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OPTIMAL CHOICES AMONG ALTERNATIVE TECHNOLOGIES WITH STOCHASTIC YIELD

Quite often, farmers have to choose among discrete alternatives with uncertain outcomes. This situation occurs, for example, when farmers consider adoption of new technologies such as the introduction of new farm machinery or construction of irrigation systems. To be sure, understanding the process of technological adoption requires the explicit recognition of uncertainty and risk aversion. The purpose of this paper is to advance a new approach for investigating this process. It will be demonstrated under conditions that describe available empirical evidence that this approach is vastly superior to current approaches.

Current Approaches

The major approach employed in the analysis of farmers' behavior under uncertainty is the expected utility approach. Since its introduction by von Neumann and Morgenstern, this approach has been refined and extended to explain the behavior of economic agents.[1] Of special importance are the measures of risk aversion suggested by Arrow and Pratt and the theory of stochastic dominance of Hadar and Russel, Rothschild and Stiglitz, and Hanoch and Levy.

Sandmo has introduced a very general model which does not require specification of either the utility function or the distribution of the random variable. His model assumes multiplicative risk and is limited to problems with only one random variable. Sandmo's model has been modified by Feder and Feder and O'Mara to analyze the adoption of new technologies in less-developed countries. While these theoretical models are very interesting and insightful, their weakness lies in their strength; namely, they are too general for empirical use. Quantitative analysis requires more detailed specifications.

When the distribution of the random variable is characterized by only two parameters or the utility function is quadratic, Tobin has demonstrated that the expected utility can be introduced as a function of the mean and the variance. In Markowitz, this function is linear in the mean and the variance. Markowitz has shown that this linear specification is justified when the utility function is quadratic or when the expected utility is approximated using the first two elements of a Taylor series for a normally distributed random variable. Freund was the first to apply the Markowitz mean-variance criterion to an agricultural programming problem with linear technologies. Freund also proved that the linear mean-variance formulation can be derived for an exponential utility function for normally distributed random variables.

The behavior of farmers under uncertainty is studied frequently using the mean-variance (E-V) analysis. It is especially common to apply this methodology assuming that farmers maximize a linear combination of the mean and the variance (Anderson and Dillon, Pope and Just, Rae, Lin et al., and Wiens). This last formulation corresponds to maximization of expected utility of income when either the utility function is quadratic or the random variable is normally distributed and the measure of absolute risk aversion is constant. These two alternative assumptions are quite restrictive. Quadratic utility is unacceptable for theoretical reasons since it implies increasing absolute risk aversion (Arrow, Pratt). On the other hand, many natural phenomena are described by probability distributions which are not normal. For example, Day has shown that crop yields can be best represented by the Pearson family type III distributions. Moreover, the gamma distribution (which is a Pearson type III distribution) was found to be very accurate in representing precipitation and drought occurrence (Starr, Rudman, and Whipple).

Another method of risk analysis, consistent with the expected utility framework, is the use of stochastic dominance rules. Anderson applied this technique for comparison of alternative agricultural techniques in less-developed countries. While this method has the advantage of not requiring knowledge of the farmers' utility functions, its practical use is restricted to comparison of discrete alternatives.

While the expected utility framework is dominant in the analysis of behavior under uncertainty, a significant amount of effort has been devoted to the application of safety rules. This alternative approach reflects the concern of decision-makers about being at the lower end of their profit distribution. Several safety rules have been suggested in the literature. These are rules of thumb that may not correspond to any specification of the Bernoullian utility function. One of these rules is the safety principle suggested by Roy. It involves minimization of the probability that profit falls below a disaster level. Another rule is the safety-first rule suggested by Telser; it involves maximization of expected profits subject to a minimum probability of disaster. The third rule is the safety-first principle (suggested by Kataoka) which involves the maximization of the minimum level of profit that can be assured with a certain significance level (probability). All the safety rules can be applied without much difficulty to cases with normally distributed profits. Roumasset has applied this approach for an analysis of choice of rice hybrids in the Philippines. While these rules are easy to apply and may be useful in certain situations, their wide use cannot be recommended because of their shaky theoretical foundation.

Among the approaches mentioned above, the linear mean-variance approach is the most operational in terms of computational convenience. It is especially

desirable when yields are distributed normally. However, in cases where yields are not normally distributed (Day), the use of the Tobin-Markowitz framework corresponds to the imposition of a quadratic utility function with all its associated limitations.

