

## ***Optimal Control of a Nuclear Power Reactor Core with a Coupled Nuclear Thermo-hydrodynamics Model***

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### **-Synopsis**

An optimal control is given for regulating power distribution in a nuclear power reactor which has cylindrical geometry. The space dependence of the system is described by expanding space dependent variables by Helmholtz modes. Results are obtained through the principle of optimality and are described by the Riccati-type algebraic equation that the optimal feedback coefficients should satisfy. Use of an integral equation as the system equation makes it possible to deal with actual controlling apparatuses: control rods or rod clusters.

### **1. Introduction**

Growth of nuclear power reactors in size and in power density makes its power distribution easy to transform in profile with small local variation or nuclear parameters, and a coupled nuclear thermo-hydrodynamic model is necessary to be regarded for a power regulating purpose.

In the last decade, efforts have been devoted to synthesize the optimal control for spatially distributed cores.

Weaver and Vanasse<sup>(1)</sup> devised a technique to determine the optimal feedback coefficients for a system described in the frequency region, and have applied their method to a nodally represented core and also to a coupled core.

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Stacey<sup>(2)</sup> and Hsu<sup>(3)</sup> used a partial differential form as the neutron governing equation, and applied the calculus of variations to obtain the control. Stacey's work provides a method to calculate an optimal open loop control by the direct method of calculus of variations, which yields an algebraic equation to be numerically solved. Hsu developed Pontryagin's Maximum Principle<sup>(4)</sup> for distributed systems, and applied his method to a regulator problem and a minimum time problem. Also a Liapunov functional was constructed to verify the stability of a system, and was further utilized to find a suboptimal control law.

Wiberg<sup>(5)</sup> treated the optimal control of xenon spatial oscillation to apply the modern control theory for lumped parameter systems, and the optimal feedback coefficients are given formally as a solution of a regulator problem. A numerical example is presented for a very simple case.

Kyong<sup>(6)</sup> applied the function space method to a regulator problem of a prompt-neutron equation, and developed a method for solving an integral equation numerically, which was derived from an open loop optimal control problem for general linear systems, initially formulated in an abstract space by Balakrishnan<sup>(7)</sup>.

Kuroda and Makino<sup>(8)</sup> dealt with a terminal cost problem with control energy constraint by the principle of optimality. They also applied the results by Lions<sup>(9)</sup> for a regulator problem of a linear one-group neutron model without precursor.

Koga<sup>(10)</sup> obtained an optimal solution for a model which takes only the precursor density as the state variable but neglecting dynamics of neutron flux because of its quickly decaying feature. The function space method is used there to derive the equation that the optimal open loop control should satisfy and some numerical characteristics of the equation were found after numerical studies.

A terminal cost problem which transfers a core state to another for a one-group neutron equation associated with precursor density was studied by Iwazumi and Koga<sup>(11)</sup>. A feedback solution was obtained through the Kaplan mode<sup>(12)</sup> expansion technique as well as by the function space method. An idea was there devised to circumvent a space dependence of core parameters that makes it difficult to obtain the Kaplan mode analytically.

All these studies treated systems described in differential equations with which difficulties arise to treat effect of control rods which are pointwisely concentrated in the core. In this report this difficulty is circumvented by reducing differential equations to an integral equation.

## 2. Reduction of System Equation to an Integral Equation

Here a thermo-hydrodynamic equation coupled through a steady state neutron equation is chosen as a governing equation. The reactor at the rated power should be regulated in the vicinity of the steady state with respect to the coolant outlet temperature and the neutron flux level over the reactor core.

Here we consider a single channel reactor(Appendix). The governing equations with respect to fluctuated temperature  $\theta$  and precursor density  $C$  are described as

$$\dot{C} = -\lambda C + \beta \nu \Sigma_f \phi \quad (1)$$

$$\dot{\theta} = -\gamma \theta + \kappa \Sigma_f \phi, \quad (2)$$

where  $\phi$  is the neutron flux density. The precursor density is considered to be of one group and the distributive neutron dynamics in a single channel is assumed to be far quicker compared with the thermo-hydrodynamics of the moderator and the control motion. Reduction of parameters  $\gamma$  and  $\kappa$  is presented in Appendix.

