently low entry. As a result, the welfare drops.

Next, consider the impact of π on the value of the intermediary, $W_I - W_{NI}$. Clearly, the welfare in absence of the intermediary, W_{NI}, increases in π as the investment cutoff θ_I^{NI} decreases in π—when the buyers only rely on the public signal, a more precise signal increases the incentives for investment in product quality. But as shown above, the welfare under intermediary, W_I, may also increase in π—either because the optimal policy switches from partial to full disclosure (and entry remains efficient) or the buyers' information on

product quality (i.e., considering both the public and the intermediary's signals) improves even if the intermediary offers partial disclosure. In both cases, however, the gains in W_I is (weakly) less than that in W_{NI}. For values of k where there is a switch from partial to full disclosure, the cutoff type for investment remains larger than its counterpart in the absence of intermediary (i.e., $\theta_I^f > \theta_I^{NI}$). And for values of k where partial disclosure remains optimal, the investment cutoffs are the same with and without the intermediary (i.e., $\tilde{\theta}_I = \theta_I^{NI}$).

In this context, two remarks are in order. First, the above argument implies that there is a threshold level in the precision of the public signal above which the intermediary's presence may be detrimental to social welfare (particularly, when the entry cost is sufficiently high). Second, our analysis also shed light on the merits of the regulation that mandates full disclosure from the intermediary. Suppose that $k \in (k_1(\pi), k_2(\pi))$, the parameter space in which partial disclosure is optimal and the full disclosure regulation has a bite. Recall that under partial disclosure, the entry is always efficient. Therefore, full disclosure cannot improve on the entry incentives. But full disclosure regulation can still improve welfare by enhancing investment incentives, provided that $k \in (k_1(\pi), k^*)$, and hence entry is still efficient. However, for $k \in (k^*, k_2(\pi))$, under full disclosure, the intermediary induces only the relatively high types of the seller to enter market as maintaining the full entry is too costly under such regulation. (In other words, the full disclosure regulation induces a regime change from $(\theta_E, \theta_I) = (0, \tilde{\theta}_I)$ to (θ_I^i, θ_I^i).) As shown in the proof of Proposition 6 (ii), such regime change leads to a welfare loss due to a discrete downward jump in the extent of entry. Thus, the full disclosure regulation does not necessarily leads to welfare gains.

6.3. **Conclusion.** We present a model of certification intermediary where the intermediary's policy influences the seller's investments in product quality as well as his decision on market entry. In our setting, the presence of an intermediary creates a novel trade-off: it improves the seller's incentives to investment in quality upon entry but mutes his entry incentives ex-ante. The intermediary faces a canonical monopoly problem where a high certification fee facilitates rent extraction from the seller but reduces the demand for certification service at the first place, due to distortions in both entry and investment incentives of the seller. We argue that a partial disclosure policy may be optimal as it can allow the intermediary to charge a relatively high certification fee without causing a large distortion in entry and investment.

A key insight that emerges from our model is that the informativeness of the intermediary's signal varies non-monotonically with the public signal's precision. When the public signal is relatively noisy, the intermediary's disclosure policy behaves as a substitute to the public signal—intermediary's report becomes less informative as the public signal becomes more precise. But when the public signal becomes sufficiently precise, the intermediary's signal may complement its public counterpart as the intermediary resorts to full disclosure. Our model also indicates that under high entry cost the optimal certification policy with full disclosure may call for restricted entry. As a result, an increase in the precision of the public signal may reduce social welfare. Therefore, in markets with certification intermediaries commonplace interventions such as mandatory disclosure requirements or provision of additional public information that are geared towards alleviating the information asymmetry may be counterproductive and should be used with caution.

7. APPENDIX

This appendix contains the proofs omitted in the text.

Proof of Proposition 1. Step 1. As for any Θ_E and Θ_I, $V(1; \theta, \Theta_E,\ \Theta_I)$ is increasing in θ whereas $V(0; \theta, \Theta_E,\ \Theta_I)$ is constant, in any equilibrium, the seller's investment decision must follow a cutoff strategy as the seller invests if and only if $V(1; \theta, \Theta_E,\ \Theta_I) \geq V(0; \theta, \Theta_E,\ \Theta_I)$.

Let θ' be the investment cutoff type, that is, $\Theta_I = [\theta', 1]$. For brevity of notation, denote

$$v_i(\theta') = v_i(\Theta_E = [0,1],\ \Theta_I = [\theta', 1]),$$

and

$$\mathbb{E}(v \mid z;\ \theta') = \mathbb{E}(v \mid z;\ \Theta_E = [0,1],\ \Theta_I = [\theta', 1]).$$

We need to show that there exists a unique θ_I^{NI} such that (i) $v_1(\theta_I^{NI}) - v_0(\theta_I^{NI}) = \frac{c}{\theta_I^{NI}}$ and (ii) $V(0; \theta, \Theta_E,\ \Theta_I) = v_0(\theta_I^{NI}) - k > 0$.

Step 2. It is useful to note that $\Pr(z = 1 \mid I) = \sum_{k \in \{0,1\}} \Pr(z = 1 \mid v = k) P(v = k \mid I)$; so,

$$\Pr(z = 1 \mid I = 1) = \pi\alpha + (1 - \pi)(1 - \alpha), \text{ and } \Pr(z = 1 \mid I = 0) = 1/2.$$

Also,

$$\mathbb{E}(v \mid z;\ \theta') = \Pr(v = 1 \mid z;\ \theta') = \frac{\Pr(z \mid v = 1)\Pr(v = 1 \mid \theta')}{\Pr(z \mid \theta')}.$$

We thus have

$$\mathbb{E}(v \mid z = 1;\ \theta') = \frac{\pi(\alpha - \theta'\Delta)}{\pi(\alpha - \theta'\Delta) + (1 - \pi)(1 - \alpha + \theta'\Delta)},$$

and

$$\mathbb{E}(v \mid z = 0;\ \theta') = \frac{(1 - \pi)(\alpha - \theta'\Delta)}{\pi(1 - \alpha + \theta'\Delta) + (1 - \pi)(\alpha - \theta'\Delta)}.$$

Step 3. Note that

$$\frac{\partial}{\partial\theta'}\mathbb{E}(v \mid z = 1;\ \theta') = -\frac{(1 - \pi)\pi\Delta}{(1 - (\pi - \alpha(2\pi - 1) + (2\pi - 1)\theta'\Delta))^2} < 0,$$

and

$$\frac{\partial}{\partial\theta'}\mathbb{E}(v \mid z = 0;\ \theta') = -\frac{\alpha(1 - \pi)\pi\Delta}{(\pi - \alpha(2\pi - 1) + (2\pi - 1)\theta'\Delta)^2} < 0.$$

This implies that both v_1 and v_0 are decreasing functions of θ'. Note that $v_0(1) = \frac{1}{2}$. So, $v_0(\theta') \geq \frac{1}{2} > k$ for any investment cutoff type $\theta' \in [0, 1]$. Hence, all types of the seller enter for any investment cutoff θ'.

