

Optimal Control of Power Split for a Hybrid Electric Refuse Vehicle

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Abstract—An optimal power split strategy in a hybrid electric refuse truck is presented. Using Pontryagin’s Minimum Principle, a set of solution candidates is found and evaluated in order to find the optimal control strategy. Simulation results are shown to demonstrate the effectiveness of the strategy.

Index Terms—optimal control, hybrid electric vehicles

I. INTRODUCTION

HYBRIDIZATION can offer significant advantages in terms of fuel economy when applied to heavy-duty trucks or buses [1], [2]. In fact, due to the high weight of these vehicles, the ability to regenerate kinetic and potential energy with electric braking is highly beneficial in certain conditions (e.g., start-stop driving cycles); another advantage is the mechanical decoupling of the engine from the road, which allows operating the engine at the highest efficiency conditions.

Among medium- and heavy-duty trucks, urban buses and refuse hauling vehicles are natural candidates for hybridization, because of their typical stop-and-go driving cycles.

The objective of this work is to present an analytical formulation of the energy management problem for a series hybrid electric refuse collection truck, with the goal of reducing fuel consumption.

Control is essential to exploit the benefits of hybrid electric powertrains; correct repartition of the load between the two on-board energy sources (fuel and electrical buffer) allows for substantial reduction of the overall fuel consumption. By its nature, the problem of fuel consumption reduction is global, and an optimal solution can be found only if the entire driving cycle is known a priori, for example using Dynamic Programming [3], [4]. Since this is impossible in typical automotive applications, only sub-optimal solutions can be found, using a variety of methods. Using rule-based or fuzzy control [5], [6], the instantaneous repartition of load is determined by pre-established rules, derived using engineering judgment and a substantial amount of testing; the technique can be made robust and suitable for

production environment, but the results may not be optimal. A more elegant alternative is the equivalent consumption minimization strategy (ECMS) [7], [8], [9], which associates the discharge (resp. recharge) of the electrical energy buffer to a future increase (resp. decrease) of fuel consumption. In this way, the global minimization problem is transformed in a local minimization one. The method can give very good results, but the equivalence factors that allow for the transformation of electrical energy into future fuel consumption must be determined with optimization techniques, and are related to the driving cycles that the vehicle follows (therefore, the factors that minimize the fuel consumption over an urban cycle are different than those that would be needed in a highway cycle). Analytical optimal control techniques have also been applied in the past, assuming known driving cycles [10] or special cases, such as constant power request [11].

This work presents an analytical solution for a case of variable load profile, based on Pontryagin’s minimum principle. This solution is computationally more efficient than dynamic programming, and is potentially implementable on-line.

II. HYBRID ELECTRIC POWERTRAIN MODELING

A longitudinal vehicle dynamics and powertrain model has been developed [12] for the prediction of performance and fuel economy, and the optimization of energy management. The simulator is implemented using Simulink and its specialized blockset SimDriveline, which introduces “physical” modeling templates. These are used to build a simulator composed of self-contained blocks representing physical components, which can be connected together to form a powertrain according to its physical layout. All powertrain components in the simulator are modeled using steady-state efficiency maps, coupled with a lumped-parameter dynamic model that limits the speed variations according to inertia, dissipative resistances and other factors, depending on the specific components.

The model has been validated by comparing the simulation results with the experimental data obtained from a prototype of the vehicle, built by Oshkosh Corporation. This series hybrid electric vehicle is characterized by the architecture shown in Figure 1: an internal combustion engine is coupled to an electrical generator to produce electrical power, which can be used in the traction machines or stored in the rechargeable energy storage system (RESS), which

Manuscript received March 13, 2008. This work was supported by the U.S. Department of Energy, the National Renewable Energy Laboratory (NREL) and Oshkosh Corporation as part of the AHHS project.

