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Optimal control and parameter selection problems in forest stand management

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Abstract

A typical stand forestry problem for determining an optimal silvicultural regime is demonstrated in this paper, where an optimal number of trees at planting and a harvesting strategy over the crop rotation are resolved. The traditional approach for determining such strategies is to formulate a dynamic programming problem where the stages and decision variables are pre-determined in a forward recurrent procedure and thus eliminating any possibilities of an exhaustive search. This type of formulation is largely driven by the 'curse of dimensionality'. Our formulation is a combined optimal control and parameter selection problem using Pontryagin's maximum principle where, the optimal control part determines the harvesting strategy, and the parameter selection determines the initial number of trees planted. Dynamical models are developed as the building blocks of the optimal control and parameter selection formulation using *Eucalyptus nitens* data from intensively managed stands, courtesy of North Forests Products, Tasmania, Australia.

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1 Introduction

Trees in nature have a way of managing themselves in relation to the availability of resources such as nutrients, water, light and space. In a young forest stand, trees channel most of these resources into height growth such that they get as much light as possible. Once canopy closure occurs, all the other trees below average height will die because of an inability to photosynthesis as fast as those exposed to unhindered light. With time, resources become limiting to the residual stand, leading to an overall slow growth and consequently competition mortality. The mortality process tends to affect the weaker trees first, making resources available to the fit trees. In intensively managed forests, mortality is mimicked by a management practice of 'thinning', giving the forester an efficient means of achieving a desired forest structure and subsequently products, *i.e.* pulpwood and sawlog in production forestry. A thinning practice is a means of controlling the stand density where some trees are harvested and the residual trees left to take advantage of the available nutrients, water, light and space. In forest science, the art of manipulating a forest in such a manner is called silvicultural practice and a recipe for managing a forest stand, silvicultural regime. A rudimentary silvicultural regime will have a specified initial planting rate, when and how much to thin, and the final crop number retained until clearfell where every tree in the stand is harvested.

For many years, forest analysts have sought mathematical ways of determining optimal silvicultural regimes, a pursuit that is fraught with a complex interaction of the type of trees species, soils, geographical location, climate and so on. Another key problem, is the capability of accounting for growth behaviour following thinning for the growth functions used in a stand optimisation formulation problem. Chen *et al.*, [1] attributed some of the failure to use of inappropriate growth functions, although no clear leads were given as to how to remedy the situation. Knoebel *et al.*, [2] illustrated a way of accounting for growth following thinning by estimating a different set of parameters for the Sullivan and Clutter basal area model [3] before thinning, after the first thinning, and after the second and subsequent thinnings. The problem with such an approach is that any change in the timing of thinning or the amount thinned, would lead to erroneous predictions of basal area yield.

Even if the growth functions were appropriate, the optimisation problems could not be formulated in such a way as to enable an exhaustive search, making it difficult to determine whether optimality was being achieved. In the real world, it is not so much of optimality that is sought after but rather a guarantee that the implemented result would improve the status quo and what better way than to do an exhaustive search. Dynamic programming (DP), an enumerative method for multi-stage optimisation has been the choice among other search techniques because of its appropriateness, but plagued by the 'curse of dimensionality' [4]. This curse arises

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from a need to define all possible states at each decision stage of a DP formulation, which may be computer-memory demanding, even for small DP problems [5]. Despite efforts of enhancing DP formulations with other heuristics, the problem still remains [6].

In this paper we utilise a mathematical model structure that enables us to account for growth responses following thinning and demonstrate an optimisation formulation that addresses the issue of exhaustive search without the downside of the curse of dimensionality. Additionally, we demonstrate the ability of determining an optimal system parameter, in particular, the initial stand density, combined with the optimisation formulation. The data used to develop the mathematical growth functions were provided by North Forests Products, Tasmania, Australia.

2 Data and state equations

The data used for model development came from the North Forests Burnie's Eucalypt Tree Farm growth plots for three species. In this paper we only looked at the *Eucalyptus nitens* growth plots. The plots were established at an age of five years and measurement commenced in the same year of establishment. Re-measurement was carried out the following year and then biennially thereafter until the age of 15 years. Dynamical models were developed to simulate and predict growth. The choice to use dynamical models was largely due to their mathematical description which is convenient for designing control systems [7] and also their flexibility to accurately predict growth following thinning [8]. These dynamical models formed the state equations of the optimisation model discussed in the latter subsections.