New Approach

A new framework, which is equally convenient and includes as a special case the mean-variance analysis, will be referred to as the exponential utility, moment-generating function approach (EUMGF). This approach assumes an exponential utility function which implies constant absolute risk aversion rather than increasing absolute risk aversion (Arrow, Pratt). This utility function can be conveniently applied in conjunction with all distributions which have moment-generating functions. Moreover, in the context of comparisons between uncertain prospects, Hammond has derived conditions under which the exponential utility function will lead to identical results as those obtained for utility functions which exhibit decreasing absolute risk aversion.

The conditions isolated by Hammond are likely to be met in choices between old and modern techniques. In most situations, modern techniques (for example, the use of chemical fertilizers or high-yielding varieties) result in higher expected profits than those generated by existing techniques; but the yield distributions associated with the former are less concentrated. The probability of earning very low profits or very high profits for the modern technique is higher than for the old technique. For these features, it is indeed plausible that the cumulative distribution of profits under the modern technique (T_1) crosses the cumulative distribution of profits under the old technique (T_0) from above (figure 1). For these properties, Hammond's
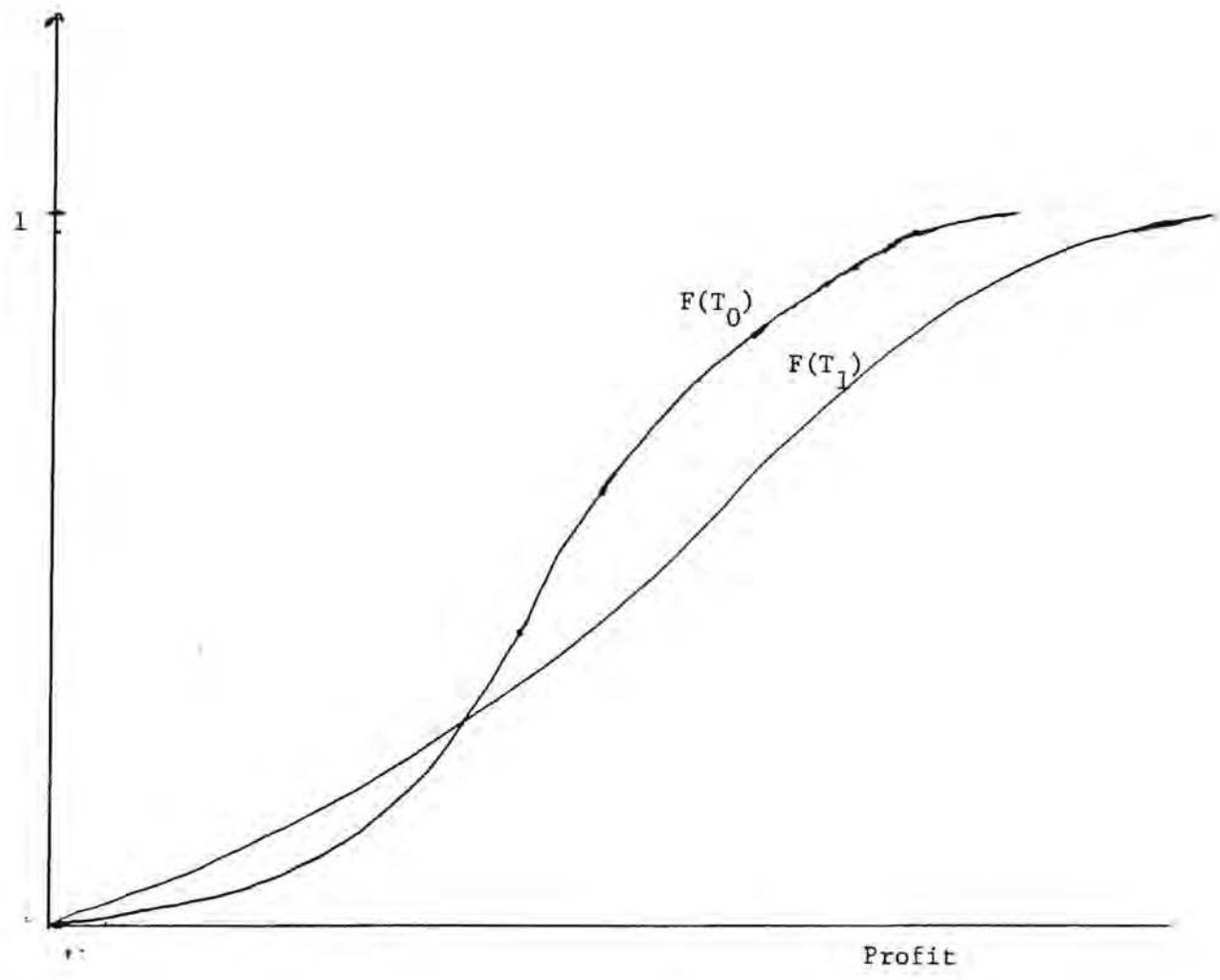
Figure 1--Cumulative Distributions of Profits under Old (T_0) and Modern (T_1) Techniques.