Step 4. Now we show that there exists a unique type cutoff for investment. Let $\psi(\theta') = v_1(\theta') - v_0(\theta')$ denote the private value of investment in the presence of adverse selection due to asymmetric information about the product's quality. This value depends on the investment cutoff θ'. Using the above expressions, we obtain

$$\frac{\partial}{\partial\theta'}\psi(\theta') = \frac{1}{(D(1 - D))^2}\left[\pi(1 - \pi)(\Delta(2\pi - 1))^2(2\alpha_1 - 1 - 2\theta'\Delta)\right].$$

where D is a linear function of θ'. Note that $\alpha \geq \alpha - \theta'\Delta \geq 1/2$ since $2\alpha - 1 - 2\theta'\Delta \geq 0$. As a result, we have $\frac{\partial}{\partial \theta'}\psi(\theta') \geq 0$. Furthermore, we have $\psi(1) = (2\pi - 1)^2 \Delta =: \bar{c}$, say.

Step 5. So, if $c > \bar{c}$, $c/\theta' > \psi(\theta')$ for all θ' and none of the types invests. Otherwise, $\psi(\theta' = 1) > c$ and $\psi(\theta') < c/\theta'$ as $\theta' \to 0$. Hence, there exists a cutoff type, θ_I^{NI} (say), such that $\psi(\theta_I^{NI}) = c/\theta_I^{NI}$. Moreover, as $\psi(\theta')$ is increasing in θ' and $\psi(1) = \bar{c} < \Delta$, we must have $\psi(\theta') < \Delta$ for all θ'. So, $\theta_I^{NI} > \theta_I^{FB}$ (otherwise, we must have $\psi(\theta_I^{NI}) = c/\theta_I^{NI} > c/\theta_I^{FB} = \Delta$, which is a contradiction).

Step 6. Finally, note that

$$\frac{\partial}{\partial \pi}\psi(\theta') = \frac{1}{(D(1-D))^2}A(1-A)(2\pi - 1)\Delta > 0,$$

where $A = \alpha(1 - \theta') + \frac{1}{2}\theta' \in \left(\frac{1}{2}, 1\right)$. Hence, θ_I^{NI} is decreasing in π. ∎

Proof of Lemma 2. Recall that depending on the intermediary's price, we can have two cases: (I) $p \leq p_E$ (*Full entry regime*): In this case, all types enter (i.e., $\theta_E = 0$) and type θ invests if and only if:

$$(7) \qquad \alpha(1-p) - \frac{c}{\theta} \geq \frac{1}{2}(1-p), \text{ or, } \theta \geq \theta_I^f := \frac{c}{(1-p)\Delta},$$

with the superscript f denotes the full entry regime.

(II) $p \in (p_E, \bar{p}]$ (*Investment-type-only entry regime*): In this case, it is suboptimal for a seller to enter if he is not going to invest on quality; all entrants invest and we have $\Theta_I = \Theta_E$. Hence, type θ enters and invests if

$$(8) \qquad \alpha(1-p) - \frac{c}{\theta} - k \geq 0, \text{ or, } \theta \geq \theta_I^i := \frac{c}{\alpha(1-p) - k},$$

and all types $\theta < \theta_I^i \, (= \theta_E)$ stay out, where the superscript i denotes the investment-type-only entry regime.

Now, using (7) and (8), it is routine to derive the following first-order conditions:

$$(9) \qquad p_E^* = \arg\max_{p < p_E} \Pi_E = \begin{cases} 1 - \sqrt{\frac{c}{\alpha}} & \text{if } k < k_E := \sqrt{\frac{c}{4\alpha}} \\ 1 - 2k & \text{otherwise} \end{cases},$$

and

$$(10) \quad p_I^* = \arg\max_{p > p_E} \Pi_I = \max\left\{1 - \frac{1}{\alpha}\left(k + \sqrt{c(\alpha - k)}\right), 1 - 2k\right\}$$

$$= \begin{cases} 1 - \frac{1}{\alpha}\left(k + \sqrt{c(\alpha - k)}\right) & \text{if } k > k_I := \frac{1}{8\Delta^2}\left(-c + \sqrt{c^2 + 16c\alpha\Delta^2}\right) \\ 1 - 2k & \text{otherwise} \end{cases}.$$

As $c < \Delta$ from Assumption 1 (i), we obtain $k_E < k_I$. Now, the proof directly follows from equations (7), (8), (9), (10) and the fact that there exists a unique $k^* \in (k_I, 1/2)$ such that $\Pi_I^* \geq \Pi_E^*$ iff $k \geq k^*$. The proof of this claim is given in the following steps:

Step 1: Recall from Assumption 1 (ii) that $k \in \left[\frac{c}{2\Delta}, \frac{1}{2}\right]$. Observe that at $k = \frac{1}{2}$, $\Pi_I^* = \overline{\Pi}_I >$ $\Pi_E^* = 0$ (note that at $k = \frac{1}{2}$, $\overline{\Pi}_I = (\sqrt{\Delta} - \sqrt{c})^2 > 0$ as $\Delta > c$ by Assumption 1) and at $k = \frac{c}{2\Delta}$, $\Pi_I^* = \underline{\Pi}_I = 0 < \Pi_E^* = \underline{\Pi}_E$. As both Π_I^*s are continuous functions of k, they must intersect at some $k = k^*$, say. In what follows, we argue that k^* is unique and $k > k_I$.

Step 2: *We must have $k^* > k_I$; and at k^*, we have $\Pi_E^* = \underline{\Pi}_E = \overline{\Pi}_I = \Pi_I^*$.* The proof is as follows: Note that $\overline{\Pi}_E > \overline{\Pi}_I \ \forall k \in \left(\frac{c}{2\Delta}, \frac{1}{2}\right)$. To see this, observe that the above inequality can be simplified as:

$$2\sqrt{c\alpha} < k + 2\sqrt{c\left(\alpha - k\right)} \ \forall k \in \left(\frac{c}{2\Delta}, \frac{1}{2}\right).$$

Now, $f\left(k\right) := k + 2\sqrt{c\left(\alpha - k\right)}$ is an increasing function of k ($f' = 1 - \sqrt{c/(\alpha - k)} > 0$ as $\alpha - k > c$) where $f\left(0\right) = 2\sqrt{c\alpha}$. Furthermore, we must have $\overline{\Pi}_I \geq \underline{\Pi}_I \ \forall k \in \left(\frac{c}{2\Delta}, \frac{1}{2}\right)$ as the left-hand side is the value under unconstrained optimum (notice that by definition of k_I, the equality holds only under $k = k_I$). Combining the two, we get $\overline{\Pi}_E > \overline{\Pi}_I \geq \underline{\Pi}_I \ \forall k \in \left(\frac{c}{2\Delta}, \frac{1}{2}\right)$.

