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Optimal Corporate Leniency Programs*

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Abstract

This study characterizes the corporate leniency policy that minimizes the frequency with which collusion occurs. Though it can be optimal to provide only partial leniency, plausible sufficient conditions are provided whereby the antitrust authority should waive all penalties for the first firm to come forward. It is also shown that restrictions should be placed on when amnesty is awarded, though it can be optimal to award amnesty even when the antitrust authority is very likely to win the case without insider testimony.

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1 Introduction

One of the most important policy developments in U.S. antitrust policy in recent decades is the revision of the Corporate Leniency Program by the Department of Justice (DOJ) in 1993. Originally instituted in 1978, this program allows corporations and individuals, who were engaging in illegal antitrust activity (such as price-fixing), to receive amnesty from government penalties. This means that a corporation can avoid government fines, while individuals escape fines and prison sentences. The 1993 revision made it possible for amnesty to be awarded even when an investigation had been started and made it a condition that the DOJ "has not received information about the illegal activity being reported from any other source." This means that amnesty is limited to one firm per cartel.[1] While it is difficult to assess the role of the leniency program on causing cartels to collapse or deterring cartels from forming, we do know that it has been widely used. Notable examples include Rhône-Poulenc in the vitamins case, Christie's in the fine arts auctions case, and Carbide/Graphite in the graphite electrodes case.[2]

In light of the usage and apparent success of these programs in many industrialized countries,[3] there have been policy discussions about the best way in which to design a leniency program. Towards that objective, it is crucial that we understand how these programs influence the incentives of firms to collude. For example, the Antitrust Criminal Penalty Enforcement and Reform Act of 2004 expanded leniency in the U.S. by extending it to civil damages. Amnesty now not only means avoiding government fines but also facing a liability of only single, rather than the usual treble, damages. Does more leniency necessarily make collusion more difficult to sustain?

Recent academic research - such as Aubert, Kovacic, and Rey (2003), Motta and Polo (2003), Spagnolo (2003), Feess and Walzl (2004), and Motchenkova (2004) - has proven constructive in shedding light on the incentives of firms and how it impacts the design of a leniency program. However, most of this work is predicated upon a strong and restrictive assumption which is that the probability of the cartel being detected and successfully prosecuted (without use of the leniency program) is fixed over time. That this assumption may be problematic is reflected in that these theories produce the counterfactual that a cartel will never use the leniency program.[4] The formation of a cartel requires that it is not optimal for firms to use the leniency program and, given the environment is fixed, that will be true in all future periods as well.[5] Of course, there are many cartels that were formed after 1993 and eventually

[1] For a statement of the conditions for corporate leniency, go to `www.usDOJ.gov/atr/public/guidelines/ lencorp.htm`

[2] A good review of the status of leniency programs is provided in "Hard Core Cartels" (2003). A critical description of the U.S. program can be found in Kobayashi (2001).

[3] Leniency programs were introduced by the European Commission in 1996 and an increasing number of members of the European Union have some form of leniency program.

[4] An exception to be discussed later is Motta and Polo (2003) which does allow the probability of detection and conviction to change over time.

[5] Motchenkova (2004) does have a result that the cartel forms and later uses the leniency program but the analysis is predicated upon two problematic assumptions. First, a firm can react instantaneously to another firm applying for amnesty. Second, collusion is not endogenized in the usual

applied for and received amnesty. The natural explanation is that the probability of successful prosecution changed over time; it was low when the cartel formed and later was sufficiently high to warrant foregoing collusion and applying for amnesty. This raises the concern that a model in which this probability is fixed may result in biased recommendations regarding the optimal design of a leniency policy.

The first contribution of this paper is to allow the probability of discovery and successful prosecution to change over time. This is shown to introduce a substantive new force into the analysis which alters the calculus underlying optimal policy design. The model of this paper produces three ways in which a leniency policy influences the frequency of collusion. I refer to these as the Deviator Amnesty Effect, the Cartel Amnesty Effect, and the Race to the Courthouse Effect. As is typical in a repeated game model of collusion, the stability of collusion is impacted through an equilibrium condition: the expected payoff from continuing to collude exceeds the payoff to a firm from cheating on the cartel. The Deviator Amnesty Effect operates through the payoff to cheating; it captures the reduction in fines from entering the leniency program at the same time that a firm undercuts the collusive price. As the Deviator Amnesty Effect enhances the payoff to cheating, it serves to make collusion more difficult to sustain.

The other two effects work through the expected payoff to colluding. In that colluding firms recognize that they may use the leniency program in the future (when the probability of detection is high), a more lenient program reduces the size of penalties in the event that a firm receives leniency. This is the Cartel Amnesty Effect and means that more leniency raises the expected payoff from continuing to collude.

The Race to the Courthouse Effect is the most subtle and it actually causes a more lenient program to *increase* the expected present value of penalties from continuing to collude. Though it is always an equilibrium for all firms to apply for amnesty, it can also be an equilibrium for no firm to do so and, furthermore, this equilibrium is Pareto superior (since only one firm can receive amnesty and use of the program results in conviction for sure). Such an equilibrium exists when leniency is minimal; in that case, the "amnesty game" is a coordination game. Since a more lenient policy increases the appeal of a firm applying for amnesty (in particular when all other firms do not), it can destabilize the equilibrium in which all firms do not use the program and make it a dominant strategy to apply for amnesty - enough leniency can turn the amnesty game into a Prisoners' Dilemma. Thus, more leniency can result in all firms applying for amnesty so that expected penalties are actually higher with a program that waives a higher fraction of penalties. The Race to the Courthouse Effect results in a more lenient policy lowering the expected payoff from colluding - making collusion more difficult - and is a countervailing force to the Cartel Amnesty Effect.

The analysis of this paper shows that when leniency is sufficiently great - that is, a high fraction of fines are waived to the first firm to come forward - only the Deviator Amnesty Effect and the Cartel Amnesty Effect are operative. It will generally be

equilibrium manner and, as a result, firms can collude even though there is a deterministic time in the future at which collusion collapses.

shown that the former effect is larger so that a more lenient policy serves to reduce cartel stability, at least in that part of the policy space. When instead leniency is sufficiently mild, only the Cartel Amnesty Effect and Race to the Courthouse Effect are operative as a deviator would not apply for leniency. How these two effects net out depends on the specifics of the model so, in this part of the policy space, a more lenient policy can either raise or lower cartel stability. In fact, I show that there are cases in which a policy of partial leniency is optimal. However, plausible sufficient conditions are provided whereby it is optimal to waive all penalties for the first firm to come forward.

The second innovation of this paper is to introduce and explore a new dimension to the policy space. I consider the conditions under which the antitrust authority awards amnesty to the first firm to come forward. Previous work has assumed that the first application is always approved. In contrast, the U.S. Corporate Leniency Policy places conditions on when leniency is given. I find that it is indeed optimal to set restrictions on when amnesty is awarded and, more specifically, it should not be provided when the antitrust authority's case is sufficiently strong. However, it is always optimal to award amnesty when the probability that the authorities will win the case without insider testimony is less than 1/2 and, quite surprisingly, it can be optimal to do so even when the chances of a conviction are very high.

The model is outlined in Section 2, after which I provide a brief literature review. A collusive equilibrium is characterized in Section 3 where firms take as given the leniency policy. The antitrust authority's optimal leniency policy is then characterized in Section 4. The policy space of the antitrust authority is expanded in Section 5 to include *when* leniency is given and an optimal policy is described. Section 6 concludes.

2 Model

Consider an industry with $n \in \{2, 3, \ldots\}$ firms. Consistent with previous work, a simple Prisoners' Dilemma formulation is specified in light of the richness with regards to detection and amnesty. If firms "collude" then each earns profit of π^c; if firms "compete" then each earns profit of π^{nc}; and if a firm "deviates" when the other firms are colluding then the deviator earns profit of π^d. Assume $\pi^d > \pi^c > \pi^{nc}$. In sustaining collusion, the grim trigger strategy is used so that any deviation results in the competitive solution (that is, static Nash equilibrium) forever. The incentive compatibility constraint (ICC) for collusion is then:

$$\frac{\pi^c}{1-\delta} \geq \pi^d + \delta \left(\frac{\pi^{nc}}{1-\delta} \right).$$

The next step is to augment this structure by assuming that the antitrust authority (AA) might detect and prosecute collusion. In that event, collusion stops forever and each firm pays a fine of $F > 0$.[6] In each period, there is a probability $\omega \in (0, 1]$ of

[6] I conjecture that results are robust to allowing collusion to re-start in the future as long as the

the AA launching an investigation.[7] If it does not then the probability of the cartel being discovered is zero. If an investigation is launched then ρ denotes the probability of detection and conviction and it has a twice differentiable cdf $G : [0,1] \rightarrow [0,1]$. ρ is realized at the beginning of a period (when there is an investigation) and is common knowledge to the firms before deciding how to act regarding whether or not to collude and whether or not to seek amnesty. If a firm applies to the leniency program then the cartel is detected and convicted for sure and it pays an expected penalty of $\left(\frac{m-1}{m}\right) F + \left(\frac{1}{m}\right) \theta F$ where m is the total number of firms (simultaneously) applying for amnesty and $\theta \in [0,1]$. A firm that did not apply for amnesty pays F. Hence, $1-\theta$ is the fraction of fines waived by being accepted into the leniency program. If no firm seeks amnesty then, with probability ρ, the cartel is detected and each firms pays F and earns π^{nc} in all future periods and, with probability $1-\rho$, the cartel is not detected and the game moves forward.[8] Thus, ρ is the probability of firms paying penalties when the authorities do not have a cartel member as a witness.

As long as the cartel remains intact - so collusion has not collapsed voluntarily or through prosecution - this process begins anew. Thus, in each period there might be an investigation and, if there is one, the authorities are able to accumulate some amount of evidence which is summarized by ρ. The events of investigation and realization of ρ are assumed to be independently and identically distributed (*iid*) over time. It is then natural to think of each period as being relatively long, perhaps on the order of a year.[9]

If firms colluded in the previous period, the simplifying assumption is made that there is a possibility of detection in the current period whether or not firms collude. If firms did not collude in the previous period (which means one or more firms chose not to collude), it is assumed there is no likelihood of being detected. This structure has the simplifying property that the only situation in which firms assign a future probability of being detected is when they are currently colluding. If I instead assumed that, upon cartel dissolution, the cartel might be detected in the future, it is natural to suppose that this probability is eventually declining but then the problem becomes non-stationary and unnecessarily complex.

Some Related Work There are two primary issues related to the literature on corporate leniency programs: *deterrence* and *desistance*. Leniency programs can *deter* cartel formation either by making it unprofitable (collusion is stable but expected penalties are such as to make it preferable not to collude) or making collusion unstable (incentive compatibility constraints are not satisfied). Even if it is initially profitable

time until that happens is sufficiently long. In the context of their model, robustness was reported in Motta and Polo (2003).

[7] Or this could be any event which results in the prospect of conviction.

[8] For a model of collusion in which the probability of conviction is endogenous to firm behavior, see Harrington (2004, 2005) and Harrington and Chen (2004).

