Optimal Control and the Calculus of Variations

ENID R. PINCH

Department of Mathematics University of Manchester

Oxford New York Tokyo OXFORD UNIVERSITY PRESS 1993

Contents

1	Introduction	1
	The maxima and minima of functions	1
	The calculus of variations	4
	Optimal control	9
2	Optimization in \mathbb{R}^n	13
	Functions of one variable	13
	Critical points, end-points, and points of discontinuity	16
	Functions of several variables	18
	Minimization with constraints	22
	A geometrical interpretation	25
	Distinguishing maxima from minima	28
3	The calculus of variations	33
	The fixed end-point problem	33
	Problems in which the end-points are not fixed	41
	Finding minimizing curves	46
	Isoperimetric problems	54
	Sufficiency conditions	58
	Fields of extremals	59
	Hilbert's invariant integral	62
	Semi-fields and the Jacobi condition	66
4	Optimal control I: Theory	70
	Introduction	70
	Control of a simple first-order system	70
	Systems governed by ordinary differential equations	72
	The optimal control problem	74
	The Pontryagin maximum principle	80
	Optimal control to target curves	100
5	Optimal control II: Applications	103
	Introduction	103
	Time-optimal control of linear systems	103
	Optimal control to target curves	139
	Singular controls	149

со	NTI	EN'	тs
----	-----	-----	----

Fuel-optimal control	151	
Problems where the cost depends on $\mathbf{x}(t_1)$	159	
Linear systems with quadratic cost	163	
The steady-state Riccati equation	168	
The calculus of variations revisited	170	
6 Proof of the maximum principle of Pontryagin	175	
Convex sets in \mathbb{R}^n	176	
The linearized state equations	182	
The behaviour of H on an optimal path	184	
Sufficiency conditions for optimal control	206	
Appendix: Answers and hints for the exercises		
Bibliography		
Index		

viii