As we know Π_E^* intersects Π_I^* at k^*, it must be the case that at k^*, $\Pi_E^* = \underline{\Pi}_E = \Pi_I^*$. So, $k^* \in (k_E, 1/2)$. Now, recall that $k_E < k_I$, and so, for all $k \in (k_E, k_I]$, $\Pi_E^* = \underline{\Pi}_E > \underline{\Pi}_I = \Pi_I^*$ (as $\alpha > \Delta$). As we know $\underline{\Pi}_E$ intersects Π_I^* at k^*, it must be the case that $k^* > k_I$ and at k^*, we have $\Pi_E^* = \underline{\Pi}_E = \overline{\Pi}_I = \Pi_I^*$.

Step 3: Recall that at $k = k_I$, $\underline{\Pi}_E > \overline{\Pi}_I = \underline{\Pi}_I$ (as $\Delta < \alpha$) and at $k = 1/2$, $\underline{\Pi}_E < \overline{\Pi}_I$. Moreover $\underline{\Pi}_E$ is concave in k ($\partial^2 \underline{\Pi}_E / \partial k^2 = -c/k^3 < 0$) and $\overline{\Pi}_I$ is convex in k ($\partial^2 \overline{\Pi}_I / \partial k^2 = \sqrt{c}/(2(\alpha - k)^{3/2}) > 0$). Hence k^* must be unique. The proof is by contradiction. Suppose $\underline{\Pi}_E = \overline{\Pi}_I$ at multiple $k \in (k_I, 1/2)$ and let k' and k'' be the smallest and the largest solutions. As $\underline{\Pi}_E\left(k\right) > \overline{\Pi}_I\left(k\right)$ for $k \in (k_I, k')$, and $\underline{\Pi}_E\left(k\right) < \overline{\Pi}_I\left(k\right)$ for $k \in (k'', 1/2)$, we have

$$\underline{\Pi}'_E\left(k'\right) < \overline{\Pi}'_I\left(k'\right) \ \text{and} \ \underline{\Pi}'_E\left(k''\right) > \overline{\Pi}'_I\left(k''\right).$$

But as $\underline{\Pi}_E$ is concave in k, we have $\underline{\Pi}'_E\left(k'\right) > \overline{\Pi}'_E\left(k''\right)$. So, we must have

$$\overline{\Pi}'_I\left(k'\right) > \underline{\Pi}'_E\left(k'\right) > \overline{\Pi}'_E\left(k''\right) > \overline{\Pi}'_I\left(k''\right).$$

But this inequality contradicts the fact that $\overline{\Pi}_I$ is convex in k (as we must have $\overline{\Pi}'_I\left(k'\right) < \overline{\Pi}'_I\left(k''\right)$). ∎

Proof of Lemma 3. It is instructive to argue part (ii) first. This claim follows directly from the fact that both the low and high quality sellers must find it sequentially rational to use the intermediary if pooling between qualities is feasible at the first place.

We prove part (i) by contradiction. In any equilibrium where only some types enter, it must be the case that entry is profitable only if the seller invests. Now, we already argued that both types must use the intermediary in equilibrium. So, the intermediary's price cannot exceed t_0. Hence, for any entry cutoff θ_I, the intermediary's payoff is $(1 - \theta_I)\,t_0$. Moreover, the marginal type must be willing to enter whereas it must be unprofitable to enter if the firm does not undertake any investment. Hence, the optimal policy must satisfy the following constraints: (i) Bayes Rationality $(BR): \alpha t_1 + (1 - \alpha)\,t_0 = \alpha$; (b) Entry for marginal type $(E_\theta): \alpha - k - c/\theta_I \geq t_0$; (c) Essentiality of investment $(E_I): t_0 \geq \frac{1}{2}\,(t_0 + t_1) - k$. Hence, the intermediary's problem is:

$$\max_{t_0,\ t_1,\ \theta_I}\ (1-\theta_I)\,t_0 \quad s.t.\ (BR),\ (E_\theta),\ (E_I).$$

Consider a relaxed program by ignoring (BR) and (E_I). The optimal $\theta_I = \sqrt{c/(\alpha-k)}$ which is same as in Proposition 2. Now $t_0 = \alpha - k - \sqrt{c(\alpha-k)}$. From (BR), we have $t_1 = 1 - \frac{1-\alpha}{\alpha}t_0$. So, from (E_I) we obtain that the solution is feasible if $k\Delta > \frac{1}{2}\sqrt{c(\alpha-k)}$. That is, if k is sufficiently large, the solution to the original program is same as full disclosure solution. But otherwise, the payoff must be (weakly) lower than that in the full disclosure case. Hence, if partial disclosure is optimal, it must be the case that all types enter. ∎

Proof of Lemma 4. (i) We solve $\mathcal{P}$ in the following two steps:

Step 1: Consider the relaxed problem $\mathcal{P}_R$ obtained from $\mathcal{P}$ by ignoring (L_1) and (U_0), and replacing (IC) by

$$\theta_I = \frac{c}{(t_1-t_0)\,\Delta} \quad (IC_R).$$

Step 1a: Using (IC_R) and (BR) constraints to solve for t_is, we obtain:

$$(11) \qquad t_0\,(\theta_I) = c + \alpha\,(1-\theta_I) + \frac{\theta_I}{2} - \frac{c\alpha}{\theta_I\Delta},\ t_1\,(\theta_I) = c + \alpha\,(1-\theta_I) + \frac{\theta_I}{2} + \frac{c\,(1-\alpha)}{\theta_I\Delta}.$$

So, $\mathcal{P}_R$ boils down to:

$$\max_{\theta_I\in[0,1]}\ p\,(\theta_I) = \min\left\{\frac{1}{2}\,(t_0\,(\theta_I) + t_1\,(\theta_I)) - k, t_0\,(\theta_I)\right\}.$$

Step 1b: Notice the following: (i) both $t_0\,(\theta_I)$ and $t_0\,(\theta_I)+t_1\,(\theta_I)$ are concave in θ_I; hence, so is $p\,(\theta_I)$. (ii) $\left(\frac{1}{2}\,\{t_0\,(\theta_I)+t_1\,(\theta_I)\} - k\right) - t_0\,(\theta_I)$ is decreasing in θ_I and has a unique root $\frac{c}{2k\Delta}$. Hence,

$$p\,(\theta_I) = \begin{cases} t_0\,(\theta_I) & if\ \theta_I < \frac{c}{2k\Delta} \\ \frac{1}{2}\,\{t_0\,(\theta_I)+t_1\,(\theta_I)\} - k & otherwise \end{cases}$$

(iii) And finally,

$$\arg\max_{\theta_I} t_0\,(\theta_I) = \frac{1}{\Delta}\sqrt{c\alpha},\ \text{and}\ \arg\max_{\theta_I}\frac{1}{2}\,\{t_0\,(\theta_I)+t_1\,(\theta_I)\} - k = \sqrt{\frac{c}{\Delta}}.$$

where $\sqrt{\frac{c}{\Delta}} < \frac{1}{\Delta}\sqrt{c\alpha}$.