Two independent dynamics Eqs.(1) and (2) are coupled through the quickly following neutron system steered by control absorber, namely,

$$\nabla^2 \phi + \frac{1}{M^2} (k_{\infty} - 1) \phi = -(\alpha - \alpha \theta) \Phi + \bar{\lambda} C \quad (3)$$

associated with a boundary condition,

$$\phi(\omega) = 0, \quad \omega \in \partial\Omega \quad (4)$$

The variables are assigned by following meanings,

$$\begin{aligned} M^2 = \Sigma_a / D_2 & : \text{migration area,} \\ k_{\infty} = \nu \Sigma_f / \Sigma_a & : \text{multiplication factor for an infinite} \\ & \text{medium,} \\ \Phi & : \text{nominal neutron flux density,} \\ \alpha & : \text{moderator temperature coefficient,} \\ \bar{\lambda} & : \lambda / \Sigma_a \end{aligned}$$

The differential equation(3) together with the boundary condition (4) can be transposed into the explicit form using Helmholtz mode expansion technique as

$$\phi(\omega) = \sum_{i=1}^{\infty} \varphi_i(\omega) \frac{1}{\eta_i} \int_{\Omega} \varphi_i(\omega') \{(\alpha - \alpha \theta) \Phi - \bar{\lambda} C\}_{(\omega')} d\omega', \quad (5)$$

where  $\varphi_i(\omega)$  is the Helmholtz eigenfunction corresponding to the eigenvalue

$$\eta_i = -M^2 / (k_{\infty} - 1 - B_i^2 M^2). \quad (6)$$

Substitution of this expression into the state equations (1) and (2) gives a system equation with respect to the state values  $c(\omega, t)$  and  $\theta(\omega, t)$ , and the control value  $u(\omega, t)$ , in an evolution equation form with an integral operator as a spatial operator, instead of the differential operator, viz,

$$\frac{\partial}{\partial t} \vec{T}(\omega, t) = -\vec{\lambda} \vec{T}(\omega, t) + \vec{b} \int_{\Omega} \vec{m}_T^t(\omega|\omega') \vec{T}(\omega', t) d\omega' \quad (7)$$

$$- \vec{b} \int_{\Omega} m_u(\omega|\omega') u(\omega', t) d\omega', \quad (7)$$

where  $\vec{T}$  is the state vector of  $c$  and  $\theta$ , as

$$\vec{T} = \text{col.}(c, \theta), \quad (8)$$

$\vec{\lambda}$ , a diagonal matrix

$$\vec{\lambda} = \text{diag.}(\lambda, \gamma) \quad (9)$$

and  $\vec{b}$  is given by

$$\vec{b} = \text{col.}(\beta \nu \Sigma_f, \kappa \Sigma_f). \quad (10)$$

The kernels of the integral operator,  $\vec{m}_T(\omega|\omega')$  and  $m_u(\omega|\omega')$ , are given by

$$\{\vec{m}_T(\omega|\omega')\}^1 = - \sum_{i=1}^{\infty} \frac{1}{\eta_i} \varphi_i(\omega) \varphi_i(\omega') \alpha(\omega') \Phi(\omega'), \quad (11)$$

$$\{\vec{m}_T(\omega|\omega')\}^2 = - \sum_{i=1}^{\infty} \frac{1}{\eta_i} \varphi_i(\omega) \varphi_i(\omega') \bar{\lambda}(\omega'), \quad (12)$$

and

$$m_u(\omega|\omega') = \sum_{i=1}^{\infty} \frac{1}{\eta_i} \varphi_i(\omega) \varphi_i(\omega') \Phi(\omega'), \quad (13)$$

respectively.

