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Optimal consumption and investment strategies with stochastic interest rates

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Abstract

We characterize the solution to the consumption and investment problem of a power utility investor in a continuous-time dynamically complete market with stochastic changes in the opportunity set. Under stochastic interest rates the investor optimally hedges against changes in the term structure of interest rates by investing in a coupon bond, or portfolio of bonds, with a payment schedule that equals the forward-expected (i.e. certainty equivalent) consumption pattern. Numerical experiments with two different specifications of the term structure dynamics (the Vasicek model and a three-factor non-Markovian Heath–Jarrow–Morton model) suggest that the hedge portfolio is more sensitive to the form of the term structure than to the dynamics of interest rates.

JEL classification: G11
Keywords: Dynamic asset allocation; Hedging; Term structure of interest rates

1. Introduction

Since the pathbreaking papers of Merton (1969, 1971, 1973) it has been recognized that long-term investors want to hedge stochastic changes in investment opportunities, such as changes in interest rates, excess returns, volatilities, and inflation rates. The main contribution of this paper is to enhance the understanding of how investors with

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constant relative risk aversion (CRRA) preferences for consumption (and, possibly, terminal wealth) should optimally hedge interest rate risk. We demonstrate that the optimal hedge against changes in interest rates is obtained by investing in a coupon bond, or portfolio of bonds, with a payment schedule that precisely equals the certainty equivalents of the future optimal consumption rates. Furthermore, we study the importance for interest rate hedging of both the current form and the dynamics of the term structure. In a numerical example we compare the solutions for a standard one-factor Vasicek and a three-factor model where the term structure can exhibit three kinds of changes: a parallel shift, a slope change, and a curvature change. Our findings suggest that the form of the initial term structure is of crucial importance for the optimal future consumption plan and, hence, important for the relevant interest rate hedge, while the specific dynamics of the term structure is of minor importance.

As shown by Heath et al. (1992), any dynamic interest rate model is fully specified by the current term structure and the forward rate volatilities. Therefore, the Heath–Jarrow–Morton (HJM) modeling framework is natural for the purpose of comparing the separate effects of the current term structure and the dynamics of the term structure on the optimal interest rate hedging strategy. The HJM class nests all Markovian interest rate models, such as the Vasicek model. However, models outside this Markovian class also frequently arise within the HJM modeling framework. This is, for example, the case for the three-factor model considered in our numerical example.

Given that we want to compute optimal investment strategies in possibly non-Markovian models, we first derive a general, exact characterization of both optimal consumption and portfolio choice in a framework that also allows for non-Markovian dynamics of asset prices and the term structure of interest rates, but requires dynamically complete markets. This characterization generalizes recent results in specialized Markovian settings (Liu, 1999; Wachter, 2002a). For the special case where interest rates have Gaussian, but still potentially non-Markovian, HJM dynamics, we obtain the explicit solution for the optimal consumption and investment strategies that we use for studying the impact of the current form and the dynamics of the term structure on hedging demand. To our knowledge, this paper provides the first explicit solution to an intertemporal consumption and investment problem where the dynamics of the opportunity set is non-Markovian and the investor has non-logarithmic utility.

There has recently been a number of studies of optimal investment strategies with specific assumptions on the dynamics of interest rates. Brennan and Xia (2000) and Sørensen (1999) consider the investment problem of a CRRA utility investor with utility from terminal wealth only. They assume complete markets and show that in the case where the term structure of interest rates is described by a Vasicek-type model and market prices on risk (and expected excess returns) are constant, the optimal hedge portfolio is the zero-coupon bond that expires at the investment horizon. This particular result is also obtained as a special case within the framework of this paper. Liu (1999) provides similar insight using the one-factor square-root model of Cox et al. (1985).

A few papers have addressed the portfolio problem under stochastic interest rates for investors with utility over consumption. In a general complete-market setting,