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is a pack of supercapacitors. These are also used to store the energy deriving from regenerative braking (obtained by operating the traction machines as generators). Despite their relatively low energy density, supercapacitors were chosen instead of batteries for their much higher power density, and longer operating life.

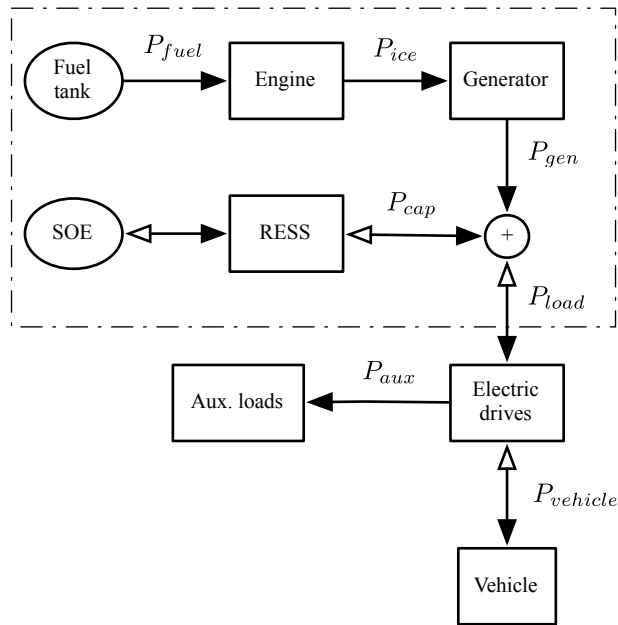


Figure 1. Series hybrid electric architecture (filled arrowheads: positive power flow; empty: negative power flow; no head: non-admissible power flow direction)

Special driving cycles developed in a previous phase of the same project [13], representative of typical operating conditions, are used to test the vehicle both experimentally and in simulation. The cycles include velocity profiles as well as load profiles (i.e., hydraulic power needed to load, pack, and dump refuses) and payload (amount of additional weight due to refuse collection during the cycle). Three standard cycles (*Route 1*, *Route 2* and *Route 3*) are used to represent different phases of the refuse collection in urban and suburban areas. Together, the cycles cover a significant range of typical vehicle operation. The model validation was conducted by replacing all driver and control actions in the simulator with the corresponding measurements obtained in the experimental vehicle, and then comparing the outputs. From the results shown in Tables I and II, we conclude that the accuracy of the model is acceptable for the purposes of the present paper.

III. OPTIMAL CONTROL: PROBLEM STATEMENT

As stated earlier, the objective of the supervisory energy management strategy is to determine the values of the power split between the engine and the reversible energy storage

Table I
VALIDATION OF THE SIMULATOR: FUEL CONSUMPTION

Driving cycles	Error
Route 1	3.68%
Route 2	-3.11%
Route 3	2%

Table II
VALIDATION OF THE SIMULATOR: SELECT DYNAMIC VARIABLES DURING CYCLE *Route 2*

Variable	RMS error
Vehicle speed	0.8%
Engine torque	6%
Electric bus power	2.9%
Capacitors State of Energy	3.4%

system (RESS) that minimize the total fuel consumption during a driving cycle. The series hybrid electric configuration in Figure 1 is considered.

The internal combustion engine is the primary energy converter and produces the mechanical power P_{ice} using the fuel power $P_{fuel} = Q_{lhw}\dot{m}_f$ (Q_{lhw} is the fuel lower heating value, i.e. its energy content for unit of mass; \dot{m}_f is the fuel mass flow rate). The electrical generator transforms the mechanical power from the engine into the electrical power P_{gen} . The rechargeable energy storage system (RESS) is a pack of electrochemical supercapacitors that deliver the power P_{cap} . The electrical power from the generator and from the capacitors is summed electrically in the bus and is used to drive the traction motors and the other vehicle loads. The total power that these machines require is determined from the accelerator pedal position and the vehicle velocity, and can be considered a known quantity (i.e., a problem parameter). The power from the generator and from the capacitor is such that:

$$P_{gen}(t) + P_{cap}(t) = P_{load}(t) \quad \forall t \in [t_0, t_f] \quad (1)$$

having indicated with $[t_0, t_f]$ the optimization interval. Given the load power, there are (in principle) infinite combinations of capacitor and generator power that satisfy (1). Once a value is attributed to the capacitor power, the generator power is automatically determined using (1). The engine power is directly related to the generator power.