### 3. Optimal Regulator Problem; Fully Distributed Control

Let us consider a problem to minimize a functional of  $\vec{T}$  and  $u$  as

$$J = \int_t^{\infty} H(\vec{T}(\omega, \tau), u(\omega, \tau)) d\tau, \quad (14)$$

choosing the control function  $u(\omega, \tau)$  suitably under the constraint, Eq.(7). The  $H$  function in Eq.(14) is also a functional of the form

$$H(\vec{T}, u) = \frac{1}{2} \iint_{\Omega} \vec{T}^t(\omega, t) \vec{Q}(\omega, \omega') \vec{T}(\omega', t) d\omega' d\omega + \frac{\gamma}{2} \int_{\Omega} \{u(\omega, t)\}^2 d\omega \quad (15)$$

The  $\gamma^2$  variable is a parameter which should be adjusted according to the controlling object. The  $Q$  in the first term is a symmetric weighting matrix function which is considered to be dependent both on  $\omega$  and  $\omega'$  also symmetrically, though we shall not consider the spatial dependence. This is because we want to make the reduction clearly.

The minimization problem as stated leads to a problem of the calculus of variations making use of the principle of optimality.

Let us introduce a functional  $\pi$  of  $\vec{T}$  associated with a parameter  $t$  as

$$\pi(\vec{T}(t), t) = \min_{u(\tau)} \int_t^{\infty} H(\vec{T}, u, \tau) d\tau. \quad (16)$$

Following the principle of optimality, Eq.(16) is to be rewritten in a separated form:

$$\pi(\vec{T}(t), t) = \min_{u(\tau)} \left[ \int_t^{t+\sigma} H(\vec{T}, u, \tau) d\tau + \pi(\vec{T}(t+\sigma), t+\sigma) \right] \quad (17)$$

Gathering both sides into one and taking account of the minimization operation, we have

$$\min_{u(\tau)} \left[ \int_t^{t+\sigma} H d\tau + \pi\{\vec{T}(t+\sigma), t+\sigma\} - \pi\{\vec{T}(t), t\} \right] = 0. \quad (18)$$

Retaining the relation(18) to the limit  $\sigma \rightarrow 0$ , we obtain

$$\min_{u(t)} \left[ H(\vec{T}(t), u(t)) + \frac{d}{dt} \pi\{\vec{T}(t), t\} \right] = 0, \quad (19)$$

for the optimal control  $u$  at an arbitrary calendar time  $t$ .

The total time derivative of  $\pi$  is written as

$$\frac{d\pi}{dt} = \frac{\partial \pi}{\partial t} + \frac{\delta \pi}{\delta \vec{T}} \frac{\partial \vec{T}}{\partial t} \quad (20)$$

where the second term in the right side together means the Fréchet derivative of  $\pi$  with respect to the variable  $\partial \vec{T} / \partial t$

Because only with the stationary case we are concerned, the *a priori* condition

$$\frac{\partial \pi}{\partial t} = 0 \quad (21)$$

should be retained. Then substitution of Eq.(20) together with Eq.(21) into Eq.(18) yields

$$\min_{u(t)} \left[ H\{\vec{T}(t), u(t)\} + \frac{\delta \pi}{\delta \vec{T}} \frac{\partial \vec{T}}{\partial t} \right] = 0. \quad (22)$$

This is the fundamental equation after which we are going to obtain the optimal feedback law analogously to the well known Hamilton-Jacobi theory for lumped parameter systems.

From intuition, suggested from many stimulated former works on

various quadratic performance problems of linear lumped parameter systems, let us put a form of the functional  $\pi\{\vec{T}(t)\}$  to be

$$\pi\{\vec{T}(t)\} = \frac{1}{2} \iint_{\Omega} \left\{ \frac{\partial \vec{T}}{\partial t}(\omega, t) \right\}^t \vec{P}(\omega, \omega') \vec{T}(\omega') d\omega' d\omega \quad (23)$$

by a square matrix function  $\vec{P}(\omega, \omega')$ , which reduces to

$$\begin{aligned} \frac{\delta \pi}{\delta \vec{T}} \frac{\partial \vec{T}}{\partial t} &= \frac{1}{2} \iint_{\Omega} \left\{ \frac{\partial \vec{T}}{\partial t}(\omega, t) \right\}^t \vec{P}(\omega, \omega') \vec{T}(\omega', t) d\omega' d\omega \\ &+ \frac{1}{2} \iint_{\Omega} \vec{T}^t(\omega, t) \vec{P}(\omega, \omega') \frac{\partial \vec{T}}{\partial t}(\omega', t) d\omega' d\omega \end{aligned} \quad (24)$$

