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Optimal Constrained Layer Damping of Beams: Experimental and Numerical Studies

This article deals with the optimal damping of beams constrained by viscoelastic layers when only one or several portions of the beam are covered. The design variables are the dimensions and locations of the viscoelastic layers and the objective function is the maximum damping factor. The discrete design variable optimization problem is solved using a genetic algorithm. Numerical results for minimum and maximum damping are compared to experimental results. This is done for a various number of materials and beams. © *1995 John Wiley & Sons, Inc.*

INTRODUCTION

Structural vibration control is a major design problem for a variety of structures. This control may be approached in several ways such as active attenuators, structural damping, etc. In most cases the designer's objective is to minimize vibration amplitudes in a wide frequency range to prevent damage by fatigue. For this purpose, two main processes may be followed by engineers. One is the use of composite materials that generally exhibit excellent material damping properties one or two orders higher than most common metals; the other is to damp the structure itself by viscoelastic material coatings or to insert passive dampers at the most efficient locations. Each of these solutions has it own advantages, but it may be pointed out that the second method may often be used without any change in the design requirements. An efficient technique to damp beams or other structures is the use of viscoelastic constrained layers glued on the surface (Fig. 1). This article is concerned with the optimal damping of beams partially covered by constrained viscoelastic layers. The optimization problem is the determination of the sizes and the locations of these specific dampers. This article combines a numerical study of optimized partial coverage of a constrained viscoelastic layer on a beam with experimental results for the predicted configurations of maximum and minimum damping.

There exist too few references for optimal constrained layer damping. The present article can be considered as the continuation of two previous articles. Marcelin et al. (1992) dealt with a similar problem but only with a numerical approach; conventional nonlinear programming optimization was used but was not very efficient. Indeed, as the design variables that represent the constrained viscoelastic layers positions are not continuous, the objective function has no derivatives and the classical mathematical approaches are invalid. The second article of Marcelin and Trompette (1994) was devoted to optimal location of plate

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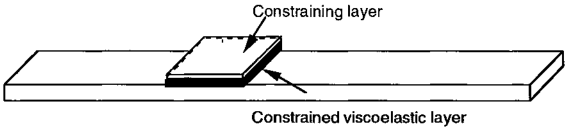

FIGURE 1 Beam damped by viscoelastic material.

damped parts by use of a genetic algorithm; but there was only numerical results and no comparison with experiments was achieved. It was shown how stochastic optimization methods as genetic algorithms offer a new and attractive way for solving this kind of question. In another article from Nokes and Nelson (1968), theoretical and experimental studies were also presented on partially covered beams, but without an optimization step. The major conclusion of Nokes and Nelson (1968) was the following: it is not necessary to cover the whole beam to achieve adequate damping in a structure. The tests of Nokes and Nelson indicated a peak in the damping at about 60% coverage (of the central portion) of a free–free beam. Here both partial covering, optimization, and for the first time, experimental validation are considered. The design variables are the dimensions and the locations of all the viscoelastic layers. Special beam finite elements are used to represent the behavior of the sandwich parts of the beam, and a genetic search is used for the optimization problem (Marcelin and Trompette, 1994). The same problem, with similar techniques and similar results, was dealt with in Hajela and Lin (1991), but without any experimental approach and for additional design variables corresponding to the thicknesses of the viscoelastic layers and the constraining layers. Hajela and Lin (1991) show that genetic search methods are well suited for such generically difficult design. Applications in the design of isotropic and composite beams for maximum damping and minimum weight are shown in Hajela and Lin (1991).

OPTIMIZATION METHOD

The dynamic behavior of partially covered beams is obtained from a modal model. The homogeneous parts of the beam are discretized by conventional C1 finite elements (FEs) and the heterogeneous or sandwich ones by specific FEs designed to represent accurately the viscoelastic core shear damping effect. The finite elements of the damped and undamped parts must be as compatible as possible. Such elements have been used previously for plates in Marcelin and Trompette (1994).