The state of the system is represented by the amount of charge present in the capacitors, proportional to their voltage; $x(t)$ denotes the capacitor voltage at time t . The control input $u(t)$ is the current through the capacitors.

The optimal control problem of minimizing the total fuel consumption can be stated as follows:

Problem 1: (optimal control problem): Find $u(t)$ such that the cost function

$$J = \phi(t_f) + \int_{t_0}^{t_f} P_{fuel}(u(t), P_{load}(t), x(t)) dt \quad (2)$$

is minimized, subject to the following constraints:

$$P_{cap,min}(t) \leq P_{cap}(t) \leq P_{cap,max}(t) \quad \forall t \in [t_0, t_f] \quad (3)$$

$$P_{gen,min}(t) \leq P_{gen}(t) \leq P_{gen,max}(t) \quad \forall t \in [t_0, t_f] \quad (4)$$

$$x_{min}(t) \leq x(t) \leq x_{max}(t) \quad \forall t \in [t_0, t_f] \quad (5)$$

The vehicle has to be charge-sustaining, which means that the state of energy at the end of a driving cycle should be the same as it was at the beginning. This condition is imposed as a soft constraint, i.e. by adding the terminal cost

$$\phi(t_f) = (x(t_0) - x(t_f))^2 \quad (6)$$

to the global cost function.

IV. CONTROL-ORIENTED MODEL

A simplified control-oriented model is needed to formulate in detail and solve the optimal control problem. For the vehicle architecture taken into consideration, it is important to model accurately the power flow in the supercapacitors and the engine-generator set.

A. Supercapacitors

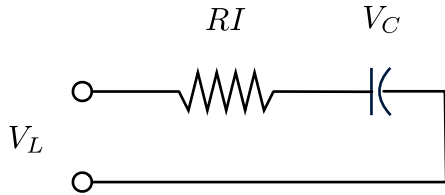


Figure 2. Circuit model of supercapacitor pack

The circuit model of the capacitors is shown in Figure 2; the resistance R and the capacitance C represent the equivalent of a large number of cells connected in series and in parallel. The total amount of energy stored in a capacitor is $E_{cap} = \frac{1}{2}CV_C^2$ and the instantaneous state of energy can be defined as

$$\xi(t) = \frac{E_{cap}(t)}{E_{cap,max}} = \frac{\frac{1}{2}CV_C^2(t)}{\frac{1}{2}CV_{C,max}^2} = \left(\frac{V_C(t)}{V_{C,max}} \right)^2 \quad (7)$$

The capacitance voltage V_C is selected as the system state variable, and the current u flowing through the capacitors as the control input. u is positive during discharge, and negative during recharge. The state equation is therefore:

$$\dot{x} = \dot{V}_C = -\frac{1}{C}u \quad (8)$$

The power that the capacitors exchange with the bus is given by the voltage and current across the capacitance, reduced by the losses due to the internal resistance:

$$P_{cap} = V_L u = V_C u - R u^2 = x u - R u^2 \quad (9)$$

B. Engine and generator

The internal combustion engine is rigidly connected to the electrical generator and therefore they can be regarded as a single component, called genset, which transforms the fuel power into electrical power. The fact that there is no mechanical connection between the engine and the vehicle wheels is advantageous because it makes the engine speed a free variable. For this reason, it is possible to operate the genset at the speed of maximum efficiency for each power level (i.e., along the line shown in Figure 3); the corresponding fuel consumption is a function of the electric power and, as shown in Figure 4, can be expressed with acceptable approximation in terms of fuel power as