Step 1c: The observations (i) to (iii) imply the following: (a) if $\frac{1}{\Delta}\sqrt{c\alpha} < \frac{c}{2k\Delta}$, i.e., if $k < \frac{1}{2}\sqrt{\frac{c}{\alpha}}$,

$$\max\ p\,(\theta_I) = \max\ t_0\,(\theta_I),$$

(b) if $\frac{c}{2k\Delta} < \sqrt{\frac{c}{\Delta}}$, i.e., $k > \frac{1}{2}\sqrt{\frac{c}{\Delta}}$,

$$\max\ p\,(\theta_I) = \max\ \frac{1}{2}\,\{t_0\,(\theta_I)+t_1\,(\theta_I)\} - k,$$

(c) otherwise,

$$\max\ p\,(\theta_I) = t_0\left(\frac{c}{2k\Delta}\right).$$

Hence, the solution to the relaxed problem is as follows:

(12)
$$\hat{\theta}_I = \begin{cases} \frac{1}{\Delta}\sqrt{c\alpha} & if \ k < \frac{1}{2}\sqrt{\frac{c}{\alpha}} \\ \frac{c}{2k\Delta} & if \ k \in \left[\frac{1}{2}\sqrt{\frac{c}{\alpha}}, \frac{1}{2}\sqrt{\frac{c}{\Delta}}\right] \\ \sqrt{\frac{c}{\Delta}} & if \ k > \frac{1}{2}\sqrt{\frac{c}{\Delta}} \end{cases}.$$

It is routine to check that $\hat{\theta}_I$ is decreasing in k.

Step 2: Consider the original problem $\mathcal{P}$. Note that the solution to $\mathcal{P}_R$ satisfies (IC) as $\frac{1}{\Delta}\sqrt{c\alpha} < 1$ (by Assumption 1). So, it remains to check when $\hat{\theta}_I$ may violate the constraints (L_1) and (U_0) and what is the solution to $\mathcal{P}$ in such a scenario. Notice that:

$$\underline{t}_1(\theta_I) = \frac{\pi^2 v(\theta_I)}{\pi v(\theta_I) + (1-\pi)(1-v(\theta_I))} + \frac{(1-\pi)^2 v(\theta_I)}{(1-\pi)v(\theta_I) + \pi(1-v(\theta_I))},$$

and

$$\bar{t}_0(\theta_I) = \frac{\pi(1-\pi)v(\theta_I)}{(1-\pi)v(\theta_I) + \pi(1-v(\theta_I))} + \frac{(1-\pi)\pi v(\theta_I)}{\pi v(\theta_I) + (1-\pi)(1-v(\theta_I))},$$

where, as defined earlier, $v(\theta_I) = \frac{1}{2}\theta_I + \alpha(1-\theta_I)$.

Step 2a: For $\theta_I \leq \frac{1}{\Delta}\sqrt{c\alpha}$, it is routine to check that (i) $t_1(\theta_I) - \underline{t}_1(\theta_I)$ is (strictly) decreasing and $t_0(\theta_I) - \bar{t}_0(\theta_I)$ is (strictly) increasing in θ_I. (ii) $\underline{t}_1(\theta_I)$ is increasing in π and for all θ_I, $\underline{t}_1(\theta_I) \to 1$ as $\pi \to 1$. (iii) $\bar{t}_0(\theta_I)$ is decreasing in π and for all θ_I, $\bar{t}_0(\theta_I) \to 0$ as $\pi \to 1$.

Step 2b: Note that $t_0(\theta_I) \to -\infty$ and $t_1(\theta_I) \to \infty$ as $\theta_I \to 0$. Hence, using the observations (i) to (iii) in *Step 2a* above, we can claim the following:

(A) Either $t_1(\theta_I) \geq \underline{t}_1(\theta_I)$ for all $\theta_I \in (0, \frac{1}{\Delta}\sqrt{c\alpha}]$, or there exists a unique $\theta \in (0, \frac{1}{\Delta}\sqrt{c\alpha}]$, θ_I^1 (say), such that $t_1(\theta_I^1) = \underline{t}_1(\theta_I^1)$ and $t_1(\theta_I) \geq \underline{t}_1(\theta_I)$ iff $\theta_I \leq \theta_I^1$. Moreover, θ_I^1 is continuous and decreasing in π as $\underline{t}_1(\theta_I)$ is increasing in π for all θ_I whereas $t_1(\theta_I)$ is independent of π.

(B) Either $t_0(\theta_I) \leq \bar{t}_0(\theta_I)$ for all $\theta_I \in (0, \frac{1}{\Delta}\sqrt{c\alpha}]$, or there exists a unique $\theta \in (0, \frac{1}{\Delta}\sqrt{c\alpha}]$, θ_I^0 (say), such that $t_0(\theta_I^0) = \bar{t}_0(\theta_I^0)$ and $t_0(\theta_I) \leq \bar{t}_0(\theta_I)$ iff $\theta_I \leq \theta_I^0$. Moreover, θ_I^0 is continuous and decreasing in π as $\bar{t}_0(\theta_I)$ is decreasing in π for all θ_I whereas $t_0(\theta_I)$ is independent of π.

(C) If θ_I^0 exists, so does θ_I^1, and vice versa. Moreover, it must be that $\theta_I^1 = \theta_I^0$. To see this consider the term

$$\phi(\theta_I) := v(\theta_I)(t_1(\theta_I) - \underline{t}_1(\theta_I)) + (1 - v(\theta_I))(t_0(\theta_I) - \bar{t}_0(\theta_I)).$$

But notice that by (BR),

$$\phi(\theta_I) = v(\theta_I) - \left[v(\theta_I)\underline{t}_1(\theta_I) + (1 - v(\theta_I))\bar{t}_0(\theta_I)\right]$$

$$= v(\theta_I) - \left\{v(\theta_I)\mathbb{E}_{z|v=1}\mathbb{E}_v\left[v \mid z, \Theta_E = [0,1], \ \Theta_I = [\theta_I, 1]\right] \\ + (1 - v(\theta_I))\mathbb{E}_{z|v=0}\mathbb{E}_v\left[v \mid z, \Theta_E = [0,1], \ \Theta_I = [\theta_I, 1]\right]\right\}$$

$$= v(\theta_I) - v(\theta_I) = 0.$$

Hence, if θ_I^0 exists such that $t_0(\theta_I^0) - \bar{t}_0(\theta_I^0) = 0$, it must be that $t_1(\theta_I^0) - \underline{t}_1(\theta_I^0) = 0$; i.e., $\theta_I^0 = \theta_I^1$.

