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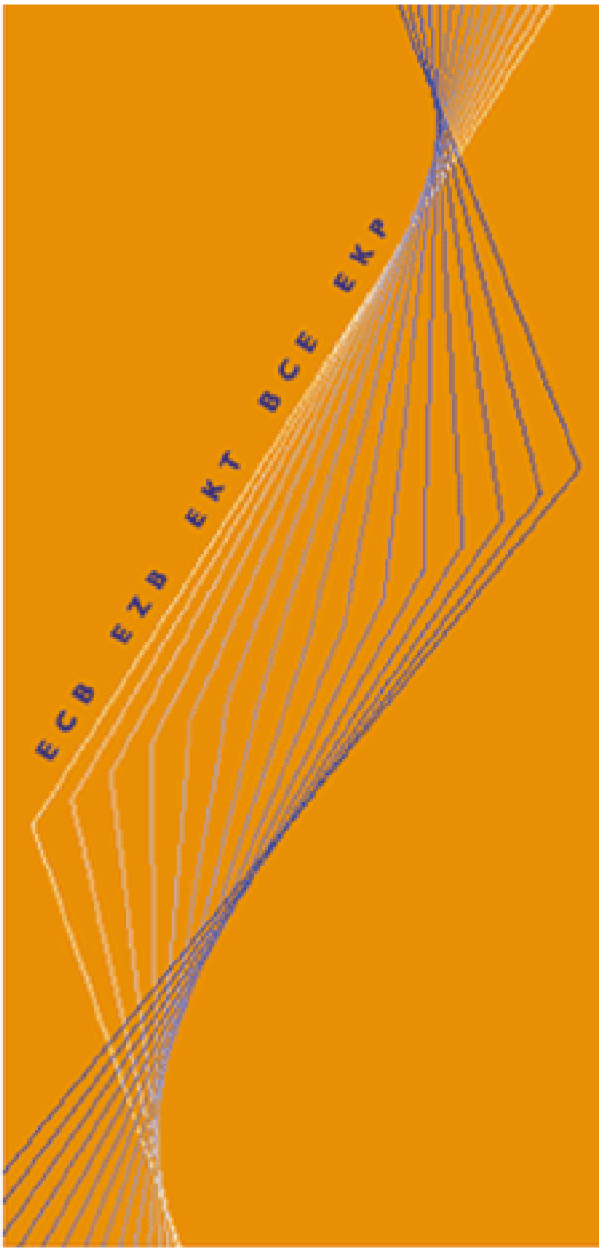

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OPTIMAL CONTRACTS IN A DYNAMIC COSTLY STATE VERIFICATION MODEL

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** European Central Bank, email: cyril.monnet@ecb.int

*** Federal Reserve Bank of Dallas, email: erwan.quintin@dal.frb.org

Address	**Kaiserstrasse 29**
	D-60311 Frankfurt am Main
	Germany
Postal address	**Postfach 16 03 19**
	D-60066 Frankfurt am Main
	Germany
Telephone	**+49 69 1344 0**
Internet	**http://www.ecb.int**
Fax	**+49 69 1344 6000**
Telex	**411 144 ecb d**

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Abstract

This paper describes optimal contracts in a dynamic costly state verification model with stochastic monitoring. An agent operates a risky project on behalf of a principal over several periods. Each period, the principal can observe the revenues from the project provided he incurs a fixed cost. We show that an optimal contract exists with the property that, in each period and for every possible revenue announcement by the agent, either the principal claims the entire proceeds from the project or promises to claim nothing in the future. This structure of payments enables the principal to minimize audit costs over the duration of the project. Those optimal contracts are such that the agent's expected income rises with time. Moreover, except in at most one period, the principal claims the entire returns of the project whenever audit occurs. We also provide conditions under which all optimal contracts must satisfy these properties.

Keywords: Dynamic Contracts; Theory of Uncertainty and Information; Costly State Verification; Monitoring.
JEL classification: D8; C7

Non Technical Summary

Most financial contracts have a dynamic component. For instance, this has been observed by Kaplin and Stromberg (2000) in a study on venture capital contracts. In particular, while the obligations stated in the contract are naturally contingent on the realized state, this contingency evolves with time.

In this theoretical paper we study this evolution and the reasons for its existence. To do so, we describe optimal contracts in a dynamic costly state verification model with stochastic monitoring. The basic framework is as follows. An agent operates a risky project on behalf of a principal over several periods. Each period, the principal can observe the revenues from the project provided he incurs a fixed cost. This very stylized framework has the main ingredients to be applied to bank financing decision.

We show that an optimal contract exists such that, at any history, either the principal claims the project's entire revenues or promises to claim nothing in the future. In particular, the agent's expected income rises with time. Moreover, except in at most one period, the principal claims all revenues when audit occurs. We provide conditions under which all optimal contracts satisfy these properties.

The results highlight some of the features of contracts observed in facts. If the firm always yields a positive outcome the result predicts the following inter-temporal structure of an optimal contract. The payment will heavily depend on the outcome of the firm in early stages, but will converge to fixed payments of standard debt contracts later on. Indeed, we observe that debt contracts are generally written when firms do not involve too much risk. However, when the uncertainty on the viability of the project is important, the optimal contract will not converge toward a standard debt contract in each period, but to a transfer of most of the revenue to the agent. This features is very much reminiscent of the special relationship between venture capitalists and project holders described in Kaplan and Stromberg (2000).

1 Introduction

Most financial contracts have a dynamic component. For instance, this has been observed by Kaplin and Stromberg (2000) in a study on venture capital contracts. In particular, they found evidences that "the state-contingency (i.e. the use of performance benchmarks and vesting) is significantly higher in earlier stage, pre-revenue financings compared to later stage, post-revenue ones. State contingencies are also greater in first venture capital rounds compared to subsequent ones". Therefore, while the obligations stated in the contract are naturally contingent on the realized state, this contingency evolves with time. In this paper we study this evolution and the reasons for its existence.

In particular, we describe optimal contracts in a dynamic version of the costly state verification model of Townsend (1979) with stochastic monitoring. More precisely, we extend the environment of Border and Sobel (1987) who study stochastic monitoring in the static case. An agent operates a risky project over several periods on behalf of a principal. The principal can audit the project's return in any period, provided he pays a fixed cost. We allow for stochastic audit strategies. Within this framework, we characterize Pareto optimal dynamic contracts.

We find that an optimal contract always exists with the feature that at any given history of the project, either the agent transfers the project's entire proceeds to the principal, or the principal promises to require no further transfers in the future. With time therefore, a greater share of the project's outcome is allocated to the agent, and the agent's expected income rises. Also, these contracts have the property that, in all but at most one period, the principal seizes the project's entire proceeds whenever audit takes place. We provide a simple condition under which all optimal contracts must satisfy those properties.

Our results are founded on a simple intuition. Whenever the principal has a choice between letting the agent keep a portion of the project's current revenues or promising an equivalent amount in the future, the latter strategy always dominates (at least weakly). Indeed, this strategy enables the principal to economize on audit costs over several periods. This result relies on two main elements. First, the agent is always willing to make a larger payment today in exchange for an actuarially fair receipt in the future. Second, in any given period, expected audit costs decrease as the income promised ex-ante to the agent rises. Intuitively, an increase in the agent's promised income weakens incentive compatibility constraints, allowing the principal to reduce the probability of an audit, hence audit costs. This implies that front-loading payments is always weakly optimal.

We find the costly state verification model to be well-suited for our goal as it has been used as a workhorse for many studies of financial contracts (see for instance, Townsend (1979), Gale and Hellwig (1985) or Williamson (1987)). While we assume for simplicity that no funds must be invested to operate the project, this feature can easily be incorporated in the model and contracts can then be thought of as standard lending arrangements[1]. In this context, the results highlight some of the features of contracts observed in facts. If the firm always yields a positive outcome the result predicts the following inter-temporal structure of an optimal contract. The payment will heavily depend on the outcome of the firm in early stages, but will converge to fixed payments of standard debt contracts later on. Indeed, we observe that debt contracts are generally written when firms do not involve too much risk. However, when the uncertainty on the viability of the project is important, the optimal contract will not converge toward a standard debt contract in each period, but to a transfer of most of the revenue to the agent. This is reminiscent of the special relationship between venture capitalists and project holders as described for instance in Kaplan and Stromberg (2000).