$$P_{fuel} = Q_{lhv} \dot{m}_f = m_0 + m_1 P_{gen} \quad (10)$$

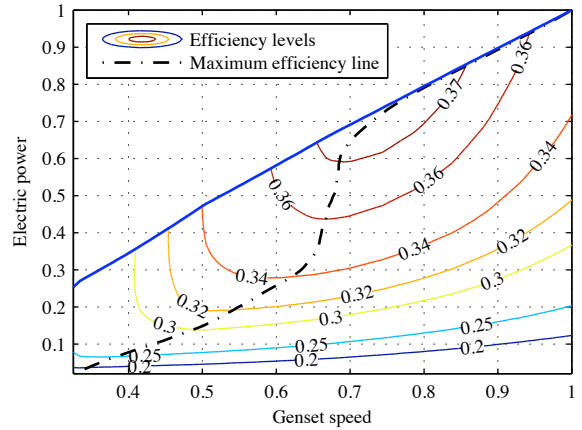


Figure 3. Map of the overall efficiency of the engine-generator assembly, with line of maximum efficiency. Both axes are normalized with respect to their maximum value.

C. Load power

The load power represents the sum of generator and capacitor power. It is used in the traction motors, for moving the vehicle, and in the auxiliary load motors, for operating the refuse collection accessories. The power flow in these components is modeled in the vehicle simulator as a function of the vehicle speed and of parameters such as mass, aerodynamic resistance, rolling resistance, auxiliary load power. However, it is not part of the control model: the instantaneous value of load power $P_{load}(t)$ is considered a known parameter (calculated in the simulation model). An example of speed and load power during urban driving conditions is shown in Figure 5.

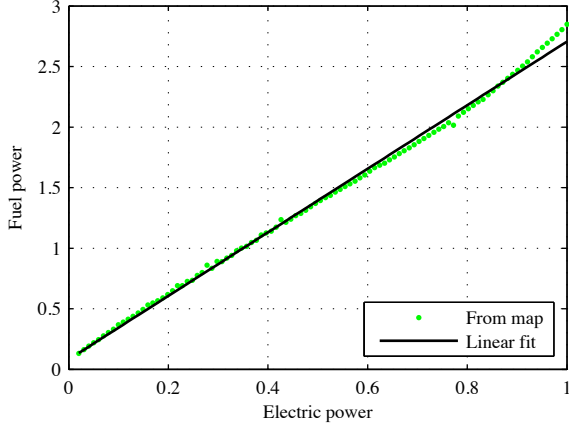


Figure 4. Fuel consumption as a function of the net electrical power delivered by the generator (values corresponding to the maximum efficiency line of Figure 3). Both axes are normalized with respect to the maximum electric power.

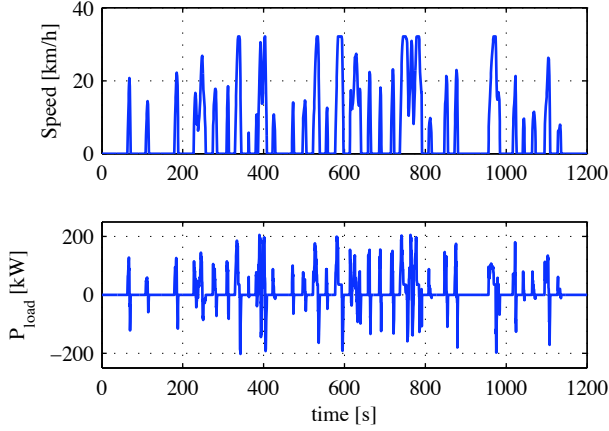


Figure 5. One of the test driving cycles (Route 2)