[9] If one considers shorter periods, it would be natural to suppose that ρ is persistent, as it evolves over time in accordance with the development of the authority's case. For analytical tractability, I chose to make ρ *iid* though it would clearly be interesting to explore what happens if ρ is instead Markovian.

and incentive compatible to form a cartel, leniency programs can cause collusion to *desist* by expanding the set of future states for which the cartel collapses. Much of the literature focuses on the issue of deterrence, while I presume a cartel forms and instead focus on desistance. Obviously, desistance is pertinent to deterrence since greater desistance makes collusion less profitable and, at some point, desistance is sufficiently great so as to result in ex ante deterrence of cartel formation. However, that is not an issue explored here.

Similar to my model, previous work has examined the implication of leniency programs in the context of the Prisoners' Dilemma.[10] The papers of, for example, Aubert, Kovacic, and Rey (2003) and Spagnolo (2003) are largely complementary to the current one. In the context of a fixed environment (that is, the probability of conviction without use of the leniency program is fixed), they explore the use of financial rewards (and not just relief from penalties). Spagnolo (2003) shows that, if there is a budget-balancing constraint, a first-best solution can be achieved by giving the first firm to come forward a reward equal to the fines levied on the remaining firms. The first-best solution means deterring collusion with zero resources expended on enforcement (that is, the probability of conviction without a firm coming forward is zero). A unique and interesting feature of Aubert et al (2003) is that they consider the incentives of the employees of colluding firms to come forward and take advantage of rewards. Leniency programs have an impact by affecting the compensation that a colluding firm must provide to employees to deter them from going to the authorities.

Papers such as Spagnolo (2003) assume the probability of successful prosecution is fixed. There the focus is on deterring cartel formation by inducing firms to apply for amnesty even when there is no investigation. Of course, in equilibrium, colluding firms do not use the leniency program so the program does not impact the payoff to colluding. Hence, referring back to the three effects described in the Introduction, only the Deviator Amnesty Effect is operative in models with a fixed probability. It follows that more leniency necessarily makes cheating more attractive and thus collusion less stable. When the probability is fixed, a maximally lenient policy is then optimal and, in fact, Spagnolo (2003) establishes the desirability of offering a reward to the first firm to come forward.

In contrast, Motta and Polo (2003) allows the probability to change over time but it is restricted to take only two values, one of which is zero.[11] Furthermore, it is assumed that conviction is not possible when a firm cheats on the cartel and, therefore, the Deviator Amnesty Effect is absent from their analysis. The Cartel Amnesty Effect is present, however, since colluding firms may anticipate using the leniency program in the future when the probability of detection and conviction takes the higher value. As shown later, restricting the probability to take only two possible values proves to be with some loss of generality as it rules out the Race to the Courthouse Effect. The exclusive force in the model of Motta and Polo (2003) is then the Cartel Amnesty Effect. Though leniency then adds to the profitability

[10] Using numerical analysis for the Bertrand price game, Chen and Harrington (2005) explore the impact of corporate leniency programs on the cartel price path.

[11] Feess and Walzl (2004) also allow the probability of detection to be random but the set-up is static and thus collusion is not endogenized.

of collusion, it can be desirable to provide maximal leniency because having cartel members come forward raises the probability of the investigation being successful and thus collusion being terminated. If, however, cartel formation could be deterred when leniency is not provided then a policy of no leniency is optimal. Thus, the optimal policy is either waiving all or no penalties.

In sum, previous work has encompassed either the Cartel Amnesty Effect or the Deviator Amnesty Effect. The model of this paper is the first to embody both of those forces and is the first to have the Race to the Courthouse Effect. The confluence of these three forces are shown to lead to a more subtle and complex analysis.

3 Collusive Equilibrium

The analysis focuses on a class of subgame perfect equilibria with the following properties: i) a deviation is punished by infinite reversion to static Nash equilibrium;[12] and ii) it has a cut-off property. More specifically, consider a cut-off strategy - characterized by ρ^o - of the following form:

i) if $\rho \in [0, \rho^o]$ then a firm colludes

ii) if $\rho \in (\rho^o, 1]$ then a firm competes and, in that case, applies to the amnesty program iff $\rho \in (\theta, 1]$.

A collusive (cut-off) equilibrium exists iff $\exists \rho^o \in (0, 1]$ such that the above is a symmetric subgame perfect equilibrium. ρ^o is determined as part of the equilibrium.

In evaluating the optimality of this strategy for an individual firm, first note that the case of no investigation is identical to that when there is an investigation and $\rho = 0$. There is then no loss of generality in only examining optimality when there is an investigation.[13] It is never optimal to collude and apply for amnesty since applying for amnesty shuts down the cartel in which case deviation is surely preferred. Next note that the cut-off strategy is clearly optimal when $\rho \in (\rho^o, 1]$; given all other firms compete, competing is optimal. As regards the prescribed behavior with respect to the leniency program, given all other firms do not apply for amnesty when $\rho \leq \theta$, a firm's expected penalty is ρF which is less than the penalty from applying for amnesty, θF, so it is optimal not to apply for amnesty when $\rho \leq \theta$.[14] When $\rho > \theta$, all other firms apply for amnesty in which case it is clearly optimal for a firm to do so as well since it reduces its expected penalty from F to $\left(\frac{n-1+\theta}{n}\right) F$.

The only remaining issue is the optimality of the strategy when, in the event of an investigation, $\rho \in [0, \rho^o]$. The incentive compatibility constraint (ICC) is:

$$\pi^c + \delta (1 - \rho) E [V | \rho^o, \theta] + \delta \rho (W - F) \geq \pi^d + \delta W - \delta \min \{\rho, \theta\} F$$

[12] All that is important is that the punishment payoff is independent of θ and ρ^o (a variable which is defined below).

[13] That investigations are probabilistic does enter the expected collusive payoff, however, and our analysis takes account of it.

[14] There is also an equilibrium in which all firms apply for amnesty. I have made the usual equilibrium selection based on Pareto dominance.

where $W \equiv \frac{\pi^{nc}}{1-\delta}$ and $E[V|\rho^o,\theta]$ is the expected future payoff from continued collusion. I explicitly denote the dependence of $E[V|\rho^o,\theta]$ on the cut-off, ρ^o, and the policy choice of the AA, θ. It is defined recursively by:

$$E[V|\rho^o,\theta] = (1-\omega)V(0,\rho^o,\theta) + \omega\int_0^1 V(\rho,\rho^o,\theta)\,dG(\rho), \tag{1}$$

where

$$V(\rho,\rho^o,\theta) = \begin{cases} \pi^c + \delta(1-\rho)E[V|\rho^o,\theta] + \delta\rho(W-F) & \text{if } 0 \le \rho \le \rho^o \\ W - \rho\delta F & \text{if } \rho^o < \rho \text{ and } \rho \le \theta \\ W - \left(\frac{n-1+\theta}{n}\right)\delta F & \text{if } \rho^o < \rho \text{ and } \theta < \rho \end{cases}.$$

When $\rho \le \rho^o$, firms continue colluding and if they escape conviction then they go on to collude again tomorrow and receive a future expected payoff of $E[V|\rho^o,\theta]$. If $\rho^o < \rho$ then the cartel collapses - so firms earn the non-collusive payoff W - and incur expected discounted penalties of $\rho\delta F$ (when $\rho \le \theta$ so that no firm applies for leniency) or $\left(\frac{n-1+\theta}{n}\right)\delta F$ (when $\theta < \rho$ so that all firms apply for leniency).

(1) can then be re-stated as

$$\begin{aligned} E[V|\rho^o,\theta] &= (1-\omega)\{\pi^c + \delta E[V|\rho^o,\theta]\} \\ &\quad +\omega\left\{\int_0^{\rho^o}\{\pi^c + \delta(1-\rho)E[V|\rho^o,\theta] + \delta\rho(W-F)\}\,dG(\rho)\right. \\ &\quad +\int_{\rho^o}^{\max\{\rho^o,\theta\}}(W-\rho\delta F)\,dG(\rho) \\ &\quad \left.+[1-G(\max\{\rho^o,\theta\})]\left[W - \left(\frac{n-1+\theta}{n}\right)\delta F\right]\right\}, \end{aligned} \tag{2}$$

and solving for $E[V|\rho^o,\theta]$ yields:

$$\begin{aligned} E[V|\rho^o,\theta] &= \left[1-\delta(1-\omega) - \delta\omega\int_0^{\rho^o}(1-\rho)\,dG(\rho)\right]^{-1} \times \\ &\quad \left\{(1-\omega)\pi^c + \omega\int_0^{\rho^o}[\pi^c + \delta\rho(W-F)]\,dG(\rho) + \omega\int_{\rho^o}^{\max\{\rho^o,\theta\}}(W-\rho\delta F)\,dG(\rho)\right. \\ &\quad \left.+\omega[1-G(\max\{\rho^o,\theta\})]\left[W - \left(\frac{n-1+\theta}{n}\right)\delta F\right]\right\}. \end{aligned} \tag{3}$$

The ICC can be represented as:

$$\Phi(\rho,\rho^o,\theta) \equiv \pi^c + \delta(1-\rho)E[V|\rho^o,\theta] + \delta\rho(W-F) - \pi^d - \delta W + \delta\min\{\rho,\theta\}F \ge 0. \tag{4}$$

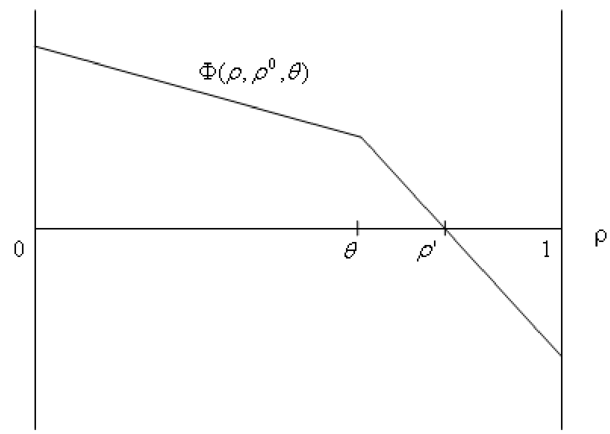
$\Phi(\rho,\rho^o,\theta)$ is a continuous function of ρ and is differentiable everywhere except at $\rho=\theta$. If $E[V|\rho^o,\theta] > W$ then $\Phi(\rho,\rho^o,\theta)$ is decreasing in ρ:

$$\frac{\partial\Phi(\rho,\rho^o,\theta)}{\partial\rho} = \begin{cases} -\delta\{E[V|\rho^o,\theta] - W\} & \text{if } \rho \le \theta \\ -\delta\{E[V|\rho^o,\theta] - W\} - \delta F & \text{if } \theta < \rho \end{cases}$$

Note that $E[V|\rho^o,\theta] > W$ must hold at a collusive cut-off equilibrium. If this inequality were not true then the expected collusive payoff is less than that from not colluding which means cartel formation is non-optimal. In characterizing collusive equilibria, it'll be presumed that $E[V|\rho^o,\theta] > W$. Finally, note that $\Phi(1,\rho^o,\theta) < 0$ because if the probability of detection is one in the current period then the future entails no collusion in which case deviation is clearly optimal. Hence, collusion does not occur when $\rho \simeq 1$.

