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Optimal Conformal Polynomial Projections for Croatia According to the Airy/Jordan Criterion

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Abstract: *The paper describes optimal conformal polynomial projections for Croatia according to the Airy/Jordan criterion. A brief introduction of history and theory of conformal mapping is followed by descriptions of conformal polynomial projections and their current application. The paper considers polynomials of degrees 1 to 10. Since there are conditions in which the 1st degree polynomial becomes the famous Mercator projection, it was not considered specifically for Croatian territory. The area of Croatia was defined as a union of national territory and the continental shelf. Area definition data were taken from the Euro Global Map 1:1 000 000 for Croatia, as well as from two maritime delimitation treaties. Such an irregular area was approximated with a regular grid consisting of 11 934 ellipsoidal trapezoids 2' large. The Airy/Jordan criterion for the optimal projection is defined as minimum of weighted mean of Airy/Jordan measure of distortion in points. The value of the Airy/Jordan criterion is calculated from all 11 934 centres of ellipsoidal trapezoids, while the weights are equal to areas of corresponding ellipsoidal trapezoids. The minimum is obtained by Nelder and Mead's method, as implemented in the fminsearch function of the MATLAB package. Maps of Croatia representing the distribution of distortions are given for polynomial degrees 2 to 6 and 10. Increasing the polynomial degree results in better projections considering the criterion, and the 6th degree polynomial provides a good ratio of formula complexity and criterion value.*

Key words*: conformal polynomial projections, Croatia, Airy/Jordan criterion*

1 Brief Historical Overview of Conformal Mapping Development

J. H. Lambert (1772) sets and solves the task of mapping a sphere and a rotational ellipsoid into a plane in a contemporary, analytical way. Prior to Lambert's work, known conformal projections were limited to the normal aspect conformal cylindrical projection of sphere (Mercator's projection) and the stereographic projection of sphere. On the basis of differential equations of mapping one surface onto another, Lambert discovered numerous map projections, notably the conformal conical projection, the transverse cylindrical conformal projection and equivalent projections. However, as Lambert himself wrote, solving a general task leads to an infinite number of different map projections.

Conformal mapping equations were set by Lambert as differential equations, which he solved on the basis of additional conditions, e.g. shapes of curves representing meridians and parallels in the plane. When considering cylindrical, conical, azimuthal and circular projections, those curves are straight lines of circles. Nevertheless, Lambert continued by considering the general solution in which the curve shape is arbitrary. He considered such a general solution in the form of infinite series and gave them priority even when a closed solution of differential equations can be found.

Lambert (1772) also represented a way of defining conformal mapping by using complex variable functions proposed by J. L. Lagrange, after Lambert had described the problem to him. In order to formulate the conformal mapping problem, Lagrange used the J. le R. d'Alembert

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Optimalne konformne polinomne projekcije za Hrvatsku po Airy/Jordanovu kriteriju

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Sažetak: *U radu su prikazane optimalne konformne polinomne projekcije za Hrvatsku po Airy/Jordanovu kriteriju. Nakon kratkog uvoda u povijest i teoriju konformnog preslikavanja, opisane su konformne polinomne projekcije i njihova dosadašnja primjena. U ovom radu razmatrani su polinomi stupnja 1. do 10. Kako polinom 1. stupnja uz određene uvjete prelazi u poznatu Mercatorovu projekciju, on nije posebno razmatran za područje Hrvatske. Definirano je područje Hrvatske kao unija državnog teritorija i epikontinentalnog pojasa. Podaci za definiciju područja preuzeti su iz karte Euro Global Map 1:1 000 000 za Hrvatsku, te iz dva sporazuma o razgraničenju na moru. Takvo nepravilno područje aproksimirano je s pravilnom mrežom od 11 934 elipsoidna trapeza veličine 2'. Definiran je Airy/Jordanov kriterij za optimalnu projekciju kao minimum opće sredine Airyjeve, odnosno Jordanove ocjene za deformacije u točki. Opća sredina računa se iz svih 11 934 središta elipsoidnih trapeza, a težine su jednake površinama odgovarajućih elipsoidnih trapeza. Minimum opće sredine nalazi se metodom Neldera i Meada kako je implementirana u funkciji fminsearch paketa MATLAB. Za stupanj polinoma 2. do 6. i 10. dane su karte Hrvatske s rasporedom i veličinom deformacija. Povećanje stupnja polinoma dovodi do sve boljih projekcija u smislu zadanog kriterija, a polinom 6. stupnja daje dobar odnos između složenosti formula i vrijednosti kriterija.*

Ključne riječi: *konformne polinomne projekcije, Hrvatska, Airy/Jordanov kriterij*

1. Kratki povijesni pregled razvoja konformnih preslikavanja

J. H. Lambert (1772) na suvremen, analitički način postavlja i rješava zadatak preslikavanja sfere i rotacijskog elipsoida u ravninu. Prije radova Lamberta, poznate konformne projekcije bile su uspravna konformna cilindrična projekcija sfere (Mercatorova projekcija) i stereografska projekcija sfere. Lambert na temelju diferencijalnih jednadžbi preslikavanja jedne plohe na drugu pronalazi veći broj kartografskih projekcija od kojih se posebno ističu konformna konusna projekcija, poprečna cilindrična konformna projekcija i ekvivalentne projekcije. Međutim, kako i sam Lambert kaže, rješavanje općenito postavljenog zadatka vodi do beskonačnog broja različitih projekcija.

Jednadžbe konformnog preslikavanja Lambert zadaje u obliku diferencijalnih jednadžbi koje rješava na temelju dodatnih uvjeta, npr. oblika krivulja kojima su prikazani meridijani i paralele u ravnini. Kod razmatranja cilindričnih, konusnih, azimutalnih i kružnih projekcija oblici tih krivulja su pravac ili kružnica. Međutim, Lambert se na zadržava samo na tome, već razmatra opće rješenje kod kojega oblik tih krivulja može biti proizvoljan. Takvo općenito rješenje razmatra u obliku beskonačnih redova, i daje im prednost čak i kad je moguće naći zatvoreno rješenje diferencijalnih jednadžbi.

Lambert (1772) prikazuje i način zadavanja konformnog preslikavanja s pomoću funkcije kompleksne varijable koje je predložio J. L. Lagrange, nakon što mu je Lambert opisao problem. Lagrange za formulaciju problema konformnog preslikavanja upotrebljava metodu method to express a differential equation system as a complex variable function (Lagrange 1779). For the second part of defining the conformal mapping task, Lambert also considered the general solution in the form of infinite series. Considering that both methods require the application of series, the rest of Lambert's work uses the definition of conformal mapping in the form of a system of two differential equations.

As can be seen from Lambert's work, he set the task of conformal mapping, explored some special cases (nowadays famous map projections with wide applications), but also considered the general solution in the form of infinite series, so he can be considered the originator of the idea that map projections can be defined in the form of series.

Lambert mostly considered mapping sphere and rotational ellipsoid into a plane. Lagrange generalized Lambert's idea to mapping a general rotational surface into a plane and used complex variable functions to describe conformal mapping. C. F. Gauss (1828) gave a general solution to conformal mapping of any surface onto any other surface, and used complex variable functions, same as Lagrange.

2 Conformal Projections Expressed by Complex Variable Polynomials

Conformal polynomial projections of a certain finite degree, as an approach to solving conformal mapping in the form of finite series, have not been researched and applied to the area of Croatia yet. Conformal polynomial projections are those for which the mapping of the surface of a rotational ellipsoid into a plane is expressed with complex variable polynomials. In order to be conformal, mapping has to use complex variables that are composed of isometric coordinates on both surfaces and to be analytical (Frančula 2004). Cartesian rectangular coordinates are isometric in a plane, while the isometric latitude and the longitude are isometric on the surface of a rotational ellipsoid (see e.g. Frančula 2004).

According to Canters (2002), in 1932 Driencourt and Laborde proposed complex variable polynomials in order to find more favourable projections with regard to area shape. Reilly (1973) described the procedure for finding a favourable conformal projection for New Zealand. Frankić (1982) explored optimal projections for the area of Canada. Some of them are conformal polynomial projections. Nestorov (1996) sought optimal conformal projections for the area of Socialist Federal Republic of Yugoslavia. González López (1995) used complex variable polynomials to find conformal projections for Chile and the Mediterranean Sea. Snyder's adaptive projections for American states, also in the form of polynomials, are well-known (Snyder, 1987). Driencourt and Laborde, Reilly (1973), González López (1995) and Snyder (1987) did not used isometric coordinates on a rotational ellipsoid, but planar coordinates in a known conformal projection, most often the Mercator or stereographic projection. Complex variable polynomials can be used to conformally map any surface into another surface, as long as both surfaces are expressed using isometric coordinates.

Conformal mapping of a rotational ellipsoid into a plane, expressed with a complex variable polynomial is as follows:

$\omega = \sum_{j=0}^{n} C_j z^j$ where

$\omega = x + iy$, $C_j = a_j + ib_j$ and $z = q + i\lambda$ and

$q = \ln\left[\tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right)\left(\frac{1 - e\sin\varphi}{1 + e\sin\varphi}\right)^{\frac{e}{2}}\right]$ is the isometric latitude

on a rotational ellipsoid.

In previous formulae, x and y are planar coordinates, q is the isometric latitude, φ and λ are geodetic coordinates on a rotational ellipsoid, e is the first numeric eccentricity of the rotational ellipsoid, and a_j and b_j are unknown coefficients, the value of which is determined on the basis of additional conditions.

The left Cartesian coordinate system is defined in the plane with the positive part of the x axis directed upward.

Free parameters a_0 and b_0 represent plane translation and are arbitrary, in this research $a_0 = b_0 = 0$.

Isometric coordinate origin on the rotational ellipsoid is going to be placed approximately in the centre of the observed area, i.e. in the point with geographic coordinates $\varphi_0 = 44°$ and $\lambda_0 = 16°$. Therefore, in previous formulae q and λ are determined as:

$q = q - q_0$, where

$q_0 = \ln\left[\tan\left(\frac{\pi}{4} + \frac{\varphi_0}{2}\right)\left(\frac{1 - e\sin\varphi_0}{1 + e\sin\varphi_0}\right)^{\frac{e}{2}}\right]$ and $\varphi_0 = 44°$, and

$\lambda = \lambda - \lambda_0$, where $\lambda_0 = 16°$.

Thus, the point with coordinates $\varphi_0 = 44°$ and $\lambda_0 = 16°$ is going to be mapped into the origin of the planar coordinate system.

The direction and orientation of coordinate axes in the plane also need to be defined. The y axis is going to have the direction of a tangent on the image of the parallel through the origin. Due to conformality, the x axis is going to be in the direction of tangent on the image of the meridian through the origin. Analytically, the condition is defined as

$$\left(\frac{\partial x}{\partial \lambda}\right)_0 = 0.$$

Orientation is determined in such a way that the y coordinate increases with λ in the origin, i.e.

$$\left(\frac{\partial y}{\partial \lambda}\right)_0 > 0.$$

J. le R. d'Alemberta s pomoću koje sustav diferencijalnih jednadžbi zapisuje u obliku funkcije kompleksne varijable (Lagrange 1779). Za taj drugi način postavljanja zadatka konformnog preslikavanja, Lambert također opće rješenje razmatra u obliku beskonačnih redova. S obzirom da oba načina zahtijevaju primjenu redova, Lambert u ostatku rada upotrebljava definiciju konformnog preslikavaja u obliku sustava dviju diferencijalnih jednadžbi.