Step 2c: Define

$$\bar{\theta}_I = \begin{cases} \theta_I^0 & \text{if exists} \\ \frac{1}{\Delta}\sqrt{c\alpha} & \text{otherwise} \end{cases} \cdot$$

As $p(\theta_I)$ is concave, if $\hat{\theta}_I$ is not feasible under (L_1) and (U_0), $p(\theta_I)$ is maximized at the largest feasible $\theta_I < \hat{\theta}_I$. That is, the solution to $\mathcal{P}$ is $\tilde{\theta}_I = \min\{\hat{\theta}_I, \bar{\theta}_I\}$. Finally, $\bar{\theta}_I$ is decreasing in π as $\theta_I^0 \left(= \theta_I^1\right)$ is.

(ii) As $\theta_I^0 \left(= \theta_I^1\right)$ is continuous and decreasing in π, there exists a $\underline{\pi}$ such that for all $\pi < \underline{\pi}$, $\bar{\theta}_I = \frac{1}{\Delta}\sqrt{c\alpha}$, and hence, $\tilde{\theta}_I = \hat{\theta}_I$. Similarly, there exists a $\overline{\pi} > \underline{\pi}$ such that for all $\pi > \overline{\pi}$, $\bar{\theta}_I < \sqrt{c/\Delta}$, and hence, $\tilde{\theta}_I = \bar{\theta}_I$. For $\pi \in [\underline{\pi}, \overline{\pi}]$, $\bar{\theta}_I \in \left[\sqrt{\frac{c}{\Delta}}, \frac{1}{\Delta}\sqrt{c\alpha}\right]$. As $\hat{\theta}_I$ is decreasing in k, for the expression for $\hat{\theta}_I$ we obtain that there exists a $k_\pi \in \left[\sqrt{\frac{c}{4\alpha}}, \sqrt{\frac{c}{4\Delta}}\right]$ such that $\hat{\theta}_I \leq \bar{\theta}_I$ iff $k \geq k_\pi$. Hence, $\tilde{\theta}_I = \min\{\hat{\theta}_I, \bar{\theta}_I\} = \bar{\theta}_I$ iff $k < k_\pi$. Finally, as $\hat{\theta}_I$ is decreasing in k (but independent of π) whereas $\bar{\theta}_I$ is independent of k but decreasing in π, k_π must be increasing in π.

(iii) This observation directly follows from the fact at any feasible solution to $\mathcal{P}$, (BR) and (IC) must hold. ∎

Proof of Proposition 4. **Step 1**. From Lemma 2, we obtain:

$$\Pi^* = \begin{cases} \alpha + c - 2\sqrt{c\alpha} & \text{if } k < \sqrt{\frac{c}{4\alpha}} \\ \alpha(1-2k)\left(1-\frac{c}{2\alpha k}\right) & \text{if } k \in \left[\sqrt{\frac{c}{4\alpha}}, k^*\right] \\ \alpha + c - 2\sqrt{c(\alpha-k)} - k & \text{if } k > k^* \end{cases} \cdot$$

Step 2. If $\pi < \overline{\pi}$, by Proposition 4 we know that there exists a $k_\pi \in \left[\sqrt{\frac{c}{4\alpha}}, \sqrt{\frac{c}{4\Delta}}\right]$ such that the solution to the program $\mathcal{P}$ yields:

$$\tilde{\theta}_I = \begin{cases} \bar{\theta}_I & \text{if } k < k_\pi \\ \frac{c}{2k\Delta} & \text{if } k \in \left[k_\pi, \sqrt{\frac{c}{4\Delta}}\right] \\ \sqrt{\frac{c}{\Delta}} & \text{if } k > \sqrt{\frac{c}{4\Delta}} \end{cases} ,$$

Recall that $\tilde{\theta}_I$ is continuous as $\bar{\theta}_I = \hat{\theta}_I(k_\pi) = \frac{c}{2k_\pi\Delta}$. Let Π_P be the value associated with the program $\mathcal{P}$. So, we have:

$$\Pi_P(k) = \begin{cases} \alpha(1-2k_\pi)\left(1-\frac{c}{2\alpha k_\pi}\right) & \text{if } k < k_\pi \\ \alpha(1-2k)\left(1-\frac{c}{2\alpha k}\right) & \text{if } k \in \left[k_\pi, \sqrt{\frac{c}{4\Delta}}\right] \\ \alpha + c - 2\sqrt{c\Delta} - k & \text{if } k > \sqrt{\frac{c}{4\Delta}} \end{cases} \cdot$$

Note that as $k^* > \sqrt{\frac{c}{4\Delta}}$ (as $\underline{\Pi}_E - \bar{\Pi}_I$ is decreasing in k and positive at $k = \sqrt{\frac{c}{4\Delta}}$, while $\underline{\Pi}_E = \bar{\Pi}_I$ at k^*), $\Pi_P(k) > \Pi^*$ iff $k > \sqrt{\frac{c}{4\Delta}}$.

Step 3. If $\pi > \overline{\pi}$, $\tilde{\theta}_I = \bar{\theta}_I$, and, moreover, $\bar{\theta}_I < \sqrt{\frac{c}{\Delta}}$. So, we have

$$\Pi_P(k) = \begin{cases} \alpha(1-2k_1)\left(1-\frac{c}{2\alpha k_1}\right) & \text{if } k \leq k_1 \\ \frac{1}{2}\{t_0(\bar{\theta}_I) + t_1(\bar{\theta}_I)\} - k & \text{if } k > k_1 \end{cases} ,$$

where $\bar{\theta}_I = c/2k_1\Delta$. Notice that $\Pi_P(k)$ is continuous and decreasing in k, as, by definition of k_1, we have:

$$\alpha(1 - 2k_1)\left(1 - \frac{c}{2\alpha k_1}\right) = t_0(\bar{\theta}_I) = \frac{1}{2}\{t_0(\bar{\theta}_I) + t_1(\bar{\theta}_I)\} - k_1.$$

Step 4. Now, if $k_1 < k^*$, we have

$$\alpha(1 - 2k_1)\left(1 - \frac{c}{2\alpha k_1}\right) = \frac{1}{2}\{t_0(\bar{\theta}_I) + t_1(\bar{\theta}_I)\} - k_1 > \alpha + c - 2\sqrt{c(\alpha - k_1)} - k_1.$$