Figure 1 provides a representative depiction of $\Phi(\rho,\rho^o,\theta)$. A firm finds it optimal to collude iff $\rho \leq \rho'$.

Figure 1



With a collusive cut-off equilibrium, we are looking for a value for ρ^o such that it is optimal for firms to collude $\forall \rho \leq \rho^o$. Since $\Phi(\rho,\rho^o,\theta)$ is decreasing in ρ, a necessary and sufficient condition for $\widetilde{\rho}$ to be a cut-off solution is $\Phi(\widetilde{\rho},\widetilde{\rho},\theta) \geq 0$. If $\Phi(\widetilde{\rho},\widetilde{\rho},\theta) < 0$ then firms do not find collusion optimal when $\rho = \widetilde{\rho}$ which is inconsistent with equilibrium. If $\Phi(\widetilde{\rho},\widetilde{\rho},\theta) \geq 0$ then, since $\Phi(\rho,\rho^o,\theta)$ is decreasing in ρ, it implies

$$\Phi(\rho,\widetilde{\rho},\theta) \geq 0 \ \forall \rho \leq \widetilde{\rho}.$$

Hence, all firms find collusion optimal when $\rho \leq \widetilde{\rho}$.

As a selection device, I'll suppose that firms settle upon the best collusive cut-off equilibrium which means the one with the highest cut-off as then collusion occurs with the highest frequency. An optimal collusive (cut-off) equilibrium is characterized by:

$$\overline{\rho}(\theta) \equiv \max\{\widetilde{\rho} : \Phi(\widetilde{\rho},\widetilde{\rho},\theta) \geq 0\}.$$

By the continuity of $\Phi(\widetilde{\rho},\widetilde{\rho},\theta)$ in $\widetilde{\rho}$, note that

$$\Phi(\overline{\rho}(\theta),\overline{\rho}(\theta),\theta) = 0.$$

Any solution to $\Phi(\widetilde{\rho},\widetilde{\rho},\theta) = 0$ is a collusive equilibrium and thus an optimal collusive equilibrium is the largest such solution:

$$\overline{\rho}(\theta) \equiv \max\{\widetilde{\rho} : \Phi(\widetilde{\rho},\widetilde{\rho},\theta) = 0\}.$$

Since

$$\Phi(1,1,\theta) = \pi^c + \delta(W - F) - \pi^d - \delta W + \delta\theta F = \left(\pi^c - \pi^d\right) - \delta(1-\theta) F < 0.$$

then, if it exists, $\overline{\rho}(\theta) < 1$.

In the remainder of the paper, it is assumed that a collusive equilibrium exists. The next result provides sufficient conditions for that to be true.

Theorem 1 *If ω is sufficiently close to zero and δ is sufficiently close to one then an optimal collusive cut-off equilibrium exists $\forall\theta$.*

Proof. The task is to derive sufficient conditions for which, $\forall\theta$, $\exists\widetilde{\rho} \in (0,1)$ (which can depend on θ) such that $\Phi(\widetilde{\rho},\widetilde{\rho},\theta) > 0$. Given that $\Phi(1,1,\theta) < 0$, continuity of $\Phi(\widetilde{\rho},\widetilde{\rho},\theta)$ implies that $\forall\theta\ \exists\overline{\rho}(\theta) \in (0,1)$ such that $\Phi(\rho',\rho',\theta) \leqq 0$ as $\rho' \geqq \overline{\rho}(\theta)$.

First note that if ω is close to zero then $E[V|\widetilde{\rho},\theta]$ is approximated by:

$$E[V|\widetilde{\rho},\theta] \simeq \left(\frac{\pi^c}{1-\delta}\right).$$

Using this approximation and after substituting for W, the (strict) ICC is:

$$\pi^c + \delta(1-\widetilde{\rho})\left(\frac{\pi^c}{1-\delta}\right) + \delta\widetilde{\rho}\left[\left(\frac{\pi^{nc}}{1-\delta}\right) - F\right] - \pi^d - \delta\left(\frac{\pi^{nc}}{1-\delta}\right) + \delta\min\{\widetilde{\rho},\theta\} F > 0.$$

Multiplying through by $1-\delta$,

$$(1-\delta)\pi^c + \delta(1-\widetilde{\rho})\pi^c + \delta\widetilde{\rho}\left[\pi^{nc} - (1-\delta)F\right] - (1-\delta)\pi^d - \delta\pi^{nc} + \delta(1-\delta)\min\{\widetilde{\rho},\theta\} F > 0.$$

Letting $\delta \to 1$,

$$(1-\widetilde{\rho})\pi^c + \widetilde{\rho}\pi^{nc} - \pi^{nc} > 0 \Leftrightarrow (1-\widetilde{\rho})\pi^c > (1-\widetilde{\rho})\pi^{nc}.$$

Since $\pi^c > \pi^{nc}$, this condition holds if $\widetilde{\rho} < 1$. ■

Leniency programs can have an influence by making cartel formation unprofitable or, when it is still optimal to form a cartel, to expand the set of circumstances under which a cartel collapses. The focus of this paper is on the latter implication. $\overline{\rho}(\theta)$ is an index of cartel stability and I will explore how leniency policy influences it. Such a focus is relevant if firms are sufficiently patient and the probability of an investigation, ω, and the penalty in the event of discovery and conviction, F, are sufficiently low so that it is always profitable to form a cartel.

4 Optimal Leniency Policy

The antitrust authority (AA) wants to choose a leniency policy, which is a value for $\theta \in [0,1]$, that minimizes the frequency of collusion.[15] As the probability of collusion

[15] One could augment this objective function by having the AA care about how much fines are collected. Greater leniency may reduce fines paid conditional on guilt though, by making conviction easier, it could raise expected fines. My preference is to keep the analysis focused on deterring collusion for that is the real objective of antitrust law and policy. Along the lines of Motta and Polo (2003), one could also introduce antitrust expenditure which impacts ω and $G(\cdot)$. By making prosecution easier, a leniency policy can allow expenditure to move from prosecution to detection. At this point, I'm keeping the analysis focused on how leniency influences the incentive to collude.

in any period is $G(\overline{\rho}(\theta))$, it is minimized by minimizing $\overline{\rho}(\theta)$. Define θ^* as an optimal policy:

$$\theta^* \in \arg\min \overline{\rho}(\theta).$$

This is a potentially complex and poorly behaved problem. θ is to be chosen to minimize the largest solution to $\Phi(\widetilde{\rho}, \widetilde{\rho}, \theta) = 0$. Substituting (3) for $E[V | \rho^o, \theta]$ in

$$\pi^c + \delta(1-\rho) E[V | \rho^o, \theta] + \delta\rho(W - F) - \pi^d - \delta W + \delta \min\{\rho, \theta\} F = 0$$

reveals its complexity. And while $\Phi(\widetilde{\rho}, \widetilde{\rho}, \theta)$ is continuous with respect to $\widetilde{\rho}$, solutions to $\Phi(\widetilde{\rho}, \widetilde{\rho}, \theta) = 0$ need not be; hence, $\overline{\rho}(\theta)$ need not be continuous. The existence of θ^* is then not generally assured.

Prior to deriving results, let us explore how θ can impact the (necessary) condition defining $\overline{\rho}(\theta)$:

$$\Phi(\overline{\rho}(\theta), \overline{\rho}(\theta), \theta) = 0. \tag{5}$$

First note that the payoff to cheating for the marginal type, $\rho = \overline{\rho}(\theta)$, enters (5). Hence, changing θ affects (5) through the deviator's payoff only if the deviator uses the leniency program when $\rho = \overline{\rho}(\theta)$ which is true when $\theta < \overline{\rho}(\theta)$. In contrast, a change in θ always influences the collusive payoff since the leniency program will be used in the future with positive probability, regardless of the current value of ρ.

Begin by considering values for θ such that $\theta < \overline{\rho}(\theta)$.[16] A firm that considers cheating when $\rho = \overline{\rho}(\theta)$ will use the leniency program since doing so lowers the expected fine from $\overline{\rho}(\theta) F$ to θF. Hence, lowering θ (making the program more lenient) reduces the penalty paid by a deviator and thereby increases the payoff to cheating. This effect, referred to as the Deviator Amnesty Effect, serves to make collusion more difficult. It is also true, however, that lowering θ raises the collusive payoff and thus makes collusion easier. Though θ doesn't affect the current period collusive profit, it does influence the future expected collusive payoff, $E[V | \rho^o, \theta]$. Firms realize that, in some future period, ρ could exceed $\overline{\rho}(\theta)$ in which case the likelihood of detection is sufficiently high that they stop colluding - for the usual reason that the future expected lifetime is sufficiently low - and, since $\theta < \rho$ (which follows from $\theta < \overline{\rho}(\theta) < \rho$), each firm applies for leniency. This means that the expected fine at that time is $\left(\frac{n-1+\theta}{n}\right) F$ which is increasing in θ. This effect, which is referred to as the Cartel Amnesty Effect, has the implication that a lower value for θ lowers the future expected discounted penalty from continuing to collude.[17]

As the Deviator Amnesty Effect raises the payoff to cheating and the Cartel Amnesty Effect raises the payoff to continuing to collude, they work in opposite directions regarding the incentives to collude. However, it is not difficult to see that the Deviator Amnesty Effect is larger so that collusion is made more difficult with a lower value for θ.[18] The reason is that the marginal effect of θ on the deviator's

[16] As it can be shown that $0 < \overline{\rho}(0)$, this case is relevant when leniency is sufficiently great.

[17] Note that this effect only emerges when ρ can change over time as, if ρ is fixed, then if firms collude and don't use the leniency program in the current period, they'll not use it in any future period either. In that case, leniency only serves to raise the payoff to cheating.

[18] There is then a local minimum at $\theta = 0$.

fine is $-F$ since it would be the only firm applying for leniency and it would use it now, while a cartel member anticipates using it in the future with some probability (so the marginal effect is smaller) and, when it does use it, it anticipates all cartel members using it so, at that time, the marginal effect is $-(F/n)$. In sum, if these were the only two effects operating then it is intuitively clear that collusion is made more difficult with a more lenient policy in which case a policy of maximal leniency would be optimal.

Now consider values for θ such that $\overline{\rho}(\theta) < \theta$.[19] A marginal change in θ has *no* effect on the deviator's payoff because the marginal type, $\rho = \overline{\rho}(\theta)$, would not use the leniency program, preferring to receive an expected fine of $\overline{\rho}(\theta) F$ rather than a certain fine of θF. The Deviator Amnesty Effect is then absent. The impact of changing θ on the ICC comes down to its impact on the future expected collusive payoff, $E[V | \rho^o, \theta]$. If the Cartel Amnesty Effect was the only force at work then lowering θ would raise the collusive payoff and since it leaves the deviator's payoff unaffected, collusion is made easier. A more lenient policy would then enhance cartel stability, contrary to the intent of the policy.

