Optimal control of beam structures by shape memory wires

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Abstract

The paper studies the optimal control of shape memory wires. These wires can be used as actuators in smart structures in order to alter the structural shape according to changing environmental conditions. As a first step, a very simple smart structure is analyzed, viz. the combination of an elastic beam with a shape memory wire. An extension of the dynamic Müller-Achenbach model, accounting for the possibility of electric heating, is used for the description of the shape memory behavior. This - together with the beam bending equation - leads to a coupled system of nonlinear ODEs. The resulting optimal control problem is solved by a direct method, thus determining the heating function for the wire that is necessary to produce a desired beam shape. Finally, an accelerated version is presented, which might be the basis for future real-time control applications.

1 Introduction

Shape memory alloys have long since been recognized as smart materials. Their range of technical applications, however, has mostly remained restricted to cases where the material simply reacts to temperature changes by switching between two configurations. The full potential of "intelligence", viz. an application as an active control device or actuator in smart structures, is only at the beginning of being exploited.

SMA wires perform a strong contraction upon heating. If the heating

is done electrically in a controlled manner, one can realize a continuous spectrum of deformation states. This can be used to control the structural shape in such a way as to optimally adapt to changing environmental conditions, e.g. an airfoil that adapts to changing flow conditions [1].

The basis for such a control application is a model that gives a good reproduction of the observed shape memory behavior. In the next section, we will give a short review of the Müller-Achenbach model [2, 3], which is capable of describing the time dependency of the wire deformation on physically motivated grounds. We added the possibility of electric heating in order to use it for a control application in the sense above.

In a previous work [4], we have used this model to describe the coupling of SMA wires to an elastic beam, which will serve as a prototype of a smart structure. The resulting equations are briefly presented in section 3.

In section 4, we give the results of an optimal control problem, where we calculate the electric heating function for the wire necessary to adjust the beam to a desired shape. The optimal control problem is solved by a direct approach using NUDOCCCS, a code developed by Büskens [5, 6, 7]. We implemented an additional integration algorithm, which considerably accelerated the computations. The method thus shows promise to be useful in future real-time control applications.

2   A Model for Shape Memory Wires

In this section, we shall give a short review of a shape memory model which is well suited for control applications. It has been originally developed by Müller and Achenbach and describes the time-dependent thermomechanical behavior of an SMA wire. It is motivated by experimental evidence of the underlying micromechanics, and the resulting mathematical structure is given by a set of differential-algebraic equations, making it fit nicely into modern mathematical theories of optimal control problems.

The main feature of shape memory alloys used for actoric applications is the capability of strong contraction upon heating. This on first glance surprising behavior is explained by inspection of the micromechanics during such a process.

The basic mechanism is a phase transition in the crystal lattice structure. In the onedimensional case, a lattice particle does either exist as the highly symmetric austenite phase A, or as a sheared version hereof. This, we call a martensitic twin phase and denote it by M_+ or M_-, depending on the direction of shear. In the absence of external loads, martensite is stable at low temperature and austenite at a higher one.

Observation of a specimen during the phase transition reveals a structure of alternating layers of austenite and martensite.
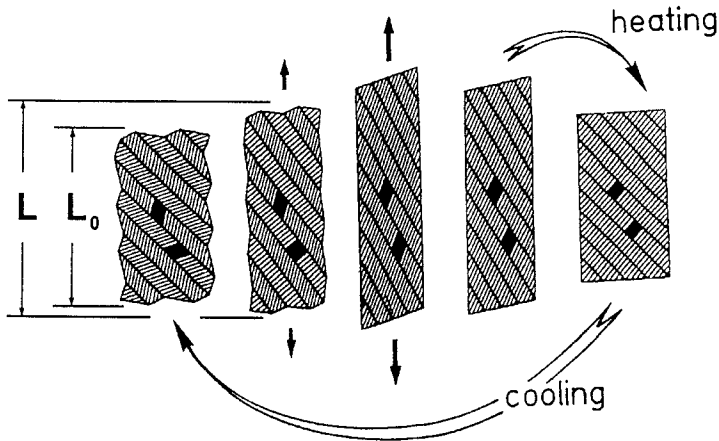
Figure 1 *Layer structure of an SMA specimen and shape memory effect ·*