V. APPLICATION OF PONTRYAGIN'S MINIMUM PRINCIPLE

Pontryagin's minimum principle [14] is used to solve the optimal control problem. Given a dynamic system with state equation

$$\dot{x} = f(x, u) \quad (11)$$

and a cost function

$$J = \phi(x(t_f), t_f) + \int_{t_0}^{t_f} L(x, u) dt \quad (12)$$

subject to the terminal conditions on the state (if they exist) $\Psi(x(t_f), t_f) = 0$, the minimum principle states that the optimal control law $u^*(t)$ must satisfy the following necessary conditions:

- 1) $u^*(t)$ minimizes at each instant of time the Hamiltonian of the system $H(t, u(t), x(t), \lambda(t)) = \lambda'f + L$,

where λ is a vector of adjoint state variables (with the same dimension as the state vector x);

- 2) the co-state dynamic equation is $\dot{\lambda} = -\frac{\partial H}{\partial x}|_{u^*, x^*}$
- 3) the terminal conditions on the co-state are given by $\lambda^*(t_f) = \frac{\partial(\Phi(t_f))}{\partial t_f}|_{*, t_f}$, where $\Phi(t_f) = \phi(t_f) + \nu'\Psi(t_f)$ is the sum of the state terminal conditions Ψ (with the arbitrary multiplier ν) and the adjoint terminal conditions ϕ .

In the system described, the state equation is (8). The control input is the capacitor current u . The instantaneous cost is the fuel power: $L = Q_{lhv}\dot{m}_f$.

The terminal cost $\phi(t_f)$ expresses the fact that the voltage (i.e. state of charge) of the capacitors at the end of the simulation should be close to the one at the beginning:

$$\phi(t_f) = (x(t_0) - x(t_f))^2 \quad (13)$$

The weighting factor w is a free parameter. Since the constraint on the state of charge is imposed using ϕ , there is no explicit terminal constraint on the state variable, i.e. $\Psi = 0$.

Taking into account (8) and (10), the Hamiltonian $H = \lambda\dot{x} + L$ can be written as

$$H = -\frac{\lambda}{C}u + m_0 + m_1P_{gen} \quad (14)$$

The generator net (electrical) power P_{gen} is expressed as a function of control and state variables using (1) and (9):

$$H = -\frac{\lambda}{C}u + m_0 + m_1P_{load} - m_1xu + m_1Ru^2 \quad (15)$$

The minimization of the Hamiltonian (15) can be done either analytically or numerically, since it is an instantaneous minimization; the local constraints: (3), (4), and (5) must be taken into account in defining the range of acceptable values of u to consider as solution candidates.

In particular, given the expression of the capacitor power (9), the limits on the value of current can be expressed as a function of the power limits as follows:

$$u_{max} = \frac{x}{2R} - \frac{1}{2R}\sqrt{x^2 - 4RP_1} \quad (16)$$

$$u_{min} = \frac{x}{2R} - \frac{1}{2R}\sqrt{x^2 - 4RP_2} \quad (17)$$

where

$$P_1 = \min(P_{cap,max}, P_{load} - P_{gen,min}) \quad (18)$$

and

$$P_2 = \max(P_{cap,min}, P_{load} - P_{gen,max}) \quad (19)$$

P_1 and P_2 take into account the conditions (3), (4); the state of energy constraints (5) are taken into account by

setting $P_{cap,min}$ or $P_{cap,max}$ to zero when the capacitor voltage x reaches its maximum or minimum value.