The substitution of Eq.(24) and the system equation(7) into Eq.(23) gives

$$\begin{aligned} &\min_{u(t)} \left[ \iint_{\Omega} \vec{T}^t(\omega, t) \vec{Q}(\omega, \omega') \vec{T}(\omega') d\omega' d\omega + r^2 \int_{\Omega} \{u(\omega, t)\}^2 d\omega \right. \\ &\quad + \iint_{\Omega} \left\{ \frac{\partial \vec{T}}{\partial t}(\omega, t) \right\}^t \vec{P}(\omega, \omega') \vec{T}(\omega', t) d\omega' d\omega \\ &\quad \left. + \iint_{\Omega} \vec{T}^t(\omega, t) \vec{P}(\omega, \omega') \left\{ \frac{\partial \vec{T}}{\partial t}(\omega', t) \right\} d\omega' d\omega \right] \\ &= \min_{u(t)} \left[ \iint_{\Omega} \vec{T}^t(\omega, t) \vec{Q}(\omega, \omega'') \vec{T}(\omega'', t) d\omega'' d\omega + r^2 \int_{\Omega} \{u(\omega)\}^2 d\omega \right. \\ &\quad - \iint_{\Omega} \vec{T}^t(\omega, t) \left\{ \vec{\lambda} \vec{P}(\omega, \omega'') + \vec{P}(\omega, \omega'') \vec{\lambda} \right\} \vec{T}(\omega'', t) d\omega'' d\omega \\ &\quad + \iint_{\Omega} \vec{T}^t(\omega, t) \left[ \int_{\Omega} \vec{m}_T(\omega'|\omega) \vec{b}^t \vec{P}(\omega', \omega'') d\omega' \right. \\ &\quad \left. + \int_{\Omega} \vec{P}(\omega, \omega') \vec{b} \left\{ \vec{m}_T(\omega'|\omega'') \right\}^t d\omega' \right] \vec{T}(\omega'', t) d\omega'' d\omega \\ &\quad \left. - 2 \iint_{\Omega} u(\omega) \left[ \int_{\Omega} \vec{b}^t m_u(\omega'|\omega) \vec{P}(\omega', \omega'') d\omega' \right] \vec{T}(\omega'', t) d\omega'' d\omega \right] \\ &= 0. \end{aligned} \quad (25)$$

Because no restriction is posed on the control  $u(\omega, t)$ , the minimum of Eq.(25) should be attained at  $u = u^*(\omega, t)$  which makes the functional derivative of the operand in Eq.(25) be zero, the optimal control should be given by

$$u^*(\omega, t) = r^{-2} \int_{\Omega} \left[ \vec{b}^t m_u(\omega'|\omega) \vec{P}(\omega', \omega'') \right] \vec{T}(\omega'', t) d\omega'', \quad (26)$$

as a necessary condition for the optimality. Again substituting this intermediate result for  $u^*(\omega, t)$  into the stationary Hamilton-Jacobi equation(25), an equation with respect to  $P$  function which is as yet undetermined, is obtained as

$$-r^2 \iint_{\Omega} \left\{ \vec{P}(\omega_2, \omega_1) \right\}^t \left[ \int_{\Omega} m_u(\omega_2|\omega_5) \vec{b} \cdot \vec{b}^t m_u(\omega_4|\omega_5) d\omega_5 \right] \vec{P}(\omega_4, \omega_3) d\omega_4 d\omega_2$$

$$\begin{aligned}
 & + \int_{\Omega} \vec{m}_T(\omega_2|\omega_1) \vec{b}^t \vec{P}(\omega_2, \omega_3) d\omega_2 + \int_{\Omega} \vec{P}(\omega_1, \omega_2) \vec{b} \{\vec{m}_T(\omega_2|\omega_3)\}^t d\omega_2 \\
 & - \vec{\lambda} \vec{P}(\omega_1, \omega_3) - \vec{P}(\omega_1, \omega_3) \vec{\lambda} + \vec{Q}(\omega_1, \omega_3) = 0. \quad (27)
 \end{aligned}$$