The equilibrium equations associated to structural damping are:

$$[-\omega^2|M| + |K_r| + j|K_i|]\,\{u_0 e^{j\omega t}\} = \{F_0 e^{j\omega t}\}. \quad (1)$$

Because of time dependent stress–strain relations, the representation (1) of the structural damping is simple only for periodic excitations. For such excitations a viscoelastic material behavior may be represented by a complex Young's modulus: $E_v = E_{vr} + jE_{vi}$. This means that the damping introduced by viscoelastic constrained layers is a special case of structural damping. It can be assumed that the real and the imaginary part of E_v, the storage and the loss modulus, are frequency dependent. This hypothesis is often experimentally verified for E_{vr}. The frequency dependence of E_{vi} might be taken into account but without adding anything to the main results of the optimization process that does not depend on this parameter; this is the reason why it is not considered hereafter.

Frequencies and mode shapes of the undamped associated structures can be considered as a good and simple modal basis to be used for predicting the dynamic behavior of the corresponding damped structure. ω_i and φ_i, $i = 1, n$ are the *undamped* frequencies and corresponding mode shapes obtained from the matrix equation:

$$(-\omega^2|M| + |K|)\{x\} = \{0\}. \quad (2)$$

Performing the usual transformation $\{x\} = |F|\{q\}$, and premultiplying by $|F|^T$, Eq. (3) is obtained for free vibrations:

$$(-\omega^2|m_{\text{diag}}| + (|k_{\text{rdiag}}| + j|k_i|)\{q\} = \{\varphi\}. \quad (3)$$

Because of the orthogonality of the modes, $|m|$ and $|k_r|$ are diagonal matrices, but not $|k_i|$. Generally for beams the frequencies are well separated, so the full damping matrices can be considered as diagonal dominant. In these conditions the modal system (3) is the sum of n uncoupled equations. It follows from the preceding that in a modal response, a good approximation of the structural loss factor for the optimization may be easily calculated from (3), so the objective function to be maximized has the general form:

$$\eta = \frac{E_d}{E_S} \quad (4)$$

in which E_S is noted as the elastic strain energy and E_d the dissipation energy. Due to the above hypothesis, the damping can be written:

$$\eta = \frac{\sum_{k=nb\text{ of retained modes}} \alpha_k \langle \varphi_k \rangle |k_i| \{\varphi_k\}}{\sum_{k=nb\text{ of retained modes}} \alpha_k k_k}. \quad (5)$$

α_k is a weighting modal factor given by the user. It is pointed out here that the use of the undamped modes simplify readily the calculation of the objective function. The denominator of (5) is invariant during all the optimization process.

Because the design variables are the locations and the dimensions of the viscoelastic parts, the optimization problem is obviously a discrete one. So to maximize the damping factor a genetic algorithm is used (Goldberg, 1989). Genetic algorithms are based on the principles of natural selection and survival of the fittest. The genetic analogy is maintained in the terminology used in the method. An initial population is generated by random selection of the individual bits in a binary string of given length. The strings represent, directly or indirectly, the design variables in the objective function. Groups are formed, initially at random, to compose families of strings, each family containing a single set of parameters comprising a design. The fitness of each group is then evaluated and assessed against the objective function. The strings in the best families are given favorable weightings in a selection process whereby pairs of strings (parents) are chosen, combined by a crossover process. It is useful also to introduce an element of mutation whereby some bits are switched to encourage the development of new genetic material. The incidence of mutation is controlled by the user through the prescription of a mutation probability. After each cycle of selection, crossover, and possibly mutation, the fitness of each family is again assessed by converting the binary strings to decimal digits (decoding) and evaluating the objective function. The cycle then continues into the next generation. The process is terminated when convergence is detected or when the specified maximum number of generations is reached. Genetic algorithms are particularly well suited to represent simply the dimensions and the locations of the viscoelastic layers. In the present work, heterogeneous beam elements may be coded by 1 and homogeneous beam elements by 0, so a design point (a chromosome) is an n binary number in which n is the number of finite elements. All the details are given in Marcelin and Trompette (1994). In the study by Marcelin et al. (1995) about optimization of composite beam structures, they show that genetic algorithms are a very attractive and efficient way to optimize damping of mechanical structures. In the article by Marcelin et al. (1995), the optimization problem is to find an optimal stacking sequence of composite materials to maximize a modal damping factor.

