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Optimal currency crises*

Franklin Allen
Wharton School, University of Pennsylvania, Philadelphia, PA 19104
and
Douglas Gale
New York University, Economics Department, New York, NY 10003

Abstract

Flawed government policies have been offered as an explanation for currency crises in most of the previous literature. With few exceptions, the role of the banking system is ignored. Empirical evidence suggests that in recent decades banking crises and currency crises have been linked. A model is developed here where the "twin" crises result from low asset returns. Large movements in exchange rates are desirable to the extent that they allow better risk-sharing between a country's bank depositors and the international bond market. The rationale for using short-term debt denominated in a foreign reserve currency is also investigated.

1 Introduction

The large movements in exchange rates that occurred in many South East Asian countries in 1997 have revived interest in the topic of currency crises. In many of the early models of currency crises, such as Krugman (1979), currency crises occur because of inconsistent and unsustainable government policies (see Flood and Marion (1998) for a survey of the literature on currency crises). These models were designed to explain the problems experienced

* *Correspondence to*: Professor Franklin Allen, Wharton School, University of Pennsylvania, Philadelphia, PA 19104, email: allenf@wharton.upenn.edu. This paper was prepared for the November 1999 Carnegie-Rochester Conference on Public Policy. We are grateful to our discussant Nancy Marion and other participants in the conference. We also thank Paolo Pesenti, Anjan Thakor, and participants at seminars and presentations at the Federal Reserve Bank of New York, New York University, Indiana University, the University of Frankfurt, and the University of Pennsylvania for helpful comments. Financial support from the Wharton Financial Institutions Center, the C.V. Starr Center for Applied Economics at NYU, and the NSF is gratefully acknowledged.

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by a number of Latin American countries in the 1970s and early 1980s. In the recent South East Asian crises, by contrast, many of the countries which experienced problems had pursued macroeconomic policies that were consistent and sustainable. This characteristic of the recent crises has prompted a reexamination of theoretical models of currency crises.

The other characteristic of the South East Asian crises that has received considerable attention is that the banking systems of these countries also experienced crises. In an important paper, Kaminsky and Reinhart (1999) have investigated the relationship between banking crises and currency crises. They find that in the 1970s, when financial systems were highly regulated in many countries, currency crises were not accompanied by banking crises. However, after the financial liberalization that occurred during the 1980s, currency crises and banking crises became intertwined. The usual sequence of events is that initial problems in the banking sector are followed by a currency crisis and this in turn exacerbates and deepens the banking crisis. Although banking crises typically precede currency crises, the common cause of both is usually a fall in asset values due to a recession or a weak economy. Often the fall is part of a boom-bust cycle that follows financial liberalization. It appears to be rare that banking and currency crises occur when economic fundamentals are sound.

Despite the apparent inter-relationship between currency crises and banking crises in recent episodes, the literatures on the two topics have for the most part developed separately. Important exceptions are Chang and Velasco (1998a,b). The first paper develops a model of currency and banking crises based on the Diamond and Dybvig (1983) model of bank runs. Chang and Velasco introduce money as an argument in the utility function. A central bank controls the ratio of currency to consumption. Different exchange-rate regimes correspond to different rules for regulating the currency-consumption ratio. There is no aggregate uncertainty in these models: banking and currency crises are "sunspot" phenomena. In other words, there are at least two equilibria, a "good" equilibrium in which early consumers receive the proceeds from short-term assets and late consumers receive the proceeds from long-term assets and a "bad" equilibrium in which everybody believes a crisis will occur and these beliefs are self-fulfilling. Chang and Velasco (1998a) show that the existence of the bad equilibrium depends on the exchange-rate regime in force. In some regimes, only the good equilibrium exists; in other regimes there exists a bad equilibrium in addition to the good equilibrium. The selection of the good or the bad equilibrium is not modeled. In Chang and Velasco (1998b) a similar model is used to consider recent crises in emerging markets. Again there is no aggregate uncertainty and crises are sunspot phenomena.