Equation(27) is for a two dimensional square matrix and does not change in the form after transposing the whole equation or interchanging the variable  $\omega_1$  and  $\omega_3$ , and therefore the solution  $\vec{P}(\omega, \omega')$  should be symmetric not only in the form but also with respect to the two arguments. This equation is analogous to the Riccati-type differential equation which appears in feedback control problems for lumped systems.

#### 4. Optimal Regulator Problem; Rod Cluster Control

A large reactor may be operated by many control rods which are driven being grouped in a few clusters according to a prescribed partitioning pattern. This operation allows us to restrict the control function within the class that is expressed by

$$u(\omega, t) = \sum_{m=1}^M u_m(t) v_m(\omega), \quad (28)$$

where  $v_m(\omega), m=1, \dots, M$  are prescribed distributions chosen so that the control rod pattern may be well described.

Substitution of Eq.(28) into Eq.(7) gives a modification,

$$\begin{aligned}
 \frac{\partial}{\partial t} \vec{T}(\omega, t) = & -A \vec{T}(\omega, t) + \vec{b} \int_{\Omega} \{\vec{m}_T(\omega|\omega')\}^t \vec{T}(\omega', t) d\omega' \\
 & - \vec{b} \{\vec{m}_u(\omega)\}^t \vec{u}(t), \quad (29)
 \end{aligned}$$

where the modified part  $\vec{m}_u(\omega)$  is an  $M$ -dimensional column vector function and is given by

$$\{\vec{m}_u(\omega)\}^m = \int_{\Omega} m_u(\omega|\omega') v_m(\omega') d\omega' \quad (30)$$

and

$$u(t) = \text{col.} \{u_1(t), u_2(t), \dots, u_M(t)\}. \quad (31)$$

The integrand of the cost functional (15) should also be modified into

$$\begin{aligned}
 H = & \frac{1}{2} \iint_{\Omega} \{\vec{T}(\omega, t)\}^t \vec{Q}(\omega, \omega') \vec{T}(\omega', t) d\omega' d\omega \\
 & + \{\vec{u}(t)\}^t \vec{R} \vec{u}(t), \quad (32)
 \end{aligned}$$

where the positive definite square matrix  $\vec{R}$  is introduced instead of

$r^2$  in Eq.(15). Introduction of  $\vec{R}$  allows us to evaluate the controlling effort on each controlling mode separately.

Parallel reduction along the previous section provides a stationary Riccati equation

$$\begin{aligned}
 & - \iint_{\Omega} \{ \vec{P}(\omega_2, \omega_1) \}^t \vec{b} \{ \vec{m}_u(\omega_2) \}^t \vec{R}^{-1} \vec{m}_u(\omega_4) \vec{b}^t \vec{P}(\omega_4, \omega_3) d\omega_4 d\omega_2 \\
 & + \int_{\Omega} [ \vec{m}_T(\omega_2 | \omega_1) \vec{b}^t \vec{P}(\omega_2, \omega_3) + \vec{P}(\omega_1, \omega_2) \vec{b} \{ \vec{m}_T(\omega_2 | \omega_3) \}^t ] d\omega_2 \\
 & - \vec{\lambda} \vec{P}(\omega_1, \omega_3) - \vec{P}(\omega_1, \omega_3) \vec{\lambda} + Q(\omega_1, \omega_3) = 0, \quad (33)
 \end{aligned}$$

and an optimum feedback law

$$\vec{u}(t) = \vec{R}^{-1} \iint_{\Omega} \vec{m}_u(\omega_1) \vec{b}^t \vec{P}(\omega_1, \omega_2) \vec{I}(\omega_2, t) d\omega_2 d\omega_1, \quad (34)$$

for a solution of Eq.(33).