Kako se vidi iz njegovog rada, Lambert je postavio zadatak konformnog preslikavanja, istražio neke specijalne slučajeve (koji su danas poznate kartografske projekcije sa svojom širokom primjenom), ali i razmatrao opće rješenje u obliku beskonačnih redova, pa ga se može smatrati začetnikom ideje da se zakonitosti kartografske projekcije daju u obliku redova.

Lambert se uglavnom zadržavao na razmatranju preslikavanja sfere i rotacijskog elipsoida u ravninu. Lagrange je Lambertovu ideju poopćio na preslikavanje općenite rotacijske plohe u ravninu, te upotrijebio funkcije kompleksne varijable za opis konformnog preslikavanja. C. F. Gauss (1828) daje opće rješenje konformnog preslikavanja bilo koje plohe na bilo koju drugu plohu, a kao i Lagrange, koristi se funkcijama kompleksne varijable.

2. Konformne projekcije izražene polinomima kompleksne varijable

Konformne polinomne projekcije određenog konačnog stupnja, kao jedan pristup rješenju konformnog preslikavanja u obliku konačnih redova, do sada nisu istraživane i primjenjivane za područje Hrvatske. Pod pojmom konformnih polinomnih projekcija smatrat će se preslikavanje plohe rotacijskog elipsoida u ravninu izraženo polinomima kompleksne varijable. Da bi preslikavanje bilo konformno, kompleksne varijable moraju biti sastavljene od izometrijskih koordinata na obje plohe, tj. takvih koordinata koje za jednaki prirast koordinata daju jednake priraste udaljenosti na plohi (Frančula 2004). U ravnini su Kartezijeve pravokutne koordinate izometrijske, a na plohi rotacijskog elipsoida to su izometrijska širina i geografska dužina (vidi npr. Frančula 2004).

Prema Cantersu (2002) još su 1932. godine Driencourt i Laborde predložili polinome kompleksne varijable sa ciljem iznalaženja povoljnijih projekcija s obzirom na oblik područja. Reilly (1973) opisuje postupak iznalaženja povoljne konformne projekcije za Novi Zeland. Frankić (1982) istražuje optimalne projekcije za područje Kanade. Između ostalog, nalazi i konformne projekcije koristeći polinome kao funkcije izometrijskih koordinata. Nestorov (1996) traži optimalne konformne projekcije za područje SFR Jugoslavije. González López (1995) upotrebljava polinome kompleksne varijable za nalaženje konformnih projekcija za Čile i Sredozemno more. Poznate su Snyderove adaptabilne projekcije za američke države također dobivene u obliku polinoma (Snyder, 1987). Driencourt i Laborde, Reilly (1973), González López (1995) i Snyder (1987) ne upotrebljavaju polinome izometrijskih koordinata na rotacijskom elipsoidu već ravninske koordinate u nekoj poznatoj konformnoj projekciji, najčešće Mercatorovoj ili stereografskoj. Polinomi kompleksne varijable mogu se upotrijebiti za konformno preslikavanje bilo koje plohe na neku drugu plohu sve dok su ispunjeni uvjeti da su koordinate na obje plohe izometrijske.

Konformno preslikavanje rotacijskog elipsoida u ravninu izraženo polinomom kompleksne varijable glasi:

$\omega = \sum_{j=0}^{n} C_j z^j$ gdje su

$\omega = x + iy$, $C_j = a_j + ib_j$ i $z = q + i\lambda$ te

$q = \ln\left(\tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right)\left(\frac{1 - e\sin\varphi}{1 + e\sin\varphi}\right)^{\frac{e}{2}}\right)$ izometrijska širina na rotacijskom elipsoidu.

U prethodnim formulama *x* i *y* su ravninske koordinate, *q* je izometrijska širina, φ i λ su geodetske koordinate na rotacijskom elipsoidu, *e* je prvi numerički ekscentricitet rotacijskog elipsoida, a a_j i b_j su nepoznati koeficijenti čija se vrijednost određuje na temelju dodatnih uvjeta.

U ravnini se definira lijevi pravokutni Kartezijev koordinatni sustav s pozitivnim dijelom osi x usmjerenim prema gore.

Slobodni članovi a_0 i b_0 predstavljaju translaciju ravnine i mogu se izabrati po volji, a u ovim istraživanjima uzet će se da je $a_0 = b_0 = 0$.

Ishodište izometrijskih koordinata na rotacijskom elipsoidu postavit će se približno u središte promatranog područja, tj. u točku s geografskim koordinatama $\varphi_0 = 44°$ i $\lambda_0 = 16°$. Dakle, u prethodnim formulama *q* i λ su zadani kao:

$q = q - q_0$, gdje su

$q_0 = \ln\left(\tan\left(\frac{\pi}{4} + \frac{\varphi_0}{2}\right)\left(\frac{1 - e\sin\varphi_0}{1 + e\sin\varphi_0}\right)^{\frac{e}{2}}\right)$ i $\varphi_0 = 44°$ te

$\lambda = \lambda - \lambda_0$, gdje je $\lambda_0 = 16°$.

Dakle, točka s koordinatama $\varphi_0 = 44°$ i $\lambda_0 = 16°$ preslikat će se u ishodište koordinatnog sustava u ravnini.

Preostaje još definirati smjer i orijentaciju koordinatnih osi u ravnini. Zadat će se da u ishodištu os y ima smjer tangente na sliku paralele. Zbog konformnosti tada će os x biti u smjeru tangente na sliku meridijana. Analitički taj se uvjet definira kao

$$\left(\frac{\partial x}{\partial \lambda}\right)_0 = 0.$$

Orijentaciju zadajemo tako da koordinata *y* raste s λ u ishodištu, odnosno

$$\left(\frac{\partial y}{\partial \lambda}\right)_0 > 0.$$

Planar coordinates are obtained from the following expression:

$x = \mathrm{Re}\left(\sum_{j=1}^{n} C_j z^j\right)$ and $y = \mathrm{Im}\left(\sum_{j=1}^{n} C_j z^j\right)$, i.e. by splitting the function ω into the real and the imaginary part.

The linear scale is gained from general expressions for conformal projection scale (Frančula 2004):

$c = \frac{\sqrt{E}}{M} = \frac{\sqrt{G}}{N\cos\varphi}$, where E and G are Gauss coefficients defined as:

$$E = \left(\frac{\partial x}{\partial \varphi}\right)^2 + \left(\frac{\partial y}{\partial \varphi}\right)^2 \text{ and } G = \left(\frac{\partial x}{\partial \lambda}\right)^2 + \left(\frac{\partial y}{\partial \lambda}\right)^2.$$

It is easy to notice how calculating the scale according to the expression $c = \frac{\sqrt{G}}{N\cos\varphi}$ is simpler because one avoids having to derive the complex function $f(q(\varphi))$.

2.1 1st degree conformal polynomial projections

A 1st degree conformal polynomial projection is determined with the formula:

$$(x+iy)_I = a_0 + ib_0 + (a_1 + ib_1)(q + i\lambda).$$

Splitting it into the real and imaginary parts results in planar coordinate equations:

$(x)_I = a_0 + a_1 q - b_1\lambda$ and $(y)_I = b_0 + b_1 q + a_1\lambda$.

Defined conditions $a_0 = b_0 = 0$, $\left(\frac{\partial x}{\partial \lambda}\right)_0 = 0$ and $\left(\frac{\partial y}{\partial \lambda}\right)_0 > 0$ should be applied.

From the condition $\left(\frac{\partial x}{\partial \lambda}\right)_0 = 0$ we get $\left(\frac{\partial x}{\partial \lambda}\right)_0 = -b_1 = 0$, and the condition $\left(\frac{\partial y}{\partial \lambda}\right)_0 > 0$ is going to be satisfied when $\left(\frac{\partial y}{\partial \lambda}\right)_0 = a_1 > 0$.

Final formulae are as follows:

$(x)_I = a_1 q$ and $(y)_I = a_1\lambda$.

In order to obtain the expression for scale, we first have to calculate partial derivatives:

$\left(\frac{\partial x}{\partial \lambda}\right)_I = 0$ and $\left(\frac{\partial y}{\partial \lambda}\right)_I = a_1$. The Gauss value G now equals:

$G = \left(\frac{\partial x}{\partial \lambda}\right)_I^2 + \left(\frac{\partial y}{\partial \lambda}\right)_I^2 = a_1^2$, and the linear scale is determined according to the expression:

$$c = \frac{\sqrt{G}}{N\cos\varphi} = \frac{a_1}{N\cos\varphi}.$$

A note has to be made; these expressions define the Mercator projection. Since properties of the Mercator projection are well-known (Borčić 1955, Frančula 2004), they are not going to be explored further in this research.

2.2 2nd degree conformal polynomial projections

A 2nd degree conformal polynomial projection is determined with following formulae:

$$(x+iy)_{II} = (x+iy)_I + (a_2 + ib_2)(q + i\lambda)^2.$$

After developing and splitting into real and imaginary parts, it follows:

$(x)_{II} = a_0 - b_1\lambda + a_1 q - a_2\lambda^2 - 2b_2\lambda q + a_2 q^2$ and

$(y)_{II} = b_0 + a_1\lambda + b_1 q - b_2\lambda^2 + 2a_2\lambda q + b_2 q^2.$

From the condition $\left(\frac{\partial x}{\partial \lambda}\right)_0 = 0$ we get $\left(\frac{\partial x}{\partial \lambda}\right)_0 = -b_1 - 2a_2\lambda - 2b_2 q = 0$, i.e. again $b_1 = 0$ because $q = 0$ and $\left(\frac{\partial y}{\partial \lambda}\right)_0 > 0$ in the origin. The condition $\left(\frac{\partial y}{\partial \lambda}\right)_0 > 0$ is going to be satisfied when $\left(\frac{\partial y}{\partial \lambda}\right)_0 = a_1 - 2b_2\lambda + 2a_2 q > 0$, i.e. again $a_1 > 0$ because $q = 0$ and $\lambda = 0$. These conditions are going to be the same for higher-degree polynomials because their derivatives do not contain additional free members.

The final formulae are as follows:

$(x)_{II} = (x)_I - a_2\lambda^2 - 2b_2\lambda q + a_2 q^2$ and

$(y)_{II} = (y)_I - b_2\lambda^2 + 2a_2\lambda q + b_2 q^2.$

In order to calculate the linear scale, we first have to calculate partial derivatives:

$\left(\frac{\partial x}{\partial \lambda}\right)_{II} = \left(\frac{\partial x}{\partial \lambda}\right)_I - 2a_2\lambda - 2b_2 q$ and

$\left(\frac{\partial y}{\partial \lambda}\right)_{II} = \left(\frac{\partial y}{\partial \lambda}\right)_I - 2b_2\lambda + 2a_2 q.$

2.3 3rd degree conformal polynomial projections

A 3rd degree conformal polynomial projection is determined with the formula:

$$(x+iy)_{III} = (x+iy)_{II} + (a_3 + ib_3)(q + i\lambda)^3.$$

By developing and splitting into the real and imaginary parts, we get planar coordinate equations, and by

Ravninske koordinate u takvim projekcijama dobiju se iz izraza:

$x = \mathrm{Re}\left(\sum_{j=1}^{n} C_j z^j\right)$ i $y = \mathrm{Im}\left(\sum_{j=1}^{n} C_j z^j\right)$, odnosno razdvajanjem funkcije ω na realni i imaginarni dio.