Also, there are many possible interpretations for the stylized framework we study in this paper. Border and Sobel (1987) point out that this type of monitoring problem arises in the context of income tax collection. One could also think of the agent as a manager who operates a risky project on behalf of a corporation. The contracts we describe should then be interpreted as compensation schemes.

Environments with costly state verification were first studied by Townsend (1979). He shows that debt contracts are optimal whenever the principal is constrained to use deterministic audit strategies. However, he points out that stochastic auditing strategies are optimal in most cases. Despite this observation, most studies in this literature exclude stochastic strategies, including the contributions of Diamond (1984), Gale and Hellwig (1985) and Williamson (1987). Exceptions include Border and Sobel (1987), Mookherjee and Png (1989) and Bernanke and Gertler (1989) who study optimal stochastic monitoring strategies in static problems.

[1] See, for instance, Boyd and Smith (1994).

The use of deterministic audit in a static environment has been motivated by Boyd and Smith (1994). They provide some quantitative analysis that suggests that gains from stochastic strategies may be small. They also point out that in optimal stochastic contracts, the principal may not claim the entire proceeds of the project when audit occurs. This feature, they argue, leads to stipulations for states of bankruptcy that are at odds with what is typically observed in lending arrangements. It is a corollary of our main result that, in optimal dynamic arrangements, agents do not retain anything when audit occurs, except in at most one period. Furthermore, whether the quantitative findings of Boyd and Smith (1994) extend to the dynamic case remains an open question. The findings we report should make addressing this open question more manageable: they suggest that the complexity of the problem does not increase markedly with the project's horizon.

Comparatively little work has been done in dynamic costly state verification environments. Chang (1990) establishes a version of our main result in a costly monitoring problem with two periods and two possible revenue realizations, when the principal is constrained to use deterministic auditing strategies, and when audit costs increase strictly with the project's return. We relax all these assumptions. Also related are Smith and Wang (1998) and Wang (1999) who study dynamic risk-sharing arrangements when verification is costly. There, audit strategies are restricted to be deterministic. One exception to this restriction in a dynamic setting is the work of Aiyagari and Alvarez (1995) who study optimal unemployment insurance schemes with costly state verification.

Our result also has a close counterpart in the limited enforcement literature. Albuquerque and Hopenhayn (1997) show that when enforcement is limited and both parties are risk-neutral, optimal dynamic lending contracts stipulate that the borrower transfers all the project's revenues to the lender until the borrowing constraint no longer binds.[2]

We begin by describing the environment and defining optimal contracts. We then turn to studying dynamic properties of optimal contracts, in the two-period case first and then in the general, finite horizon case. Considering the two-period case helps us develop some intuition. Arguments become lengthier in the general case. A final section provides several extensions of our main result.

[2]Cooley, Marimon and Quadrini (2000) establish a similar result in a related model.

2 The Model

Consider a finitely repeated version of the standard costly state verification model (see Townsend, 1979 or Border and Sobel, 1987) in which both the principal and the agent are risk neutral and do not discount future payoffs.[3]

Time is discrete and there is a finite number of periods indexed by $s = 1, 2, ..., T$. A principal is endowed with a risky project which, if operated, yields a stochastic revenue z^s in each period s, where $z^s \in \mathcal{Z} = \{z_1, z_2, ..., z_n\}$ and $0 \leq z_1 < z_2 ... < z_n$. Revenue realizations are independent but identically distributed across periods.[4] For all i, we denote by h_i the probability that z_i is realized in any given period, so that $\sum_{i=1}^{n} h_i = 1$.

To implement the project, the principal must hire an agent. Each period, the revenue realization is freely observed by the agent, who then makes an announcement. The principal can verify (audit) the revenue realization only if she pays a cost $\gamma > 0$.

If the project is undertaken, its outcome is divided between the agent and the principal according to a contract. To define a contract we borrow from Border and Sobel (1987) and Spear and Srivastava (1987). We adopt Border and Sobel's notation to facilitate comparisons. A contract is a finite stage game between the principal and the agent. Formally, denote by $\mathcal{H}^s$ the set of all possible histories at date s. Histories include all past announcements by the agent, the list of previous periods in which verification took place, and the result of those verifications. A contract consists for each period s of a message space $\mathcal{M}^s$, an audit function $p^s : \mathcal{M}^s \times \mathcal{H}^s \rightarrow [0, 1]$, a transfer function $t^s : \mathcal{M}^s \times \mathcal{H}^s \rightarrow \mathbb{R}$, and a penalty function $f^s : \mathcal{Z} \times \mathcal{M}^s \times \mathcal{H}^s \rightarrow \mathbb{R}$.[5]

Given history $h \in \mathcal{H}^s$ and an announcement $m \in \mathcal{M}^s$, the principal commits to verifying the project's revenue realization with probability $p^s(m, h)$. Upon verification, the principal seizes $f^s(z, m, h)$ from the agent's wealth z. If verification does not occur, the agent must transfer $t^s(m, h)$ to the principal.

We assume that transfers cannot be negative or exceed the project's realization. Therefore,

[3] We relax several of these assumptions in section 5. In particular, our results continue to hold when both parties discount flows at the same rate, or provided the agent is more patient than the principal. Section 5 also extends our results to the case where the project's horizon is infinite.

[4] The assumption of independence simplifies the exposition and is not essential for our main result. The first part of Theorem 1 can easily be extended to the case where revenue realizations follow a Markov process.

[5] For notational simplicity, we assume here that the message space does not depend on past events.

for all s and $h \in \mathcal{H}^s$, there must be at least one message $m \in \mathcal{M}^s$ with $0 \leq t^s(m, h) \leq z_1$, and contracts must be such that $0 \leq f^s(z, m, h_s) \leq z$ for all revenues z and possible announcements m. Border and Sobel (1987) point out that audit costs could always be made vanishingly small if the principal had the ability to impose arbitrarily large penalties on agents, or could offer arbitrarily large rewards. Section 5 considers the case where there are finite but arbitrary bounds on transfers in each period.

The principal first proposes a contract to the agent. The agent can choose to reject the contract, in which case the project is not undertaken. For simplicity and without loss of generality, we will assume that it is always optimal for the project to be undertaken, so that the principal always offers an acceptable contract to the agent.

Given a contract, a strategy for the agent is a function that specifies an announcement for each period, each possible history of events, and each possible revenue realization. For simplicity, we only consider pure announcement strategies.

Given a contract $C = \{\mathcal{M}^s, p^s, t^s, f^s\}_{s=1}^T$ and a strategy $m = \{m^s\}_{s=1}^T$ for the agent, denote the expected payoffs for the principal and the agent for the subgame starting with history h by $\Pi(h, C, m)$ and $V(h, C, m)$, respectively.[6] We will consider subgame perfect Nash equilibria for the overall game. In particular, we require that the agent's announcement strategy m satisfy:

$$V(h, C, m) \geq V(h, C, \bar{m})$$

for all possible histories h and alternative announcement strategies $\bar{m} \neq m$. The principal simply implements the specifications of the contract at any given history.

We show in appendix A that the revelation principle continues to hold in our model so that we can restrict our analysis to contracts that use direct revelation. Therefore, we will only consider contracts where $\mathcal{M}^s = \mathcal{Z}$ for all periods s and such that it is optimal for the agent to truthfully announce his wealth at any history.

In the context of a given history h and in a given period s, we follow Border and Sobel (1987) and write $p_i = p^s(z_i, h)$, $t_i = t^s(z_i, h)$ and $f_{ij} = f^s(z_i, z_j, h)$. Note that for any i such that $p_i = 0$, f_{ii} is indeterminate but also irrelevant. The same remark applies to

[6] These continuation payoffs depend on, among other things, the transition function for histories induced by strategy m and audit functions p^s.

t_i when $p_i = 1$. Again, we follow Border and Sobel and adopt the following convention: for all i, if $p_i = 0$ then $f_{ii} = z_i$, and if $p_i = 1$ then $t_i = z_i$.

3 Optimal contracts

An optimal contract is a contract that uses direct revelation such that no other contract in this class is accepted by the agent and Pareto dominates the initial one. That is, no other contract in this class gives the principal a higher ex-ante expected net income without reducing the agent's ex-ante expected income.