conditions imply that, if the modern technology is adopted under an exponential utility function, it will also be adopted under utility functions which exhibit decreasing absolute risk aversion (DARA) as long as the degree of risk aversion for the DARA function is smaller than the absolute risk aversion (say, r) of the former. Similarly, if the old technology is chosen under constant absolute risk aversion, it will also be adopted under DARA when the measure of risk aversion for DARA is always greater than r.

Model

Suppose a farmer must decide whether to continue employing his existing technology, T_0, or to adopt a new technology, T_1. Each technology is presumed to have deterministic variable costs of c_0 and c_1 and stochastic yields, Y_0 and Y_1, respectively. It is assumed that the yield is a function of the climatic conditions (N) and the technology (T), or

(1) $$Y = f(N, T),$$

and the costs are only a function of the technology used, or

(2) $$c = f(T).$$

The net profit is thus

(3) $$R = PY - c$$

where P is the fixed output price.

Given the above specifications, the new technology will be adopted if

(4) $$E_1 U(R_1) > E_0 U(R_0)$$

where E is the expectation operator and U is the utility function. Note that the expectations are over the respective probability distribution since the yield distributions of the two technologies may differ not only in their parameters but also may belong to different families.

At this juncture, the major features of the EUMGF approach are introduced. The exponential utility function is

(5)
$$U(R) = -e^{-rR}$$

where r is the measure of absolute risk aversion. Substituting (5) and (3) into (4) provides the basis for introducing the moment-generating function and, thus, obtains a compact analytical solution. That is,

(6)
$$E_1 \left[-e^{-r(Y_1 P - c_1)} \right] > E_0 \left[-e^{-r(Y_0 P - c_0)} \right]$$

which is equivalent to

(7)
$$e^{rc_1} M_1(-rP) < e^{rc_0} M_0(-rP)$$

where

(8)
$$M(t) = E \, e^{tY}$$

is the moment-generating function of the yield probability distribution.

Alternative Yield Distributions

As noted above, the mean-variance analysis is a special case of the EUMGF approach. Specifically, if the distribution of yields under both technologies is normal, the EUMGF reduces to the linear mean-variance framework. That is, the moment-generating function of the normal distribution is

(9)
$$M(t) = \exp\left(\mu t + \sigma^2 \frac{t^2}{2} \right).$$

Substituting (9) into (6) and simplifying,

$$(10) \quad \left[P (\mu_1 - \mu_0) - (c_1 - c_0) \right] - \left[\frac{r}{2} P^2 (\sigma_1^2 - \sigma_0^2) \right] > 0$$

or, in other words, the increase in the expected profit should exceed the increase in the variance times half the risk-aversion measure. This result is obtained as a necessary condition when the objective is to maximize the expected value (E) less $r/2$ times the variance, i.e., $E - (r/2) V$.

The above result is not surprising; it was originally derived more than 25 years ago by Freund. Extending the results of Freund, Pratt has shown that, for "small lotteries" with any distribution and any utility function,

$$(11) \quad RP(\ell) = \frac{r}{2} V(\ell)$$

or, equivalently,

$$(12) \quad CE(\ell) = E(\ell) - \frac{r}{2} V(\ell)$$

where $RP(\ell)$ is the risk premium of the lottery, $V(\ell)$ is the variance of the lottery profit, $CE(\ell)$ is the certainty equivalent of the lottery, $E(\ell)$ is the expected lottery profit, and r is the measure of absolute risk aversion ($r = -u''/u'$). Equations (11) and (12) will prove useful in comparisons of the mean-variance approach with more general forms of EUMGA.