The sequence of pictures in Figure 1 illustrates the behavior of these layers in a tensile experiment. Initially, at low temperature, the body is in a martensitic state, half of the layers M_+, the other half M_-. Application of a tensile external load first causes the layers to straighten, and, at a critical load level, the M_--layers flip into the M_+-phase, thus contributing to a considerable length change. Upon removal of the load, the M_--layers do not flip back into their original phase, but when the specimen is heated, all the layers transform into the unsheared austenitic phase. This causes the body to shorten again and thus gives rise to the well-known shape memory effect. Subsequent cooling finally completes the cycle by having the martensitic twins occur again.

In order to describe the above behavior, the model takes the metallic layers as basic elements. The total length change of the wire is calculated as the sum of the length changes of the individual layers

$$D = N \left(x_A \langle \Delta_A \rangle + x_+ \langle \Delta_+ \rangle + x_- \langle \Delta_- \rangle \right) . \tag{1}$$

N is the total number of lattice layers, x_A, x_+ and x_- denote the volume fractions of the corresponding phases, and the bracketed quantities are the expectation values of the length changes in the phases. They are calculated from statistical thermodynamics, e.g.

$$\langle \Delta_A \rangle = \frac{\int_{-\Delta_s}^{+\Delta_s} \Delta \exp -\frac{\Phi(\Delta, P)}{kT} \, d\Delta}{\int_{-\Delta_s}^{+\Delta_s} \exp -\frac{\Phi(\Delta, P)}{kT} \, d\Delta} . \tag{2}$$

In equation (2), T is the wire temperature, k is Boltzmann's constant and $\Phi(\Delta, P)$ is the potential energy seen by a layer. It depends on the layer shear length Δ and is given by a triple-well function with each well

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corresponding to one of the three phases, see Figure 2. In the presence of an external load, this function has to be superposed by the work done by the load, which in the onedimensional case simply is $-P\Delta$.
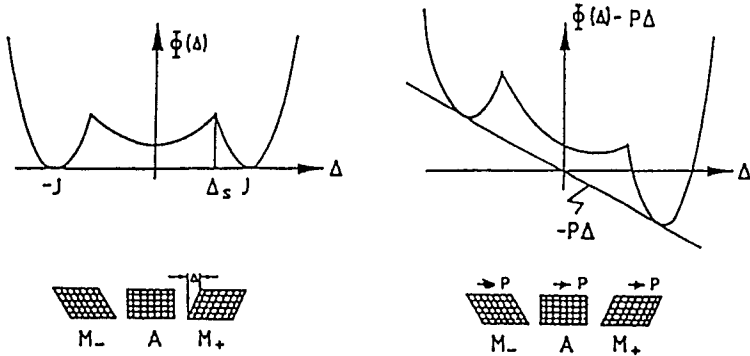
Figure 2 *Three phases of a lattice particle and corresponding potential energy.*

For the determination of the phase fractions, the model assumes the shear lengths of the layers to fluctuate about their equilibrium values in the potential wells. Occasionally, the layers are able to overcome the barriers between the wells, and this gives rise to time rates of change for the phase fractions x_+ and x_- according to

$$\dot{x}_+ = -x_+ p^{+A} + x_A p^{A+}$$
$$\dot{x}_- = -x_- p^{-A} + x_A p^{A-} . \tag{3}$$

The quantities $p^{\alpha\beta}$ are the transition probabilities from phase α to phase β, which also can be calculated from statistical thermodynamics, e.g.

$$p^{+A} = \sqrt{\frac{kT}{2\pi m}} \frac{\exp -\frac{\Phi(\Delta_s, P)}{kT}}{\int_{+\Delta_s}^{+\infty} \exp -\frac{\Phi(\Delta, P)}{kT} d\Delta} \exp \frac{A(1 - 2x_A)}{kT}. \tag{4}$$

In equation (4), A is an interfacial energy coefficient responsible for the alloy's hysteretical behavior and m is the mass of a layer.