It is useful to formulate the optimal contract problem in a recursive fashion, following Spear and Srivastava (1987). At the beginning of a given period and at given history, denote by V the agent's expected income according to the contract. V is the agent's continuation payoff introduced in the previous section, evaluated at the strategy consisting of truthfully revealing the project's revenue realization at every possible history. Also, let $\bar{V} = \sum_{i=1}^{n} h_i z_i$ denote the expected revenue from the project in any given period. Now define, for $0 \leq V \leq \bar{V}$:

$$\Pi_T(V) = \max_{(p_i,t_i,f_{ij})_{i,j}} \sum_{i=1}^{n} [p_i(f_{ii} - \gamma) + (1 - p_i)t_i]h_i$$

subject to:

$$\begin{aligned} 0 \leq t_i &\leq z_i \text{ for all } i \\ 0 \leq f_{ii}, f_{ij} &\leq z_i \text{ for all } i, j \\ (1 - p_i)(z_i - t_i) + p_i(z_i - f_{ii}) &\geq (1 - p_j)(z_i - t_j) + p_j(z_i - f_{ij}) \\ & \quad \text{for all } i, j \text{ with } t_j \leq z_i \\ \sum_{i=1}^{n} [(1 - p_i)(z_i - t_i) + p_i(z_i - f_{ii})]h_i &= V \end{aligned}$$

This problem yields the maximum expected income for the principal in the last period subject to our assumed bounds on transfers, incentive compatibility and promise keeping. Incentive compatibility requires that the agent be better off announcing the true state of the project than any other state whose associated transfer is below the realized level of revenues.

The last equality says that the agent's expected income must equal promised income V. Note that the principal cannot promise an expected income larger than $\bar{V}$ in the last period since transfers are required to be non-negative. Define, recursively, for $s < T$ and $V \leq (T+1-s)\bar{V}$,

$$\Pi_s(V) = \max_{(p_i,t_i,f_{ij},V_i^t,V_{ij}^f)_{i,j}} \sum_{i=1}^{n} [p_i(f_{ii} - \gamma + \Pi_{s+1}(V_{ii}^f)) + (1-p_i)(t_i + \Pi_{s+1}(V_i^t))]h_i$$

subject to:

$$\begin{aligned}
0 \leq t_i &\leq z_i \text{ for all } i \\
0 \leq f_{ii}, f_{ij} &\leq z_i \text{ for all } i, j \\
0 \leq V_i^t, V_{ij}^f &\leq (T-s)\bar{V} \\
(1-p_i)(z_i - t_i + V_i^t) + p_i(z_i - f_{ii} + V_{ii}^f) &\geq (1-p_j)(z_i - t_j + V_j^t) + p_j(z_i - f_{ij} + V_{ij}^f) \\
&\quad \text{for all } i, j \text{ with } t_j \leq z_i \\
\sum_{i=1}^{n} [(1-p_i)(z_i - t_i + V_i^t) + p_i(z_i - f_{ii} + V_{ii}^f)]h_i &= V
\end{aligned}$$

where (V^t, V^f) is the vector of promised future incomes for the agent, depending on whether an audit takes place. More precisely, if the agent announces revenue z_i and audit does not take place, he transfers t_i to the principal today, but the principal promises to deliver the agent expected income V_i^t in the future. If audit takes place and the true state of the project turns out to be j, the agent transfers f_{ji} to the principal today, but is promised expected income V_{ji}^f in the future. Promised incomes cannot be made negative since transfers cannot exceed the revenue realization in any state. Furthermore, promised incomes cannot exceed $(T-s)\bar{V}$ since transfers are constrained to be non-negative.

For all s, Π_s gives the maximum feasible expected net income for the principal over the remaining duration of the project as of period s, given the agent's promised income. One can argue as in Spear and Srivastava (1987) that an optimal T-period contract that gives the agent ex-ante expected income $V_0 \geq 0$ must give the principal an expected ex-ante income of $\Pi_1(V_0)$. This condition, however, is not sufficient. It may be possible to strictly raise both the agent's and the principal's ex-ante net income. In other words, at any optimal contract,

V_0 must be in the following set:

$$\mathcal{P} = \{V \in [0, T.\bar{V}] : V' \in (V, T.\bar{V}] \Longrightarrow \Pi_1(V') < \Pi_1(V)\}$$

Note that this set is not empty since it contains at least $V_0 = T.\bar{V}$. In that case, the unique optimal contract is trivial. The agent does not make any transfer to the principal for the duration of the project, and audit never occurs.[7] Throughout the remainder of this paper we will assume that V_0 is in set $\mathcal{P}$. We now make use of our recursive formulation to characterize optimal contracts in this environment.

Note that it is optimal for the principal to set $f_{ij} = z_i$ and $V_{ij}^f = 0$ in all periods and histories whenever $i \neq j$. This is because these transfers and promises do not enter the principal's objective function. Since these specifications make incentive compatibility constraints as weak as possible, it is optimal to use them. We can then define $f_i \equiv f_{ii}$ and $V_i^f \equiv V_{ii}^f$ without ambiguity. The main result of the paper can now be stated.

Theorem 1. *For any $V_0 \in \mathcal{P}$, there exists an optimal contract that gives the agent ex-ante expected income V_0 and satisfies for all periods $s \in \{1, \ldots T-1\}$, histories, and $i \in \{1, \ldots n\}$:*

1. $t_i = z_i$ *or* $V_i^t = (T-s)\bar{V}$,

2. $f_i = z_i$ *or* $V_i^f = (T-s)\bar{V}$.

Furthermore, all optimal contracts satisfy those properties when $z_1 = 0$.

These two properties of optimal contracts mean that in any period and any state, the principal either claims all the project's revenues today, or promises to claim nothing in the future. The intuition for this result is simple. Consider a feasible contract such that in a given period $s < T$ and at a given history, both $t_i < z_i$ and $V_i^t < (T-s)\bar{V}$ for some $i \in \{1, \ldots n\}$. Then it is possible to raise both t_i and V_i^t, leaving $t_i - V_i^t$ unchanged so that the agent is indifferent. Incentive compatibility continues to hold at the original vector of audit probabilities, and the next period begins with a higher promised income for the agent, which may enable the principal to economize on audit costs in the future.

[7]For an example where $\mathcal{P}$ is larger than $\{T.\bar{V}\}$, see appendix B.

The argument we use consists of showing that the latter conjecture is always correct in a weak sense and is correct in a strict sense when $z_1 = 0$. To do this, we show that when the agent's promised income rises, it is always possible for the principal to deliver the extra income to the agent without increasing audit costs. When $z_1 = 0$, audit costs decrease strictly when the agent's promised income rises.

Before turning to the details of the argument, we record several immediate consequences of theorem 1. Given a contract, let t^s and f^s be the stipulated vector of transfers from the agent to the principal in period s, as a function of the realized history. Properties 1 and 2 of theorem 1 imply that the expectation taken over all possible histories up to (and including) date s of those transfers decreases over time, i.e., letting E_1 denote the expectation operator as of the beginning of the first period of the project:

Corollary 1. *There exists an optimal contract such that $E_1(t^s)$ and $E_1(f^s)$ both decrease with s. Furthermore, all contracts satisfy those properties when $z_1 = 0$.*

In particular, the agent's income rises over time in expected terms. Moreover,

Corollary 2. *There exists an optimal contract such that the event $f_i < z_i$ for some i occurs at most once. Furthermore, all optimal contracts satisfy this property when $z_1 = 0$.*

To see this, note that if at a particular history in period $s \in \{1, \ldots T-1\}$ it turns out that $f_i < z_i$ for some $i \in \{1, \ldots n\}$, it must then be that $V_i^f = (T-s)\bar{V}$. This in turn implies that audit never occurs with positive probability in subsequent periods. As a result, the principal claims all the revenues when audit takes place, except possibly in one period. As discussed for instance by Boyd and Smith (1994), static optimal stochastic contracts have the feature that transfers are made as small as possible when audit occurs. As a result, "optimal contracts with stochastic monitoring often exhibit a form of debt forgiveness and may call for the initiation of bankruptcy even though the borrower is able to repay in full" (p.541). Dynamic optimal contracts need not have the feature that stipulated transfers are the same for all announcements that do not trigger audit, and it is therefore difficult to extend their notion of debt forgiveness to our dynamic context. Nevertheless, corollary 2 says that, except in at most one period, agents do not retain anything when audit occurs.