But at $k = 1/2$, we must also have

$$\frac{1}{2}\{t_0(\bar{\theta}_I) + t_1(\bar{\theta}_I)\} - k < \alpha + c - 2\sqrt{c(\alpha - k)} - k.$$

To see this, notice that

$$\frac{1}{2}\{t_0(\bar{\theta}_I) + t_1(\bar{\theta}_I)\} - k < \frac{1}{2}\left\{t_0\left(\sqrt{\frac{c}{\Delta}}\right) + t_1\left(\sqrt{\frac{c}{\Delta}}\right)\right\} - k,$$

and

$$\frac{1}{2}\left\{t_0\left(\sqrt{\frac{c}{\Delta}}\right) + t_1\left(\sqrt{\frac{c}{\Delta}}\right)\right\} - k = \alpha + c - 2\sqrt{c(\alpha - k)} - k \text{ for } k = 1/2.$$

So, there exists a cutoff k_2 where $k_2 > k^* > k_1$ and is the unique solution to

$$\frac{1}{2}\{t_0(\bar{\theta}_I) + t_1(\bar{\theta}_I)\} = \alpha + c - 2\sqrt{c(\alpha - k)},$$

such that $\Pi_P(k) > \Pi^*$ iff $k \in (k_1, k_2)$.

Step 5. As $\bar{\theta}_I$ is decreasing in π, k_1 is increasing in π. And, from the equation above it follows that k_2 is decreasing in π. Thus, there exists a threshold for π, π^{FD} (say) such that for $\pi > \pi^{FD}$, $k_1 \geq k^*$ and $\Pi_P(k) \leq \Pi^*$ for all k. $\blacksquare$

Proof of Proposition 5. Consider any equilibrium where partial disclosure is optimal and the solution to the intermediary's problem is $(\tilde{t}_0, \tilde{t}_1, \tilde{\theta}_I)$. Notice that:

$$
\begin{aligned}
t_1(\theta_I) &= \mathbb{E}_{(x,z)|v=i}\mathbb{E}_v\left[v \mid x, z, \Theta_E = [0,1], \Theta_I = [\tilde{\theta}_I, 1]\right] \\
&= \sum_{x,z}\Pr[v = 1 \mid x, z, \tilde{\theta}_I] \times \Pr[x, z \mid v = 1] \\
&= \Pr[v = 1 \mid x_1, z = 1, \tilde{\theta}_I] \times \Pr[x_1, z = 1 \mid v = 1] \\
&\qquad + \Pr[v = 1 \mid x_1, z = 0, \tilde{\theta}_I] \times \Pr[x_1, z = 0 \mid v = 1] \\
&= \Pr[v = 1 \mid x_1, z = 1, \tilde{\theta}_I]\pi + \Pr[v = 1 \mid x_1, z = 0, \tilde{\theta}_I](1 - \pi).
\end{aligned}
$$

Now, consider a disclosure policy where the set of signals is $X = \{x_1, x_0\}$ and $\Pr(x_1 \mid v = 1) = 1$ and $\Pr(x_1 \mid v = 0) = \rho$. Under this policy, for a given ρ, we have:

$$\Pr\left[v = 1 \mid x_1, \ z = 1, \ \tilde{\theta}_I\right] \ = \ \frac{\Pr\left[x_1, \ z = 1 \mid v = 1, \ \tilde{\theta}_I\right]\Pr\left[v = 1 \mid \tilde{\theta}_I\right]}{\sum_v \Pr\left[x_1, \ z = 1 \mid v, \ \tilde{\theta}_I\right]\Pr\left[v \mid \tilde{\theta}_I\right]}$$

$$= \ \frac{\pi v(\tilde{\theta}_I)}{\pi v(\tilde{\theta}_I) + \rho\,(1 - \pi)\left(1 - v(\tilde{\theta}_I)\right)},$$

and, similarly,

$$\Pr\left[v = 1 \mid x_1, \ z = 0, \ \tilde{\theta}_I\right] = \frac{(1 - \pi)\,v(\tilde{\theta}_I)}{(1 - \pi)\,v(\tilde{\theta}_I) + \rho\pi\left(1 - v(\tilde{\theta}_I)\right)}.$$

We thus have

$$t_1\,(\theta_I) = \frac{\pi^2 v(\tilde{\theta}_I)}{\pi v(\tilde{\theta}_I) + \rho\,(1 - \pi)\left(1 - v(\tilde{\theta}_I)\right)} + \frac{(1 - \pi)^2\,v(\tilde{\theta}_I)}{(1 - \pi)\,v(\tilde{\theta}_I) + \rho\pi\left(1 - v(\tilde{\theta}_I)\right)},$$

and ρ must solve:

$$(13) \qquad \tilde{t}_1 = \frac{\pi^2 v(\tilde{\theta}_I)}{\pi v(\tilde{\theta}_I) + \rho\,(1 - \pi)\left(1 - v(\tilde{\theta}_I)\right)} + \frac{(1 - \pi)^2\,v(\tilde{\theta}_I)}{(1 - \pi)\,v(\tilde{\theta}_I) + \rho\pi\left(1 - v(\tilde{\theta}_I)\right)}.$$

We claim that for any $\tilde{t}_1 \in [\underline{t}_1(\tilde{\theta}_I), 1]$, there exists a value of ρ that satisfies (13), where

$$\underline{t}_1\left(\tilde{\theta}_I\right) \ = \ \mathbb{E}_{z\mid v=1}\mathbb{E}_v\left[v \mid z, \Theta_E = [0, 1], \ \Theta_I = [\tilde{\theta}_I, 1]\right]$$

$$= \ \frac{\pi^2 v(\tilde{\theta}_I)}{\pi v(\tilde{\theta}_I) + (1 - \pi)\left(1 - v(\tilde{\theta}_I)\right)} + \frac{(1 - \pi)^2\,v(\tilde{\theta}_I)}{(1 - \pi)\,v(\tilde{\theta}_I) + \pi\left(1 - v(\tilde{\theta}_I)\right)}.$$

But this is trivial as ρ takes all values from 0 to 1, and hence the right-hand side takes all values from $\underline{t}_1(\tilde{\theta}_I)$ to 1. The corresponding value of $\tilde{t}_0$ could also be induced by the same value of ρ as $\tilde{t}_1$ uniquely pins down $\tilde{t}_0$ by (BR), which can be written as

$$\tilde{t}_0 = \frac{v(\tilde{\theta}_I)}{1 - v(\tilde{\theta}_I)}\,(1 - \tilde{t}_1).$$

∎

Proof of Proposition 6. *Part* (i): **Step 1.** From Proposition 4, we know that if $\pi < \bar{\pi}$, $(k_1(\pi), k_2(\pi)) = \left(\sqrt{\frac{c}{4\Delta}}, \frac{1}{2}\right)$, and the following holds: (*a*) for all $k < \sqrt{\frac{c}{4\Delta}}$, full disclosure is optimal and the associated $\theta_E = 0$ and $\theta_I = \theta_I^f$, (*b*) for all $k \geq \sqrt{\frac{c}{4\Delta}}$, partial disclosure is optimal and the associated $\theta_E = 0$ and $\theta_I = \hat{\theta}_I$. Moreover, from the proof of Proposition 4 (part (ii)), we know that , $\tilde{\theta}_I = \hat{\theta}_I$ for all $k > k_\pi \geq \sqrt{\frac{c}{4\Delta}}$. Since both θ_I^f and $\hat{\theta}_I$ are independent of π, W_I is invariant to π.