Linearno mjerilo preslikavanja dobije se iz općih izraza za mjerilo u konformnim projekcijama (Frančula 2004):

$c = \frac{\sqrt{E}}{M} = \frac{\sqrt{G}}{N\cos\varphi}$, gdje su E i G Gaussove veličine definirane ovako:

$$E = \left(\frac{\partial x}{\partial \varphi}\right)^2 + \left(\frac{\partial y}{\partial \varphi}\right)^2 \text{ i } G = \left(\frac{\partial x}{\partial \lambda}\right)^2 + \left(\frac{\partial y}{\partial \lambda}\right)^2.$$

Lako se primjeti da je računanje mjerila prema izrazu $c = \frac{\sqrt{G}}{N\cos\varphi}$ jednostavnije jer se izbjegava derivacija složene funkcije oblika $f(q(\varphi))$.

2.1. Konformne polinomne projekcije 1. stupnja

Konformna polinomna projekcija 1. stupnja zadana je formulom:

$$(x+iy)_I = a_0 + ib_0 + (a_1 + ib_1)(q + i\lambda).$$

Rastavljanjem na realni i imaginarni dio dobiju se jednadžbe ravninskih koordinata:

$$(x)_I = a_0 + a_1 q - b_1\lambda \text{ i } (y)_I = b_0 + b_1 q + a_1\lambda.$$

Uvrstimo naprijed definirane uvjete $a_0 = b_0 = 0$, $\left(\frac{\partial x}{\partial \lambda}\right)_0 = 0$ i $\left(\frac{\partial y}{\partial \lambda}\right)_0 > 0$.

Iz uvjeta $\left(\frac{\partial x}{\partial \lambda}\right)_0 = 0$ dobije se $\left(\frac{\partial x}{\partial \lambda}\right)_0 = -b_1 = 0$, a uvjet $\left(\frac{\partial y}{\partial \lambda}\right)_0 > 0$ bit će zadovoljen kada je $\left(\frac{\partial y}{\partial \lambda}\right)_0 = a_1 > 0$.

Konačne formule glase:

$$(x)_I = a_1 q \text{ i } (y)_I = a_1\lambda.$$

Da bismo dobili izraz za mjerilo treba najprije izračunati parcijalne derivacije:

$\left(\frac{\partial x}{\partial \lambda}\right)_I = 0$ i $\left(\frac{\partial y}{\partial \lambda}\right)_I = a_1$. Gaussova veličina G sada iznosi:

$G = \left(\frac{\partial x}{\partial \lambda}\right)_I^2 + \left(\frac{\partial y}{\partial \lambda}\right)_I^2 = a_1^2$, a linearno mjerilo određuje se prema izrazu:

$$c = \frac{\sqrt{G}}{N\cos\varphi} = \frac{a_1}{N\cos\varphi}.$$

Ovdje treba primjetiti da je tim izrazima zadana Mercatorova projekcija. Kako su svojstva Mercatorove projekcije dobro poznata (Borčić 1955, Frančula 2004), ovdje se ona neće dalje istraživati.

2.2. Konformne polinomne projekcije 2. stupnja

Konformna polinomna projekcija 2. stupnja zadana je formulama:

$$(x+iy)_{II} = (x+iy)_I + (a_2 + ib_2)(q + i\lambda)^2.$$

Nakon razvijanja i razdvajanja realnog i imaginarnog dijela jednadžbe dobije se:

$$(x)_{II} = a_0 - b_1\lambda + a_1 q - a_2\lambda^2 - 2b_2\lambda q + a_2 q^2 \text{ i}$$

$$(y)_{II} = b_0 + a_1\lambda + b_1 q - b_2\lambda^2 + 2a_2\lambda q + b_2 q^2.$$

Iz uvjeta $\left(\frac{\partial x}{\partial \lambda}\right)_0 = 0$ dobije se

$\left(\frac{\partial x}{\partial \lambda}\right)_0 = -b_1 - 2a_2\lambda - 2b_2 q = 0$, odnosno ponovo $b_1 = 0$ jer su u ishodištu $q = 0$ i $\lambda = 0$. Uvjet $\left(\frac{\partial y}{\partial \lambda}\right)_0 > 0$ bit će zadovoljen kada je $\left(\frac{\partial y}{\partial \lambda}\right)_0 = a_1 - 2b_2\lambda + 2a_2 q > 0$, odnosno ponovo $a_1 > 0$ zbog $q = 0$ i $\lambda = 0$. Ti uvjeti bit će jednaki i za polinome viših stupnjeva jer njihove derivacije ne sadrže dodatne slobodne članove.

Konačne formule glase:

$$(x)_{II} = (x)_I - a_2\lambda^2 - 2b_2\lambda q + a_2 q^2 \text{ i}$$

$$(y)_{II} = (y)_I - b_2\lambda^2 + 2a_2\lambda q + b_2 q^2.$$

Da bismo izračunali linearno mjerilo treba najprije izračunati parcijalne derivacije:

$$\left(\frac{\partial x}{\partial \lambda}\right)_{II} = \left(\frac{\partial x}{\partial \lambda}\right)_I - 2a_2\lambda - 2b_2 q \text{ i}$$

$$\left(\frac{\partial y}{\partial \lambda}\right)_{II} = \left(\frac{\partial y}{\partial \lambda}\right)_I - 2b_2\lambda + 2a_2 q.$$

2.3. Konformne polinomne projekcije 3. stupnja

Konformna polinomna projekcija 3. stupnja zadana je formulom:

$$(x+iy)_{III} = (x+iy)_{II} + (a_3 + ib_3)(q + i\lambda)^3.$$

Raspisivanjem i rastavljanjem na realni i imaginarni dio dobiju se jednadžbe ravninskih koordinata, te uvođenjem zadanih uvjeta konačne formule za ravninske koordinate glase:

introducing the conditions given, final formulae for planar coordinates are as follows:

$$(x)_{III} = (x)_{II} + b_3\lambda^3 - 3a_3\lambda^2 q - 3b_3\lambda q^2 + a_3 q^3 \text{ and}$$

$$(y)_{III} = (y)_{II} - a_3\lambda^3 - 3b_3\lambda^2 q + 3a_3\lambda q^2 + b_3 q^3.$$

Partial derivatives required for calculating the linear scale are as follows:

$$\left(\frac{\partial x}{\partial \lambda}\right)_{III} = \left(\frac{\partial x}{\partial \lambda}\right)_{II} + 3b_3\lambda^2 - 6a_3\lambda q - 3b_3 q^2$$

and

$$\left(\frac{\partial y}{\partial \lambda}\right)_{III} = \left(\frac{\partial y}{\partial \lambda}\right)_{II} - 3a_3\lambda^2 - 6b_3\lambda q + 3a_3 q^2.$$

2.4 4th degree conformal polynomial projections

A 4th degree conformal polynomial projection is determined with following formulae:

$$(x+iy)_{IV} = (x+iy)_{III} + (a_4 + ib_4)(q+i\lambda)^4.$$

After expanding, splitting into the real and imaginary parts and applying initial conditions, formulae for planar coordinates are as follows:

$$(x)_{IV} = (x)_{III} + a_4\lambda^4 + 4b_4\lambda^3 q - 6a_4\lambda^2 q^2 - 4b_4\lambda q^3 + a_4 q^4$$

and

$$(y)_{IV} = (y)_{III} + b_4\lambda^4 - 4a_4\lambda^3 q - 6b_4\lambda^2 q^2 + 4a_4\lambda q^3 + b_4 q^4.$$

Partial derivatives required for calculating the linear scale are as follows:

$$\left(\frac{\partial x}{\partial \lambda}\right)_{IV} = \left(\frac{\partial x}{\partial \lambda}\right)_{III} + 4a_4\lambda^3 + 12b_4\lambda^2 q - 12a_4\lambda q^2 - 4b_4 q^3$$

and

$$\left(\frac{\partial y}{\partial \lambda}\right)_{IV} = \left(\frac{\partial y}{\partial \lambda}\right)_{III} + 4b_4\lambda^3 - 12a_4\lambda^2 q - 12b_4\lambda q^2 + 4a_4 q^3.$$

2.5 5th degree conformal polynomial projection

A 5th degree conformal polynomial projection is determined with following formulae:

$$(x+iy)_V = (x+iy)_{IV} + (a_5 + ib_5)(q+i\lambda)^5.$$

After expanding, splitting into real and imaginary parts and applying the initial conditions, formulae for planar coordinates are as follows:

$$(x)_V = (x)_{IV} - b_5\lambda^5 + 5a_5\lambda^4 q + 10b_5\lambda^3 q^2 - 10a_5\lambda^2 q^3 - 5b_5\lambda q^4 + a5q^5$$

and

$$(y)_V = (y)_{IV} + a_5\lambda^5 + 5b_5\lambda^4 q - 10a_5\lambda^3 q^2 - 10b_5\lambda^2 q^3 + 5a_5\lambda q^4 + b_5 q^5.$$

Partial derivatives required for calculating the linear scale are as follows:

$$\left(\frac{\partial x}{\partial \lambda}\right)_V = \left(\frac{\partial x}{\partial \lambda}\right)_{IV} - 5b_5\lambda^4 + 20a_5\lambda^3 q + 30b_5\lambda^2 q^2 - 20a_5\lambda q^3 - 5b_5 q^4$$

and

$$\left(\frac{\partial y}{\partial \lambda}\right)_V = \left(\frac{\partial y}{\partial \lambda}\right)_{IV} + 5a_5\lambda^4 + 20b_5\lambda^3 q - 30a_5\lambda^2 q^2 - 20b_5\lambda q^3 + 5a_5 q^4.$$

2.6 6th degree conformal polynomial projections

A 6th degree conformal polynomial projection is determined with following formulae:

$$(x+iy)_{VI} = (x+iy)_V + (a_6 + ib_6)(q+i\lambda)^6.$$

After expanding, splitting into imaginary and real parts and applying initial conditions, formulae for planar coordinates are as follows:

$$(x)_{VI} = (x)_V - a_6\lambda^6 - 6b_6\lambda^5 q + 15a_6\lambda^4 q^2 + 20b_6\lambda^3 q^3 - 15a_6\lambda^2 q^4 - 6b_6\lambda q^5 + a_6 q^6$$

and

$$(y)_{VI} = (y)_V - b_6\lambda^6 + 6a_6\lambda^5 q + 15b_6\lambda^4 q^2 - 20a_6\lambda^3 q^3 - 15b_6\lambda^2 q^4 + 6a_6\lambda q^5 + b_6 q^6.$$

Partial derivatives required for calculating the linear scale are as follows:

$$\left(\frac{\partial x}{\partial \lambda}\right)_{VI} = \left(\frac{\partial x}{\partial \lambda}\right)_V - 6a_6\lambda^5 - 30b_6\lambda^4 q + 60a_6\lambda^3 q^2 + 60b_6\lambda^2 q^3 - 30a_6\lambda q^4 - 6b_6 q^5$$

and

$$\left(\frac{\partial y}{\partial \lambda}\right)_{VI} = \left(\frac{\partial y}{\partial \lambda}\right)_V - 6b_6\lambda^5 + 30a_6\lambda^4 q + 60b_6\lambda^3 q^2 - 60a_6\lambda^2 q^3 - 30b_6\lambda q^4 + 6a_6 q^5.$$

Formulae for higher-degree conformal polynomial projections are derived in the same way.