Finally, theorem 1 can also be invoked to rewrite and simplify our dynamic problem. To see this, fix $s \in \{1, \dots T\}$ and for all i, let $r_i^t = t_i - V_i^t$ and $r_i^f = f_i - V_i^f$. Define for all i and $-(T+1-s)\bar{V} \leq r \leq z_i$,

$$F_i^s(r) = \max_{t,V} t + \Pi_{s+1}(V)$$

subject to:

$$\begin{aligned} 0 \leq t &\leq z_i \\ 0 \leq V &\leq (T-s)\bar{V} \\ t - V &= r \end{aligned}$$

Loosely speaking, F_i^s gives the optimal way to implement r in period s and state i from the point of view of the principal. Theorem 1 implies that a solution to this problem is obtained by setting $t = \max\{z_i, r + (T-s)\bar{V}\}$. Note that for all periods s and states i, F_i^s is continuous and increasing.

We can now rewrite the recursive contracting problem as follows[8] for all s and $V \leq (T+1-s)\bar{V}$:

$$\Pi_s(V) = \max_{(p_i, r_i^t, r_i^f)_{i=1}^n} \sum_{i=1}^n [(1-p_i)F_i^s(r_i^t) + p_i(F_i^s(r_i^f) - \gamma)]h_i$$

subject to:

$$\begin{aligned} -(T+1-s)\bar{V} \leq r_i^t, r_i^f &\leq z_i \text{ for all } i \\ (1-p_i)(z_i - r_i^t) + p_i(z_i - r_i^f) &\geq (1-p_j)(z_i - r_j^t) \\ &\qquad \text{for all } i, j \text{ with } t_j(r_j^t) \leq z_i \\ \sum_{i=1}^n [(1-p_i)(z_i - r_i^t) + p_i(z_i - r_i^f)]h_i &= V \end{aligned}$$

where $t_j(r_j^t) = \min(z_j, r_j + (T-s)\bar{V})$.

[8] We omit the details of the argument here. Simply note that whenever it is the case that for some i, $t_i + \Pi_{s+1}(V_i^t) < F_i^s(t_i - V_i^t)$, it is possible to raise the transfer and the continuation payoff by the same amount so as to increase the principal's expected income. Incentive compatibility continues to hold since raising a transfer can only reduce the number of constraints one needs to check.

This problem closely resembles the static problem. The two key differences are that the principal does not have a linear objective function and that this objective function depends on the state. The complexity of the problem does not increase significantly with the project's horizon, which bodes well for addressing such questions as: Are optimal transfers, audit probabilities and continuation payoffs monotonic in the project's return?

We now turn to establishing theorem 1.

4 Dynamic properties of optimal contracts

The proof of theorem 1 relies on the fact that audit costs can only decrease when the agent's share of the project's surplus rises. We first establish that this claim is correct in the last period and use that result to establish theorem 1 when $T = 2$. Arguments become lengthy in the general case and proofs are relegated to the appendix.

4.1 The last period

Given promised income V, the last period problem is a standard one-period stochastic monitoring problem. Border and Sobel (1987) characterize *audit efficient* contracts in this context. Their contracts are audit efficient in the sense that they minimize monitoring probabilities for a given level of gross revenues for the principal. Optimal contracts, as we define them, must be audit efficient in the last period.[9] Therefore, in the last period, optimal transfers and monitoring policies must satisfy the properties of audit efficient policies established by Border and Sobel. The following theorem records two of their monotonicity results:

Theorem 2. *(Border and Sobel, 1987) In the last period, optimal transfers and monitoring probabilities satisfy:*

If $i > j$,

[9] Indeed, consider any feasible contract whose stipulations are not audit efficient in the last period. There must then exist a transfer and audit policy that reduces audit costs without diminishing the principal's gross income. If this new contract leaves the agent with strictly less expected income, decrease all transfers proportionately until the agent's expected income is the same as in the original contract. As argued in the proof of lemma 2, the resulting contract is incentive compatible. This leaves the principal with the same gross income as before, but strictly lower audit costs, establishing as needed that the original contract was not optimal.

1. $t_i \geq t_j$ *with equality if and only if* $p_j = 0$,

2. $p_i \leq p_j$ *with equality if and only if* $p_i = 1$ *or* $p_j = 0$.

We now provide a necessary and sufficient condition for monitoring to occur with positive probability in the last period at a given history. Given promised income $V \in [0, \bar{V}]$, there may be many optimal audit and transfer policies that give the agent the required expected income. Consider the corresponding set of optimal audit probabilities and denote by $p_1(V)$ the smallest probability of a verification when the agent announces revenues z_1. The Theorem of the Maximum implies that the set of optimal verification probabilities in the last period, given V, is closed, so that this smallest probability is achieved by an optimal contract.

The first item of theorem 2 implies that some audit is necessary unless the contract stipulates the same transfer for all possible revenue announcements. As soon as transfers differ across states, audit must take place to ensure that the agent tells the truth. Otherwise, the agent would always report the state associated with the lowest transfer. The following result is based on the simple observation that when transfers are equated across states, this common transfer must be feasible when revenues are at z_1, their lowest possible value.

Lemma 1. *For all* $V \in [0, \bar{V}]$, $p_1(V) > 0$ *if and only if* $V < \bar{V} - z_1$.

Proof. Assume first that $V \geq \bar{V} - z_1$. Then it is feasible to set $t_i = \bar{V} - V$ and $p_i = 0$ for all i. This gives the required expected income to the agent without any monitoring cost and is thus optimal. If on the other hand $V < \bar{V} - z_1$ it is no longer possible to make t constant across states, and the first item of theorem 2 implies that some monitoring must occur. In other words, expected monitoring costs are strictly positive, and the second item of theorem 2 implies in turn that $p_1(V) > 0$. □

In particular, we obtain:

Remark 1. *If* $z_1 = 0$, $p_1(V) > 0$ *whenever* $V < \bar{V}$.

That is, when the project yields nothing in the lowest state, some monitoring is necessary except in the trivial case where no transfers from the agent to the principal are necessary.

We now turn to studying how the agent's promised income affects expected audit costs in optimal contracts. To do this, define *total surplus* in period s by:

$$W_s(V) = V + \Pi_s(V) \text{ for all } V \in [0, (T+1-s)\bar{V}].$$

This function gives the sum of the agent and the principal's expected net income, in period s, given the agent's promised income. Since expected gross revenues are $\bar{V}$ in each period, total surplus as of period s is obtained by subtracting expected audit costs over the remaining duration of the project from $(T+1-s)\bar{V}$. In particular, expected audit costs decrease with V if and only if W_s rises with V. The next result establishes that this necessary and sufficient condition for audit costs to decrease with V holds in the last period.

Lemma 2. *W_T is continuous and weakly increasing on* $[0, \bar{V}]$.

Proof. Continuity follows from the Theorem of the Maximum. Now fix $V \in (0, \bar{V})$ and take any policy vector (p, t, f) that is optimal given V. Observe that, in this last period, we can require without loss of generality that incentive compatibility hold for all $i, j \in \{1, \ldots n\}$ since $t_j > z_i$ implies that $(1 - p_j)(z_i - t_j) < 0$.

With this convention, it is easy to see that scaling down all transfers proportionately without changing p does not affect incentive compatibility. That is, $(p, \theta t, \theta f)$ is incentive compatible for any $\theta \in [0, 1]$. For $V' > V$, choose $\theta \in [0, 1]$ so that $\bar{V} - V' = \sum_i h_i((1 - p_i)\theta t_i + p_i \theta f_i)$. It is always possible to find such a θ since $\bar{V} - V' < \bar{V} - V$. The resulting contract gives the agent expected income V'. Since expected audit costs have not changed, this implies that $\Pi_T(V') - \Pi_T(V) \geq V - V'$ as needed. □

In other words, expected audit costs do not increase when the agent's expected income rises. We will obtain sharper characterizations of optimal contracts when expected audit costs decrease strictly, hence total surplus rises strictly, with the agent's promised income. Obviously, for this to happen, some monitoring must occur in the first place. This condition also turns out to be sufficient (in the last period) as we now argue.

Lemma 3. *For all $V \in [0, \bar{V})$, W_T increases strictly at V if and only if $p_1(V) > 0$.*

Proof. Given V, consider an optimal contract with $p_1 = p_1(V)$. Take any $V' \in (V, \bar{V})$ and scale down all transfers proportionately in the original contract until the agent's utility is V'. The resulting contract is feasible. However, it is not optimal if $p_1(V) > 0$. Indeed, by corollary 2 in Border and Sobel (1987), audit probabilities pin down audit efficient transfers whenever $p_1 > 0$. In other words, if (p, t, f) and (p, t', f') are audit efficient and $p_1 > 0$ then $t = t'$ and $f = f'$. The new transfers cannot be optimal at the old probabilities. This implies $\Pi_T(V') - \Pi_T(V) > V - V'$ as needed. To establish necessity, recall that $p_1(V) = 0$ implies that the transfer is constant across states. This remains true when V rises, and the scaled down contract is clearly optimal at V'. □

These properties are sufficient to establish theorem 1 when $T = 2$, which we do formally in the next section. They also enable us to start the induction argument we use to treat the general case.