## 5. Optimal Regulator Problem; A Few Regulating Rods

Let us consider a reactor equipped with a few regulating rods separately located in a core. Effect of slight motion of the control rods is described by

$$f(\omega, t) = \sum_{n=1}^N u_n(t) \delta(\omega - \omega_n) \quad (35)$$

Substituting this equation into Eq.(7) instead of Eq.(28), the same results with Eqs.(33) and (34) are obtained except for the definition of  $\vec{m}_u(\omega)$ , which should be substituted by

$$\{ \vec{m}_u(\omega) \}^n = m_u(\omega | \omega_n), \quad n = 1, \dots, N. \quad (36)$$

## 6. Reduction of the Space-Dependent Riccati-type Equation to a Set of Algebraic Equations

One method to solve the Riccati-type equation(33) can be found in a literature by Lions<sup>(9)</sup> which has expanded the function  $\vec{P}(\omega, \omega')$  in terms of eigenfunctions of the system operator. These eigenfunctions coincide with the "natural mode" introduced by Kaplan and are known to require much effort to obtain numerical solution for existing reactors.

We can expand the  $\vec{P}$  function also in terms of the Helmholtz mode, which is given *a priori* for a specific core geometry, as



$$\vec{P}(\omega, \omega') = \sum_{i,j=1}^{\infty} \varphi_i(\omega) \vec{P}_{ij} \varphi_j(\omega'). \quad (37)$$

Substituting this form into Eq.(33) and multiplying it by  $\varphi_i(\omega)$  and  $\varphi_k$  from the left and from the right respectively, we obtain a series of equations

$$-\sum_{j,l=1}^{\infty} \{\vec{P}_{ij}\}^t \vec{K}_{jle} \vec{P}_{lk} + \sum_f \frac{1}{\eta_i} \sum_{l=1}^{\infty} \{\vec{B}_{li}\}^t \vec{P}_{lk} + \sum_f \frac{1}{\eta_k} \sum_{l=1}^{\infty} \vec{P}_{ik} \vec{B}_{lk} - \lambda \vec{P}_{ik} - \vec{P}_{ik} \lambda + Q_{ik} = 0, \quad \begin{matrix} i=1, 2, \dots \\ k=1, 2, \dots, \end{matrix} \quad (38)$$

where

$$\vec{K}_{jle} = \vec{b} \sum_{n=1}^N \frac{\omega_j^n \omega_l^n}{r_n^2 \eta_j \eta_l} \vec{b}^t \quad (39)$$

with

$$\omega_j^n = \varphi_j(\omega_n) \Phi(\omega_n), \quad (40)$$

and

$$\vec{B}_{lk} = \begin{bmatrix} \beta \nu \tilde{b}^{lk} & \beta \nu \bar{\lambda} \\ \kappa \tilde{b}^{lk} & \kappa \bar{\lambda} \end{bmatrix} \quad (41)$$

with

$$\tilde{b}^{lk} = \int_{\Omega} a(\omega) \Phi(\omega) \varphi_l(\omega) \varphi_k(\omega) d\omega \quad (42)$$

and

$$\vec{Q}_{ik} = \iint_{\Omega} \varphi_i(\omega_1) \vec{Q}(\omega_1, \omega_3) \varphi_k(\omega_3) d\omega_3 d\omega_1. \quad (43)$$

## 7. Results and Discussions

Thus a Riccati type equation in an integral equation form has been obtained for a system of power distribution in a nuclear power reactor core. This integral equation expression of the system has enabled us to describe actual control-rod operations which are specifically limited from the structures of the reactors.

The Riccati type equation for the optimal feedback control described in terms of distributed parameter system has been reduced to a set of coupled algebraic equations using the Helmholtz mode expansion. Different from the result by Lion, which uses the eigenfunction expansion and there the system is divided into decoupled subsystems, our equations retain couplings and we must deal with the whole system together instead of decoupled equations in numerical calculation.