As indicated above, there is sufficient evidence to indicate that yields are better approximated by skewed distributions such as gamma. The density function for the gamma distribution is

$$(13) \quad f(Y) = \frac{\lambda^\alpha}{\Gamma(\alpha)} Y^{\alpha-1} e^{-\lambda Y}, \quad Y > 0$$

where $\Gamma(\alpha)$ is the complete gamma function and α and λ are the parameters of the distribution. The mean and the variance of the gamma distribution are given by

(14)
$$\mu = \frac{\alpha}{\lambda}$$

and

(15)
$$\sigma^2 = \frac{\alpha}{\lambda^2} .$$

The moment-generating function in this case is

(16)
$$M(t) = \frac{1}{(1 - t/\lambda)^\alpha} .$$

Substituting (16) into (7) and taking the logarithms of both sides yields a condition (stated in monetary units) where the second technology is preferred,[2] i.e.,

(17)
$$\frac{\alpha_1 \ln (1 + rP/\lambda_1)}{r} - c_1 > \frac{\alpha_0 \ln (1 + rP/\lambda_0)}{r} - c_0.$$

The right-hand side of (17) is the certainty equivalent (CE) of profit resulting from using the first technology, and the left-hand side is the CE resulting from adopting the second. Since the gamma distribution has only two parameters, condition (17) can be expressed [using (14) and (15)] in terms of the respective means and variances of yield, i.e.,

(18)
$$\frac{1}{r} \cdot \left(\frac{\mu_2}{\sigma_2}\right)^2 \ln \left(1 + \frac{\sigma_2^2}{\mu_2} rP\right) - c_2 > \frac{1}{r} \left(\frac{\mu_1}{\sigma_1}\right)^2 \ln \left(1 + \frac{\sigma_1^2}{\mu_1} rP\right) - c_1.$$

Comparison of Normal- and Gamma-Yield Distributions

To understand how different specifications of the yield distribution affect the choice of technology, consider two yields with the same mean and variance but with different distributions. The CE of technology with mean μ and variance σ^2 under normal distribution (CE_n) is given by

$$(19) \qquad CE_n(T) = P\mu - \left(\frac{r}{2} \right) P^2 \sigma^2 - c,$$

and the CE of a similar technology but with a gamma distribution (CE_g) is given by

$$(20) \qquad CE_g(T) = \frac{1}{r} \left(\frac{\mu}{\sigma} \right)^2 \ln \left(1 + \frac{\sigma^2}{\mu} rP \right) - c.$$

Using the Taylor expansion of a logarithmic function, (20) becomes[3]

$$(21) \qquad CE_g(T) = \frac{1}{r} \left(\frac{\mu}{\sigma} \right)^2 \sum_{j=1}^{\infty} (-1)^{j-1} \frac{1}{j} \left(\frac{\sigma^2}{\mu} rP \right)^j - c.$$

The difference between the two CEs is thus given by

$$(22) \qquad CE_g(T) - CE_n(T) = \frac{1}{r} \left(\frac{\mu}{\sigma} \right)^2 \sum_{j=3}^{\infty} (-1)^{j-1} \frac{1}{j} \left(\frac{\sigma^2}{\mu} rP \right)^j.$$

Assuming that all elements from $j = 4$ on are insignificant, expression (22) reduces to

$$(23) \qquad CE_g(T) - CE_n(T) \approx \frac{1}{3} \frac{\sigma^4 r^2 P^3}{\mu} > 0.$$

It is clear from (23) that a random yield with gamma distribution is preferred to that with normal distribution if both have the same mean and

variance and equal production cost. This result clarifies the limitations of the frequently assumed normal distribution for decision-making purposes in cases where the actual distribution is skewed. As shown in (23), this mis-specification leads to underestimation of the benefit from the random yield and may also result in erroneous decision making. In particular, if a linear mean-variance framework is employed assuming incorrectly that yields are normally distributed when, in fact, they are gamma distributed, the resulting choices are generally too conservative.

The importance of the above result will be illustrated by a simple example and by an empirical example. For the simple illustration, suppose the existing technology has mean μ and standard deviation σ, and a new technology is introduced with the same production cost and a random yield which is k times the existing technology yield. Using (19) and (20), the CEs of the new technology under both the normal and gamma distributions are

$$(24) \qquad CE_n (T_k) = k \left(P\mu - \frac{k}{2} rP^2 \sigma^2 \right) - c$$

$$(25) \qquad CE_g (T_k) = \frac{1}{r} \left(\frac{\mu}{\sigma} \right)^2 \ln \left(1 + \frac{k\sigma^2}{\mu} rP \right) - c.$$

These two CEs are depicted as functions of k, the technology multiplier, in figure 2.