4.2 The two-period case

Assume that $T = 2$. One feasible strategy consists of giving the agent income $\frac{V_0}{2}$ in each period and repeating the optimal static contract twice. Appendix B provides a simple example in which the resulting contract is not optimal. In general, the principal can economize on overall audit costs by shifting transfers from the agent forward. We now formalize this intuition.

A two-period contract is optimal given $V_0 \in \mathcal{P}$ provided it solves:

$$\max_{(p_i, t_i, f_i, V_i^t, V_i^f)_{i=1}^n} \sum_{i=1}^n [(1 - p_i)(t_i + \Pi_T(V_i^t)) + p_i(f_i - \gamma + \Pi_T(V_i^f))]h_i$$

subject to:

$$\begin{aligned} 0 \le t_i, f_i &\le z_i \text{ for all } i \\ 0 \le V_i^t, V_i^f &\le \bar{V} \text{ for all } i \\ (1 - p_i)(z_i - t_i + V_i^t) + p_i(z_i - f_i + V_i^f) &\ge (1 - p_j)(z_i - t_j + V_j^f) \text{ for all } i, j \text{ with } t_j \le z_i \\ \sum_{i=1}^n [(1 - p_i)(z_i - t_i + V_i^t) + p_i(z_i - f_i + V_i^f)]h_i &= V_0 \end{aligned}$$

The following result is a version of theorem 1 in the case where $T = 2$.

Lemma 4. *There exists a solution to the two-period problem stated above such that for all i,*

1. $t_i = z_i$ *or* $V_i^t = \bar{V}$,

2. $f_i = z_i$ *or* $V_i^f = \bar{V}$.

Furthermore, all solutions satisfy those properties when $z_1 = 0$.

Proof. Consider a feasible contract that does not satisfy the first property of the lemma. Then it is possible for some i to raise t_i and V_i^t by the same amount. This raises $t_i + \Pi(V_i^t)$ (weakly) by lemma 2. Indeed, the principal's expected gross income does not change while audit costs can only decrease in the last period. Since $t_i - V_i^t$ is unaffected and raising a transfer can only reduce the number of truth-telling constraints one has to check, incentive compatibility continues to hold. The second item is established in a similar fashion. The last part of the lemma follows immediately from lemma 3 and remark 1. □

The argument becomes lengthier in the general case, although the intuition carries through. It is the purpose of the next section to establish that regardless of the project's horizon, increasing current transfers and the agent's continuation payoffs can only reduce overall audit costs.

4.3 The general case

In this section, we first generalize lemma 2 and prove that an increase in the agent's expected income does not raise audit costs. Then we show that when the project yields nothing in the worst state, audit costs decrease strictly when the agent's expected income rises. Results presented in this section are demonstrated in the appendix.

Lemma 5. *For all $s \in \{1, \dots T\}$, W_s is continuous and weakly increasing.*

The proof given in the appendix is by induction. We have already established the desired result when $s = T$. Now pick any $s < T$ and assume that the result holds for period $s + 1$. We show that when the principal must deliver more expected income to the agent in period

s, he can do so while leaving audit probabilities unchanged. The additional income can take the form of lower transfers in period s or higher continuation payoffs. By the induction hypothesis, the corresponding decline in the principal's expected income is no greater than the increase in the agent's expected income.

The only difficulty that arises is that changes in transfers change the agent's incentives. Decreasing certain transfers might increase the set of falsely low reports the agent can make. In other words, the set of active incentive compatibility constraints may increase. In the static case, one can assume without loss of generality that contracts always satisfy all incentive compatibility constraints. Due to the presence of continuation payoffs in the dynamic case, this simplifying assumption is no longer valid. We get around this difficulty by arguing that when the increase in promised income is small enough, one can deliver the extra income without changing the set of active incentive compatibility constraints.

Lemma 5 suffices to establish the first part of theorem 1. For the second part, we need a stronger result.

Lemma 6. *If $z_1 = 0$, then W_s is strictly increasing for all $s \in \{1, \ldots T\}$.*

The proof of lemma 6 requires a lengthy argument, although much of it builds on the intuition of the previous section. In the two-period case, we were able to rely directly on the results of Border and Sobel (1987) since the problem solved in the last period is a special case of the problem they consider. While we cannot replicate this method in the general case, our strategy consists of reducing the problem until the static argument can be applied almost immediately.

The proof is again by induction and we argue once again that in any period a small increase in the agent's expected income can be delivered without changing audit probabilities. If the principal is able to deliver the extra income by raising continuation payoffs, the induction hypothesis guarantees that audit costs will decrease strictly in the future. We argue that, as a result, we can restrict our attention to contracts where all the continuation payoffs are at their maximum before the increase in the agent's expected income. But this problem - with all the continuation payoffs at their maximum $(T-s)\bar{V}$ - is similar to the problem considered by Border and Sobel (1987) where possible outcomes of the project have been redefined as

$\tilde{z}_i = z_i + (T-s)\bar{V}$ for all $i \in \{1, \dots n\}$. In fact, we are able to argue that their result that optimal audit probabilities pin down optimal transfers holds in this slightly modified context. The desired result is then established as in the proof of lemma 3.

Lemmata 5 and 6 can now be invoked to demonstrate theorem 1.

5 Extensions

This section elaborates on the role of several assumptions we made in the previous analysis. We begin by relaxing the assumption that transfers must be non-negative and cannot exceed the project's return in a given period. We then consider the case where the principal and the agent discount future flows and the case where the project's horizon is infinite.

5.1 Arbitrary bounds on transfers

Assume that the constraints on transfers are replaced by:

$$-\delta_L \leq t_i, f_i \leq z_i + \delta_R \text{ for all } i$$

where $\delta_L, \delta_R \geq 0$, for simplicity, are assumed independent of the time period and the realized state. Correspondingly, the set of constraints on continuation utilities in period s becomes:

$$-(T-s)\delta_R \leq V_i^t, V_i^f \leq (T-s)(\bar{V} + \delta_L) \text{ for all } i \in \{1, \dots n\}$$

Note that it is now possible for the principal to face a strictly negative expected payoff at a particular history since $\bar{V} + \delta_L > \bar{V}$ whenever $\delta_L > 0$. The set of expected incomes the agent may receive in an optimal contract becomes:

$$\mathcal{P} = \{V \in [0, T.(\bar{V} + \delta_L)] : V' \in \big(V, T.(\bar{V} + \delta_L)\big] \Longrightarrow \Pi_1(V') < \Pi_1(V)\}$$

The first half of theorem 1 holds as before:

Proposition 1. *For any $V_0 \in \mathcal{P}$, there exists an optimal contract that gives the agent ex-ante* *expected income V_0 and satisfies the following properties for all periods $s \in \{1, \dots T-1\}$, histories, and $i \in \{1, \dots n\}$:*

1. $t_i = z_i + \delta_R$ *or* $V_i^t = (T-s)(\bar{V} + \delta_L)$,

2. $f_i = z_i + \delta_R$ *or* $V_i^f = (T-s)(\bar{V} + \delta_L)$.

Proof. It suffices to show that total surplus rises weakly with V in period s whenever $V < (T-s)(\bar{V} + \delta_L)$, which can be done by replicating the proof of lemma 5. □

It need no longer be the case, however, that *all* optimal contracts satisfy these two properties, even when $z_1 = 0$. To see this, consider for instance the last period of the contract and assume that the agent's continuation utility lies in $(\bar{V} - \delta_R, \bar{V} + \delta_L)$. Then, simple algebra shows that it is possible to make transfers state-independent and, therefore, to eliminate audit costs. In particular, raising the agent's continuation utility does not reduce audit costs. Nevertheless, as this analysis suggests, a version of the second half of lemma 4 continues to hold:

Proposition 2. *Assume $T = 2$. Then all optimal contracts satisfy, for all $i \in \{1, \dots n\}$:*

- $t_i = z_i + \delta_R$ *or* $V_i^t \geq \bar{V} - \delta_R - z_1$,

- $f_i = z_i + \delta_R$ *or* $V_i^f \geq \bar{V} - \delta_R - z_1$.