In [8], a variant of the model is introduced, which is based on an approximative evaluation of the integrals giving the possibility of high speed computations. The paper also gives a good overview of some features of the model like the strong temperature dependence of its load-deformation behavior. In two further papers [9, 4], an extension of the model has been given incorporating the possibility of electric heating, which is crucial for an application as actuator.

This extension enters the balance of energy, which reads

$$mc\dot{T} = \alpha (T - T_E(t)) + j(t) - \dot{x}_+ H_+(P) - \dot{x}_- H_-(P) - A(1 - 2x_A)\dot{x}_A. \tag{5}$$

It is readily interpreted as follows: the temperature in the wire changes due to

- heat exchange with the environment at temperature $T_E(t)$,

- the Joule heating $j(t)$ produced by the electric current and

- the latent heats of the phase transitions. The third and fourth term on the R.H.S. of (5) $H_\pm(P)$ represent the reversible parts hereof, and the last term is the irreversible part due to creation and annihilation of interfaces between austenitic and martensitic layers.

c is the specific heat and α the thermal conductivity coefficient.

The equations (3) and (5) together with (1) constitute a system of nonlinearly coupled ODEs and an algebraic relation. Together with appropriate initial conditions and prescribed heating function $j(t)$, it can be solved for the resulting length change $D(t)$ or, by inversion of (1), the load $P(t)$. One of the two has to be known, however, and it follows from the coupling to the remaining structure. To specify this point, we shall proceed with the formulation of the bending problem of an elastic beam coupled to an SMA wire. This is a very simple smart structure, but it will serve as an illustration for the applied optimal control method.

3   A Simple Smart Structure - Elastic Beam and SMA Wire

For simplicity, we will confine attention to linear elementary beam theory, adopting the Euler-Bernoulli hypothesis and the assumption of small deflections. As our main objective is to study the optimal control problem, we shall only treat the case of a single wire coupled to a beam, see Figure 3.
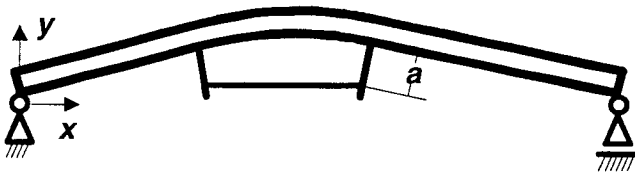

Figure 3 *Prototype of a smart structure - SMA wire coupled to an elastic beam.*

For a detailed description of the extension to two or more wires and the resulting solution algorithm, see the previous work in [4]. The resulting system reads

$$
\begin{aligned}
EIv''(x,t) &= a\,P(t)\left[H(x-x_L)-H(x-x_R)\right] \\
\dot{x}_+(t) &= \dot{x}_+(x_\pm(t),T(t),P(t)) \\
\dot{x}_-(t) &= \dot{x}_-(x_\pm(t),T(t),P(t)) \\
\dot{T}(t) &= \dot{T}(x_\pm(t),T(t),P(t),j(t),T_E(t)) \\
P(t) &= P(x_\pm(t),T(t),D(t))\ .
\end{aligned}
\tag{6}
$$

Here, EI is the bending rigidity, $v(x,t)$ is the transversal displacement, and the R.H.S. of $(6)_1$ is the moment exerted by the SMA wire. It consists of a couple force given by the wire force $P(t)$ and the lever a. $H(x-x_L)$ is the Heaviside step function, which, avoiding the introduction of a large number of integration intervals, is particularly useful in the case of several wires. x_L and x_R are the coordinates at the left and right support of the wire.

The shape of the beam can be calculated by a straightforward integration to give

$$
v(x,t) = \frac{a\,P(t)}{2EI}\left[(x-x_L)^2 H(x-x_L)-(x-x_R)^2 H(x-x_R)\right]+C_1 x+C_2\ ,
\tag{7}
$$

with C_1 and C_2 to be determined by the boundary conditions

$$
v(0)=0 \qquad\qquad \text{and} \qquad\qquad v(L)=0\ .
\tag{8}
$$

In the case of a simply supported, statically determinate beam, the coupling of beam and SMA wire provides another relation between $P(t)$ and $D(t)$, viz.

$$
D(t) = -\frac{a^2 L_0 P(t)}{EI}\ .
\tag{9}
$$

Thus, we can completely eliminate $P(t)$ and $D(t)$ from (6), and this case leaves us with a system of only three ODEs, viz. $(6)_{2-4}$.