The value of $H(t)$ depends - at each instant of time - on $\lambda(t)$, which derives from the simultaneous solution of the state and co-state dynamic equations:

$$\dot{x}(t) = -\frac{1}{C}u \quad (20)$$

$$\dot{\lambda}(t) = -\frac{\partial H}{\partial x} = -m_1u \quad (21)$$

$$u(t) = \arg \min H(u(t), x(t), P_{road}(t)), u \in \mathcal{U} \quad (22)$$

where $\mathcal{U} = [u_{min}, \dots, u_{max}]$ is the set of admissible solutions. The state and co-state equations must also satisfy the split terminal conditions

$$x(t_0) = x_0 \quad (23)$$

and

$$\lambda(t_f) = \left. \frac{\partial \phi}{\partial x} \right|_{t_f} = 2(x(t_f) - x(t_0)) \quad (24)$$

The implementation of the optimal control strategy is done in simulation by defining a vector of N admissible values of the control variable u , equally spaced in the interval $[u_{min}, u_{max}]$. The Hamiltonian function H is calculated, according to (15), for each of these values. The instantaneous values of the state $x(t)$ and the co-state $\lambda(t)$ are obtained by integrating the dynamic equations, starting from the initial values x_0 and λ_0 (λ_0 is chosen arbitrarily). At each instant t , the value of the control that minimizes the Hamiltonian $H(u(t), P_{load}(t))$ is then chosen as the optimal control action $u^*(t)$. The fact that $H(u)$ is a continuous quadratic function of u for $u \in [u_{min}, u_{max}]$ ensures that it has a unique minimum. Therefore, this value of u satisfies the first two necessary conditions, by construction. Whether the third condition (terminal condition on λ) is satisfied or not can only be determined after applying the strategy for the entire optimization interval $[t_0, t_f]$, by verifying that (24) holds. If this is not true, then the initial value λ_0 should be modified and the optimization procedure repeated until (24) is satisfied.

The off-line implementation of this control strategy is therefore relatively straightforward and, using an iterative procedure to find the correct value of λ_0 , can give the optimal control sequence for a known driving cycle. In practice, λ_0 represents the only parameter that needs to be tuned. For charge sustaining operation ($x_f = x_0$), the value of λ_0 should be selected in order to obtain $\lambda_f = 2(x_f - x_0) \simeq 0$.

The solution obtained in this way satisfies all necessary conditions set by Pontryagin's minimum principle. The fact that the entire set of solution candidates is considered implies that the only one among them that satisfies the necessary conditions is indeed the optimal solution, in the limits given

by the discretization of the set of solution candidates and by the modeling assumptions.

VI. RESULTS OF THE IMPLEMENTATION

The procedure just described is applied in simulation to the same driving cycles that were used to validate the simulator (see Section 2). The value of λ_0 is selected iteratively so that the capacitor voltage x at the end of the simulation is equal to the initial value: $x(t_f) = x(t_0)$, and that the co-state terminal condition (24) is satisfied: $\lambda(t_f) = 2(x(t_f) - x(t_0)) = 0$. The value of λ_0 that satisfies these conditions is different for the different cycles, as it depends on the cycle characteristics; it represents the only parameter needed for tuning the strategy for a specific cycle. In order to select the correct value of λ_0 for each of the cycles considered, simulations were repeated varying its value. The effect of the parameter λ_0 on the net variation of capacitor voltage (i.e., state of charge) is shown in Figure 6. As it can be observed, it changes the behavior of the vehicle from charge-increasing to charge-depleting: there exists one value for each cycle for which the vehicle is charge sustaining. This effect can be justified by looking at the Hamiltonian (14) as the sum of the terms: one represents the fuel power (i.e. the fuel consumption); the other is proportional (via $\lambda(t)$) to the current u , and can be interpreted as the fuel consumption equivalent to the use of the capacitors. Varying the value of $\lambda(t)$ changes the value of u for which $H(\lambda, u)$ is minimum, or, in other words, the cost of using the electrical power source, in terms of equivalent fuel consumption.