3 Optimal Conformal Polynomial Projections

An optimal conformal polynomial projection is going to be the one which gives the smallest value of a given criteria for a given geographic area. In addition to its administrative territory, Croatia also lays its economic rights and interests in the area of the continental shelf. Thus, this research considers the area of Croatia as its administrative territory together with its continental shelf.

$$(x)_{III} = (x)_{II} + b_3\lambda^3 - 3a_3\lambda^2 q - 3b_3\lambda q^2 + a_3 q^3 \quad \text{i}$$

$$(y)_{III} = (y)_{II} - a_3\lambda^3 - 3b_3\lambda^2 q + 3a_3\lambda q^2 + b_3 q^3.$$

Parcijalne derivacije potrebne za računanje linearnog mjerila glase:

$$\left(\frac{\partial x}{\partial \lambda}\right)_{III} = \left(\frac{\partial x}{\partial \lambda}\right)_{II} + 3b_3\lambda^2 - 6a_3\lambda q - 3b_3 q^2$$

i

$$\left(\frac{\partial y}{\partial \lambda}\right)_{III} = \left(\frac{\partial y}{\partial \lambda}\right)_{II} - 3a_3\lambda^2 - 6b_3\lambda q + 3a_3 q^2.$$

2.4. Konformne polinomne projekcije 4. stupnja

Konformna polinomna projekcija 4. stupnja zadana je formulama:

$$(x+iy)_{IV} = (x+iy)_{III} + (a_4 + ib_4)(q + i\lambda)^4.$$

Nakon raspisivanja, rastavljanja na realni i imaginarni dio i primjene početnih uvjeta formule za ravninske koordinate glase:

$$(x)_{IV} = (x)_{III} + a_4\lambda^4 + 4b_4\lambda^3 q - 6a_4\lambda^2 q^2 - 4b_4\lambda q^3 + a_4 q^4$$

i

$$(y)_{IV} = (y)_{III} + b_4\lambda^4 - 4a_4\lambda^3 q - 6b_4\lambda^2 q^2 + 4a_4\lambda q^3 + b_4 q^4.$$

Parcijalne derivacije potrebne za računanje linearnog mjerila glase:

$$\left(\frac{\partial x}{\partial \lambda}\right)_{IV} = \left(\frac{\partial x}{\partial \lambda}\right)_{III} + 4a_4\lambda^3 + 12b_4\lambda^2 q - 12a_4\lambda q^2 - 4b_4 q^3 \quad \text{i}$$

$$\left(\frac{\partial y}{\partial \lambda}\right)_{IV} = \left(\frac{\partial y}{\partial \lambda}\right)_{III} + 4b_4\lambda^3 - 12a_4\lambda^2 q - 12b_4\lambda q^2 + 4a_4 q^3.$$

2.5. Konformne polinomne projekcije 5. stupnja

Konformna polinomna projekcija 5. stupnja zadana je formulama:

$$(x+iy)_V = (x+iy)_{IV} + (a_5 + ib_5)(q + i\lambda)^5.$$

Nakon raspisivanja, rastavljanja na realni i imaginarni dio i primjene početnih uvjeta formule za ravninske koordinate glase:

$$(x)_V = (x)_{IV} - b_5\lambda^5 + 5a_5\lambda^4 q + 10b_5\lambda^3 q^2 - 10a_5\lambda^2 q^3 - \\ - 5b_5\lambda q^4 + a_5 q^5$$

i

$$(y)_V = (y)_{IV} + a_5\lambda^5 + 5b_5\lambda^4 q - 10a_5\lambda^3 q^2 - 10b_5\lambda^2 q^3 + \\ + 5a_5\lambda q^4 + b^{5} q^5 .$$

Parcijalne derivacije potrebne za računanje linearnog mjerila glase:

$$\left(\frac{\partial x}{\partial \lambda}\right)_{V} = \left(\frac{\partial x}{\partial \lambda}\right)_{IV} - 5b_5\lambda^4 + 20a_5\lambda^3 q + 30b_5\lambda^2 q^2 - \\ - 20a_5\lambda q^3 - 5b_5 q^4 \quad \text{i}$$

$$\left(\frac{\partial y}{\partial \lambda}\right)_{V} = \left(\frac{\partial y}{\partial \lambda}\right)_{IV} + 5a_5\lambda^4 + 20b_5\lambda^3 q - 30a_5\lambda^2 q^2 \\ - 20b_5\lambda q^3 + 5a_5 q^4 .$$

2.6. Konformne polinomne projekcije 6. stupnja

Konformna polinomna projekcija 6. stupnja zadana je formulama:

$$(x+iy)_{VI} = (x+iy)_{V} + (a_6 + ib_6)(q + i\lambda)^6.$$

Nakon raspisivanja, rastavljanja na realni i imaginarni dio i primjene početnih uvjeta formule za ravninske koordinate glase:

$$(x)_{VI} = (x)_V - a_6\lambda^6 - 6b_6\lambda^5 q + 15a_6\lambda^4 q^2 + 20b_6\lambda^3 q^3 - \\ - 15a_6\lambda^2 q^4 - 6b_6\lambda q^5 + a_6 q^6$$

i

$$(y)_{VI} = (y)_V - b_6\lambda^6 + 6a_6\lambda^5 q + 15b_6\lambda^4 q^2 - 20a_6\lambda^3 q^3 - \\ - 15b_6\lambda^2 q^4 + 6a_6\lambda q^5 + b_6 q^6 .$$

Parcijalne derivacije potrebne za računanje linearnog mjerila glase:

$$\left(\frac{\partial x}{\partial \lambda}\right)_{VI} = \left(\frac{\partial x}{\partial \lambda}\right)_{V} - 6a_6\lambda^5 - 30b_6\lambda^4 q + 60a_6\lambda^3 q^2 + \\ + 60b_6\lambda^2 q^3 - 30a_6\lambda q^4 - 6b_6 q^5$$

i

$$\left(\frac{\partial y}{\partial \lambda}\right)_{VI} = \left(\frac{\partial y}{\partial \lambda}\right)_{V} - 6b_6\lambda^5 + 30a_6\lambda^4 q + 60b_6\lambda^3 q^2 - \\ - 60a_6\lambda^2 q^3 - 30b_6\lambda q^4 + 6a_6 q^5 .$$

Na isti način izvode se formule za konformne polinomne projekcije viših stupnjeva.

3. Optimalne konformne polinomne projekcije

Optimalnom konformnom polinomnom projekcijom smatrat će se ona koja na zadanom geografskom području daje najmanju vrijednost zadanog kriterija. Hrvatska osim državnog teritorija svoja gospodarska prava i interese polaže i na područje epikontinetnalnog pojasa. Iz tog razloga kao područje Hrvatske u ovom radu uzima se unija državnog teritorija i epikontinentalnoga morskog pojasa.

Kriterij koji je upotrijebljen je Airy/Jordanov kriterij za konformne projekcije. Airyjeva i Jordanova ocjena

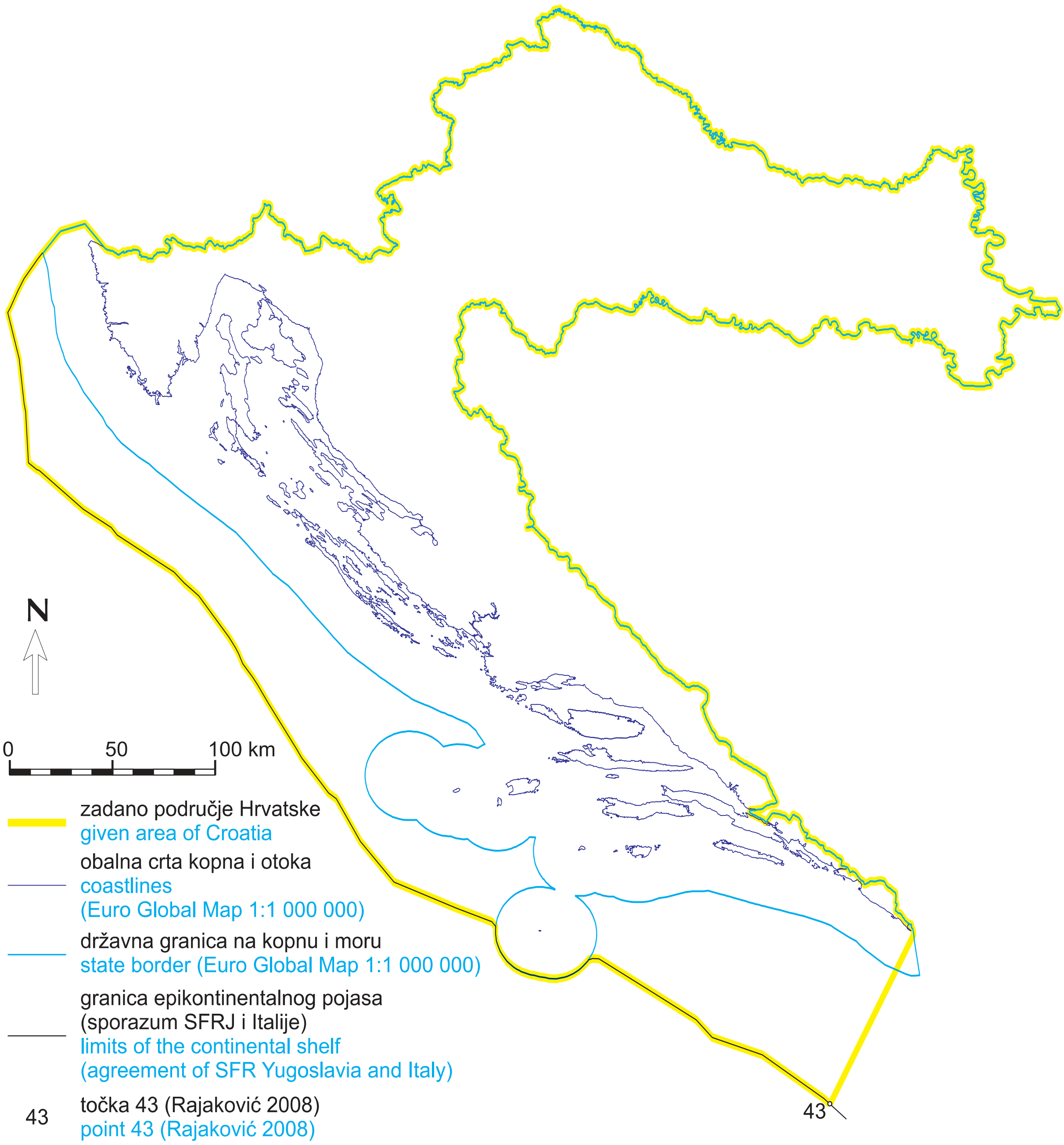

Fig. 1. Data defining the area of Croatia

Slika 1. Podaci kojima je definirano područje Hrvatske

The criterion used was the Airy/Jordan criterion for conformal projections. The Airy and Jordan distortion mesure is often used for evaluating distortions over the area (Canters 2002, Frankić 1982, Reilly 1973, Frančula 1971, Airy 1861).

There are various approaches to finding the minimum of a function determined with a criterion. For example, Frankić (1982) and Reilly (1973) used a linearization of the function and the least squares method in an iterative procedure for finding a minimum. Canters (2002) applied the simplex method according to Nelder and Mead (1965). The simplex method as implemented within the *fminsearch* function in the MATLAB (URL1) program was applied in this research.