But there is yet another effect when $\overline{\rho}(\theta) < \theta$ which can instead cause the collusive payoff to be *increasing* in θ. This effect is the most subtle and is referred to as the Race to the Courthouse Effect. Recall that the cartel stops colluding when $\rho > \overline{\rho}(\theta)$ but only uses the leniency program when $\rho > \theta\,(> \overline{\rho}(\theta))$. Let us consider a cartel member's expected penalty in the event that $\rho > \overline{\rho}(\theta)$:

$$\text{Expected Penalty } = \begin{cases} \rho F & \text{if } \rho \in (\overline{\rho}(\theta), \theta] \\ \left(\frac{n-1+\theta}{n}\right) F & \text{if } \rho \in (\theta, 1] \end{cases}$$

Since $\left(\frac{n-1+\theta}{n}\right) F > \theta F$, there is a discontinuity at $\rho = \theta$. As soon as ρ exceeds θ, it's optimal for a firm to apply for leniency even if no other firm does so and it is certainly optimal to do so if one or more firms do so. This comparison applies to all firms so behavior switches from no firms applying for leniency when $\rho \in (\overline{\rho}(\theta), \theta]$ to all firms doing so when $\rho \in (\theta, 1]$. This results in a jump in the expected penalty from θF to $\left(\frac{n-1+\theta}{n}\right) F$. Now consider what happens to firms' incentives - in the case that $\rho \simeq \theta'$ - when θ is reduced from θ' to $\theta' - \varepsilon$ where $\varepsilon > 0$ and small. When $\theta = \theta'$, if $\rho \in (\theta' - \varepsilon, \theta')$ then the cartel stops colluding but no firm applies for leniency. Thus, each firm earns W and has an expected fine of approximately $\theta' F$. Now consider reducing θ to $\theta' - \varepsilon$. When $\rho \in (\theta' - \varepsilon, \theta')$, the cartel stops colluding and, given that the policy is more lenient, all firms now apply for leniency. Hence, a more lenient policy has *raised* the expected penalty for the marginal type from approximately $\theta' F$ to approximately $\left(\frac{n-1+\theta'}{n}\right) F$. This additional effect of lowering θ increases the expected future penalty and thereby lowers the expected collusive payoff. The Cartel Amnesty Effect and the Race to the Courthouse Effect are then counter-acting. Given that the cheating payoff is unaffected, the effect of a more lenient policy is not obvious when $\overline{\rho}(\theta) < \theta$.

[19] As it can be shown that $\overline{\rho}(1) < 1$, this case is relevant when leniency is sufficiently mild.

In sum, there are three distinct effects by which the leniency program impacts the incentives to collude and thereby the frequency of collusion. More structure is required in order to sign the effect of θ on the ease of collusion. The next result provides a condition on the hazard rate in order for a lower value of θ to reduce $\overline{\rho}(\theta)$ and thereby make collusion less frequent. This then serves as a sufficient condition for maximal leniency to be optimal.

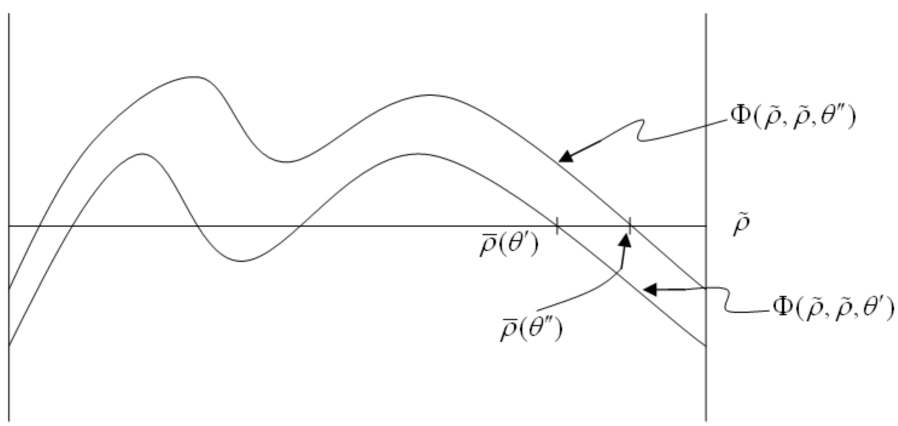
Theorem 2 *If* $\frac{G'(\rho)}{1-G(\rho)} > \frac{1}{(n-1)(1-\rho)}$ $\forall \rho \in [0,1)$ *then* θ^* *exists and* $\theta^* = 0$.

Proof. The method of proof is to show that $\overline{\rho}(\theta)$ is increasing in θ. By the assumption of a collusive equilibrium existing, $\exists \widetilde{\rho} \in (0,1)$ such that $\Phi(\widetilde{\rho}, \widetilde{\rho}, \theta) = 0$ $\forall \theta$. Recall that

$$\overline{\rho}(\theta) \equiv \max \{\widetilde{\rho} : \Phi(\widetilde{\rho}, \widetilde{\rho}, \theta) = 0\},$$

and $\Phi(\widetilde{\rho}, \widetilde{\rho}, \theta) \leqq 0$ as $\widetilde{\rho} \geqq \overline{\rho}(\theta)$. It will be shown that $\Phi(\widetilde{\rho}, \widetilde{\rho}, \theta)$ is increasing in θ. Hence, if $\theta'' > \theta'$ then $\Phi(\overline{\rho}(\theta'), \overline{\rho}(\theta'), \theta') = 0$ implies $\Phi(\overline{\rho}(\theta'), \overline{\rho}(\theta'), \theta'') > 0$ which implies, by continuity and $\Phi(1,1,\theta) < 0$, that $\overline{\rho}(\theta'') > \overline{\rho}(\theta')$. This is depicted in Figure 2.

Figure 2



Note that $\Phi(\widetilde{\rho}, \widetilde{\rho}, \theta)$ is the minimum of two functions:

$$\Phi(\widetilde{\rho}, \widetilde{\rho}, \theta) = \min \{\psi(\widetilde{\rho}, \theta) + \delta \rho F, \psi(\widetilde{\rho}, \theta) + \delta \theta F\}$$

where

$$\psi(\widetilde{\rho}, \theta) \equiv \pi^c + \delta(1-\widetilde{\rho}) E[V | \widetilde{\rho}, \theta] + \delta \widetilde{\rho}(W - F) - \pi^d - \delta W.$$

As the minimum operator is non-decreasing in its arguments then, if each of those two functions is increasing in θ, it follows that $\Phi(\widetilde{\rho}, \widetilde{\rho}, \theta)$ is also increasing in θ. As

$\psi(\widetilde{\rho},\theta)$ is differentiable then $\Phi(\rho,\rho^{o},\theta)$ is differentiable $\forall\rho\neq\theta$. Hence, this derivative is well-defined:

$$\frac{\partial\Phi(\widetilde{\rho},\widetilde{\rho},\theta)}{\partial\theta}=\begin{cases}\delta(1-\widetilde{\rho})\left[\frac{\partial E[V|\widetilde{\rho},\theta]}{\partial\theta}\right] & \text{if } \widetilde{\rho}<\theta\\ \delta(1-\widetilde{\rho})\left[\frac{\partial E[V|\widetilde{\rho},\theta]}{\partial\theta}\right]+\delta F & \text{if } \theta<\widetilde{\rho}\end{cases}\tag{6}$$

If (6) is positive then $\Phi(\widetilde{\rho},\widetilde{\rho},\theta)$ is increasing in θ.

To evaluate (6), $\frac{\partial E[V|\widetilde{\rho},\theta]}{\partial\theta}$ must be evaluated. Using (3), if $(\rho^{o}=)\,\widetilde{\rho}<\theta$ then

$$\begin{aligned}\frac{\partial E[V|\widetilde{\rho},\theta]}{\partial\theta} &= \left[1-\delta(1-\omega)-\delta\omega\int_0^{\widetilde{\rho}}(1-\rho)\,dG(\rho)\right]^{-1}\times\\ &\quad\left\{\omega(W-\theta\delta F)G'(\theta)-\omega G'(\theta)\left[W-\left(\frac{n-1+\theta}{n}\right)\delta F\right]\right.\\ &\quad\left.-\omega[1-G(\theta)]\left(\frac{1}{n}\right)\delta F\right\}\end{aligned}$$

which is positive iff

$$\omega\left\{G'(\theta)\left[W-\theta\delta F-W+\left(\frac{n-1+\theta}{n}\right)\delta F\right]-[1-G(\theta)]\left(\frac{1}{n}\right)\delta F\right\}>0.$$

Simplifying and multiplying through by n,

$$\begin{aligned}\omega\delta F\left\{G'(\theta)(n-1)(1-\theta)-[1-G(\theta)]\right\} &> 0\Leftrightarrow\\ \omega\delta F\left\{\frac{G'(\theta)}{1-G(\theta)}-\frac{1}{(n-1)(1-\theta)}\right\} &> 0\end{aligned}$$

which holds by assumption. Thus, the expected collusive payoff is increasing in θ when $\widetilde{\rho}<\theta$ which implies $\Phi(\widetilde{\rho},\widetilde{\rho},\theta)$ is increasing in θ when $\widetilde{\rho}<\theta$.

If $\theta<\widetilde{\rho}\,(=\rho^{o})$ then

$$\frac{\partial E[V|\widetilde{\rho},\theta]}{\partial\theta}=-\left[\frac{\omega[1-G(\widetilde{\rho})]}{1-\delta(1-\omega)-\delta\omega\int_0^{\widetilde{\rho}}(1-\rho)\,dG(\rho)}\right]\left(\frac{1}{n}\right)\delta F<0,$$

so the expected collusive payoff is decreasing in θ when $\theta<\widetilde{\rho}$. If $\theta<\widetilde{\rho}$ then $\frac{\partial\Phi(\widetilde{\rho},\widetilde{\rho},\theta)}{\partial\theta}>0$ iff

$$\delta(1-\widetilde{\rho})\left[\frac{\partial E[V|\widetilde{\rho},\theta]}{\partial\theta}\right]+\delta F>0\Leftrightarrow$$

$$-\delta(1-\widetilde{\rho})\left[\frac{\omega[1-G(\widetilde{\rho})]}{1-\delta(1-\omega)-\delta\omega\int_0^{\widetilde{\rho}}(1-\rho)\,dG(\rho)}\right]\left(\frac{1}{n}\right)\delta F+\delta F>0\Leftrightarrow$$

$$\frac{n}{\delta(1-\widetilde{\rho})}>\frac{\omega[1-G(\widetilde{\rho})]}{1-\delta(1-\omega)-\delta\omega\int_0^{\widetilde{\rho}}(1-\rho)\,dG(\rho)}.$$

Since the lhs exceeds one then a sufficient condition is that the rhs is less than or equal to one:

$$\begin{aligned} 1-\delta(1-\omega)-\delta\omega\int_0^{\widetilde{\rho}}(1-\rho)\,dG(\rho) &\geq \omega\left[1-G(\rho^o)\right]\Leftrightarrow \\ (1-\delta)(1-\omega)+\omega G(\widetilde{\rho}) &\geq \delta\omega\int_0^{\widetilde{\rho}}(1-\rho)\,dG(\rho)\Leftrightarrow \\ (1-\delta)(1-\omega)+\omega\int_0^{\widetilde{\rho}}\left[1-\delta(1-\rho)\right]dG(\rho) &\geq 0, \end{aligned}$$

which is true. Hence, if $\theta<\widetilde{\rho}$ then $\Phi(\widetilde{\rho},\widetilde{\rho},\theta)$ is increasing in θ.