EXPERIMENTS

For measurement of damping vibration in the structures there are several methods; they are explained in Ewins (1989). The best method is the impulse frequency response technique, because with an only hammer shock, different frequencies are excited and therefore one can gain several modes of vibrations. In this work damping in flexural vibration for the first mode is considered. One can apply different boundary conditions on the beam specimens, but to avoid the different effects of supports and fixations on the structure, a free–free beam is selected. The beam specimens are suspended on the nodes of the first frequency; and with a modally tuned hammer one excite the specimens. Therefore there are rigid body motions as well as the other modes of vibration. Figure 2 shows a schematic of the test and apparatus.

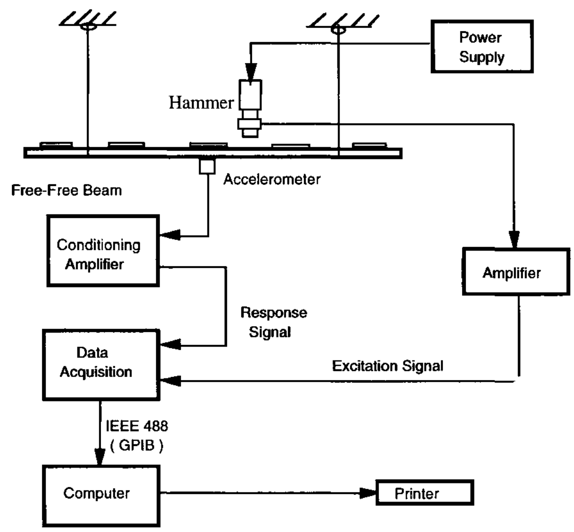

FIGURE 2 Apparatus for free–free vibration of beam.

Hammer

For exciting the beams with an impulse shock, we use a modal-tuned impact hammer. This hammer reduces the input force so that the amplitude of the beams is small enough to eliminate aerodynamic damping as a significant factor. The nonlinearity and external noise are less than the other methods of excitation. There are different tips with different hardness for exciting different beam materials. A force transducer is installed in the tip of hammer. The hammer is energized by a power supply.

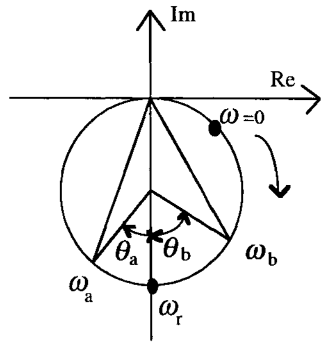

FIGURE 3 Properties of modal circle.

Accelerometer

An accelerometer is installed in the middle of the free–free beam for acquiring the acceleration of the beam at that location. An amplifier and a conditioner are used for amplifying and conditioning the signals.

Data Acquisition

The input signal of the force transducer and response signal of the accelerometer are sent to the data acquisition system. An analyzer fast Fourier transform (FFT) is installed for treatment of the data acquired.

Computer

A PC computer is used for FFT computation by curve fitting to the frequency response function. The resonant frequencies and the modal loss factors are determined with good accuracy. Zoom measurement of the frequency response near the resonant frequency improves the results. The major advantage of the curve fitting is that much more data near a resonance are used to measure damping. For minimizing the errors associated with curve fitting, a Nyquist plot is applied using a circle fit algorithm (Fig. 3). Equation (6) is used for determination of the loss factor.

$$\eta = \frac{\omega_a^2 - \omega_b^2}{\omega_r^2\left(\tan\frac{\theta_a}{2} + \tan\frac{\theta_b}{2}\right)} \tag{6}$$

where ω_a, θ_a, and ω_b, θ_b are points on the modal circle below and above the resonant frequency, ω_r. A close correlation exists between the experimental damping properties of a known aluminum specimen and the theoretical Zener thermoelastic prediction.

Two kinds of beams are considered in the numerical and experimental processes. The first beams are made of polyurethane (PU). The dimensions of these beams are the following; length 0.950 m, section 0.05 × 0.014 m. The loss factor of PU at normal temperatures is around 1%. The first free–free frequency of the PU beams is 19 Hz. The second beams are sandwich aluminum-PU-aluminum beams. The dimensions of these beams are the following; length 1.5 m, width 0.07 m, aluminum thicknesses 0.0006 m, PU height 0.0108 m. The first free–free frequency of the sandwich beams is 28 Hz. In the two cases, the constrained viscoelastic layers have a thickness of 0.0008 m; the loss factor of the viscoelastic material used is near 1%; the constraining layers are made of aluminum and their thickness is 0.0006 m.