Step 2. If $\pi \geq \pi^{FD}$, W_I depends on θ_I^f, θ_I^i, and k^*, all of which are invariant to π; and hence, so is W_I.

Part (ii): **Step 1.** First consider the case where $\pi'' < \pi^{FD}$, *i.e.*, $(k_1(\pi''), k_2(\pi'')) \neq \emptyset$. The proof is obtained by characterizing W_I in each of the following mutually exclusive and exhaustive parameter ranges for k.

Step 1a: For $k < k_1(\pi')$, full disclosure remains as the optimal policy with $\theta_E = 0$ and $\theta_I = \theta_I^f$. Hence, there is no change in W_I.

Step 1b: For $k \in [k_1(\pi'), k_1(\pi''))$, optimal policy switches from partial disclosure where $(\theta_E, \theta_I) = (0, \tilde{\theta}_I)$ to full disclosure where $(\theta_E, \theta_I) = (0, \theta_I^f)$. We claim that for $k \in [k_1(\pi'), k_1(\pi''))$ and $\pi = \pi'$, $\tilde{\theta}_I \geq \theta_I^f$. The argument is as follows. As $\pi'' < \pi^{FD}$, $k_1(\pi'') < k^*$. So, for any $k \in [k_1(\pi'), k_1(\pi'')) \subseteq [\sqrt{\frac{c}{4\Delta}}, k^*)$, $\theta_I^f = \frac{c}{2k\Delta}$, and hence, strictly decreasing in k. But $\tilde{\theta}_I$ remains constant. If $\pi' \leq \bar{\pi}$, from the proof of Proposition 4 (ii), we know that $k_1(\pi') = \sqrt{\frac{c}{4\Delta}}$ and $\tilde{\theta}_I = \sqrt{\frac{c}{\Delta}} = \frac{c}{2k_1(\pi')\Delta}$. Also, if $\pi' > \bar{\pi}$, from the proof of Proposition 4 (ii) and the proof of Proposition 4 (step 3) we have $\tilde{\theta}_I = \bar{\theta}_I = \frac{c}{2k_1(\pi')\Delta}$. Hence, for all $k \in [k_1(\pi'), k_1(\pi''))$ $\tilde{\theta}_I = \frac{c}{2k_1(\pi')\Delta} \geq \frac{c}{2k\Delta} = \theta_I^f$.

Step 1c: For $k \in [k_1(\pi''), k_2(\pi'')]$ partial disclosure remains optimum. So, we have $(\theta_E, \theta_I) = (0, \tilde{\theta}_I)$ and $\tilde{\theta}_I = \min\{\sqrt{\frac{c}{\Delta}}, \bar{\theta}_I\}$. Hence, W_I is increasing in π as θ_I is decreasing in π.

Step 1d: Finally, for $k > k_2(\pi'')$, two cases can arise: (i) for $k > k_2(\pi') > k_2(\pi'')$ full disclosure remains as the optimal policy where $(\theta_E, \theta_I) = (\theta_I^i, \theta_I^i)$, and hence there is no change in W_I for a given k. (ii) For $k \in (k_2(\pi''), k_2(\pi')]$ the optimal policy switches from partial disclosure to full disclosure and the associated type cutoffs change from $(\theta_E, \theta_I) = (0, \tilde{\theta}_I)$ to (θ_I^i, θ_I^i). But notice that $W_I(0, \tilde{\theta}_I) \geq W_I(0, \sqrt{\frac{c}{\Delta}}) > W_I(\theta_I^i, \theta_I^i)$. The first inequality follows from the fact that for such k, $\tilde{\theta}_I = \min\{\sqrt{\frac{c}{\Delta}}, \bar{\theta}_I\}$ and W_I is decreasing in θ_I. The second inequality follows as $W_I(0, \sqrt{\frac{c}{\Delta}}) = W_I(\theta_I^i, \theta_I^i)$ at $k = \frac{1}{2}$ (recall that $\theta_I^i = \sqrt{\frac{c}{\alpha-k}}$), and for $k < \frac{1}{2}$, $\frac{\partial}{\partial k}W_I(0, \sqrt{\frac{c}{\Delta}}) = -1$ whereas $\frac{\partial}{\partial k}W_I(\theta_I^i, \theta_I^i) = -1 + \frac{1}{2}(\frac{c}{\alpha-k} + \sqrt{\frac{c}{\alpha-k}}) \in (-1, 0)$.

Step 2. The case where $\pi'' \geq \pi^{FD}$ can be treated exactly as above by setting $k_1(\pi'') = k_2(\pi'') = k^*$. ∎

Below we present a set of lemmas that are useful in our proof of part (iii) of Proposition 6.

Lemma 5. *Suppose that all types enter but only type $\theta > \theta^* \in [0, 1]$ invests. Then,*
$$v_1(\theta^*) - v_0(\theta^*) = \Delta\left(\underline{t}_1(\theta^*) - \bar{t}_0(\theta^*)\right).$$

Proof. The proof follows from direct computation of the terms given in the equation above. Denote $\mathbb{E}(v \mid z; \theta^*) := \mathbb{E}(v \mid z; \Theta_E = [0, 1], \Theta_I = [\theta^*, 1])$ and recall that

$$v_i(\theta^*) := \sum_{z \in \{0,1\}} \mathbb{E}(v \mid z; \theta^*)\Pr(z \mid I = i), \quad i = 0, 1,$$

that is, the expected value of the product after z is realized. Also note that,

$$\underline{t}_1\left(\theta^*\right) - \bar{t}_0\left(\theta^*\right) = \mathbb{E}_{z|v=1}\mathbb{E}_v\left(v \mid z;\ \theta^*\right) - \mathbb{E}_{z|v=0}\mathbb{E}_v\left(v \mid z;\ \theta^*\right)$$

$$= \left[\mathbb{E}\left(v \mid z = 1;\ \theta^*\right)\pi + \mathbb{E}\left(v \mid z = 0;\ \theta^*\right)\left(1 - \pi\right)\right]$$
$$- \left[\mathbb{E}\left(v \mid z = 1;\ \theta^*\right)\left(1 - \pi\right) + \mathbb{E}\left(v \mid z = 0;\ \theta^*\right)\pi\right].$$