4   Optimal Control

For the optimal control problem, we now prescribe a beam shape that corresponds to a wire contraction of $D^*=0.7$. The initial condition for the wire is $D(0)=0.8$, and we ask for the electric heating function that is necessary to minimize

$$
\int_0^T [D(t)-D^*]^2\,dt\ .
\tag{10}
$$

We thus seek a solution that makes the beam approach its target shape in a fast, asymptotic way.

To this purpose, we have implemented the model into NUDOCCCS. The code is based on a direct approach, discretizing the original control problem and transforming it into a nonlinear optimization problem.

The control, which can be subject to box constraints, is either assumed piecewise constant or can be interpolated by higher order splines. The differential equations for the state variables, for which the same type of constraints as for the control applies, are integrated by a simple Euler method or a variety of higher order single step methods like the Runge-Kutta scheme. Several reliable optimization codes have been developed to solve the resulting NLP problem. NUDOCCCS uses the sequential quadratic programming code E04UCF from the NAG fortran library.

We started with a very fine discretization for the control ($NDISKRET = 201$). T has been chosen as $4s$. The upper four diagrams in Figure 4 show the solutions of the three differential equations $x_+(t), x_-(t)$ and $T(t)$ as well as the wire contraction $D(t)$. The lower diagram gives the calculated optimal control $j(t)$.
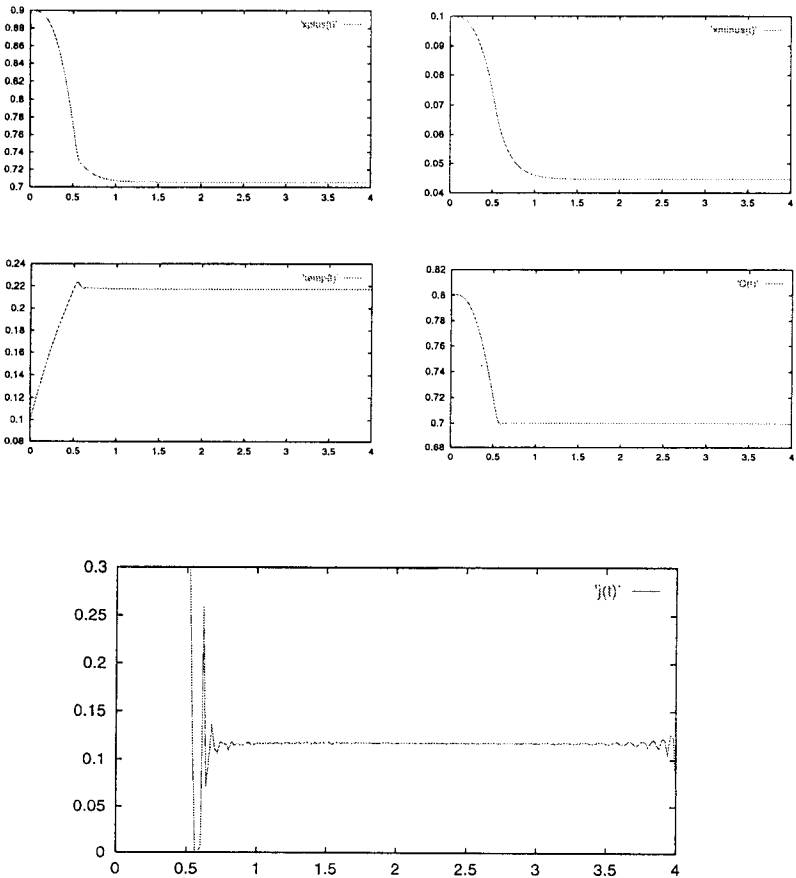

Figure 4 *Optimal control of beam shape adjustment using fine discretization for the control variables ($NDISKRET = 201$). Computation time 32.5s.*

It has been restricted to the domain $0 \leq j(t) \leq 0.3$, and it starts with a typical bang-bang behavior before it reaches a stationary value of ~ 0.11. The resolution of the bangs depends on the degree of discretization, and it is still to be studied whether the stationary value belongs to a singular branch of the solution or whether it corresponds to the mean value of an infinite series of bangs.