3.1 Area of Croatia

In order to determine optimal map projections, accurate data about boundaries of an area are not necessary. A choice of map projection for an area does not substantially depend on the high accuracy of boundary coordinates of the area, but primarily on the shape and size of the area. Data for defining the administrative area were taken from the Euro Global Map 1:1 000 000 for Croatia published by the State Geodetic Administration

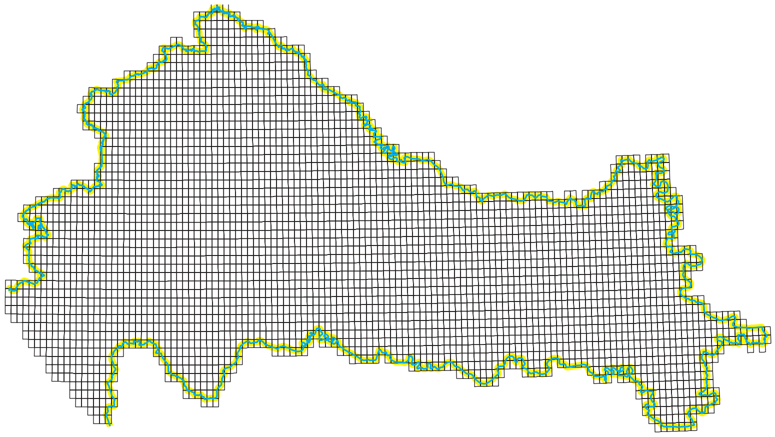

Fig. 2. Section of approximation of the given area with ellipsoidal trapezoids 2' × 2'

Slika 2. Isječak aproksimacije zadanog područja elipsoidnim trapezima veličine 2' × 2'

deformacija često se upotrebljava prilikom ocjene deformacija na nekon području (Canters 2002, Frankić 1982, Reilly 1973, Frančula 1971, Airy 1861).

Za nalaženje minimuma funkcije zadane kriterijem mogući su različiti pristupi. Primjerice, Frankić (1982) i Reilly (1973) upotrebljavaju postupak linearizacije funkcije i primjenu metode najmanjih kvadrata u iteracijskom postupku nalaženja minimuma. Canters (2002) upotrebljava metodu simpleksa po Nelderu i Meadu (1965). U ovom radu upotrijebljena je metoda simpleksa kako je implementirana funkcijom *fminsearch* u programu MATLAB (URL1).

3.1. Područje Hrvatske

Za potrebe nalaženja optimalnih kartografskih projekcija nije nužno imati vrlo točne podatke o granici područja. Izbor kartografske projekcije za neko područje ne ovisi u značajnoj mjeri o točnosti koordinata granice tog područja, već ponajprije o obliku i veličini tog područja. Podaci za definiciju državnog područja preuzeti su iz karte Euro Global Map 1:1 000 000 za Hrvatsku što ju izdaje Državna geodetska uprava (URL2). Podaci o granici epikontinentalnog pojasa preuzeti su iz Službenog lista SFRJ, Međunarodni ugovori i drugi sporazumi, br. 28/1970. Koordinate za točku broj 43 granice epikontinentalnog pojasa umjesto iz tog sporazuma preuzete su iz (Rajaković 2008) gdje su koordinate te točke određene na temelju *Protokola između Vlade Republike Hrvatske i Savezne Vlade Savezne Republike Jugoslavije o privremenom režimu uz južnu granicu između dviju država* iz 2002. godine (sl. 1). Za referentni elipsoid uzet je GRS80 i svi su podaci svedeni na taj elipsoid.

Takvo nepravilno područje aproksimirano je pravilnom mrežom od 11 934 elipsoidna trapeza veličine 2' × 2' (sl. 2). Ta je veličina izabrana na temelju istraživanja (Tutić i Lapaine 2008) gdje se pokazalo da se vrijednosti nepoznatih koeficijenata u projekciji ne razlikuju znatno od onih dobivenih još gušćom mrežom elipsoidnih trapeza.

Aproksimacija nepravilnog područja elipsoidnim trapezima praktično je izvedena kao izbor onih poligona u mreži elipsoidnih trapeza koji se preklapaju ili su unutar područja Hrvatske. Takav postupak moguće je provesti unutar programa za GIS ili prostornih baza podataka. Za potrebe ovog rada upotrijebljen je program GRASS GIS (URL3).

3.2. Airy/Jordanov kriterij za konformne projekcije

Airy/Jordanova ocjena deformacija u konformnim projekcijama na nekom području glasi (Nestorov 1996, Rajaković 2008):

$E^2 = \frac{1}{A}\int_A (c-1)^2 dA$, gdje je c linearno mjerilo, A je površina promatranog područja, a dA diferencijal površine.

Integral u ovoj formuli za nepravilna područja rijetko je moguće riješiti analitički. Zbog toga će se integral zamijeniti sumom. Tada Airy/Jordanova ocjena glasi (Tutić i Lapaine 2008):

$E^2 = \frac{1}{\sum_i \Delta A_i}\sum_{i=1}^{n}(c_i-1)^2\Delta A_i$, gdje je ΔA_i jedan (mali) dio područja, a c_i linearno mjerilo u nekoj točki tog dijela područja.

Uzme li se za ΔA_i područje elipsoidnog trapeza kojemu je središte u točki (φ_i, λ_i), a veličina po geografskoj dužini mu je $\Delta\lambda$ i po geografskoj širini $\Delta\varphi$ površina takvog područja može se odrediti po formuli (Lapaine, Lapaine 1991):

(URL2). Data about the continental shelf boundary were taken from the Official Sheet of the Socialist Federal Republic of Yugoslavia, International Contracts and Other Treaties, no. 28/1970. Coordinates for point no. 43 of the continental shelf boundary were taken from (Rajaković 2008), where they were determined on the basis of *Protocol Between the Government of the Republic of Croatia and the Federal Government of the Federal Republic of Yugoslavia About Temporary Regime for the South Boundary Between the two Countries* from 2002 (Fig. 1). GRS80 is chosen as the reference ellipsoid and all data were transformed to it.

Such an irregular area was approximated with a regular grid consisting of 11 934 ellipsoidal trapezoids of 2'×2' (Fig. 2). The size was chosen on the basis of research (Tutić and Lapaine 2008), which showed that values of unknown coefficients in a map projection for Croatia do not differ substantially from those obtained through an even denser grid of ellipsoidal trapezoids.

Approximation of the irregular area with ellipsoidal trapezoids was practically executed as a selection of those polygons in the grid of ellipsoidal trapezoids which overlap or are within the area of Croatia. Such a procedure can be carried out within a program for GIS or spatial databases. The program GRASS GIS was used for the needs of this research (URL3).

3.2 Airy/Jordan's criterion for conformal projections

Airy/Jordan's distorsion measure in conformal projection over the area is as follows (Nestorov 1996, Rajaković 2008):

$E^2 = \frac{1}{A}\int_A (c-1)^2 dA$, where c is a linear scale, A is the area of the observed territory, and dA is the area differential.

It is difficult to analytically solve the integral in this formula for irregular areas. Thus the integral will be replaced with a sum. Doing so, the Airy/Jordan's measure becomes (Tutić and Lapaine 2008):

$E^2 = \frac{1}{\sum_i \Delta A_i}\sum_{i=1}^{n}(c_i-1)^2\Delta A_i$, where ΔA_i is one (small) part of the area, and c_i is the linear scale in a point of that area part.

If ΔA_i is the area of an ellipsoidal trapezoid, the centre of which is in point (φ_i, λ_i), and size along longitude is $\Delta\lambda$ and along latitude $\Delta\varphi$, the area of such an ellipsoidal trapezoid can be determined according to the formula (Lapaine, Lapaine 1991):

$$\Delta A_i = \frac{b^2\Delta\lambda}{2}\left(\frac{\sin\varphi_i}{1-e^2\sin^2\varphi_i} + \ln\left(\frac{1+e\sin\varphi_i}{1-e\sin\varphi_i}\right)^{\frac{1}{2e}}\right)\Bigg|_{\varphi_i-\frac{\Delta\varphi}{2}}^{\varphi_i+\frac{\Delta\varphi}{2}}$$

where b is the minor semi-axis of the rotational ellipsoid and e is the first eccentricity of rotational ellipsoid. The linear scale in this case can be calculated in the point (φ_i, λ_i).

Doing so defines the Airy/Jordan's measure for conformal projections in a given area approximated with a regular grid of ellipsoidal trapezoids.

If we find for a chosen projection a set of parameter values of the projection $P = \{p_1, p_2, \ldots, p_n\}$ for which the minimal value of Airy/Jordan's measure is obtained, i.e.

$$\min_P E^2 = \min_P \frac{1}{\sum_i \Delta A_i}\sum_{i=1}^{n}(c_i-1)^2\Delta A_i,$$

the variant of the chosen projection determined with a parameter value set P is going to be referred to as the optimal projection according to the Airy/Jordan's criterion.

3.3 Optimal conformal polynomial projections of degrees 2 to 10 according to the Airy/Jordan criterion

Using MATLAB to define all given values and functions and finding the minimum results in criterion values and unknown coefficient values. Table 1 contains criterion values, and Fig. 3 represents the dependence of the criterion value on the polynomial degree.

Fig. 4 to 9 represent the absolute scale error distribution in optimal conformal polynomial projections of degrees 2 to 6 and 10. Corresponding coefficient values are given in the image description for degrees 2 to 6.

Table 1. Comparison of criterion values for researched projections

Conformal projection	Value of the Airy/Jordan criterion
2nd degree polynomial	0.000176
3rd degree polynomial	0.000109
4th degree polynomial	0.000076
5th degree polynomial	0.000075
6th degree polynomial	0.000058
7th degree polynomial	0.000051
8th degree polynomial	0.000051
9th degree polynomial	0.000046
10th degree polynomial	0.000044

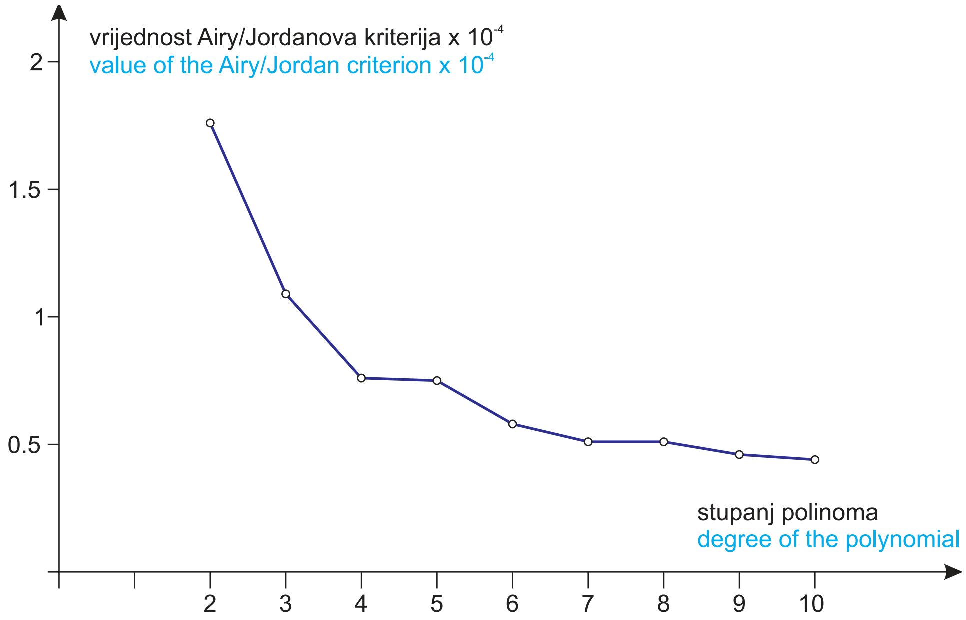

Fig. 3. Value of the Airy/Jordan criterion in relation to the degree of conformal polynomial projection
Slika 3. Vrijednost Airy/Jordanova kriterija u odnosu na stupanj konformne polinomne projekcije

$$\Delta A_i = \frac{b^2 \Delta\lambda}{2} \left(\frac{\sin\varphi_i}{1 - e^2 \sin^2 \varphi_i} + \ln \left(\frac{1 + e \sin\varphi_i}{1 - e \sin\varphi_i} \right)^{\frac{1}{2e}} \right) \Bigg|_{\varphi_i - \frac{\Delta\varphi}{2}}^{\varphi_i + \frac{\Delta\varphi}{2}}$$

gdje je b mala poluos rotacijskog elipsoida i e prvi ekscentricitet. Linearno mjerilo c_i u tom slučaju neka se računa u točki .