Now,

$$v_1\left(\theta^*\right) - v_0\left(\theta^*\right) = \sum_{i\in\{0,1\}}\left[\sum_{z\in\{0,1\}}\mathbb{E}\left(v \mid z;\ \theta^*\right)\Pr\left(z \mid I = i\right)\right],$$

and

$$\Pr\left(z = 1 \mid I = 1\right) = \sum_{v\in\{0,1\}}\Pr\left(z = 1 \mid v, I = 1\right)\Pr\left(v \mid I = 1\right)$$

$$= \pi\alpha + \left(1 - \pi\right)\left(1 - \alpha\right),$$

and

$$\Pr\left(z = 1 \mid I = 0\right) = \frac{1}{2}\pi + \frac{1}{2}\left(1 - \pi\right) = \frac{1}{2}.$$

So,

$$v_1\left(\theta^*\right) - v_0\left(\theta^*\right) = \sum_{i\in\{0,1\}}\left[\sum_{z\in\{0,1\}}\mathbb{E}\left(v \mid z;\ \theta^*\right)\Pr\left(z \mid I = i\right)\right]$$

$$= \mathbb{E}\left(v \mid z = 1;\ \theta^*\right)\pi\Delta + \mathbb{E}\left(v \mid z = 1;\ \theta^*\right)\left(\left(1 - \pi\right)\left(1 - \alpha\right) - \tfrac{1}{2}\left(1 - \pi\right)\right)$$
$$- \mathbb{E}\left(v \mid z = 0;\ \theta^*\right)\left(\pi\alpha + \left(1 - \pi\right)\left(1 - \alpha\right)\right) + \mathbb{E}\left(v \mid z = 0;\ \theta^*\right)\left(\tfrac{1}{2}\pi + \tfrac{1}{2}\left(1 - \pi\right)\right)$$

$$= \mathbb{E}\left(v \mid z = 1;\ \theta^*\right)\pi\Delta - \mathbb{E}\left(v \mid z = 1;\ \theta^*\right)\left(1 - \pi\right)\Delta$$
$$- \mathbb{E}\left(v \mid z = 0;\ \theta^*\right)\pi\Delta + \mathbb{E}\left(v \mid z = 0;\ \theta^*\right)\left(1 - \pi\right)\Delta$$

$$= \Delta\left(\underline{t}_1\left(\theta^*\right) - \bar{t}_0\left(\theta^*\right)\right).$$

Hence the proof. ∎

Lemma 6. *For $\pi \geq \bar{\pi}$ and $k \in \left[k_1\left(\pi\right), k_2\left(\pi\right)\right]$, $\tilde{\theta}_I = \theta_I^{NI}$. Also, for $\pi < \bar{\pi}$ and $k > \sqrt{\frac{c}{4\Delta}}$, $\tilde{\theta}_I = \sqrt{\frac{c}{\Delta}}$.*

Proof. For $\pi \geq \bar{\pi}$ and $k \in \left[k_1\left(\pi\right), k_2\left(\pi\right)\right]$, $\tilde{\theta}_I = \bar{\theta}_I$ where for (IC) we have

$$\Delta\left(\underline{t}_1\left(\bar{\theta}_I\right) - \bar{t}_0\left(\bar{\theta}_I\right)\right) = \frac{c}{\bar{\theta}_I}.$$

So, using Lemma 5, we have

$$(14) \qquad\qquad v_1\left(\bar{\theta}_I\right) - v_0\left(\bar{\theta}_I\right) = \Delta\left(\underline{t}_1\left(\bar{\theta}_I\right) - \bar{t}_0\left(\bar{\theta}_I\right)\right) = \frac{c}{\bar{\theta}_I}.$$

Now, θ_I^{NI} is the unique solution to

$$(15) \qquad\qquad\qquad\qquad v_1\left(\theta\right) - v_0\left(\theta\right) = \frac{c}{\theta}.$$

Hence, we must have

$$\theta_I^{NI} = \bar{\theta}_I = \tilde{\theta}_I.$$

For $\pi < \overline{\pi}$, $\tilde{\theta}_I = \hat{\theta}_I$, and hence, from equation (12) we have $\tilde{\theta}_I = \sqrt{\frac{c}{\Delta}}$ for $k > \sqrt{\frac{c}{4\Delta}}$. ∎

Lemma 7. $\pi^{NI} < \overline{\pi}$.

Proof. By definition, equation (15), i.e.,

$$v_1(\theta) - v_0(\theta) = \frac{c}{\theta}$$

has a unique solution in $[0, 1)$ if and only if $\pi > \pi^{NI}$ and no solution otherwise. Also, by definition, at $\pi = \overline{\pi}$, $\bar{\theta}_I = \sqrt{\frac{c}{\Delta}} < 1$ and $\Delta \left(\underline{t}_1 (\bar{\theta}_I) - \bar{t}_0 (\bar{\theta}_I) \right) = \frac{c}{\theta_I}$. But by equation (14) as given in the proof of Lemma 6, $\bar{\theta}_I = \sqrt{\frac{c}{\Delta}}$ is also a solution to equation (15). Since equation (15) does not have any solution for $\pi \leq \pi^{NI}$, we must have $\overline{\pi} > \pi^{NI}$. ∎

We are now ready to present the proof of part (iii) of Proposition 6.

Proof of Proposition 6. *Part* (iii). **Step 1.** W_{NI} is invariant to π if $\pi < \pi^{NI}$ but increases otherwise as θ_I^{NI} is decreasing in π.

Step 2. For $\pi < \overline{\pi}$, W_I is invariant to π since it depends on either θ_I^f or θ_I^i, and both are independent of π. Hence, for $\pi < \pi^{NI}$, $W_I - W_{NI}$ is constant (note that by Lemma 7, $\pi^{NI} < \overline{\pi}$) but for $\pi \in [\pi^{NI}, \overline{\pi})$, $W_I - W_{NI}$ is strictly decreasing in π.

Next, consider $\pi \in [\overline{\pi}, \pi^{FD})$. If $k \notin (k_1(\pi), k_2(\pi))$, W_I is invariant to π as it only depends on either θ_I^f or θ_I^i. Otherwise, $W_I = W(0, \tilde{\theta}_I) = W \left(0, \theta_I^{NI} \right) = W_{IN}$ (by Lemma 5). Hence, in the former case, $W_I - W_{NI}$ is strictly decreasing in π and in the later case $W_I - W_{NI} = 0$ for all such π.

Finally, for $\pi \geq \pi^{FD}$, W_I is still invariant to π as k^* is also independent of π. So, $W_I - W_{NI}$ is strictly decreasing in π. Hence, the proof. ∎

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