Proof. One can adapt the proof of lemma 3 to show that total surplus in the last period rises strictly whenever $V < \bar{V} - \delta_R - z_1$. Indeed, in that case, there must exist i such that $t_i > z_1 + \delta_R > t_1$ so that $p_1(V) > 0$, by remark 1. The result follows. □

Although it is natural to conjecture that, in the general case, the obvious extension of proposition 2 to cases where $T > 2$ holds as well, one cannot replicate the argument behind lemma 6 directly.[10] Checking this conjecture is left for future work.

[10] Specifically, the proof of lemma 6 carries through until the analysis of value function H. In the development of the induction argument, one can assume, without loss of generality, that both V_i^t and V_i^f exceed $(T-s)(\bar{V} - \delta_R - z_1)$. But the remainder of the argument hinges on the fact that all continuation utilities can be assumed equal, which no longer holds here.

5.2 Discounting

Assume the agent and the principal discount one-period-ahead flows at constant rates β_A and β_P, respectively. For the remainder of this section, we return to the case where $\delta_L = \delta_R = 0$. As before, let:

$$\mathcal{P} = \{V \in [0, \frac{1-\beta_A^{T-1}}{1-\beta_A}\bar{V}] : V' \in (V, \frac{1-\beta_A^{T-1}}{1-\beta_A}\bar{V}] \Longrightarrow \Pi_1(V') < \Pi_1(V)\}$$

denote the set of expected incomes the agent may receive in an optimal contract. For $s \in \{1, \ldots T\}$ define, recursively, for all $V \in [0, \frac{1-\beta_A^{T-s}}{1-\beta_A}\bar{V}]$,

$$\Pi_s(V) = \max_{(p_i,t_i,f_i,V_i^t,V_i^f)_{i=1}^n} \sum_{i=1}^n [p_i(f_i - \gamma + \beta_P \Pi_{s+1}(V_i^f)) + (1-p_i)(t_i + \beta_P \Pi_{s+1}(V_i^t))]h_i$$

subject to:

$$\begin{aligned} 0 \leq t_i, f_i &\leq z_i \text{ for all } i \\ 0 \leq V_i^t, V_i^f &\leq (T-s)\bar{V} \\ (1-p_i)(z_i - t_i + \beta_A V_i^t) + p_i(z_i - f_i + \beta_A V_i^f) &\geq (1-p_j)(z_i - t_j + \beta_A V_j^t) \\ &\quad \text{for all } i,j \text{ with } t_j \leq z_i \\ \sum_{i=1}^n [(1-p_i)(z_i - t_i + \beta_A V_i^t) + p_i(z_i - f_i + \beta_A V_i^f)]h_i &\geq V \end{aligned}$$

with $\Pi_{T+1} \equiv 0$. As before, define $W_s(V) = V + \Pi_s(V)$ for all $s \in \{1, \ldots, T\}$ and $V \in [0, \frac{1-\beta_A^{T-s}}{1-\beta_A}\bar{V}]$. As long as $\beta_A \geq \beta_P$, the proofs of lemmata 5 and 6 apply directly, and we have:

Lemma 7. *Assume $\beta_A \geq \beta_P$. Then for all s, W_s is weakly increasing, strictly increasing if $z_1 = 0$.*

As a consequence, we obtain the following version of theorem 1:

Proposition 3. *Assume that $\beta_A \geq \beta_P$. For any $V_0 \in \mathcal{P}$, there exists an optimal contract that gives the agent lifetime income V_0 and satisfies the following properties for all periods $s \in \{1, \ldots T-1\}$, histories, and $i \in \{1, \ldots n\}$:*

1. $t_i = z_i$ *or* $V_i^t = \frac{1-\beta_A^{T-s}}{1-\beta_A}\bar{V}$,

2. $f_i = z_i$ *or* $V_i^f = \frac{1-\beta_A^{T-s}}{1-\beta_A}\bar{V}$.

Furthermore, all optimal contracts satisfy those properties if $z_1 = 0$.

The intuition for these results is simple. Early payments on the part of the agent in exchange for large future payments enable the principal to economize on audit costs over the duration of the contract. As long as the agent is as patient as the principal, this strategy continues to be optimal.

5.3 Infinite horizon

Assume that $T = +\infty$ and that the agent and the principal discount future flows at geometric rates β_A and β_P, respectively. The corresponding total surplus function W is defined on $[0, \frac{\bar{V}}{1-\beta_A}]$ and set $\mathcal{P}$ becomes:

$$\mathcal{P} = \{V \in [0, \frac{\bar{V}}{1-\beta_A}] : V' \in (V, \frac{\bar{V}}{1-\beta_A}] \Longrightarrow \Pi_1(V') < \Pi_1(V)\}$$

Denote by Π the principal's net income function in this infinite case, so that $W(V) = V + \Pi(V)$ for all $V \in [0, \frac{\bar{V}}{1-\beta_A}]$. Standard dynamic programming arguments show that an optimal contract continues to exist. The first part of theorem 1 is unaffected:

Proposition 4. *Assume that* $\beta_A \geq \beta_P$. *For any* $V_0 \in \mathcal{P}$, *there exists an optimal contract that gives the agent lifetime income* V_0 *and satisfies the following properties for all periods, histories, and* $i \in \{1, \dots n\}$:

1. $t_i = z_i$ *or* $V_i^t = \frac{\bar{V}}{1-\beta_A}$,

2. $f_i = z_i$ *or* $V_i^f = \frac{\bar{V}}{1-\beta_A}$.

Proof. Consider the set $\mathcal{C}$ of continuous functions on $[0, \frac{\bar{V}}{1-\beta_A}]$ equipped with the sup-norm topology. Let T be the operator defined for all $H \in \mathcal{C}$ and $V \in [0, \frac{\bar{V}}{1-\beta_A}]$ by:

$$T(H)(V) = \max_{(p_i, t_i, f_i, V_i^t, V_i^f)_{i=1}^n} \sum_{i=1}^n [p_i(f_i - \gamma + \beta_P H(V_i^f)) + (1-p_i)(t_i + \beta_P H(V_i^t))]h_i$$

subject to:

$$\begin{aligned}
0 \le t_i, f_i &\le z_i \text{ for all } i \\
0 \le V_i^t, V_i^f &\le \frac{\bar{V}}{1-\beta_A} \\
(1-p_i)(z_i - t_i + \beta_A V_i^t) + p_i(z_i - f_i + \beta_A V_i^f) &\ge (1-p_j)(z_i - t_j + \beta_A V_j^t) \\
&\quad \text{for all } i, j \text{ with } t_j \le z_i \\
\sum_{i=1}^{n} [(1-p_i)(z_i - t_i + \beta_A V_i^t) + p_i(z_i - f_i + \beta_A V_i^f)] h_i &= V
\end{aligned}$$

Note that for all $H \in \mathcal{C}$, $T(H) \in \mathcal{C}$ by the Theorem of the Maximum. In fact, T defines a contraction on $\mathcal{C}$ since $\beta_P < 1$. Consider now the subset $\mathcal{C}_0$ of elements H of $\mathcal{C}$ such that $V \longmapsto H(V) + V$ is weakly increasing. One easily verifies that $\mathcal{C}_0$ is closed. Furthermore, using the same arguments as in the proof of lemma 5, T maps $\mathcal{C}_0$ into $\mathcal{C}_0$. It now follows that Π, the unique fixed point of T, is in $\mathcal{C}_0$, which suffices to establish the desired result. □

As in the finite case, an optimal contract thus exists such that the agent's one-period income grows in expected terms. Whether this expected income converges to $\bar{V}$ so that the agent's share of the project's revenues converges to one with probability one is an open question.

6 Conclusion

In this paper we establish a property of optimal contracts in a dynamic extension of the framework studied by Border and Sobel (1987). We find that there always exists an optimal contract such that, each period, the agent either transfers the entire revenues of the project to the principal or becomes the sole claimant of the project's revenue as of the following period. This is because by setting transfers as large as possible in any given period, the principal can economize on audit costs over the remaining duration of the project without any compensating increase in expected audit costs in the current period. When there is a positive probability that the project may yield nothing in any given period, all optimal contracts must satisfy this dynamic property.

As a corollary, the agent's income rises over time in expected terms. Furthermore, in all but possibly one period, the principal claims all the project's revenues whenever an audit takes place. Therefore, although some form of "debt forgiveness" may occur, it occurs infrequently in long-lived projects.