RESULTS

The locations of viscoelastic layers are determined from numerical optimization for minimum and then for maximum damping for the two kinds of beams. In both cases only the first free–free mode is considered and the total length of the damped parts are set equal to 25% of the length of the beams. Sixteen FEs were used to model the beams. Only a part (4) of the elements can be covered. In this case, the string length is 16, but the first four strings determine the location of the first element, the following four other strings determine the location of the second element, and so on. There are no equality or inequality constraints.

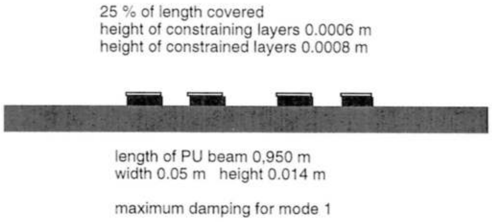

FIGURE 4 Maximum damping solution for PU beam.

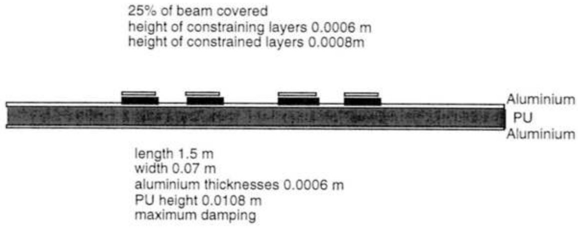

FIGURE 6 Maximum damping solution for sandwich beam.

An example of string when only four elements are covered is:

0010 0000 1000 1011

that means that elements 3 (0010), 1 (0000), 9 (1000), and 12(1011) are covered. The results for the PU beams are the following. Figure 4 gives the genetic search results for maximum damping only for mode 1. The parameters defining the genetic algorithm are the following: population size 20, number of generations 40, probability of crossover 0.6, probability of mutation 0.1. We do not use the weighting modal factor defined in Eq. (5). The damped parts are separated. In this way, flexural vibration causes more shearing strain in the viscoelastic core and thus more energy is dissipated. The experiments for this configuration give a damping factor of 1.55% (the numerical one is 1.36%). Figure 5 gives the genetic search result for minimum damping for mode 1. The experiments for this configuration give a loss factor of 0.99% (the numerical one is 0.11%). The difference between the two cases is about 50%. The optimization was not performed for modes 2 and 3; nevertheless experiments were done for modes 2 and 3. For the configuration of Fig. 4, the dampings of modes 2 and 3 are, respectively, 1.53 and 1.48%. For the configuration of Fig. 5, the dampings of modes 2 and 3 are, respectively, 0.97 and 0.93%. In conclusion, in this case the optimum results for mode 1 also seem valid for modes 2 and 3.

The results for the sandwich beams are the following. Figure 6 gives the genetic search results for maximum damping only for mode 1. The distribution of the damped parts is the same as that in the previous example. The experiments for this configuration give a damping factor of 0.45% (the numerical one is 1.17%). Figure 7 gives the genetic search result for minimum damping for mode 1. The experiments for this configuration give a loss factor of 0.06% (the numerical one is 0.22%). The difference between the two cases is very important and shows the interest of an optimal partial coverage. The physical reasons that the configuration of Fig. 4 or 6 provides maximum damping and the configuration of Fig. 5 or 7 provides minimum damping may be the following: for maximum damping, the constrained viscoelastic layers are located in the areas where shear strain energy is maximum and the layers are divided into four parts, so shear strain is increased; for minimum damping, the constrained viscoelastic layers are located at the two ends of the beams in the areas where strain energy is minimum and they are not divided.

The experimental validations follow the numerical calculations because the numerical calculations of damping give only qualitative results and allow one to see whether damping is important or not. Numerical calculations do not give the exact values so the numerical damping cannot be compared with the experimental ones. Other investigators have achieved much better agreement between numerical and experimental results. The reasons why the numerical results are at such variance with experimental results are: first, the same modal basis (the undamped modes) is used all along the calculations to guarantee the efficiency of the optimization (which need a lot of calculations); second, we have not determined

FIGURE 5 Minimum damping solution for PU beam.

FIGURE 7 Minimum damping solution for sandwich beam.

the frequency dependence of the damping of the materials that we used. Nevertheless the inaccuracy of our numerical prediction of damping does not seem to affect the accuracy of our prediction of the optimum damping configuration.