The CE for the gamma distribution is always higher than that of the normal distribution and increases at a decreasing rate with k. The CE for the normal distribution rises for relatively small k's, peaks at $k = \mu/\sigma^2 rP$, and decreases thereafter. Thus, technologies with higher multipliers are always preferred under the gamma distribution. However, they will not be preferred
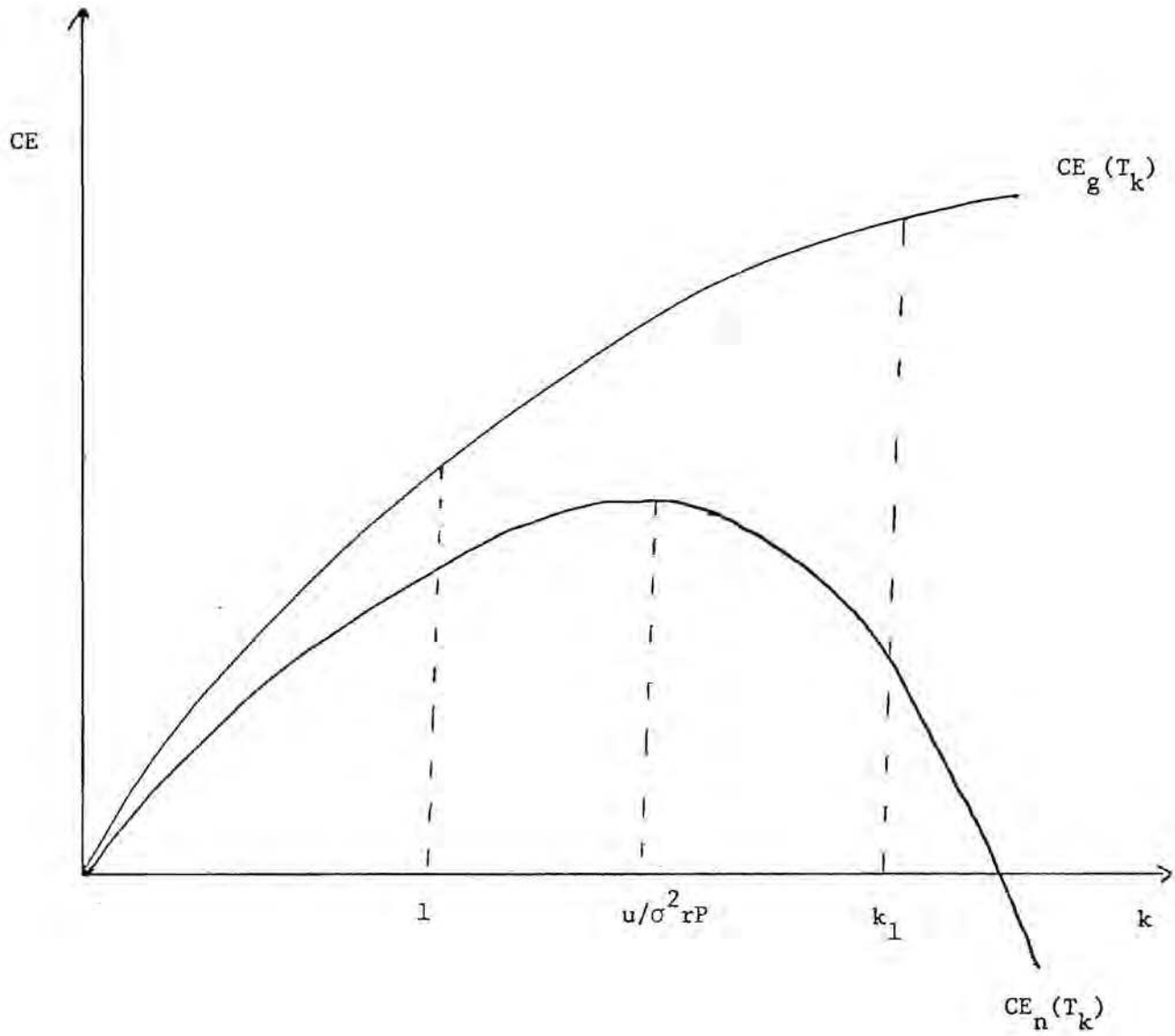
Figure 2--Certainty Equivalents Under Gamma and Normal Yield Distributions.