We also illustrate how this dynamic property of optimal contracts can be used to reduce the difficulty of solving for optimal auditing schemes. Given that the dynamic problem can be rewritten to closely resemble the static problem, an intriguing question is which of the monotonicity properties established by Border and Sobel (1987) are robust to dynamic extensions of their framework.

One could also consider the impact of relaxing the assumption that revenue realizations are independent across periods. One could assume, for instance, that the evolution of the project's revenues is governed by a first order Markov process. It is easy to see that the first part of Theorem 1 does not rely on our independence assumption, but the proof of the second part, in other words the proof of lemma 6, must be extended to accommodate this case.

Another important question concerns the introduction of risk aversion on the part of the agent. In this case two forces interact: because of risk aversion the agent favors contracts with smooth consumption profiles within and across periods, but audit costs will tend to dampen this effect as the principal prefers to front-load payment. Further work is needed to describe the interplay of these two forces.

A The revelation principle

Border and Sobel (1987) point out that standard arguments cannot be applied in their framework because the message space depends on the revenue realization: it might not be feasible for an agent with a low income to announce a high income. This section demonstrates that the revelation principle continues to hold nevertheless.

Consider any subgame perfect equilibrium with contract $(\mathcal{M}, p, t, f)$. Border and Sobel argue that we can restrict our attention to contracts that use direct revelation in the last period, i.e. for which the last period message space is $\mathcal{M}^T(z_i) = \{z \in \mathcal{Z} : t^T(z) \leq z_i\}$ for all $i \in \{1, \dots n\}$ and such that the agent chooses to announce revenues truthfully in this last period.

We proceed by backward induction. Consider the subgame starting in period $T-1$ at any given history. Date T continuation payoffs given any action in period $T-1$ are specified by the contract. We can therefore construct a one-shot game at $T-1$ whose payoffs are the agent's expected income in period $T-1$ plus the stipulated continuation payoffs, for any possible action on the part of the agent.

Now construct a new contract $(\mathcal{M}', p', t', f')$ by changing the stipulations of contract $(\mathcal{M}, p, t, f)$ in period $T-1$ as follows. For all $i \in \{1, \ldots n\}$, let $\mathcal{M}'^{T-1}(z_i) = \{z \in \mathcal{Z} : t^{T-1}(m^{T-1}(z)) \leq z_i\}$, $p'^{T-1}(z_i) = p^{T-1}(m^{T-1}(z_i))$, $t'^{T-1}(z_i) = t^{T-1}(m^{T-1}(z_i))$ and $f'^{T-1}(z_i, z_i) = f^{T-1}(z_i, m^{T-1}(z_i))$. While the new message space continues to depend on the revenue realization, it satisfies the *Nested Range Condition*, i.e.:

For any $i_1, i_2, i_3 \in \{1, \ldots n\}$, $z_{i_1} \in \mathcal{M}'^{T-1}(z_{i_3})$ whenever $z_{i_2} \in \mathcal{M}'^{T-1}(z_{i_3})$ and $z_{i_1} \in \mathcal{M}'^{T-1}(z_{i_2})$.

Using the results of Green and Laffont (1986), this implies that the modified contract together with the strategy m'^{T-1} defined by $m'^{T-1}(z_i) = z_i$ for all $i \in \{1, \ldots n\}$ constitutes a Nash equilibrium in the one-shot game we defined above for period $T-1$. Moreover, by construction, neither the principal's payoff nor the agent's is affected. By repeating this argument for earlier periods, one establishes that the revelation principle holds in our model.

B Gains from using dynamic contracts: an example

This appendix shows that T-period contracts obtained by repeating the optimal static solution T times are generally sub-optimal. In particular, optimal dynamic contracts may not be stationary. We demonstrate this with a simple two-period example.

Set $n = 2$ and assume that z equals $z_2 > 0$ with probability $\frac{1}{2}$, zero otherwise, so that $h_1 = h_2 = \frac{1}{2}$ and $\bar{V} = \frac{z_2}{2}$. We begin by characterizing the optimal static contract. From theorem 1 in Border and Sobel (1987) we know that $p_2 = 0$ in any optimal contract. Moreover, if the agent expects income $V \geq 0$, promise keeping implies:

$$\frac{z_2 - t_2}{2} = V \tag{B.1}$$

while incentive-compatibility requires:

$$z_2 - t_2 \geq (1 - p_1) z_2 \tag{B.2}$$

Since monitoring is costly, condition (B.2) must bind so that $p_1 = \frac{t_2}{z_2}$ at any optimal contract. The ex-ante likelihood that an audit will take place is $h_1 p_1 = \frac{t_2}{2z_2}$. Together with equation (B.1), this implies that optimal audit costs given V are $\left(\frac{1}{2} - \frac{V}{z_2}\right)\gamma$. Audit costs decrease strictly with V, as they must by remark 1. The principal's expected income is thus given by:

$$\Pi(V) = \bar{V} - V - \left(\frac{1}{2} - \frac{V}{z_2}\right)\gamma = \frac{z_2 - \gamma}{2} - V\left(1 - \frac{\gamma}{z_2}\right)$$

The solution to the static problem when the agent has no equity in the project (i.e. $V = 0$) is to set $(p_1, p_2) = (\frac{t_2}{z_2}, 0)$, $t_2 = z_2$ and $t_1 = f_1 = 0$, which gives the principal a payoff of $\Pi(0) = \frac{z_2 - \gamma}{2}$. Note that $\Pi'(V) > -1$ for all V, as must be the case since $z_1 = 0$. When V rises, the principal can economize on monitoring costs. Also note that $\Pi(\bar{V}) = 0$.

Now consider the two-period case. Assume that the agent's expected income over the two periods is $V_0 = \frac{\bar{V}}{2}$. Below, we further restrict parameters so that this value for V_0 maximizes the principal's overall income so that, in particular, $V_0 \in \mathcal{P}$. A feasible strategy for the principal consists of giving the agent expected income $\frac{V_0}{2} = \frac{\bar{V}}{4}$ in both periods, using the corresponding optimal static contract. This gives the principal an overall payoff of $2\Pi\left(\frac{\bar{V}}{4}\right) < 2\Pi(0)$. We will show that, generally, the principal can improve upon this myopic strategy.

Consider the following two-period contract. When the agent announces z_2 in the first period, no audit occurs, a transfer $t_2 = z_2$ is made by the agent to the principal, and the continuation utility is set to $\bar{V} = \frac{z_2}{2}$ (i.e. the agent keeps nothing in the first period but keeps everything in the second period). If the agent announces a revenue of zero today, the continuation payoff is set to zero. Incentive-compatibility holds if and only if:

$$\frac{z_2}{2} \geq (1-p_1)z_2 \Leftrightarrow \frac{1}{2} \geq (1-p_1)$$

Naturally, since auditing is costly, it is optimal to set $p_1 = \frac{1}{2}$ so that this last condition binds.

Note first that this two period contract gives the agent the desired expected income. Moreover, the principal is strictly better off with this contract than when he simply repeats the static solution twice provided the following condition holds:

$$\frac{1}{2}\left(z_2 - \frac{\gamma}{2}\right) + \frac{1}{2}\Pi(0) > 2\Pi\left(\frac{V_0}{2}\right)$$

This is the case for instance, when $\gamma = 3$ and $z_2 = 5.5$. Indeed, we then have $\frac{1}{2}(z_2 - \frac{\gamma}{2}) + \frac{1}{2}\Pi(0) = 2.625$ while $2\Pi(\frac{\bar{V}}{4}) < 2\left(\frac{z_2-\gamma}{2}\right) = 2.5$. It is also easy to find examples where the project is not profitable in the static case, but can be implemented profitably in the two-period case.

This example illustrates how setting large transfers early in exchange for smaller (possibly zero) transfers in the future can help the principal economize on audit costs. For the values above, one can solve for the optimal first-period policy numerically and check that, in fact, the contract we suggested is the unique optimal contract, and that setting $V_0 = \frac{\bar{V}}{2}$ maximizes the principal's expected net income over the duration of the project. Note that this optimal contract satisfies the two properties listed in theorem 1, as it must since $z_1 = 0$.

