Optimal control problems of fully coupled FBSDEs and viscosity solutions of Hamilton-Jacobi-Bellman equations

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Abstract. In this paper we study stochastic optimal control problems of fully coupled forward-backward stochastic differential equations (FBSDEs). The recursive cost functionals are defined by controlled fully coupled FBSDEs. We study two cases of diffusion coefficients σ of FBSDEs. We use a new method to prove that the value functions are deterministic, satisfy the dynamic programming principle (DPP), and are viscosity solutions to the associated generalized Hamilton-Jacobi-Bellman (HJB) equations. The associated generalized HJB equations are related with algebraic equations when σ depends on the second component of the solution (Y, Z) of the BSDE and doesn't depend on the control. For this we adopt Peng's BSDE method, and so in particular, the notion of stochastic backward semigroup. We emphasize that the fact that σ also depends on Z makes the stochastic control much more complicate and has as consequence that the associated HJB equation is combined with an algebraic equation, which is inspired by Wu, Yu. However, in our work we use the continuation method combined with the fixed point theorem to prove very clearly that the algebraic equation has a unique solution, and, moreover, we also give the representation for this solution. On the other hand, we also prove some new basic estimates for fully coupled FBSDEs under the monotonic assumptions. In particular, we prove under the Lipschitz and linear growth conditions that fully coupled FBSDEs have a unique solution on the small time interval, if the Lipschitz constant of σ with respect to z is sufficiently small. We also establish a generalized comparison theorem for such fully coupled FBSDEs.

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