if the yield distribution is assumed to be normal. In figure 2, for example, T_1 is inferior to T_{k_1} for the gamma distribution but is superior for the normal distribution. The importance of using the right specification increases with the measure of risk aversion. From (20) and (21) it can be derived that both CEs decrease with r, but the CE under the normality assumption decreases more rapidly. Thus, the difference between the two CEs increases with the degree of risk aversion. The above results have another implication, assuming a normal distribution of yield will result in underestimation of the Arrow-Pratt measure of risk aversion if, in fact, the real distribution is skewed.

Turning to the empirical example, we employ data reported by Roumasset concerning production technologies available to operators of the rice land in the Philippines. Four technologies are considered. One is a traditional technology, denoted by T_0, which is lowest in yield as well as lowest in cost and risk. The other three, denoted by T_1, T_2, and T_3 are modern technologies. They use high-yield variety seeds combined with cash inputs (such as fertilizer) and require more labor. Among them, T_1 has the lowest costs and yields, while T_3 has the highest yield and also the highest cost. The means and standard deviations of the yield for the four technologies, as well as output price and production cost per acre, are given in table 1.

Table 2 presents the certainty equivalents of profits under the four technologies for a farm with 1 hectare of land. These certainty equivalents are computed for the two-yield distribution specifications (i.e., normal and gamma) under different degrees of risk aversion. The measure of absolute risk aversion varies from zero (risk neutrality) to .01. The optimal choices under both distribution specifications for each level of risk aversion are given in separate columns.

TABLE 1

The Parameters of Four Technologies

Technology	T_0	T_1	T_2	T_3
Average yield per hectare	32	70	80	90
Standard deviation per hectare	5	25	30	35
Output price	16	16	16	16
Cost per hectare	106	350	410	490

Source: J. A. Roumasset, Rice and Risk: Decision Making Among Low-Income Farmers (Amsterdam: North-Holland Publishing Co., 1976), p. 54.

TABLE 2

The Certainty Equivalent Under Alternative Technologies

R	T_0		T_1		T_2		T_3		Optimal choice	
	N	Y	N	Y	N	Y	N	Y	N	Y
.000	406.0	406.0	770.0	770.0	870.0	870.0	950.0	950.0	T_3	T_3
.0001	405.6	405.6	762.0	762.0	858.4	858.6	934.3	934.5	T_3	T_3
.001	402.8	402.8	690.0	696.0	743.2	757.8	793.0	812.0	T_3	T_3
.002	399.6	399.7	602.0	629.3	639.6	690.2	636.0	705.0	T_2	T_3
.003	396.4	396.6	522.0	577.0	524.0	613.0	472.0	618.0	T_2	T_3
.004	393.2	393.6	450.0	535.8	407.2	554.1	322.0	545.0	T_1	T_2
.005	390.0	390.6	362.0	491.0	294.0	502.0	166.0	484.0	T_0	T_2
.006	386.8	387.7	282.0	455.4	178.0	457.0	9.2	431.0	T_0	T_2
.007	383.6	384.8	202.0	423.0	63.0	418.0	-147.0	384.0	T_0	T_1
.008	380.4	381.9	122.0	394.0	- 63.0	379.5	-304.0	343.7	T_0	T_1
.009	377.2	379.1	42.0	367.0	-178.3	348.0	-476.0	303.0	T_0	T_0
.010	374.0	376.4	- 29.8	345.0	-281.0	322.0	-617.0	274.0	T_0	T_0

Under both specifications, increases in risk aversion result in a gradual move to less risky technologies. This tendency is much stronger under normality. For example, for the gamma distributions, all the farms with $r \leq .003$ will adopt the most modern techniques, while only the farms with $r \leq .001$ will do so under normality. The traditional technology will be adopted by all the farms with $r \geq .005$ under normality but only by the farms with $r \geq .009$ under gamma distribution. The effect of an increase in risk aversion on the certainty equivalents is much more drastic under the normal distribution, especially for the high-risk technology. Note that the certainty equivalent of the most modern technology is <u>negative</u> and rapidly declines for $r \geq .007$.