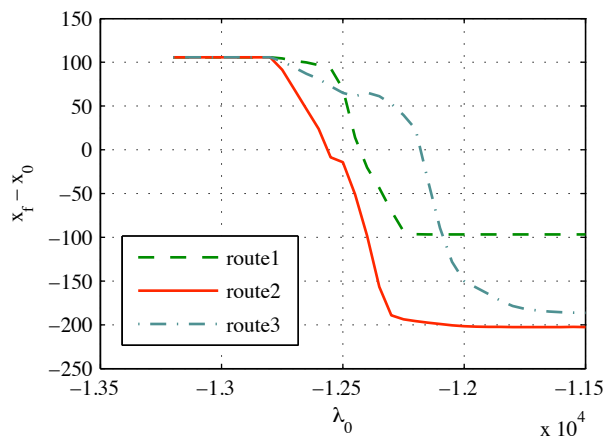


Figure 6. Effect of the parameter λ_0 on the variation of capacitor voltage between the beginning and the end of the simulation. A value of zero correspond to charge-sustaining operation and to the satisfaction of all necessary conditions for optimal control.

The results of the optimal controller defined in this way can be compared to those of the rule-based strategy implemented in the prototype vehicle, whose parameters were tuned using the test cycles. The rule-based controller gives good results, with a sensible reduction in fuel consumption

with respect to the conventional (non-hybrid) vehicle. Table III shows a comparison between the optimal control strategy described in this paper and the rule-based strategy, both tested on the same simulation model: as expected, the optimal control strategy is advantageous in terms of fuel consumption (even though the rule-based strategy is close).

Table III
DIFFERENCES IN FUEL CONSUMPTION BETWEEN THE OPTIMAL CONTROL STRATEGY AND THE RULE-BASED CONTROL.

Driving cycle	Optimal value of λ_0	Difference in fuel consumption
Route 1	-12430	-10.7%
Route 2	-12565	-5.4 %
Route 3	-12180	-7.5 %

To better understand the differences between the two strategies, the variation of the capacitor state of energy during one of the cycles is shown in Figure 7: the optimal control generates a higher variability of state of energy, i.e., it uses the capacitors in a wider range, thus maximizing the benefits of their presence.

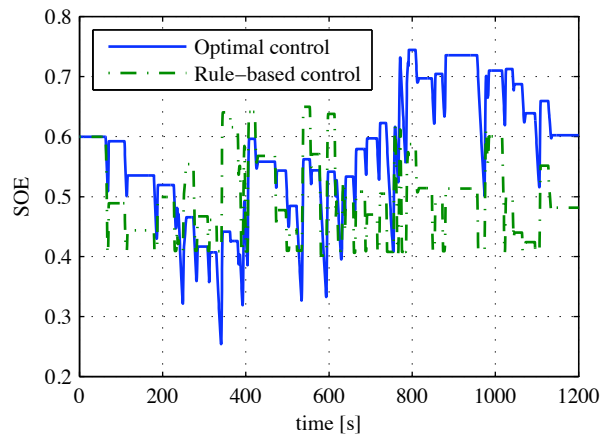


Figure 7. Comparison of capacitor state of energy (as defined in (7)) between the optimal control and the rule-based control. The driving cycle is Route 2, shown in Figure 5.

VII. CONCLUSION

An application of optimal control theory to hybrid electric vehicles has been presented, using Pontryagin's minimum principle to find the optimal solution to the energy management problem of hybrid electric vehicles. The solution reduces the global optimization problem to an instantaneous one, which can be solved iteratively. The results are optimal in the limits of the control model, and improve on the previously implemented rule-based controller.

Besides the lower fuel consumption, an advantage of the solution based on the Pontryagin's minimum principle is the fact that only one parameter (namely λ_0) is needed to tune the strategy for optimal results over a specific cycle. However, as seen in Figure 6, this parameter must be correctly determined to ensure charge-sustainability, which

is possible only using an iterative procedure. This limits the applicability of this control to simulation environment, for the solution of off-line optimization problems. However, an approximation of the optimal results can be implemented on-line if driving pattern recognition algorithms are used to select the optimal value of the tuning parameter λ_0 as a function of the current driving conditions. This will be the object of future investigation.

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