Since $\overline{\rho}(\theta)$ is increasing in θ then θ^* exists and equals zero. ■

Let us provide some intuition for the condition on the hazard rate. It was shown in the proof that if $\theta<\overline{\rho}(\theta)$ then $\overline{\rho}(\theta)$ is increasing in θ but if $\overline{\rho}(\theta)<\theta$ then $\overline{\rho}(\theta)$ is increasing in θ iff $\frac{G'(\theta)}{1-G(\theta)}>\frac{1}{(n-1)(1-\theta)}$. Thus, if this condition holds then $\overline{\rho}(\theta)$ is increasing in θ and is minimized at $\theta=0$. Re-arranging this condition, one derives:

$$(n-1)(1-\theta)\,G'(\theta)-\left[1-G(\theta)\right]>0. \tag{7}$$

Recall that, when $\theta>\overline{\rho}(\theta)$, changing θ has two effects on the expected collusive payoff and no effect on the deviator's payoff (when evaluated at $\rho=\overline{\rho}(\theta)$). Marginally lowering θ reduces the fine paid when $\rho>\theta$; this event occurs with probability $1-G(\theta)$. But, at the same time, it raises the fine paid when $\rho\simeq\theta$ as it substitutes $\left(\frac{n-1+\theta}{n}\right)F$ for θF. This latter event occurs (roughly speaking) with probability $G'(\theta)$. The impact of changing θ on the expected collusive payoff (and thereby on the ease of collusion) depends on the relative size of these two countervailing effects. In (7), the weight of $(n-1)(1-\theta)$ reflects the effect of changing θ on the fine paid for these two events. If $\rho>\theta$ then the marginal effect from lowering θ is $-\left(\frac{1}{n}\right)F$. If $\rho\simeq\theta$ then the marginal effect is $\left(\left(\frac{n-1+\theta}{n}\right)-\theta\right)F$. Dividing through by $\left(\frac{F}{n}\right)$, these weights become -1 and $(n-1)(1-\theta)$, which is consistent with (7). Thus, the condition on the hazard rate comes from balancing off these two countervailing forces on the expected collusive payoff when θ is changed.

Towards understanding when a maximally lenient policy is optimal, it is useful to consider restrictions on G such that the hazard rate condition holds. To begin, that condition is reproduced here:

$$\frac{G'(\rho)}{1-G(\rho)}>\frac{1}{(n-1)(1-\rho)}. \tag{8}$$

So as to evaluate this condition as $\rho\rightarrow1$, re-arrange it to:

$$\frac{G'(\rho)(n-1)(1-\rho)}{1-G(\rho)}>1. \tag{9}$$

Evaluate the lhs as $\rho\rightarrow1$:

$$\lim_{\rho\rightarrow1}\frac{G'(\rho)(n-1)(1-\rho)}{1-G(\rho)}=\lim_{\rho\rightarrow1}\frac{G''(\rho)(n-1)(1-\rho)-G'(\rho)(n-1)}{-G'(\rho)}=n-1\geq1.$$

Hence, (9) holds with weak inequality as $\rho \to 1$. Next, re-arrange (8) to:

$$(n-1)(1-\rho) G'(\rho) - [1 - G(\rho)] > 0.$$

Taking its derivative with respect to ρ,

$$(n-1)(1-\rho) G''(\rho) - (n-2) G'(\rho),$$

it is then negative when $G'' < 0$. Since the expression weakly holds as $\rho \to 1$ then it holds strictly $\forall \rho < 1$ when G is strictly concave. One sufficient condition for $\theta^* = 0$ is then that G is strictly concave. Given the difficulty in discovering and prosecuting cartels, a declining density function is not unreasonable.

When G is the uniform distribution, (8) becomes:

$$\frac{1}{1-\rho} > \frac{1}{(n-1)(1-\rho)} \Leftrightarrow n > 2.$$

Hence, if there are at least three firms and the probability of detection is uniformly distributed then the leniency policy that minimizes the frequency of collusion is one of maximal leniency. It is also true that $\theta^* = 0$ when the industry is a duopoly and ρ is uniformly distributed though that requires a separate proof (see Appendix A).

While there are then plausible conditions whereby maximal leniency is optimal, is it possible for it to be optimal to waive only a portion of penalties? One can indeed construct examples. By Theorem 2, we know they will require that the density function on ρ not be monotonically decreasing. In order to work with closed-form solutions, numerical analysis was performed using the following functional form: $G(\rho) = \rho^\alpha$ where α is an integer at least as great as 2. For each $\theta \in \{0, 0.01, 0.02, ..., 1\}$, $\Phi(\widetilde{\rho}, \widetilde{\rho}, \theta) = 0$ is solved for its $\alpha + 1$ roots. The largest real solution is $\overline{\rho}(\theta)$. For when

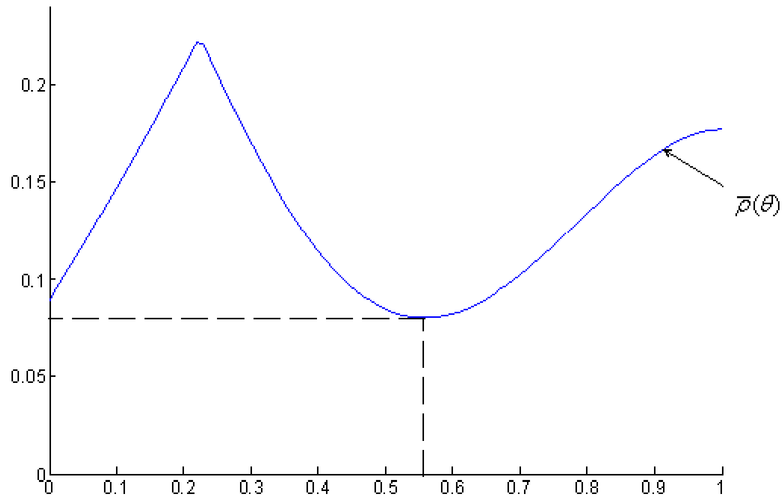
$$\left(\delta, n, F, \omega, \pi^c, \pi^{nc}, \pi^d\right) = (.99, 3, 10, .09, 1, 0, 3)$$

and $\alpha = 3$, Figure 3 depicts $\overline{\rho}(\theta)$. In accordance with the proof of Theorem 2, $\overline{\rho}(\theta)$ has a local minimum at $\theta = 0$.[20] It also has a local minimum at $\theta = .57$ and this is, in fact, the global minimum so that $\theta^* = .57$. It is also interesting to note that there is a range of values for θ, $\theta \in (.22, .57)$, such that more leniency serves to promote

[20] Recall that $\overline{\rho}(\theta)$ is increasing in θ when $\theta < \overline{\rho}(\theta)$.

collusion as reflected in a higher frequency of collusion.

Figure 3



Though this (and other unreported) examples show the possibility that partial leniency is optimal, this is admittedly an extreme example and I have been unable to find more plausible ones (and I have tried!). A density function which is rising over $[0, 1]$ strikes me as unrealistic in that it says values of ρ close to one are vastly more likely than low values. Next note that, for the above example, $\overline{\rho}(\theta^*) = .08$ which implies the probability of collusion in the first period is only $.08^3 \simeq .0005$. In comparison, $\overline{\rho}(0) = .09$ and the probability of collusion is $.09^3 \simeq .0007$. Thus, there is not much difference in the outcomes between maximal and optimal leniency. Generally, I have not found a case in which $|\overline{\rho}(\theta^*) - \overline{\rho}(0)| >> 0$.

Alternatively, let us consider more natural specifications and see to what extent less than maximal leniency is optimal. Assume the density function is symmetric and triangular on $[0, 1]$:

$$G'(\rho) = \begin{cases} 4\rho & \text{if } \rho \in [0, .5] \\ 4(1-\rho) & \text{if } \rho \in [.5, 1] \end{cases}$$

Thus, $G(\cdot)$ does not satisfy the condition in Theorem 2. 27 parameter configurations were considered:

$$\left(\delta, \pi^c, \pi^{nc}, \pi^d\right) = (.99, 1, 0, n)$$

$$(n, \omega, F) \in \{2, 4, 8\} \times \{.025, .05, .1\} \times \{3, 6, 12\}$$

In all of these cases, $\theta^* = 0$.[21] In conclusion, my informed opinion is that maximal leniency is either optimal or close to optimal under plausible assumptions.

[21] For parameter values such that a collusive equilibrium exists $\forall\theta$ then maximal leniency is the unique optimum. When instead a collusive equilibrium does not exist for some θ, it was found not to exist when θ is sufficiently low. In those cases, cartel formation can be prevented by offering sufficient leniency. There are then multiple optima and setting $\theta = 0$ is one optimum.

Though it has been assumed throughout that only the first firm to come forward receives amnesty, this assumption is without loss of generality for it is shown in Appendix B that it is optimal to only give amnesty to the first firm. The appeal of the leniency program lies in enhancing the incentive to deviate. If the other firms are colluding then a firm that cheats and applies for amnesty will necessarily be the first firm to come forward. Offering leniency to more than the first firm does not then enhance the payoff to cheating. However, it does enhance the payoff to continuing to collude since, when all firms decide to discontinue colluding and apply for amnesty, allowing more than one firm to receive it reduces expected penalties. Hence, keeping leniency limited to the first firm results in it being targeted at the firm that cheats and breaks up the cartel. At least within the framework considered here, I do not find any basis for the policy in the E.U. which provides partial leniency to the second firm to come forward. Of course, such a policy could be justified if more informants make for a stronger case which is an effect assumed away here in that conviction occurs for sure with at least one firm joining the leniency program.

5 Limiting the Acceptance of Amnesty Applications

Thus far, it has been assumed that a firm, as long as it is the first to come forward, receives leniency. Suppose, however, that amnesty is provided only when a firm delivers evidence that substantially improves the antitrust authority's case. Since ρ is the probability that the AA would successfully convict without the assistance of one of the firms, a natural way in which to model this condition is to assume that amnesty is provided if and only if ρ is sufficiently small (where the increase in the probability of conviction due to the firm's evidence is then $1-\rho$). In other words, amnesty is awarded iff $\rho < \widehat{\rho}$ where $\widehat{\rho} \in [0,1]$.

There is a natural interpretation of $\widehat{\rho}$ in light of actual leniency policies. Under Section B of the Department of Justice's Corporate Leniency Program, an applicant can qualify when an investigation has started as long as seven conditions are satisfied, one of which is: "The Division, at the time the corporation comes in, does not yet have evidence against the company that is likely to result in a sustainable conviction." Setting a value for $\widehat{\rho}$ is commensurate to specifying what it means to be "likely to result in a sustainable conviction."