Dynamical models are 'black box' or 'gray box' models that simulate/predict the dynamics of a system, usually based on a time-series dataset that consists of an input variable, u and an output variable, y. By definition, an input variable can be measured and directly controlled by an observer whereas the input variable, although it can be measured, is indirectly controlled by manipulating u. For the discrete-time model, the mathematical representation is a difference equation in orders of magnitude that relate to the complexity of the dynamics. A first-order model implies that the response variable is regressed on the previous value at time, t. A second-order model implies a response variable regressed on previous variables at times, t-1 and t-2. These models are quite common in Systems Engineering and have only been recently adopted and demonstrated in forestry applications in the last 10 years [8,9,10]. Two state equations for stand basal area and mean dominant height growth were developed for *E. nitens* . The equations were as follows:

$$sba(t) = a_1(el, si, sph)sba(t-1) + b_1(1 - a_1(el, si, sph)) \tag{1}$$

$$\begin{aligned} mdh(t) = a_2(el, sph)mdh(t-1) + a_3(a_2)mdh(t-2), \\ +b_2(1 - a_2(el, sph) - a_3(a_2)) \end{aligned} \tag{2}$$

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$$a_1 = -1.816e-5*sph+7.3817e-8*el*sph-1.0118e-6*si*sph, \quad (3)$$

$$-3.575e-6*el*si+0.9985$$

$$a_2 = 2.3775e-4*sph-3.3146e-7*el*sph, \quad (4)$$

$$+7.5876e-4*el+1.381$$

$$a_3 = -0.914a_2+0.8342 \quad (5)$$

where,

t= time in years,
sba = stand basal area in m^2ha^{-1},
mdh = mean dominant height in metres,
el = elevation in metres,
si = site index in metres,
sph = initial or residual stand density in stems ha^{-1},
b_1=75, b_2=60.

The parameters of the eqns (1) and (2), *i.e.*, a_1,a_2 and a_3 are themselves predicted from environmental variables and the initial/residual stand density. Stand density inclusion as a dependent variable in eqns (3) and (4) was strategic in the sense that it enabled the capture of changes in growth dynamics following thinning [8]. Eqns (1) and (2) were cross validated using data from stands that had been thinned at different times and rates. The differences between the predicted and actual trends were expressed in terms of mean squared error and in all cases they were low and close zero. This confirmed the flexibility of the models to accurately estimate growth trends following thinning. It should be noted that the models were developed using data from stands that had not been thinned, but with varying initial stand densities.

A stand volume equation was also developed which was used to formulate the quadratic objective functional of the optimal control problem. This equation was of a different form from the dynamical models because of its intended use. It was as follows:

$$V(t) = 0.3512*sba(t)*mdh(t) \quad (6)$$

where,

V = stand volume in m^3ha^{-1}.

The validation procedure was no different from the previous ones carried out for eqns (1) and (2). The only difference was that volume prediction was done in two stages, *i.e.* the stand basal area and mean dominant height were initially predicted from their respective models, eqns (1)and (2), and then used in eqn (6). Again the mean squared errors were low and close zero.

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3 Optimal control and parameter selection formulation

The combined optimal and selection parameter formulation in discrete-time may be generalised as follows:

$$\underset{\mathbf{u},\mathbf{z}}{minimise}\left\{ J_0(\mathbf{u},\mathbf{z}) = \phi_0(\mathbf{x}(T),\mathbf{z}) + \sum_{t_0}^{T-1} g_0(t,\mathbf{x}(t),\mathbf{u}(t),\mathbf{z}) \right\} \qquad (7)$$

subject to the dynamics,

$$\mathbf{x}(t+1) = f(t,\mathbf{x}(t),\mathbf{u}(t),\mathbf{z}), \quad t = 0, 1, \ldots, T-1 \qquad (8)$$

where,

J_0 = cost functional,
t_0, T = initial and fixed times respectively,
$\mathbf{u}$,$\mathbf{x}$ = control and state vectors respectively,
$\mathbf{z}$ = system parameter vector (or decision vector independent of time),
ϕ_0 = continuously differentiable functions,
g_0= continuously differentiable function with respect to the states and control.