A number of other recent papers have focused on the possibility of multiple equilibria. These include Flood and Garber (1984), Obstfeld (1986; 1994) and Calvo (1988). In these models governments are unable to commit to policies and this lack of commitment can give rise to multiple equilibria, at least one of which is a self-fulfilling crisis. Again, the selection of equilibrium is problematic. An exception is Morris and Shin (1998) who show that traders' lack of common knowledge about the state of the economy can lead to a unique equilibrium selection.

Kaminsky and Reinhart's (1999) finding that crises are related to economic fundamentals is consistent with work on U.S. financial crises in the nineteenth and early twentieth centuries. Gorton (1988) and Calomiris and Gorton (1991) argue that the evidence is consistent with the hypothesis that banking crises are an essential part of the business cycle rather than a sunspot phenomenon. Allen and Gale (1998) develop a model in which banking crises are generated by aggregate uncertainty about asset returns. Moreover, although equilibrium is not necessarily unique, it can be shown that crises are a feature of all equilibria of the model when asset returns are low.[1]

In the Allen-Gale model, crises can improve risk-sharing but they also involve deadweight costs if they cause projects to be prematurely liquidated. A central bank can avoid these deadweight costs and implement an optimal allocation of resources through an appropriate monetary policy. By creating fiat money and lending it to banks, the central bank can prevent the inefficient liquidation of investments while at the same time allowing optimal sharing of risks.

In this paper we extend the model of Allen and Gale (1998) to an international context and study the relationship between banking and currency crises. Section 2 begins by describing a simple one-country version of the model with three dates and two assets. As in Allen and Gale (1998), there is a large number of ex ante identical agents who discover at the intermediate date whether they require liquidity immediately or at the final date. There are two assets, a safe, short-term asset represented by a storage technology and a risky, long-term asset that pays off at the final date. At the intermediate date a leading economic indicator reveals the true return to the risky asset. If the long-term asset is liquidated at the intermediate date, there is a liquidation cost. The optimal allocation is characterized as a planner's problem with state contingent contracts.

Section 3 analyzes risk-sharing in a banking system in which banks use a noncontingent nominal deposit contract and the central bank controls the price level through its monetary policy. By adopting an appropriate monetary policy, the central bank makes the real value of the deposit contract state-contingent and the banking system uses this state-contingent contract

[1]For a discussion of other banking papers with aggregate uncertainty and how they relate, see Allen and Gale (1998).

to achieve the first-best allocation. Depositors bear risk, but it is allocated optimally between early consumers and late consumers.

Section 4 extends the model by introducing an international bond market in which the domestic country can borrow and lend at a fixed rate. The domestic country is assumed to be small relative to the rest of the world and therefore has no impact on foreign prices and interest rates. Also, since the domestic country is small relative to the global market, lenders are risk neutral.

We begin by studying the optimal allocation implemented by a planner who is allowed to trade state-contingent contracts on the international market. Since the market is risk neutral, the optimal allocation requires the (risk-averse) domestic depositors to bear no risk. Instead, all the risk is borne by the international capital market.

Next we consider a market equilibrium in which banks issue debt denominated in the domestic currency on the international bond market. Both domestic-currency debt and the domestic-deposit contracts promise a fixed amount of the domestic currency. However, the central bank controls the real value of these securities through its control of the price level. Once again, an appropriate monetary policy introduces the right amount of state-contingency into the real contracts. This allows the banking system to achieve optimal risk-sharing. In this case, access to the international market allows the banking system to eliminate all risk for domestic depositors. Banks issue a large amount of bonds denominated in the domestic currency and invest the money in bonds denominated in foreign currency. Variations in the price level cause variations in the relative value of bonds denominated in the domestic and foreign currencies, respectively. These state-contingent variations in the relative values of the bonds allow the banking system to export all the risk to the international market.

In order to achieve optimal risk-sharing, banks acquire large offsetting positions in domestic-currency bonds and foreign-currency bonds. This is consistent with the observation that the volume of trading in foreign-exchange markets is much higher than can be justified by the needs of world trade.

It is also shown that the use of short-term debt is optimal if the yield curve in the international bond market is flat or upward sloping. Providing liquidity at the intermediate date by rolling over debt is at least as good as borrowing long-term in these circumstances. This may help to rationalize the otherwise puzzling use of unhedged short-term debt in many emerging markets.