The objective of this section is to characterize the optimal value of $\widehat{\rho}$. In doing so, it'll be assumed that, when $\widehat{\rho}=1$, $\theta^* < 1$ so that some leniency is optimal and let us further presume that it is strictly preferred to no leniency.[22]

It seems quite intuitive that it'll be optimal to limit the value of ρ for which a firm is accepted into the leniency program. Recall that when $\rho > \overline{\rho}(\theta)$, firms stop colluding and, when $\rho > \theta$ as well, they all apply for leniency. Thus, for $\rho > \overline{\rho}(\theta)$, the leniency program is not affecting the payoff to cheating but only the payoff from continuing to collude. Allowing such firms to receive future leniency when conviction is almost assured without it (that is, $\rho \simeq 1$) would seem to be counter-productive as it

[22] It can be shown that a sufficient condition is that $\exists \varepsilon > 0$ such that $\frac{G'(\rho)}{1-G(\rho)} > \frac{1}{(n-1)(1-\rho)}$ $\forall \rho \in (1-\varepsilon, 1)$. This implies $\theta^* < 1$.

only serves to reduce expected future penalties from continuing to collude. Indeed, it will be shown that it is optimal to set $\widehat{\rho} < 1$; amnesty is only awarded when conviction without insider testimony is sufficiently small. What is surprising, however, is how high ρ can be and it is still optimal to provide leniency. The optimal value of $\widehat{\rho}$ will be shown to have a lower bound of $\frac{n-1}{n}$ so, for example, when there are ten firms, it is optimal to offer amnesty even when the AA would have had a 90% chance to convict.

The first step is to show that, at an optimum, $\widehat{\rho} > \max\{\overline{\rho}, \theta\}$. If $\widehat{\rho} \leq \theta$ then the leniency program is irrelevant since the only time a firm would potentially like to use the program is when $\rho > \theta$ but then it would not receive amnesty as $\rho > \theta \geq \widehat{\rho}$. Hence, if $\widehat{\rho} \leq \theta$ then the frequency of collusion is the same as when $\theta = 1$ (in which case $\widehat{\rho}$ is irrelevant). But this cannot be an optimum for the AA as it has already been supposed that no leniency is not an optimum. I conclude that, at an optimum for the AA, $\widehat{\rho} > \theta$.

The ICC is now

$$\Phi(\rho, \rho^o, \theta, \widehat{\rho}) \geq 0$$

where

$$\Phi(\rho, \rho^o, \theta, \widehat{\rho}) \equiv \begin{cases} \pi^c + \delta(1-\rho) E[V|\rho^o, \theta, \widehat{\rho}] + \delta\rho(W-F) - \pi^d - \delta W + \delta \min\{\rho, \theta\} F & \text{if } \rho < \widehat{\rho} \\ \pi^c + \delta(1-\rho) E[V|\rho^o, \theta, \widehat{\rho}] + \delta\rho(W-F) - \pi^d - \delta W + \delta\rho F & \text{if } \widehat{\rho} \leq \rho \end{cases}$$

and, given that $\theta < \widehat{\rho}$,

$$\begin{aligned} E[V|\rho^o, \theta, \widehat{\rho}] \quad = \quad & \left[1 - \delta(1-\omega) - \delta\omega \int_0^{\rho^o} (1-\rho)\, dG(\rho)\right]^{-1} \times \\ & \left\{(1-\omega)\pi^c + \omega \int_0^{\rho^o} [\pi^c + \delta\rho(W-F)]\, dG(\rho) + \omega \int_{\rho^o}^{\max\{\rho^o,\theta\}} (W - \rho\delta F)\, dG(\rho)\right. \\ & + \omega [G(\max\{\max\{\rho^o,\theta\}, \widehat{\rho}\}) - G(\max\{\rho^o,\theta\})] \left[W - \left(\frac{n-1+\theta}{n}\right)\delta F\right] \\ & \left. + \omega \int_{\max\{\max\{\rho^o,\theta\},\widehat{\rho}\}}^1 (W - \rho\delta F)\, dG(\rho)\right\}. \end{aligned}$$

$\Phi(\rho, \rho^o, \theta, \widehat{\rho})$ is continuous in ρ and, since

$$\frac{\partial \Phi(\rho, \rho^o, \theta, \widehat{\rho})}{\partial \rho} = \begin{cases} -\delta(E[V|\rho^o, \theta, \widehat{\rho}] - W) & \text{if } \rho < \theta \\ -\delta(E[V|\rho^o, \theta, \widehat{\rho}] - W) - \delta F & \text{if } \theta < \rho < \widehat{\rho} \\ -\delta(E[V|\rho^o, \theta, \widehat{\rho}] - W) & \text{if } \widehat{\rho} \leq \rho \end{cases},$$

it is also strictly decreasing in ρ.[23] By the same argument as in Section 4, a collusive equilibrium is then defined by firms colluding iff $\rho \in [0, \overline{\rho}(\theta, \widehat{\rho})]$ where

$$\overline{\rho}(\theta, \widehat{\rho}) \equiv \max\{\widetilde{\rho} : \Phi(\widetilde{\rho}, \widetilde{\rho}, \theta, \widehat{\rho}) = 0\}.$$

[23] As in Section 4, this presumes a collusive equilibrium exists which implies $E[V|\rho^o, \theta, \widehat{\rho}] > W$.

An optimum for the AA is then:

$$\left(\theta^*, \widehat{\rho}^*\right) \in \arg\min \overline{\rho}\left(\theta, \widehat{\rho}\right).$$

The next step is to argue that $\widehat{\rho}^* > \overline{\rho}\left(\theta^*, \widehat{\rho}^*\right)$. If not then, when $\rho = \overline{\rho}\left(\theta^*, \widehat{\rho}^*\right)$, a firm that considers deviating could not use the leniency program. Furthermore, a colluding firm never anticipates using the leniency program in the future because it would only do so when $\rho > \overline{\rho}\left(\theta^*, \widehat{\rho}^*\right)$ but then $\rho > \widehat{\rho}$. Therefore, if $\widehat{\rho}^* \leq \overline{\rho}\left(\theta^*, \widehat{\rho}^*\right)$ then neither the expected collusive payoff nor the payoff to deviating (at $\rho = \overline{\rho}\left(\theta^*, \widehat{\rho}^*\right)$) involve using leniency which implies the ICC is the same as when $\theta = 1$. Once again, this contradicts a policy of some leniency being optimal. Therefore, $\widehat{\rho}^* > \overline{\rho}\left(\theta^*, \widehat{\rho}^*\right)$.

To summarize, we have that:

$$\widehat{\rho}^* > \max\left\{\overline{\rho}\left(\theta^*, \widehat{\rho}^*\right), \theta^*\right\}.$$

In characterizing an optimum, the analysis can then focus on values such that $\widehat{\rho} > \max\left\{\rho^o, \theta\right\}$ in which case:

$$\Phi\left(\widetilde{\rho}, \widetilde{\rho}, \theta, \widehat{\rho}\right) = \pi^c + \delta\left(1 - \widetilde{\rho}\right) E\left[V\left|\rho^o, \theta, \widehat{\rho}\right.\right] + \delta\widetilde{\rho}\left(W - F\right) - \pi^d - \delta W + \delta\min\left\{\widetilde{\rho}, \theta\right\} F$$

and

$$\begin{aligned} E\left[V\left|\rho^o, \theta, \widehat{\rho}\right.\right] &= \left[1 - \delta\left(1 - \omega\right) - \delta\omega\int_0^{\rho^o}\left(1 - \rho\right) dG\left(\rho\right)\right]^{-1} \times \\ & \quad \left\{\left(1 - \omega\right)\pi^c + \omega\int_0^{\rho^o}\left[\pi^c + \delta\rho\left(W - F\right)\right] dG\left(\rho\right) + \omega\int_{\rho^o}^{\max\left\{\rho^o, \theta\right\}}\left(W - \rho\delta F\right) dG\left(\rho\right)\right. \\ & \quad + \omega\left[G\left(\widehat{\rho}\right) - G\left(\max\left\{\rho^o, \theta\right\}\right)\right]\left[W - \left(\frac{n - 1 + \theta}{n}\right)\delta F\right] \\ & \quad \left. + \omega\int_{\widehat{\rho}}^1\left(W - \rho\delta F\right) dG\left(\rho\right)\right\}. \end{aligned}$$

Next note that:

$$\begin{aligned} \frac{\partial\Phi\left(\widetilde{\rho}, \widetilde{\rho}, \theta, \widehat{\rho}\right)}{\partial\widehat{\rho}} &= \delta\left(1 - \widetilde{\rho}\right)\left[\frac{\partial E\left[V\left|\rho^o, \theta, \widehat{\rho}\right.\right]}{\partial\widehat{\rho}}\right] \\ &= \delta\left(1 - \widetilde{\rho}\right)\left[\frac{\omega G'\left(\widehat{\rho}\right)\delta F\left[\widehat{\rho} - \left(\frac{n-1+\theta}{n}\right)\right]}{1 - \delta\left(1 - \omega\right) - \delta\omega\int_0^{\rho^o}\left(1 - \rho\right) dG\left(\rho\right)}\right]. \end{aligned}$$

Hence,

$$sign\left\{\frac{\partial\Phi\left(\widetilde{\rho}, \widetilde{\rho}, \theta, \widehat{\rho}\right)}{\partial\widehat{\rho}}\right\} = sign\left\{\widehat{\rho} - \left(\frac{n - 1 + \theta}{n}\right)\right\},$$

from which this theorem follows.

Theorem 3 *$\widehat{\rho}^* = \frac{n-1+\theta^*}{n}$ so that it is optimal to award amnesty if and only if $\rho < \frac{n-1+\theta^*}{n}$.*

There is a simple explanation as to why leniency should only be awarded when $\rho < \frac{n-1+\theta^*}{n}$. Since $\overline{\rho}(\theta^*, \widehat{\rho}^*) < \widehat{\rho}^*$ then a marginal change in $\widehat{\rho}^*$ does not affect the payoff to cheating because a deviator (when $\rho = \overline{\rho}(\theta^*, \widehat{\rho}^*)$) would still be able to use the leniency program. As $\widehat{\rho}$ is then only affecting the anticipated future penalties from continuing to collude, it should be set to maximize those penalties and thereby tighten the ICC. The expected present value of fines for colluding firms is