Applying Pontryagin's maximum principle (PMP) [11] to solve this problem, introduces the Hamiltonian [12]. The PMP does not suffer from the curse of dimensionality, because it breaks the problem to a sequence of smaller problems. Using 'control parameterisation' [13] the combined control and parameter selection problem is approximated by a constrained nonlinear programming problem that may be solved by a standard mathematical programming algorithm. For our problem we used an optimal control software progam called MISER [14] that employs NLPQL [15] for solving mathematical programming formulations. NLPQL uses state-of-the-art sequential quadratic programming (QP) algorithm that replaces the objective functional with a quadratic approximation and the constraint functions with linear approximations. The local convergence properties of the sequential QP approach are well understood if the Lagrange multiplier estimates satisfy the second-order sufficiency conditions [11].

Therefore, our complete problem definition for maximising the volume harvested was as follows:

$$J(u) = \underset{u}{maximise} \sum_{t=0}^{T} \frac{u(t)}{sph(t)} V(t) \qquad (9)$$

where,

u = the number of trees harvested.

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In order to harvest large logs that command a higher premium, the objective functional was formulated such that the diameter size of the individual trees would be maximised as well and in the shortest possible time. Therefore the objective functional became:

$$J(u) = \overset{maximise}{u} \sum_{t=0}^{T} \frac{u(t)}{sph(t)} V(t) \frac{sba(t)}{sph(t)} \qquad (10)$$

Eqns (1) and (2) were defined as the state equations including the following:

$$sph(t) = sph(t-1) - u(t-1), \qquad (11)$$

with an inequality constraint,

$$sph(t) \geq 0, \qquad (12)$$

initial states,

$$sph(t_0) = z(1), \quad sba(t_0) = 0, \qquad (13)$$
$$mdh(t_0) = 0, \quad mdh(t_0 + 1) = 0.7 + 1.19 * mdh(t_0),$$

lower and upper bounds on the control,

$$0 \leq u(t) \geq 300, \quad \forall t \in [0, 40], \qquad (14)$$

and, lower and upper bounds for the system parameter,

$$900 \leq z(1) \geq 2000, \qquad (15)$$

4 Results and discussion

The control and parameter selection problem in eqns (10)-(15) gave a solution with an initial planting density of 1818 stems ha^{-1} and thinning at ages 28, 32 and 36 years of 56, 147, 1.3 stems ha^{-1} respectively, with of course a clearfell at age 40. The elevation and site index were set at 400m and 20m respectively. For implementation the thinnings at ages 28 and 36 would be ignored because of practical and economic reasons. The final crop at 998 stems ha$^-$1, would be deemed high, because the stand basal area and mean dominant height models are still increasing at age 40 requiring another 10 years or so before reaching their biological limits of growth reflected by the asymptotic limits of the models. Since our study was only restricted to investigating the feasibility of solving as a combined optimal control and parameter selection problem with a specified terminal time, we did not seek to find the optimal clearfelling age. It was not feasible at this stage to compare this result with what is observed in the field, as this research was initiated to provide a basis for determining optimal silvicultural

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regimes. This way, North Forest Products will not have to wait for many years analysing field data from experimental tree plots before finding alternative regimes that would improve the status quo, where thinnings are not being currently carried out.

An attempt was made to avoid thinnings of less than 100 stems ha^{-1} by introducing a second inequality constraint. This caused the problem to be ill-conditioned, *i.e.* there existed widely varying solutions that satisfied the criteria for acceptance as the optimal solution. Different trials of initial control values and large numbers of iterations still proved to be of no avail and therefore,the original formulation was adhered to.

Changing the elevation and site index in the the control and parameter selection problem gave the flexibility to look at different geographical locations. The system identification process for the state equations (1) and (2) resulted in 5 distinct site qualities being identified which had unique initialisation values for eqn (2). A GIS layer may be generated that delineates the North Forests Products timber resource area into 5 site quality zones, such that the models can be applied appropriately. Within each zone, there would be variations in elevation, making it possible to model a variety of geographical locations. Initialisation values for eqn (2) in the control and parameter selection problem, eqns (10)-(15), were based on the best site quality.