Na taj način definirano je određivanje vrijednosti Airy/Jordanove ocjene za konformne projekcije na nekom zadanom području aproksimiranom pravilnom mrežom elipsoidnih trapeza.

Ako se za izabranu projekciju nađe skup vrijednosti parametara te projekcije $P = \{p_1, p_2, \ldots, p_n\}$ za koji se postiže najmanja vrijednost Airy/Jordanove ocjene, tj.

$$\min_P E^2 = \min_P \frac{1}{\sum_i \Delta A_i} \sum_{i=1}^{n} (c_i - 1)^2 \Delta A_i,$$

varijantu izabrane projekcije određenu skupom vrijednosti parametara P nazvat ćemo optimalnom projekcijom po Airy/Jordanovom kriteriju.

3.3. Optimalne konformne polinomne projekcija 2. do 10. stupnja po Airy/Jordanovu kriteriju

Nakon što se u MATLAB-u definiraju sve zadane veličine i funkcije, te provede postupak traženja minimuma kao rezultat dobiju se vrijednosti kriterija i vrijednosti nepoznatih koeficijenata. U tablici 1. dane su dobivene vrijednosti kriterija, a na slici 3. prikazana je ovisnost vrijednosti kriterija o stupnju polinoma.

Slike 4. do 9. prikazuju veličinu i raspored deformacija u optimalnim konformnim polinomnim projekcijama stupnja 2. do 6. i 10. U opisu slike za stupnjeve 2. do 6. dane su i odgovarajuće vrijednosti koeficijenata.

Tablica 1. Usporedba vrijednosti kriterija za istražene projekcije

Konformna projekcija	Vrijednost Airy/Jordanova kriterija
Polinomna 2. stupnja	0,000176
Polinomna 3. stupnja	0,000109
Polinomna 4. stupnja	0,000076
Polinomna 5. stupnja	0,000075
Polinomna 6. stupnja	0,000058
Polinomna 7. stupnja	0,000051
Polinomna 8. stupnja	0,000051
Polinomna 9. stupnja	0,000046
Polinomna 10. stupnja	0,000044

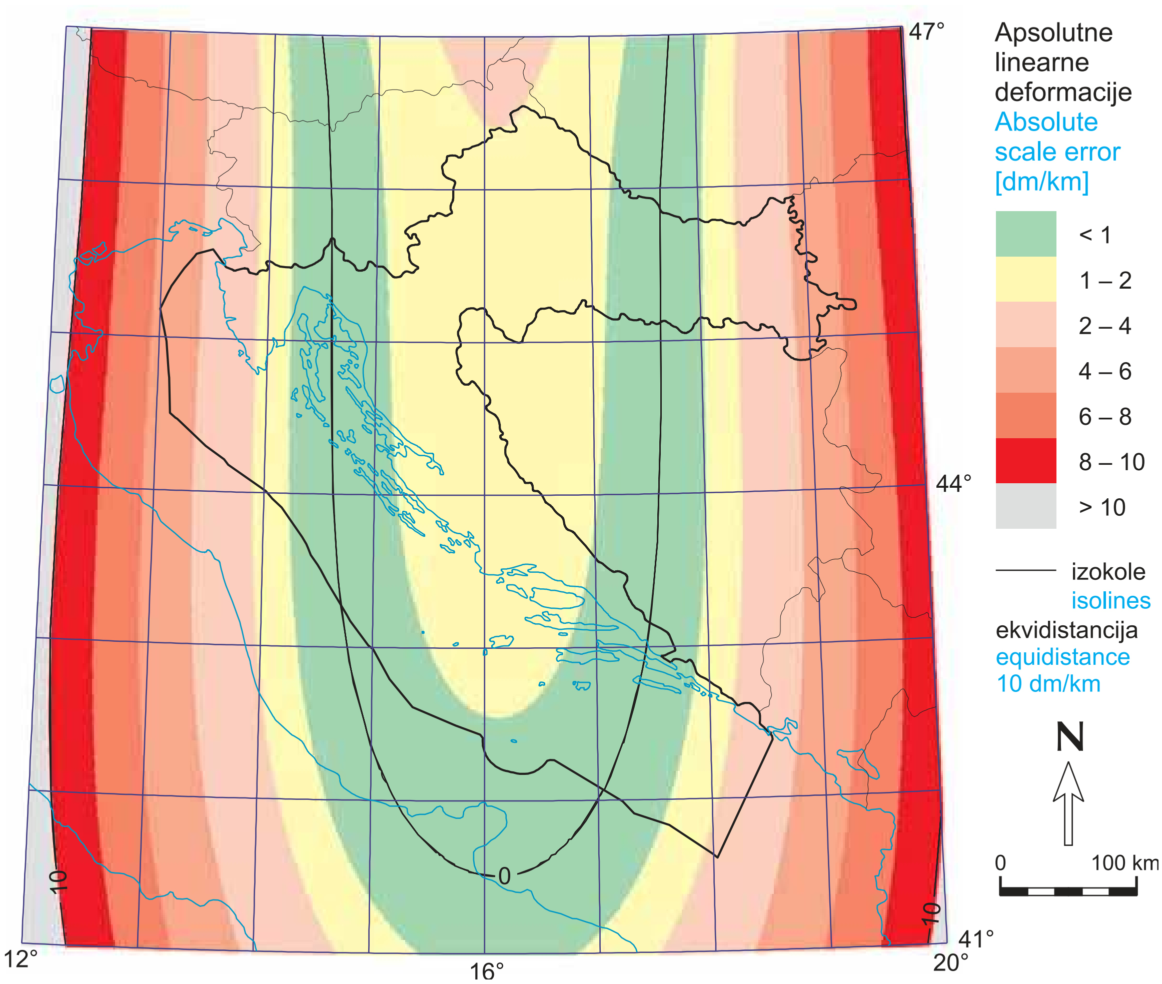

Fig. 4. Absolute scale error distribution in the optimal conformal polynomial projection of the 2nd degree according to the Airy/Jordan criterion. Coefficients are $a_1 = 4.59474 \cdot 10^6$, $a_2 = -1.59788 \cdot 10^6$ *and* $b_2 = 2.07707 \cdot 10^3$

Slika 4. Raspored i veličina deformacija u optimalnoj konformnoj polinomnoj projekciji 2. stupnja po Airy/Jordanovu kriteriju. Koeficijenti $a_1 = 4{,}59474 \cdot 10^6$, $a_2 = -1{,}59788 \cdot 10^6$ *i* $b_2 = 2{,}07707 \cdot 10^3$

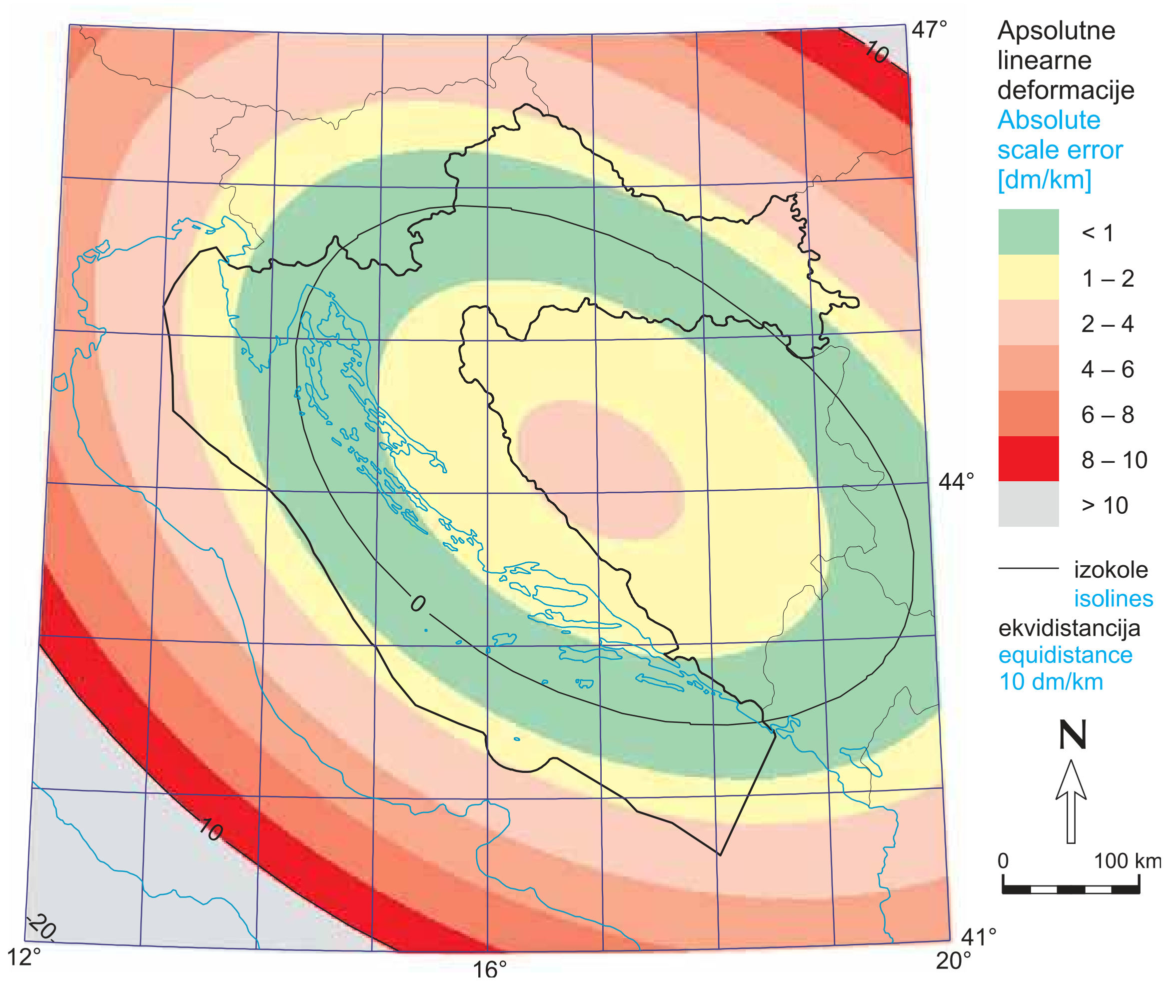

Fig. 5. Absolute scale error distribution in the optimal conformal polynomial projection of the 3rd degree according to the Airy/Jordan criterion. Coefficients are $a_1 = 4.59468 \cdot 10^6$, $a_2 = -1.60251 \cdot 10^6$, $b_2 = 9.61478 \cdot 10^3$, $a_3 = 2.05344 \cdot 10^5$ *and* $b_3 = -8.14867 \cdot 10^4$