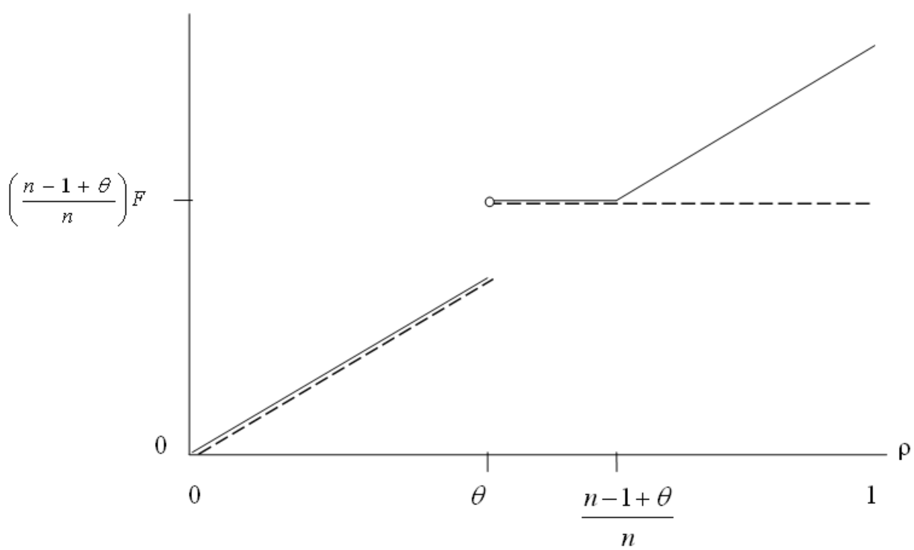
$$\frac{\omega\delta F\left\{\int_0^{\max\{\rho^o,\theta\}} \rho dG(\rho) + \left[G(\widehat{\rho}) - G(\max\{\rho^o,\theta\})\right]\left(\frac{n-1+\theta}{n}\right) + \int_{\widehat{\rho}}^1 \rho dG(\rho)\right\}}{1-\delta(1-\omega)-\delta\omega\int_0^{\rho^o}(1-\rho)\,dG(\rho)}.$$

Taking the derivative of it with respect to $\widehat{\rho}$, one derives:

$$\frac{\omega\delta F G'(\widehat{\rho})\left[\left(\frac{n-1+\theta}{n}\right)-\widehat{\rho}\right]}{1-\delta(1-\omega)-\delta\omega\int_0^{\rho^o}(1-\rho)\,dG(\rho)} \gtreqless 0 \text{ as } \widehat{\rho} \lesseqgtr \frac{n-1+\theta}{n}.$$

The cartel's expected fine is then maximized by setting $\widehat{\rho} = \frac{n-1+\theta}{n}$. Figure 4 depicts the expected penalty in a future period, depending on ρ, where the dotted line is when amnesty is always awarded and the solid line is when amnesty is optimally awarded.

Figure 4



It is then optimal for the AA to only provide leniency when its case is sufficiently weak: $\rho < \widehat{\rho}^*$. What is surprising is that $\widehat{\rho}^*$ can be quite high as it is bounded from below by $\frac{n-1}{n}$. Thus, the AA always provides amnesty as long as the probability of conviction without an insider witness is less than $\frac{n-1}{n}$. Depending on the number of firms, amnesty could be awarded even when the AA has a very good chance of conviction. Of course, one must remember that $\widehat{\rho}$ is being set so as to destabilize the cartel and not to maximize the fines paid.

Holding θ fixed, raising n causes $\widehat{\rho}$ to rise so that more firms makes for a less stringent criterion for acceptance into the leniency program. This result is due to how the number of firms affects the Race to the Courthouse Effect. Recall that the Race to the Courthouse Effect results in more leniency *raising* expected penalties as it destabilizes an equilibrium in which all cartel members do not turn witness. As the number of firms increases, the Race to the Courthouse Effect becomes stronger - which means leniency actually raises expected penalties - and a limitation on when amnesty can be awarded only reduces the power of that effect. In conclusion, while amnesty should not always be awarded, the general lesson is that the conditions for leniency could be relatively weak. At a minimum, it is optimal to award leniency when the AA's chances of winning the case are less than 50%.

Given that $\widehat{\rho}^* = \frac{n-1+\theta^*}{n}$, the AA's problem can be re-cast as choosing θ to minimize $\overline{\rho}\left(\theta, \frac{n-1+\theta}{n}\right)$. By an analogous method of proof to that used in proving Theorem 2, a sufficient condition for maximal leniency to be optimal is derived. By optimally restricting the conditions for amnesty, this sufficient condition is made weaker. The proof largely mimics that of the proof of Theorem 2 and, therefore, is relegated to Appendix C.

Theorem 4 *If* $\frac{G'(\rho)}{G\left(\frac{n-1+\rho}{n}\right)-G(\rho)} > \frac{1}{(n-1)(1-\rho)}$ $\forall \rho \in [0,1)$ *then* $(\theta^*, \widehat{\rho}^*)$ *exists and* $(\theta^*, \widehat{\rho}^*) = \left(0, \frac{n-1}{n}\right)$.

When the density function on ρ is weakly declining, the optimal policy is to waive all penalties to the first firm to come forward when $\rho < \frac{n-1}{n}$ and to not provide amnesty otherwise.

6 Concluding Remarks

Though the analysis of this paper shows that maximal leniency is not the universally optimal policy, under some plausible conditions it is indeed best to waive all penalties of the first firm to come forward. A unique feature of the modelling of the policy space is to encompass the requirement, as stated in the U.S. Corporate Leniency Program, that amnesty is awarded to the first firm only if "... at the time the corporation comes in, [the antitrust authority] does not yet have evidence against the company that is likely to result in a sustainable conviction." The analysis shows that such a provision is indeed optimal but, quite surprisingly, it can be desirable to award amnesty even when the probability of conviction is quite high. Though doing so may not be best *ex post*, it is optimal *ex ante* in order to reduce the stability of the cartel.

In reviewing the body of theoretical work, the general conclusion is strong support for leniency policies in that they show that leniency can reduce cartel stability. What is much less clear is whether there is evidence in support of this hypothesis. Though it is well-documented that many firms have used the amnesty program and it has provided valuable evidence in support of the prosecution's case, it is unknown how influential leniency programs have been in inducing cartels to collapse or deterring them from forming. The data obstacle to addressing these questions is that we only observed *discovered* cartels, so we do not know the frequency of cartels in the economy. Until we find a way in which to surmount that obstacle, the ultimate impact of leniency programs on cartel formation and the lifetime of cartels will remain an open question.

7 Appendix A

Theorem 5 *If G is a uniform distribution and $n = 2$ then $\theta^* = 0$.*

Proof. To begin, I know that if $\widetilde{\rho} > \theta$ then $\frac{\partial \Phi(\widetilde{\rho},\widetilde{\rho},\theta)}{\partial \theta} > 0$ and if $\widetilde{\rho} < \theta$ then

$$sign\left\{\frac{\partial \Phi\left(\widetilde{\rho},\widetilde{\rho},\theta\right)}{\partial \theta}\right\} = sign\left\{f\left(\theta\right)\right\}$$

where

$$f\left(\theta\right) \equiv G'\left(\theta\right)\left(n-1\right)\left(1-\theta\right) - \left[1 - G\left(\theta\right)\right].$$

Since $G\left(\rho\right) = \rho$, it follows that:

$$f\left(\theta\right) = \left(1-\theta\right) - \left(1-\theta\right) = 0.$$

Since $f\left(\theta\right) = 0\ \forall\theta$ then

$$\frac{\partial \Phi\left(\widetilde{\rho},\widetilde{\rho},\theta\right)}{\partial \theta}\begin{cases} = 0 & \text{if } \widetilde{\rho} < \theta \\ \\ > 0 & \text{if } \theta < \widetilde{\rho}\end{cases}.$$

Now let us show that $\theta^* = 0$. Define

$$\overline{\theta} \equiv \sup\left\{\theta' : \theta < \overline{\rho}\left(\theta\right)\ \ \forall\theta < \theta'\right\}.$$

$\overline{\theta}$ is the least upper bound to the set of values for θ such that $\overline{\rho}\left(\theta\right)$ lies above the 45 degree line for all lesser values.

Let us first show that $\overline{\rho}\left(\theta\right)$ is increasing in θ over $\left[0,\overline{\theta}\right)$ and thus $\overline{\rho}\left(0\right) < \overline{\rho}\left(\theta\right) \forall\theta \in \left[0,\overline{\theta}\right)$. We start with $0 < \overline{\rho}\left(0\right)$ which implies $\theta < \overline{\rho}\left(\theta\right) \forall\theta \in \left[0,\overline{\theta}\right)$. Since $\frac{\partial \Phi(\widetilde{\rho},\widetilde{\rho},\theta)}{\partial \theta} > 0$ when $\widetilde{\rho} > \theta$ then $\overline{\rho}\left(\theta\right) > \theta$ implies $\frac{\partial \Phi(\widetilde{\rho},\widetilde{\rho},\theta)}{\partial \theta} > 0\ \forall\widetilde{\rho} \geq \overline{\rho}\left(\theta\right)$. By definition of $\overline{\rho}\left(\theta\right)$, $\Phi\left(\overline{\rho}\left(\theta\right),\overline{\rho}\left(\theta\right),\theta\right) = 0$. Hence, it follows from the previous step that: if $\theta'' > \theta'$ then $\Phi\left(\overline{\rho}\left(\theta'\right),\overline{\rho}\left(\theta'\right),\theta''\right) > 0$ which implies $\overline{\rho}\left(\theta''\right) > \overline{\rho}\left(\theta'\right)$. Since $\overline{\rho}\left(\theta\right)$ is increasing in θ over $\left[0,\overline{\theta}\right)$ then $\overline{\rho}\left(0\right) < \overline{\rho}\left(\theta\right) \forall\theta \in \left[0,\overline{\theta}\right)$. Given that $\overline{\rho}\left(\theta\right)$ is increasing over $\left[0,\overline{\theta}\right)$ and maps $[0,1]$ into $[0,1]$ this implies $\lim_{\theta \to_+ \overline{\theta}} \overline{\rho}\left(\theta\right) = \overline{\theta}$.

Now consider $\theta \in \left[\overline{\theta},1\right]$. If $\overline{\rho}\left(\theta\right) \geq \theta$ then $\overline{\rho}\left(\theta\right) \geq \theta \geq \overline{\theta} > \overline{\rho}\left(0\right)$. Hence, if $\exists\theta' \in \left[\overline{\theta},1\right]$ such that $\overline{\rho}\left(\theta'\right) \leq \overline{\rho}\left(0\right)$ then it must be true that $\overline{\rho}\left(\theta'\right) < \theta'$. Since $\frac{\partial \Phi(\widetilde{\rho},\widetilde{\rho},\theta)}{\partial \theta} = 0$ when $\widetilde{\rho} < \theta$ then

$$\Phi\left(\overline{\rho}\left(\theta'\right),\overline{\rho}\left(\theta'\right),\theta'\right) \leqq 0 \text{ as } \widetilde{\rho} \geqq \overline{\rho}\left(\theta'\right)$$

implies

$$\Phi\left(\overline{\rho}\left(\theta'\right),\overline{\rho}\left(\theta'\right),\theta\right) \leqq 0 \text{ as } \widetilde{\rho} \geqq \overline{\rho}\left(\theta'\right)\ \ \forall\theta > \overline{\rho}\left(\theta'\right).$$

Hence, $\overline{\rho}\left(\theta\right) = \overline{\rho}\left(\theta'\right) \forall\theta > \overline{\rho}\left(\theta'\right)$. Recall that $\overline{\rho}\left(\theta'\right) \leq \overline{\rho}\left(0\right) < \overline{\theta}$. Hence, $\overline{\rho}\left(\theta\right) < \theta\forall\theta \in \left(\overline{\rho}\left(\theta'\right)\overline{\theta}\right)$. But we also know that $\overline{\rho}\left(\theta\right) > \theta\ \forall\theta < \overline{\theta}$. Since $\overline{\rho}\left(\cdot\right)$ is single-valued, this is a contradiction. ■

8 Appendix B

Here I show that it is never optimal to provide leniency for any firm but the first. Assume that the j^{th} firm to come forward pays a fine of $\theta^j F$ where $\theta^j \in [0,1]$. Observing the ICC, note that θ^j for $j \geq 2$ matters only when all firms stop colluding and apply for leniency. In that event, the expected penalty is $\sum_{j=1}^{n} \left(\frac{1}{n}\right) \theta^j F$. Defining $\xi \equiv \sum_{j=1}^{n} \left(\frac{1}{n}\right) \theta^j$, a leniency policy is then defined by a pair (θ, ξ) where the first firm that comes forward pays a fine of θF and, when all firms simultaneously come forward, each pays an expected fine of ξF. It only makes sense to assume $\xi \in \left[\theta, \frac{n-1+\theta}{n}\right]$.