5 Conclusion

We believe we have provided a mechanism, which if used wisely, will provide a basis for analysing and estimating optimal silvicultural regimes that have never been used before, saving North Forests Products from many years of analysing trial experimental plots in the field. Another added advantage is that the optimisation models can be used to predict a wide matrix of optimal silvicultural regimes over the whole geographical location where forestry operations are being currently carried out by North Forest Products.

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References

[1] Chen, C.M., Rose, D.W. & Leary, R.A., *How to formulate and solve optimal stand density over time problem for even-aged stands using dynamic programming*, General Technical NC-56, St. Paul, MN:USDA Forest Service, North Central Forest Experiment Station, 17p, 1980.

Paper from: *Development and Application of Computer Techniques to Environmental Studies*, CA Brebbia and P Zannetti (Editors).
ISBN 1-85312-909-7

[2] Knoebel, B.R., Burkhart, H.E. & Beck, D.E., A growth and yield model for thinned stands of yellow-poplar. *Forest Science Monograph*, 27p, 1986.

[3] Sullivan, A.D. & Clutter, J.L., A simultaneous growth and yield model for loblolly pine, *Forest Science*, **18**, pp. 76-86, 1972.

[4] Bellman, R.E., *Dynamic Programming*, Princeton, NJ:Princeton University Press, 1957.

[5] Chikumbo, O., Mareels, I.M.Y. & Turner, B.J., A stand optimisation model developed from dynamical models for determining thinning strategies, *Seventh Symposium on Systems Analysis in Forest Resources*, eds. J. Fried, L. Leefers & M. Vasievich, General Technical NC-205, St. Paul, MN:USDA Forest Service, North Central Forest Experiment Station, pp. 355-360, 1997.

[6] Pelkki, M.H. & Arthaud, G.J., Exploring enhancements to dynamic programming, *Seventh Symposium on Systems Analysis in Forest Resources*, eds. J. Fried, L. Leefers & M. Vasievich, General Technical NC-205, St. Paul, MN:USDA Forest Service, North Central Forest Experiment Station, pp. 337-342, 1997.

[7] Ljung, L., *System Identification:Theory for the User*, Prentice-Hall, Inc., Englewood Cliffs, NJ, 519p, 1987.

[8] Chikumbo, O., Mareels, I.M.Y. & Turner, B.J., Predicting stand basal area in thinned stands using a dynamical model, *Forest Ecology and Management*, **116** (1999), pp. 175-187, 1999.

[9] Chikumbo, O., James, R.N., Mareels, I.M.Y. & Turner, B.J., Mortality simulations in *Pinus radiata* plantations in the Tarawera Valley regimes trial, *Ecological Modelling*, **86**, pp. 253-258, 1996.

[10] Chikumbo, O., Mareels, I.M.Y. & Turner, B.J., Integrating the Weibull into a dynamical model to predict future diameter distributions, *Integrating Forest Information over Space and Time*, eds. B. Wood & B.J. Turner, ANUTECH, The Australian National University, pp. 94-104, 1992.

[11] Dixon, L.C.W., *Nonlinear Optimisation*, The English Universities Press Ltd., Bell & Ban Limited, Glasgow, UK, 213p., 1972.

[12] Barnett, S. & Cameron, R.G., *Introduction to mathematical Control Theory*,Claredon Press, Oxford, 404p, 1985.

[13] Teo, K.L., Wong, K.H. & Goh, C.J., Optimal maintenance of a system of machines with the weakest link dependence performance, *Optimal Control Methods and Application*,**10**, pp. 113-127, 1989.

Paper from: *Development and Application of Computer Techniques to Environmental Studies*, CA Brebbia and P Zannetti (Editors).
ISBN 1-85312-909-7

[14] Jennings, L.S., Fisher, M.E., Leo, K.L. & Goh, C.J., *MISER3 Optimal Control Software: Theory and User Manual*, EMCOSS PTY Ltd., WA, Australia, 1990.

[15] Schittkowski, K., NLPQL:A FORTRAN subroutine for solving constrained nonlinear programming problems, *Annals of Operations Research*, **5**, pp. 485-500, 1985.