The use of domestic-currency debt presents a risk to investors in the domestic country. After the contracts with foreign bondholders are written, the country has an incentive to inflate its currency and effectively expropriate the bondholders. For this reason, lenders may be reluctant to hold debt denominated in the domestic currency. Instead, they may demand debt denominated in terms of a foreign (reserve) currency which is not subject to inflation risk. Section 5 considers two variants of the model in which debt denominated in foreign currency is used. The first represents a dollarized economy in which bonds and deposit contracts are denominated in foreign currency. In certain circumstances, it is still possible to shift risk to the international market. In this case, it is the possibility of default that makes domestic debt and deposit contracts state-contingent. In the limit, domestic depositors bear no risk but costly liquidation cannot be avoided. It is generally true that banks can eliminate all risk for domestic depositors by acquiring large offsetting positions in (risky) domestic and (safe) foreign debt. However, it is typically suboptimal to eliminate all risk because of the costly liquidation this entails.

In the second variant of the model, a central bank is introduced. By writing nominal contracts in domestic currency, the amount of bankruptcy caused by the foreign denominated debt can be reduced for a given portfolio of bank assets and a given amount of real liabilities. Although risk-sharing between early and late consumers is improved, risk-sharing between depositors and the international bond market is eliminated. Given these trade-offs, the existence of a central bank and a domestic monetary system may or may not improve welfare when international debt is denominated in foreign currency.

Section 6 discusses the policy implications of the model for the role of the International Monetary Fund (IMF).

2 Optimal risk-sharing

In this section, we define the risk-sharing problem for a closed economy. Later the model will be "opened" to include an international bond market.

The basic structure of the model is drawn from Allen and Gale (1998). There are three dates $t = 0, 1, 2$. At each date, there is a single good that can be used for consumption and investment. There are two kinds of assets in the domestic economy, a safe asset and a risky asset. The safe asset is modeled as a storage technology: one unit of the good invested at date t produces one unit of the good at date $t + 1$, for $t = 0, 1$. The risky asset takes two periods to mature: x units of the good invested at date 0 yields $Rh(x)$ units of the good at date 2 where $h(x)$ is a neoclassical, decreasing-returns-to-scale production function (increasing, strictly concave, twice continuously differentiable). The random variable R has realization r and a support $[r_0, r_1]$, where $0 \leq r_0 < r_1 < \infty$. The cumulative distribution function $F(r)$ is assumed to be continuous and increasing on the support $[r_0, r_1]$. At date 1 agents observe a signal, which can be thought of as a leading economic indicator. For simplicity, it is assumed that this signal predicts with perfect accuracy the value of r that will be realized at date 2. We begin by considering the planner's problem, in which the optimal allocation is contingent on r. In subsequent sections we consider the case where it is impossible to write explicit contracts contingent on r.

There is a continuum of ex ante identical agents. Each agent has an endowment of one unit of the good at date 0 and none at dates 1 and 2. Agents are subject to a time-preference shock at date 1. A fraction of them become *early consumers*, who only value consumption at date 1 and the remainder of them become *late consumers*, who only value consumption at date 2. For simplicity, we assume that there are equal numbers of early and late consumers and that each consumer has an equal chance of belonging to each group. The size of each group is normalized to one. Thus, the agent's utility function can be written as

$$U(c_1, c_2) = u(c_1) + u(c_2)$$

where $c_t \geq 0$ is the agent's consumption at date $t = 1, 2$ and $u(\cdot)$ is a neoclassical utility function (increasing, strictly concave, twice continuously differentiable).

At date 0 all agents hold the same beliefs about the future asset returns. Uncertainty is resolved at the beginning of date 1: individual agents learn whether they are early or late consumers and the returns to the risky asset are revealed. A consumer's type is not observable, so late consumers can always imitate early consumers. Therefore, contracts explicitly contingent on this characteristic are not feasible.

Suppose that a planner were given the task of choosing an optimal risk-sharing arrangement. Since all agents are ex ante identical, it is natural for the planner to treat all agents alike and maximize their ex ante expected utility. The optimal consumption allocation will depend only on the aggregate wealth of the economy. Let (x, y) denote the optimal portfolio, where x is the investment in the risky asset and y is the investment in the safe asset. Let $(c_1(r), c_2(r))$ denote the optimal consumption allocation, where $c_t(r)$ is the consumption at date $t = 1, 2$ when r is the realization of the risky return.