The ICC is

$$\Phi(\rho, \rho^o, \theta, \xi) \equiv \pi^c + \delta(1-\rho) E[V|\rho^o, \theta, \xi] + \delta\rho(W-F) - \pi^d - \delta W + \delta \min\{\rho, \theta\} F \geq 0$$

where

$$\begin{aligned} E[V|\rho^o, \theta, \xi] &= \left[1 - \delta(1-\omega) - \delta\omega \int_0^{\rho^o} (1-\rho)\, dG(\rho)\right]^{-1} \times \\ & \left\{(1-\omega)\pi^c + \omega \int_0^{\rho^o} [\pi^c + \delta\rho(W-F)]\, dG(\rho) + \omega \int_{\rho^o}^{\max\{\rho^o,\theta\}} (W - \rho\delta F)\, dG(\rho) \right. \\ & \left. + \omega [1 - G(\max\{\rho^o, \theta\})][W - \xi\delta F] \right\}. \end{aligned}$$

Since

$$\begin{aligned} \frac{\partial \Phi(\widetilde{\rho}, \widetilde{\rho}, \theta, \xi)}{\partial \xi} &= \delta(1-\widetilde{\rho}) \left[\frac{\partial E[V|\widetilde{\rho}, \theta, \xi]}{\partial \xi}\right] \\ &= -\delta(1-\widetilde{\rho}) \left[\frac{\omega[1 - G(\max\{\rho^o, \theta\})]\delta F}{1 - \delta(1-\omega) - \delta\omega \int_0^{\rho^o} (1-\rho)\, dG(\rho)}\right] < 0, \end{aligned}$$

it is always better to set a higher value for ξ. It is then optimal to set $\xi = \frac{n-1+\theta}{n}$ and only offer leniency to the first firm to come forward.

9 Appendix C

Proof of Theorem 5. The ICC is presented as:

$$\widehat{\Phi}(\widetilde{\rho}, \widetilde{\rho}, \theta) \equiv \pi^c + \delta(1-\widetilde{\rho}) \widehat{E}[V|\rho^o, \theta] + \delta\widetilde{\rho}(W-F) - \pi^d - \delta W + \delta \min\{\widetilde{\rho}, \theta\} F$$

where:

$$\widehat{E}[V|\rho^o, \theta] \equiv E\left[V \middle| \rho^o, \theta, \widehat{\rho} = \frac{n-1+\theta}{n}\right] =$$

$$\left[1-\delta\left(1-\omega\right)-\delta\omega\int_0^{\rho^o}\left(1-\rho\right)dG\left(\rho\right)\right]^{-1}$$

$$\left\{\left(1-\omega\right)\pi^c+\omega\int_0^{\rho^o}\left[\pi^c+\delta\rho\left(W-F\right)\right]dG\left(\rho\right)+\omega\int_{\rho^o}^{\max\{\rho^o,\theta\}}\left(W-\rho\delta F\right)dG\left(\rho\right)\right.$$

$$+\omega\left[G\left(\frac{n-1+\theta}{n}\right)-G\left(\max\left\{\rho^o,\theta\right\}\right)\right]\left[W-\left(\frac{n-1+\theta}{n}\right)\delta F\right]$$

$$\left.+\omega\int_{\left(\frac{n-1+\theta}{n}\right)}^{1}\left(W-\rho\delta F\right)dG\left(\rho\right)\right\}.$$

Note that

$$\frac{\partial\widehat{\Phi}\left(\widetilde{\rho},\widetilde{\rho},\theta\right)}{\partial\theta}=\begin{cases}\delta\left(1-\widetilde{\rho}\right)\left[\frac{\partial\widehat{E}[V|\widetilde{\rho},\theta]}{\partial\theta}\right] & \text{if } \widetilde{\rho}<\theta\\ \\ \delta\left(1-\widetilde{\rho}\right)\left[\frac{\partial\widehat{E}[V|\widetilde{\rho},\theta]}{\partial\theta}\right]+\delta F & \text{if } \theta<\widetilde{\rho}\end{cases}$$

If $\theta<\rho^o$ then

$$\begin{aligned}\frac{\partial\widehat{E}\left[V\left|\widetilde{\rho},\theta\right.\right]}{\partial\theta} &= \left[1-\delta\left(1-\omega\right)-\delta\omega\int_0^{\widetilde{\rho}}\left(1-\rho\right)dG\left(\rho\right)\right]^{-1}\times\\ &\quad \omega\left\{\left(\frac{1}{n}\right)G'\left(\frac{n-1+\theta}{n}\right)\left[W-\left(\frac{n-1+\theta}{n}\right)\delta F\right]\right.\\ &\quad -\left(\frac{1}{n}\right)\left[G\left(\frac{n-1+\theta}{n}\right)-G\left(\widetilde{\rho}\right)\right]\delta F\\ &\quad \left.-\left(\frac{1}{n}\right)\left(W-\left(\frac{n-1+\theta}{n}\right)\delta F\right)G'\left(\frac{n-1+\theta}{n}\right)\right\}\\ &= -\frac{\omega\left(\frac{1}{n}\right)\left[G\left(\frac{n-1+\theta}{n}\right)-G\left(\widetilde{\rho}\right)\right]\delta F}{1-\delta\left(1-\omega\right)-\delta\omega\int_0^{\widetilde{\rho}}\left(1-\rho\right)dG\left(\rho\right)}.\end{aligned}$$

Since, at an optimum, $\widehat{\rho}^*=\left(\frac{n-1+\theta^*}{n}\right)>\overline{\rho}\left(\theta^*\right)$, then $\frac{\partial\widehat{E}[V|\widetilde{\rho},\theta]}{\partial\theta}<0$. $\frac{\partial\widehat{\Phi}(\widetilde{\rho},\widetilde{\rho},\theta)}{\partial\theta}>0$ iff

$$\delta F-\delta\left(1-\widetilde{\rho}\right)\left[\frac{\omega\left(\frac{1}{n}\right)\left(G\left(\frac{n-1+\theta}{n}\right)-G\left(\widetilde{\rho}\right)\right)\delta F}{1-\delta\left(1-\omega\right)-\delta\omega\int_0^{\widetilde{\rho}}\left(1-\rho\right)dG\left(\rho\right)}\right]>0\Leftrightarrow$$

$$\frac{n}{\delta\left(1-\widetilde{\rho}\right)}>\frac{\omega\left[G\left(\frac{n-1+\theta}{n}\right)-G\left(\widetilde{\rho}\right)\right]}{1-\delta\left(1-\omega\right)-\delta\omega\int_0^{\widetilde{\rho}}\left(1-\rho\right)dG\left(\rho\right)}.$$

Given that the lhs exceeds one then a sufficient condition is that the rhs is less than or equal to one:

$$1-\delta\left(1-\omega\right)-\delta\omega\int_0^{\widetilde{\rho}}\left(1-\rho\right)dG\left(\rho\right)\geq\omega\left[G\left(\frac{n-1+\theta}{n}\right)-G\left(\widetilde{\rho}\right)\right]\Leftrightarrow$$

$$\left(1-\delta\right)\left(1-\omega\right)+\omega G\left(\widetilde{\rho}\right)+\omega\left[1-G\left(\frac{n-1+\theta}{n}\right)\right]\geq\delta\omega\int_0^{\widetilde{\rho}}\left(1-\rho\right)dG\left(\rho\right)\Leftrightarrow$$

$$(1-\delta)(1-\omega)+\omega\int_0^{\widetilde{\rho}}[1-\delta(1-\rho)]\,dG(\rho)+\omega\left[1-G\left(\frac{n-1+\theta}{n}\right)\right]\geq 0,$$

which is true. To conclude, if $\theta<\rho^o$ then $\frac{\partial\widehat{\Phi}(\widetilde{\rho},\widetilde{\rho},\theta)}{\partial\theta}>0$.

If instead $\rho^o<\theta$ then

$$\frac{\partial\widehat{\Phi}(\widetilde{\rho},\widetilde{\rho},\theta)}{\partial\theta}=\delta(1-\widetilde{\rho})\left[\frac{\partial\widehat{E}[V\,|\widetilde{\rho},\theta]}{\partial\theta}\right].$$

In that case,

$$\begin{aligned}\widehat{E}[V\,|\widetilde{\rho},\theta] \;=\;& \left[1-\delta(1-\omega)-\delta\omega\int_0^{\widetilde{\rho}}(1-\rho)\,dG(\rho)\right]^{-1}\times\\ & \left\{(1-\omega)\pi^c+\omega\int_0^{\widetilde{\rho}}[\pi^c+\delta\rho(W-F)]\,dG(\rho)+\omega\int_{\widetilde{\rho}}^{\theta}(W-\rho\delta F)\,dG(\rho)\right.\\ & +\omega\left[G\left(\frac{n-1+\theta}{n}\right)-G(\theta)\right]\left[W-\left(\frac{n-1+\theta}{n}\right)\delta F\right]\\ & \left.+\omega\int_{\left(\frac{n-1+\theta}{n}\right)}^{1}(W-\rho\delta F)\,dG(\rho)\right\},\end{aligned}$$

so

$$\begin{aligned}\frac{\partial\widehat{E}[V\,|\widetilde{\rho},\theta]}{\partial\theta} \;=\;& \left[1-\delta(1-\omega)-\delta\omega\int_0^{\widetilde{\rho}}(1-\rho)\,dG(\rho)\right]^{-1}\times\\ & \left(\frac{1}{n}\right)\omega\delta F\left\{G'(\theta)(1-\theta)(n-1)-\left[G\left(\frac{n-1+\theta}{n}\right)-G(\theta)\right]\right\}.\end{aligned}$$

This is positive iff

$$G'(\theta)(1-\theta)(n-1)-\left[G\left(\frac{n-1+\theta}{n}\right)-G(\theta)\right]>0\Leftrightarrow$$

$$\frac{G'(\theta)}{G\left(\frac{n-1+\theta}{n}\right)-G(\theta)}>\frac{1}{(1-\theta)(n-1)}.$$

I conclude that $(\theta^*,\widehat{\rho}^*)=\left(0,\frac{n-1}{n}\right)$. ■

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