For a nuclear reactor, however, the eigenfunction under consideration is called as the natural mode which should be obtained numerically, unless all nuclear parameters are homogeneous over the core. If error is contained in the obtained natural modes, their profit of finiteness is canceled considering the labour required for obtaining this mode numerically.

Use of Helmholtz modes for expansion does not require sophisticated calculations to obtain them prior to dealing with dynamics of the reactor and it is desirable in setting up a short model for specific object which does not require high accuracy. The reduced Riccati-type equations are, however, coupled each other, since the Helmholtz modes do not have the finality which the Kaplan modes retain. The relation between the number of truncation and the accuracy should be studied numerically for specific reactors and is a further problem to be solved.

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Appendix: A Model for a Single Channel Thermo-hydrodynamics

Argument is made on an assembly illustrated in Fig.A.

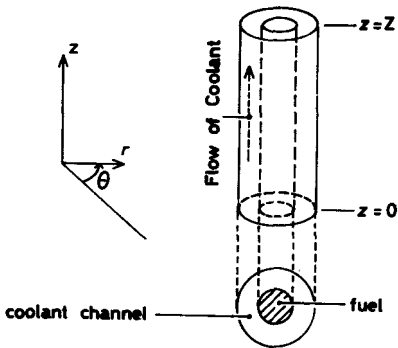


Fig.A. Single fuel assembly.

An assembly is composed of fuel pins and coolant channels, and is installed in the reactor core at  $(r, \varphi)$  parallel to the axial  $z$  direction, and a reactor core is of the assemblies. Assuming a reactor dynamics whose rate of change is within a range of controlling action, the thermo-hydrodynamics about a channel is described as

$$\rho C A \left\{ \frac{\partial}{\partial t} \theta_c(z, t) + V \frac{\partial}{\partial z} \theta_c(z, t) \right\} = q(z, t), \quad (A1)$$

where the nomenclatures are given as

- $\theta_c(z, t)$  : fractional temperature of the coolant,
- $\rho$  : density of coolant,
- $C$  : specific heat of coolant,
- $A$  : cross sectional area of the coolant channel
- $V$  : speed of coolant flow,

$q(z, t)$  : total linear heat release from the fuel pin  
belonging to the assembly.

Assuming that the temperature-increase profile of the coolant along the longitudinal direction is fixed at  $h(z)$  as

$$\theta_c(z, t) = \theta(t)h(z) + \theta_i, \quad (\text{A2})$$

where  $\theta_i$  stands for an inlet temperature. Substitution of Eq.(A2) into Eq.(A1) yields

$$h(z) \dot{\theta}(t) + V \frac{\partial}{\partial z} h(z) \theta(t) = (\rho c A)^{-1} q(z, t). \quad (\text{A3})$$

Also assuming that thermal flux distribution along the axial direction at the assembly can be factorized by  $\sin \frac{\pi z}{Z}$ , the heat release density  $q(z, t)$  is expressed as

$$q(z, t) = A_f E \Sigma_f \phi \frac{2}{\pi} \sin \frac{\pi z}{Z}, \quad (\text{A4})$$

where nomenclatures are given as

- $A_f$  : total cross sectional area of fuel pins,
- $E$  : energy released per fission,
- $\Sigma_f$  : macroscopic fission cross section,
- $\phi$  : thermal flux density averaged over the full length of the assembly.

Substituting Eq.(A4) into Eq.(A3) and integrating the both sides with weight  $h^*(z)$  over full length of the assembly, the system with respect to the representative coolant temperature  $\theta$  is obtained as

$$\dot{\theta} = -\gamma \theta + K \Sigma_f \phi, \quad (\text{A5})$$

where

$$\gamma = V \frac{\int_0^Z h^*(z) \frac{\partial}{\partial z} h(z) dz}{\int_0^Z h^*(z) h(z) dz} \quad (\text{A6})$$

and

$$K = \frac{2}{\pi} \frac{EA_f}{\rho c A} \frac{\int_0^Z h^*(z) \sin \frac{\pi z}{Z} dz}{\int_0^Z h^*(z) h(z) dz} \quad (\text{A7})$$

which have been required.