C Proofs of results in the general case

In the next two proofs, $IC(i,j)$ refers to the following incentive compatibility constraint:

$$(1-p_i)(z_i - t_i + V_i^t) + p_i(z_i - f_i + V_i^f) \geq (1-p_j)(z_i - t_j + V_j^t)$$

Proof of lemma 5. The proof is by induction. The desired properties hold for $s = T$. Fix s and assume the lemma holds for $s+1$. The Theorem of the Maximum implies that Π_s is continuous as claimed. Fix V. We need to consider several cases.

Assume first that an optimal contract at V is such that for some i, $p_i > 0$ and either $f_i > 0$ or $V_i^f < (T-s)\bar{V}$. Assume that the principal now needs to deliver expected income $V+\epsilon$ to the agent, where $\epsilon > 0$. If $f_i > 0$, set $f_i = f_i - \frac{\epsilon}{h_i p_i}$, which is feasible for ϵ small enough. If $V_i^f < (T-s)\bar{V}$, set $V_i^f = V_i^f + \frac{\epsilon}{h_i p_i}$, which, again, is feasible if ϵ is small enough. The new contract is incentive compatible because only the left-hand side of all incentive compatibility constraints may be affected, and it can only increase. This contract delivers $V+\epsilon$ without altering audit costs, and the principal's objective falls by no more than ϵ by the induction hypothesis.

We can therefore restrict our attention to cases where $f_i = 0$ and $V_i^f = (T-s)\bar{V}$ for all i. Incentive compatibility constraints now become:

$$z_i - (1-p_i)(t_i - V_i^t) + p_i(T-s)\bar{V} \geq (1-p_j)(z_i - t_j + V_j^t)$$

for all i, j with $t_j \leq z_i$.

Consider the set S of states i such that $p_i < 1$ and either $t_i > 0$ or $V_i^t < (T-s)\bar{V}$. Note that S is non-empty whenever $V < (T+1-s)\bar{V}$. For $\epsilon > 0$ small enough, we will reduce t_i or increase V_i^t in all such states, so as to deliver $V+\epsilon$, and without violating any incentive compatibility constraint. To do this, define

$$\delta = \min_{\{i,j \in S: t_j > z_i\}} t_j - z_i$$

This problem considers all pair of states (i,j) in S such that the agent is unable to announce state j when state i occurs. For any $i \in S$, so long as we do not decrease any transfer by δ or more, the set of incentive compatibility constraints we need to verify when state i occurs does not increase. Also note that $\delta > 0$ since the set of states is finite.

Now decrease t_i or increase V_i^t for all $i \in S$ so that $(1-p_i)(t_i - V_i^t)$ decreases by $\frac{\epsilon}{\sum_{i \in S} h_i}$. For ϵ small enough, this can be done in such a way that t_i does not decrease by more than $\frac{\delta}{2}$. Also, the left-hand and the right-hand sides of all incentive compatibility constraints for which both i and j are in S both rise by the same amount. When j is not in S, the right-hand side of $IC(i,j)$ does not increase while the left-hand side, if anything, rises. When i is not in S, the incentive compatibility constraint is always met since the left-hand side is calculated for transfers of zero and the maximum possible continuation utility. This establishes the lemma. □

Proof of lemma 6. The proof is again by induction. When $s = T$, the result follows from remark 1 and lemma 3. Now fix s and assume that the result holds for $s+1$. Fix V.

Assume first that there exists an optimal contract that delivers V to the agent such that $p_i > 0$ and $V_i^f < (T-s)\bar{V}$ for some $i \in \{1, \ldots n\}$. To increase V one can simply increase V_i^f. Incentive compatibility still holds and the induction hypothesis implies that W_s rises strictly as needed.

Now turn to cases where $V_i^f = (T-s)\bar{V}$ for all i. Assume $V_{i^*}^t < (T-s)\bar{V}$ for some state i^* with $p_{i^*} < 1$. We will show that it is always possible to increase $V_{i^*}^t$ without violating any incentive compatibility constraint.

Define the set S of states i such that the following holds:

$$p_i < 1 \text{ and } [t_i > 0 \text{ or } V_i^t < (T-s)\bar{V}] \quad \text{ or } \quad [p_i > 0 \text{ and } f_i > 0]$$

Clearly, $i^* \in S$. We will reduce t_i, increase V_i^t , or reduce f_i in all states in S so as to deliver $V + \epsilon > V$, without violating any incentive compatibility constraint. To do this, define

$$\delta = \min_{\{i,j \in S : t_i > z_j\}} t_i - z_j$$

As in the proof of lemma 5, $\delta > 0$ since the set of states is finite, and, for any $i \in S$, the set of constraints we need to verify does not increase so long as we don't change any transfer by δ or more.

Now decrease t_i, increase V_i^t or decrease f_i for all $i \in S$ so that $(1-p_i)(t_i - V_i^t) + p_i f_i$ decreases by $\frac{\epsilon}{\sum_{i \in S} h_i}$ for all i. For ϵ small enough, this can be done in such a way that t_i does not decrease by more than $\frac{\delta}{2}$. This can also be done in such a way that $V_{i^*}^t$ rises strictly. We only need to worry about incentive compatibility for all possible pairs (i, j) of states. When j is not in S, the right-hand side of $IC(i, j)$ does not increase, while the left-hand side, if anything, rises. When i is not in S, the left-hand side of $IC(i, j)$ is at its maximum feasible value. Now assume both i and j are in S. If j is such that it is not necessary to alter f_j, both sides of the constraint go up by the same amount and we are done. If j is such that it is necessary to change f_j, the right-hand side of $IC(i,j)$ rises by a smaller amount than the left-hand side.

Since $V_{i^*}^t$ rises strictly, the induction hypothesis implies that total surplus rose strictly as well, as needed. We have thus established that the property holds whenever V is such that there exists an optimal contract with $V_i^t < (T-s)\bar{V}$ or $V_i^f < (T-s)\bar{V}$ for some state i.

So assume that all contracts are such that all continuation utilities are at their maximum. Note that $\Pi_{s+1}((T-s)\bar{V}) = 0$. Therefore, in this final case, optimal audit and transfer vectors at date s must solve the following problem:

$$H(V) = \max_{(p,t,f)} \sum_{i=1}^{n} [p_i(f_i - \gamma) + (1-p_i)t_i]h_i$$

subject to:

$$\begin{aligned} 0 \leq t_i, f_i &\leq \tilde{z}_i - (T-s)\bar{V} \text{ for all } i \\ (1-p_i)(\tilde{z}_i - t_i) + p_i(\tilde{z}_i - f_i) &\geq (1-p_j)(\tilde{z}_i - t_j) \\ & \quad \text{for all } i,j \text{ with } t_j \leq z_i \\ \sum_{i=1}^{n}[(1-p_i)(\tilde{z}_i - t_i) + p_i(\tilde{z}_i - f_i)]h_i &= V \end{aligned}$$

where $\tilde{z}_i = z_i + (T-s)\bar{V}$, for all i. Evidently, $H(V) = \Pi_s(V)$ and $H(V') \leq \Pi_s(V')$ whenever $V' \neq V$. It will suffice, therefore, to show that $H(V) + V$ rises strictly at V.

Note that this reduced problem resembles very closely the static case studied by Border and Sobel (1987). The only difference is that transfers are constrained to be strictly smaller than $\tilde{z}_i$ in all states i. In fact, the arguments of Border and Sobel (1987) can be directly applied to show that:

Remark 2. *Solutions to the problem defined above satisfy:*

$$\text{If } i > j,\ t_i \geq t_j \text{ with equality if and only if } p_j = 0$$

Border and Sobel's second corollary also continues to hold:

Remark 3. *If* (p,t,f) *and* (p',t',f') *are audit-efficient*[11] *solutions to the problem above and* $p_1 = p'_1 > 0$*, then* $t = t'$ *and* $f = f'$

We can now replicate the proof of lemma 3. Observe that we can assume without loss of generality that all incentive compatibility constraints must hold. Indeed, if $t_j > z_i$ for a pair (i,j) of states, it must be that $t_i < t_j$ and $f_i < f_j$. Therefore (see lemma 3), scaling down all transfers in a proportional fashion does not affect incentive compatibility. For any $V' > V$, choose θ such that the scaled down transfers $(\theta t, \theta f)$ give the agent expected income V'. As long as $V < (T+1-s)\bar{V}$, remark 2 implies that $p_1 > 0$ so that, by remark 3, this feasible contract is not audit efficient. Therefore, $H(V') - H(V) > V - V'$ as needed. This completes the argument. □

[11] Recall that (p,t,f) is audit efficient if it is not possible to reduce audit probabilities without reducing the principal's expected gross income.

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