Slika 5. Raspored i veličina deformacija u optimalnoj konformnoj polinomnoj projekciji 3. stupnja po Airy/Jordanovu kriteriju. Koeficijenti $a_1 = 4{,}59468 \cdot 10^6$, $a_2 = -1{,}60251 \cdot 10^6$, $b_2 = 9{,}61478 \cdot 10^3$, $a_3 = 2{,}05344 \cdot 10^5$ *i* $b_3 = -8{,}14867 \cdot 10^4$

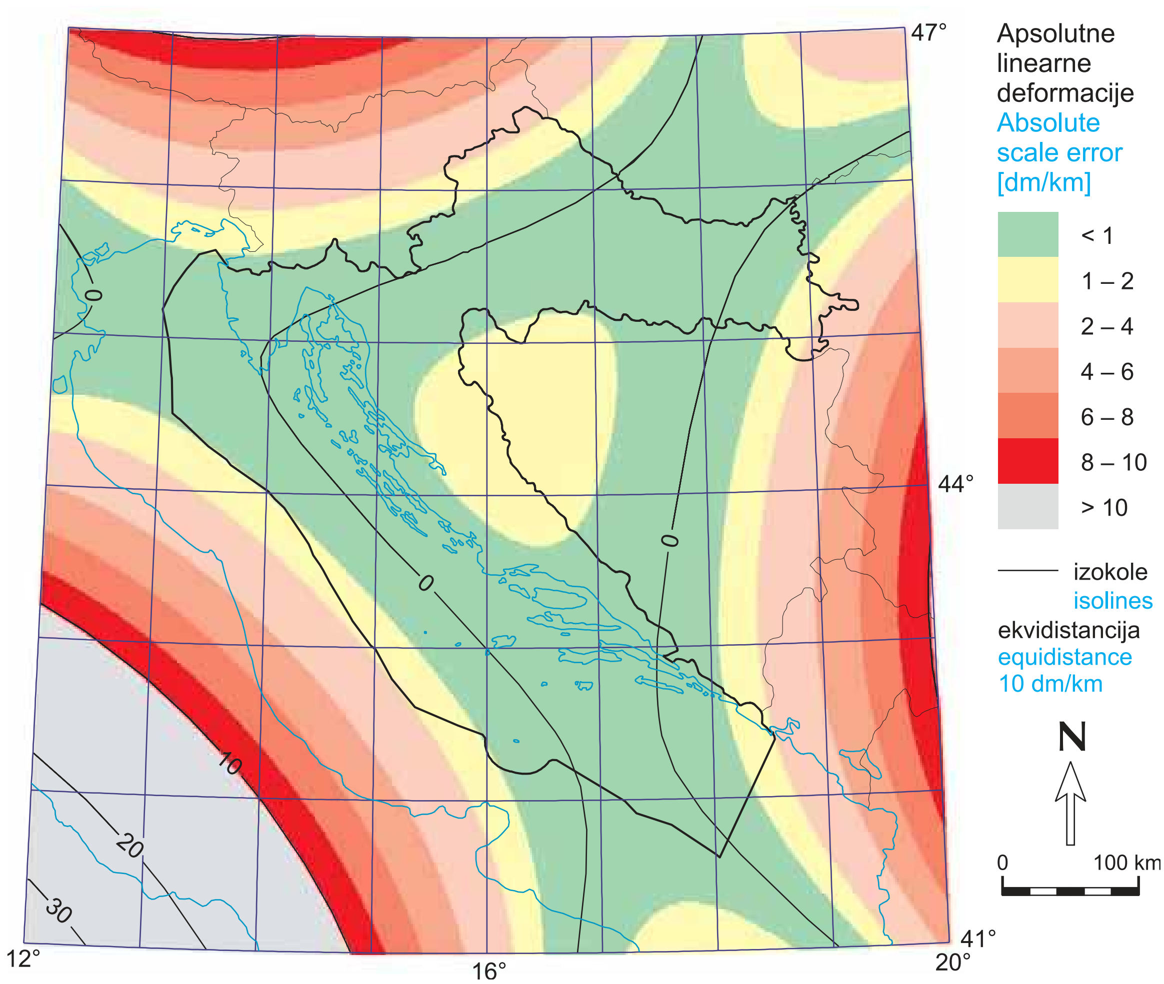

Fig. 6. Absolute scale error distribution in the optimal conformal polynomial projection of the 4th degree according to the Airy/Jordan criterion. Coefficients are $a_1 = 4.59495 \cdot 10^6$, $a_2 = -1.60233 \cdot 10^6$, $b_2 = 4.97068 \cdot 10^3$, $a_3 = 1.30868 \cdot 10^5$, $b_3 = -8.45032 \cdot 10^4$, $a_4 = 1.19404 \cdot 10^6$ *and* $b_4 = 1.45752 \cdot 10^6$

Slika 6. Raspored i veličina deformacija u optimalnoj konformnoj polinomnoj projekciji 4. stupnja po Airy/Jordanovu kriteriju. Koeficijenti $a_1 = 4{,}59495 \cdot 10^6$, $a_2 = -1{,}60233 \cdot 10^6$, $b_2 = 4{,}97068 \cdot 10^3$, $a_3 = 1{,}30868 \cdot 10^5$, $b_3 = -8{,}45032 \cdot 10^4$, $a_4 = 1{,}19404 \cdot 10^6$ *i* $b_4 = 1{,}45752 \cdot 10^6$

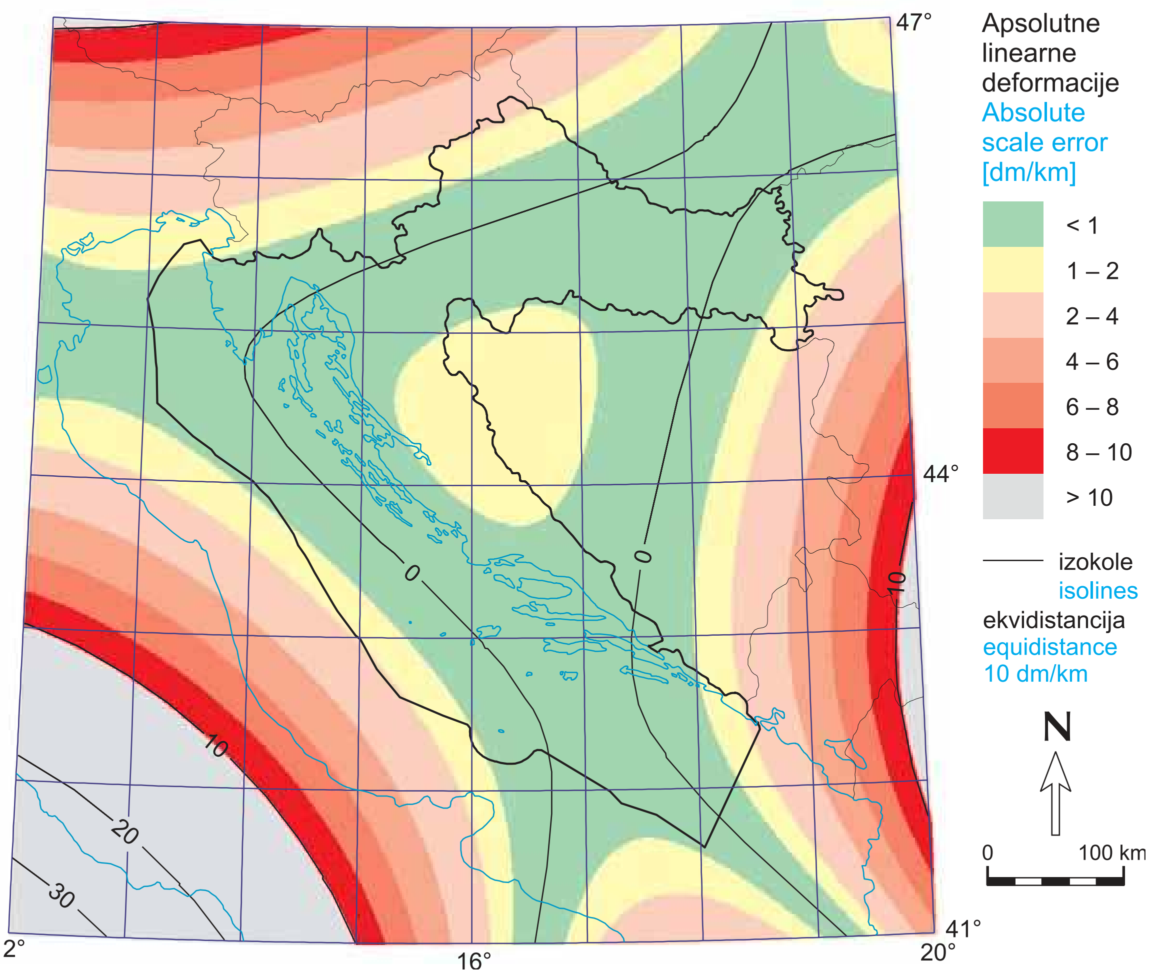

Fig. 7. Absolute scale error distribution in the optimal conformal polynomial projection of the 5th degree according to the Airy/Jordan criterion. Coefficients are $a_1 = 4.59496 \cdot 10^6$, $a_2 = -1.60273 \cdot 10^6$, $b_2 = 4.37379 \cdot 10^3$, $a_3 = 1.34363 \cdot 10^5$, $b_3 = -8.50200 \cdot 10^4$, $a_4 = 1.18477 \cdot 10^6$, $b_4 = 1.59488 \cdot 10^6$, $a_5 = -1.67331 \cdot 10^6$ *and* $b_5 = -3.81873 \cdot 10^6$

Slika 7. Raspored i veličina deformacija u optimalnoj konformnoj polinomnoj projekciji 5. stupnja po Airy/Jordanovu kriteriju. Koeficijenti $a_1 = 4{,}59496 \cdot 10^6$, $a_2 = -1{,}60273 \cdot 10^6$, $b_2 = 4{,}37379 \cdot 10^3$, $a_3 = 1{,}34363 \cdot 10^5$, $b_3 = -8{,}50200 \cdot 10^4$, $a_4 = 1{,}18477 \cdot 10^6$, $b_4 = 1{,}59488 \cdot 10^6$, $a_5 = -1{,}67331 \cdot 10^6$ *i* $b_5 = -3{,}81873 \cdot 10^6$