The shape memory equations exhibit a very stiff behavior, in particular in the regions where a sudden phase transition takes place. For an illustration of this behavior, we refer the reader to the previously mentioned works again.
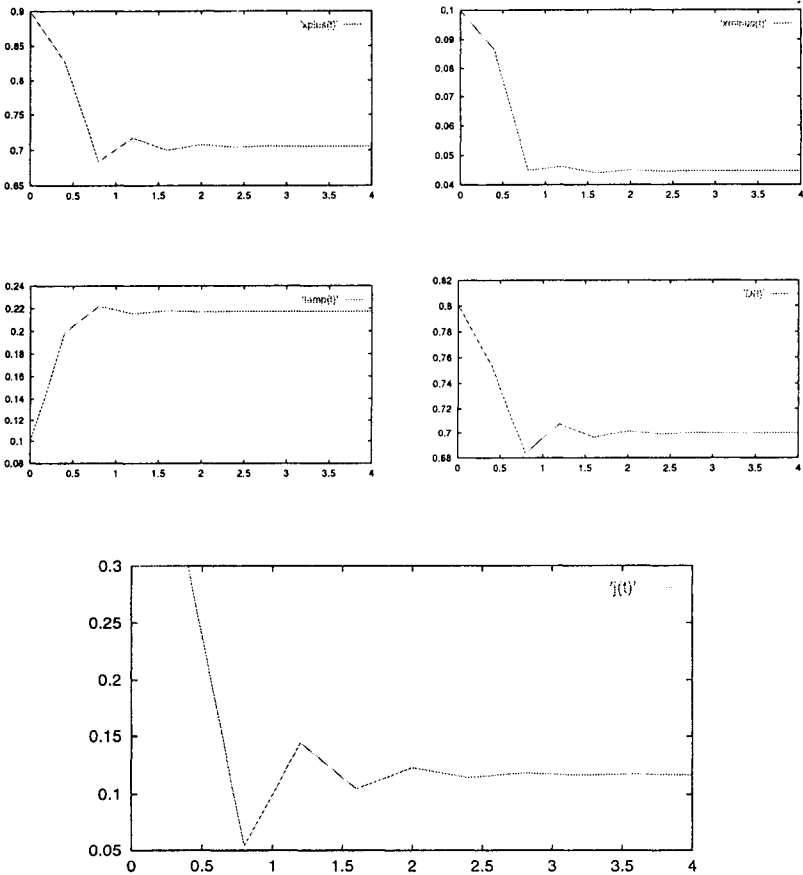
Figure 5 *Optimal control with low degree of discretization*
($NDISKRET = 11$). Computation time 1.6s.

As the original version of NUDOCCCS coupled the grid on which the ODEs are integrated to the discretization of the control, the large number of grid points had also been necessary for a precise integration. This, however

implied computation times of 32.5s on a DEC Alpha 500/333 for the control period of $T = 4s$. As the long range goal is to use the developed code for real-time control applications, we implemented a time step control scheme (Runge-Kutta-Fehlberg) for the integration of the ODEs. This enabled us to do the computation on a considerably rougher grid for the control ($NDISKRET = 11$), and the results can be seen in Figure 5. Of course, the solution is not able to reproduce the fine details of the former one, but, approximately, it gives the same behavior as before, and the computation time was reduced to 1.6s, which is now clearly below the control period.

5   Conclusions

The paper has presented a dynamic model for shape memory behavior, which has been applied to an SMA wire coupled to an elastic beam. This combination represents a simple smart structure with the SMA wire being‘ used as an actuator for beam bending. The coupled problem yields a set of nonlinear ODEs, which have been implemented into NUDOCCCS, a direct optimal control code. Subsequently, a solution for the optimal control of beam shape adjustment by electric heating of the SMA wire has been given. By modification of NUDOCCCS's integration algorithm, very fast computation times could be achieved, which promises to be the basis for future real-time control applications.

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