Special Cases

The decision rule for the choice of technology becomes much simpler for two special cases of the gamma distribution, namely, the chi-squared and exponential distributions. When the parameters of the gamma distribution are $\alpha = n/2$ and $\lambda = 1/2$, the distribution becomes chi-squared with n degrees of freedom. The moment-generating function of that distribution is

$$(26) \qquad\qquad M(t) = (1 - 2t)^{-n/2}.$$

Using the same procedure as above, it is found that T_1 is preferred to T_0 when

$$(27) \qquad\qquad \bar{Y}_1 - \bar{Y}_0 > \frac{2r\,(c_1 - c_0)}{\ln\,(1 + 2rP)}$$

where $\bar{Y}_1$ and $\bar{Y}_0$ are the average yields (and also the degrees of freedom) when the yields are distributed as chi-squared.

When the parameter α of the gamma distribution is equal to one, the distribution becomes exponential with parameter λ. The average yield, Y, is equal to $1/\lambda$. The moment-generating function of the exponential distribution is

$$(28) \qquad M(t) = \frac{\lambda}{\lambda - t} \, .$$

T_1 would be preferred to T_0 if

$$(29) \qquad r(c_1 - c_0) < \ln \frac{1 + \overline{Y}_1 \, rP}{1 + \overline{Y}_0 \, rP}$$

Other Yield Distributions

The gamma- and normal-yield distributions are simply illustrative of the analysis that can be conducted using the methodology developed here. Any continuous or discrete yield distribution with a well-behaved, moment-generating function can be analyzed. Moreover, in many cases a new technology results in stochastic yield which differs from that of the existing technology not only in the parameters but in the type of the distribution. Consider, for example, a case where the yield under the existing technology (T_0) has a continuous distribution, say, chi-squared with n_0 degrees of freedom. Under the new technology (T_1), suppose the distribution of the yield can be best approximated by a Poisson distribution with parameter λ_1. The moment-generating function of the Poisson distribution is given by

$$(30) \qquad M(t) = \exp\left[\lambda(e^t - 1)\right].$$

Following the same procedure as above using (26) and (30), it is found that T_1 is preferred to T_0 if

$$(31) \qquad r(c_1 - c_0) < \bar{Y}_1(1 - e^{-rP}) - \bar{Y}_0 \frac{1}{2} \ln(1 + 2rP)$$

where $Y_0 = n_0$ and $Y_1 = \lambda_1$ are the expected yields for the two distributions.

Conclusions

The use of the mean-variance approach to analyze farmers' behavior under uncertainty is objectionable in many situations on theoretical and empirical grounds. Nevertheless, this approach is commonly applied because of its computability. Therefore, it is desirable to develop alternative methods for risk analysis which are theoretically sound and easily applied.

One possible direction of research is to assume specific utility functions which are preferred to the quadratic utility function and to use existing results in mathematics and statistics to derive simple expressions for the expected utility of income under various distributions. Such an approach is introduced in this paper and is applied for discrete choices among alternative technologies with random yield. The utility of income was measured by the negative exponential utility function. This formulation allows the expression of expected utility using the moment-generating function of the yield. The methodology was applied to derive specific decision rules for cases where yields are gamma distributed. It has been demonstrated that the mean-variance approach may lead to significantly inferior decisions. In general, the use of the decision rule suggested here results in adopting the more risky technology in some cases where the mean-variance approach erroneously recommends against its adoption.

The utility function assumed here implies constant absolute risk aversion. This assumption is more reasonable than increasing absolute risk aversion implied by quadratic utility (and the mean-variance approach). Note, however, as Arrow has argued, most individuals have decreasing absolute risk aversion. Thus, even though the new method introduced here is an improvement over the mean-variance approach, economists remain challenged to find utility functions with decreasing absolute risk aversion which will result in general and simple expressions for expected utility.

Footnotes

[1]Few doubt the usefulness of the expected utility framework as a prescriptive theory. Kahneman and Tversky, however, have conducted some simple experiments and found that observed behavior contradicts the axioms and implications of the expected utility approach. They suggested an alternative theory for explaining behavior under uncertainty—prospect theory. At this stage, although prospect theory is in its infancy, it has substantial promise.

[2]Using the L'hospital theorem, it can be verified that condition (17) implies that, when a farmer is risk neutral ($r = 0$), he will prefer the technology with the highest expected profit.

[3]Assuming $(\sigma^2/\mu)\ rP < 1$.

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