CONCLUSION

The optimization of damping of beams by constrained viscoelastic layers, when only one or several portions of the beam are covered, was considered. Applications may exist in several areas such as the aeronautics, automobile, sports, and building industries. The comparison between the computational and the experimental results show that the proposed approach is an efficient and attractive way for minimizing vibration amplitudes.

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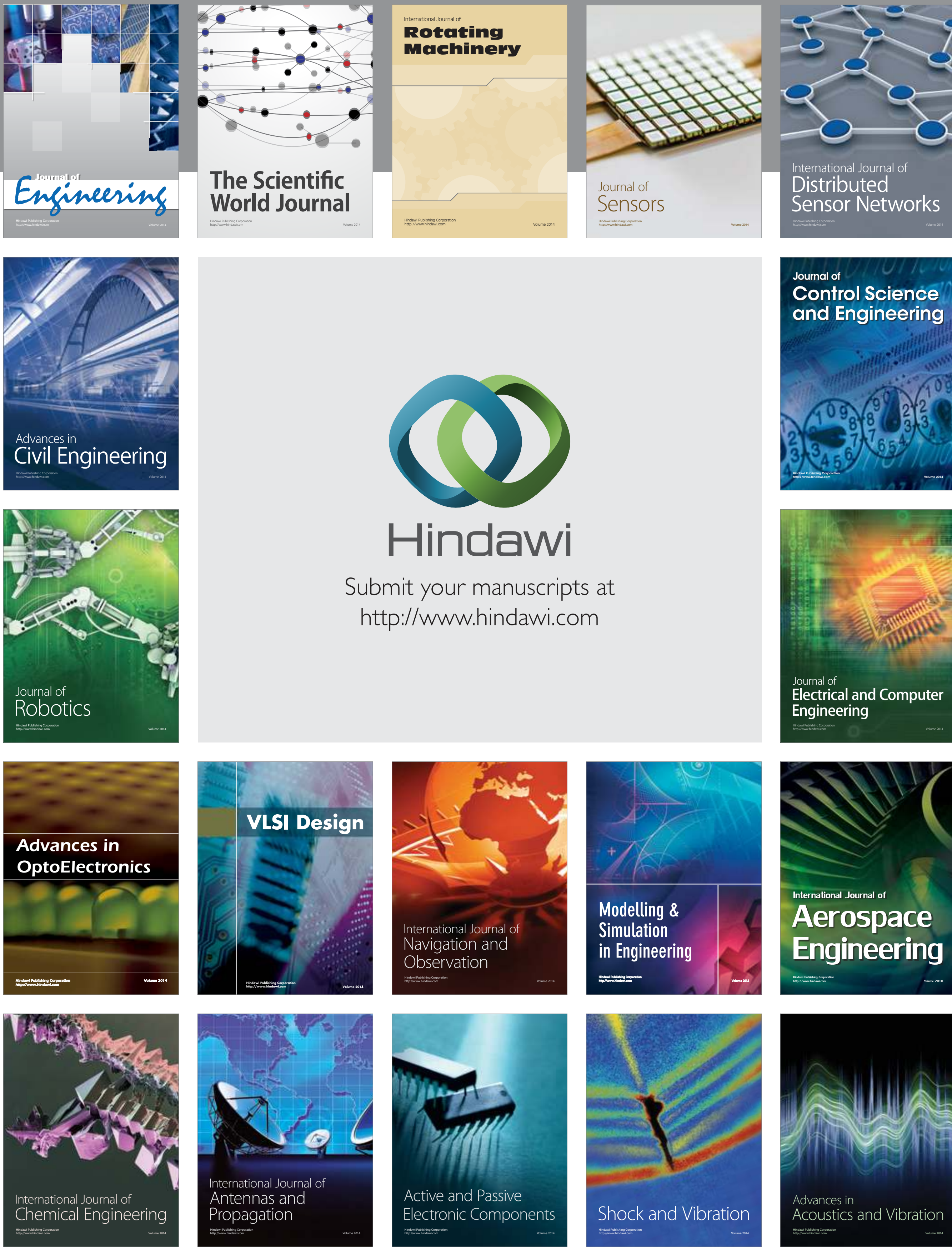
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