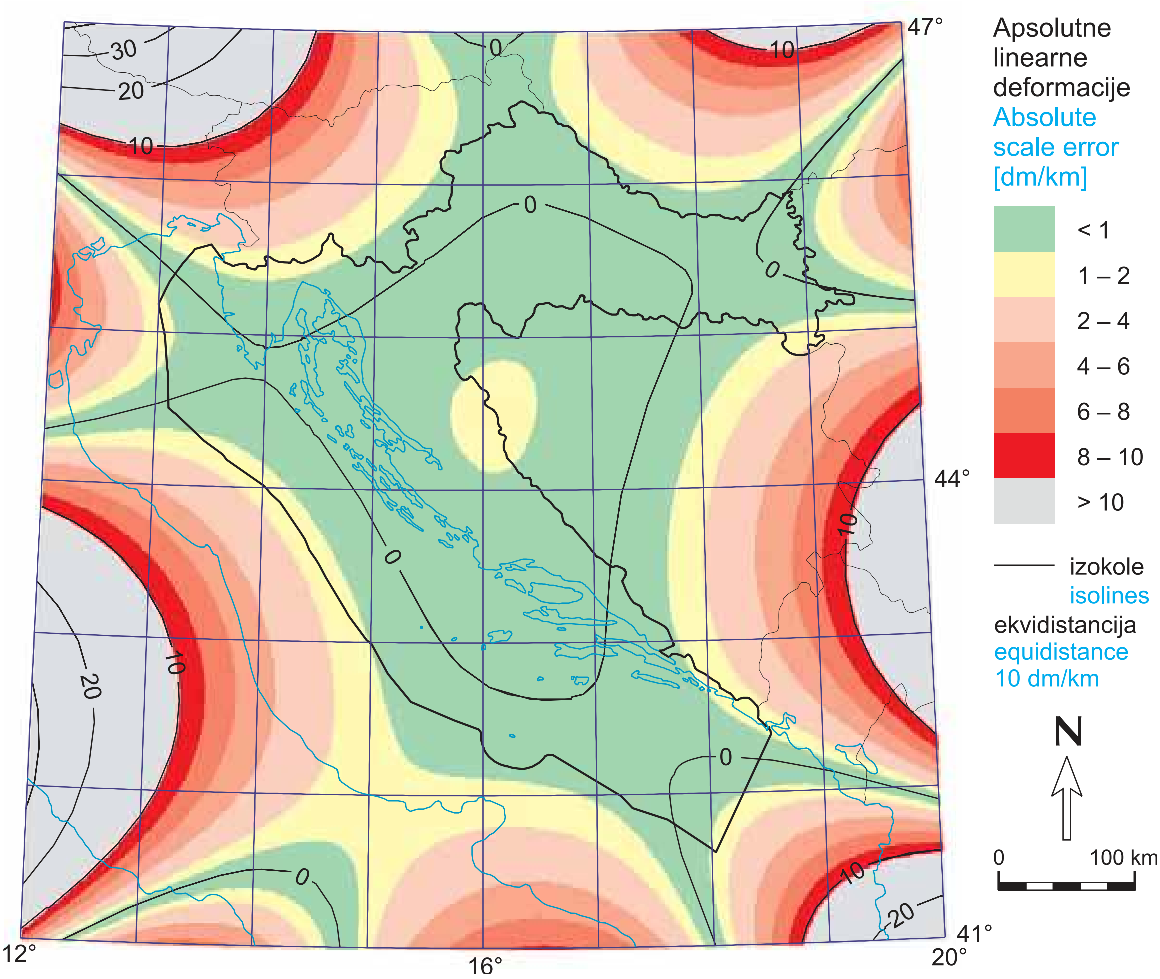

Fig. 8. Absolute scale error distribution in the optimal conformal polynomial projection of the 6th degree according to the Airy/Jordan criterion. Coefficients are $a_1 = 4.59504 \cdot 10^6$, $a_2 = -1.60038 \cdot 10^6$, $b_2 = 1.76780 \cdot 10^3$, $a_3 = 6.19324 \cdot 10^4$, $b_3 = -4.51810 \cdot 10^4$, $a_4 = 1.53766 \cdot 10^6$, $b_4 = 9.41033 \cdot 10^5$, $a_5 = 7.17668 \cdot 10^6$, $b_5 = 1.04285 \cdot 10^7$, $a_6 = -2.76147 \cdot 10^8$ *and* $b_6 = -1.33392 \cdot 10^8$

Slika 8. Raspored i veličina deformacija u optimalnoj konformnoj polinomnoj projekciji 6. stupnja po Airy/Jordanovu kriteriju. Koeficijenti $a_1 = 4{,}59504 \cdot 10^6$, $a_2 = -1{,}60038 \cdot 10^6$, $b_2 = 1{,}76780 \cdot 10^3$, $a_3 = 6{,}19324 \cdot 10^4$, $b_3 = -4{,}51810 \cdot 10^4$, $a_4 = 1{,}53766 \cdot 10^6$, $b_4 = 9{,}41033 \cdot 10^5$, $a_5 = 7{,}17668 \cdot 10^6$, $b_5 = 1{,}04285 \cdot 10^7$, $a_6 = -2{,}76147 \cdot 10^8$ *i* $b_6 = -1{,}33392 \cdot 10^8$

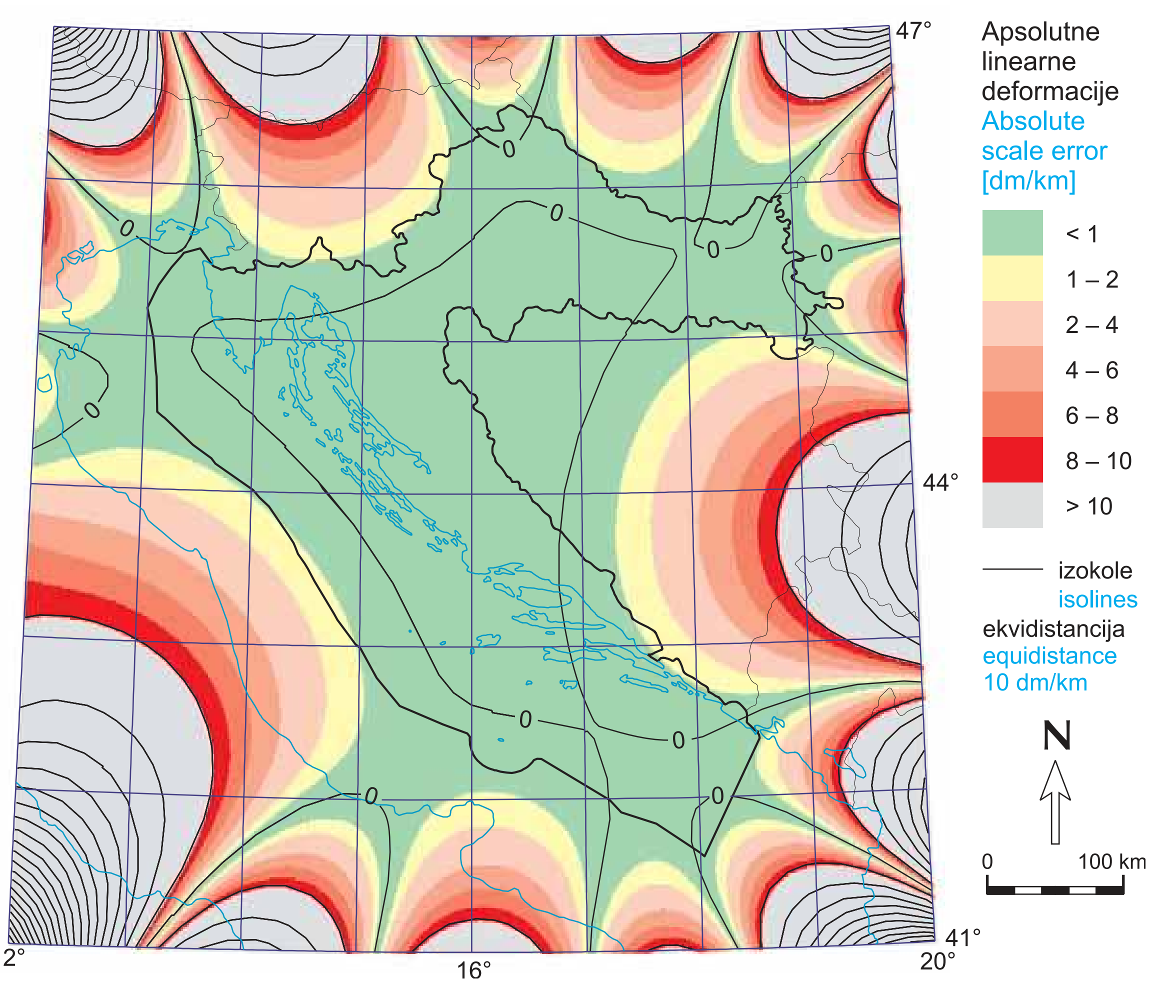

Fig. 9. Absolute scale error distribution in the optimal conformal polynomial projection of the 10th degree according to the Airy/Jordan criterion

Slika 9. Raspored i veličina deformacija u optimalnoj konformnoj polinomnoj projekciji 10. stupnja po Airy/Jordanovu kriteriju

4. Conclusion

Conformal polynomial projections have not been applied or researched for the area of Croatia yet. The methodology described in the paper was successfully applied to finding new optimal conformal polynomial projections for Croatia according to the Airy/Jordan criterion for conformal projections. Increasing the polynomial degree leads to smaller criterion values, as expected. In contrary, a larger criterion value for a higher polynomial degree would indicate the methodology used does not provide good results. The reason lies in the fact that a lower polynomial degree is a special case of the higher degree. A good ratio between formula complexity and the criterion value is the 6th degree conformal polynomial projection. None of the presented projections provides the shape of distortion isolines considerably approaching the shape of the area boundary. A probable reason lies in the shape of the area, which is very irregular. Conformal polynomial projections are usually intended to approximate the Chebyshev conformal projection (Chebyshev/Čebišev 1856), i.e. such a projection which has the linear scale constant on the area boundary. Thus, the projections presented here, especially those of the 6th and 10th degree, represent certain approaching to the Chebyshev projection.

Note

Coefficient values for optimal variants of conformal polynomial projections of degrees 7 to 10 can be obtained from the author (dtutic@geof.hr).

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4. Zaključak

Konformne polinomne projekcije do sada se nisu primjenjivale niti istraživale za područje Hrvatske. Opisana metodologija uspješno je primijenjena na nalaženje novih optimalnih konformnih polinomnih projekcija za Hrvatsku po Airy/Jordanovu kriteriju za konformne projekcije. Povećanje stupnja polinoma očekivano vodi i do manjih vrijednosti kriterija. U protivnom, kada bi vrijednost kriterija bila veća za veći stupanj polinoma, to bi bio znak da upotrijebljena metodologija ne daje dobre rezultate. Razlog leži u činjenici da je niži stupanj polinoma specijalni slučaj višeg stupnja. Dobar odnos između složenosti formula i vrijednosti kriterija predstavlja konformna polinomna projekcija 6. stupnja. Niti jedna prikazana projekcija ne daje oblik izokola koji bi se u većoj mjeri približio obliku granice područja. Vjerojatni razlog leži u obliku područja koje je vrlo nepravilno. Konformnim polinomnim projekcijama obično se nastoji aproksimirati Čebiševljeva konformna projekcija (Čebišev 1856), tj. takva projekcija u kojoj je linearno mjerilo konstantno na granici područja. U tom smislu i ovdje prikazane projekcije, posebno 6. i 10. stupnja predstavljaju određeno približenje Čebiševljevoj projekciji.

Napomena

Vrijednosti koeficijenata za optimalne varijante konformnih polinomnih projekcija 7. do 10. stupnja mogu se dobiti od autora (dtutic@geof.hr).

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