The planner's problem can be defined as follows:

$$\begin{array}{ll} \max & E_R[u(c_1(r)) + u(c_2(r))] \\ \text{s.t.} & x + y \leq 2 \\ & c_1(r) \leq y \\ & c_2(r) \leq rh(x) + y - c_1(r) \\ & c_1(r) \leq c_2(r). \end{array} \tag{1}$$

The first constraint is the budget constraint at date 0, which says that the investment in safe and risky assets must be less than or equal to the endowment. The second constraint is the budget constraint at date 1, which says that consumption at date 1 must be less than or equal to the amount of the safe asset held over from date 0. The third constraint is the budget constraint at date 2, which says that consumption at date 2 must be less than the return from the risky asset $rh(x)$ plus the amount of the safe asset $y - c_1(r)$ left over from date 1. The final constraint is the incentive constraint, which says that the late consumers (weakly) prefer their own allocation to that of the early consumers. If this constraint were violated, the late consumers would pretend to be early consumers, receive $c_1(r)$ at date 1, save it in the form of the safe asset until date 2, and then consume it.

The preferences and technology are assumed to satisfy the inequalities

$$E[r]h'(0) > 1 \tag{2}$$

and

$$u'(0) > E[u'(rh(2))rh'(2)]. \tag{3}$$

The first inequality ensures that a positive amount of the risky asset is held while the second ensures a positive amount of the safe asset is held.

In solving the planner's problem, it turns out that we can ignore the incentive constraint. To see this, we drop the constraint and solve the unconstrained problem. From the first-order conditions, we see that a necessary condition for an optimum is that the consumption of the early and late consumers be equal, unless the budget constraint $c_1(r) \leq y$ is binding, in which case it follows from the first-order conditions that $c_1(r) = y \leq c_2(r)$. Thus, the incentive constraint will always be satisfied if we optimize subject to the first three constraints only and the solution to the planner's problem is in fact the first-best allocation.

Proposition 1 *The solution $(x, y, c_1(\cdot), c_2(\cdot))$ to the planner's problem is uniquely characterized by the following conditions:*

$$c_1(r) = c_2(r) = \frac{rh(x) + y}{2} \text{ if } y \geq rh(x),$$

$$c_1(r) = y, c_2(r) = rh(x) \text{ if } y \leq rh(x),$$

$$x + y = 2$$

and

$$E[u'(c_1(r))] = E[u'(c_2(r))rh'(x)].$$

Under the maintained assumptions, the optimal portfolio must satisfy $x > 0$ and $y > 0$. The allocation is first-best efficient.

Proof. See the Appendix.

The optimal allocation is illustrated by Figure 1, which plots consumption at each of the two dates against r. At date 0, the portfolio (x, y) is chosen to equate the expected marginal utilities of early and late consumers. Suppose that at date 1 it is found that $r = 0$. The optimal allocation divides the available output y between the early and late consumers. As r increases, both early and late consumers receive equal but higher consumption levels. Eventually, asset returns reach a level $\bar{r}$ such that $\bar{r}h(x) = y$. For $r > \bar{r}$, it is no longer possible to equate consumption at the two dates. Most of the output is now produced at date 2 instead of date 1. Whereas it is technologically feasible to carry output forward through time, it is not physically possible to do the reverse. The best that can be done is to give all the output available at date 1, that is, y to the early consumers. The late consumers receive everything produced at date 2, that is, $rh(x)$.

It is a well-known result that the allocation achieved by a classical stock market is inefficient (Jacklin (1987)) in a model with individual liquidity shocks. In such a market, agents invest their individual endowments in the long and short assets to provide for consumption at dates 1 and 2, but this provides no insurance against the intertemporal preference shock. If they invest in the short asset to provide consumption at date 1, they miss out on the higher returns from the long asset. If they invest in the long asset to provide consumption at date 2, they run the risk of having to sell the asset at a low price to provide consumption at date 1. The absence of an effective market for insuring individual preference shocks means that the first-best cannot be implemented using the stock market alone.

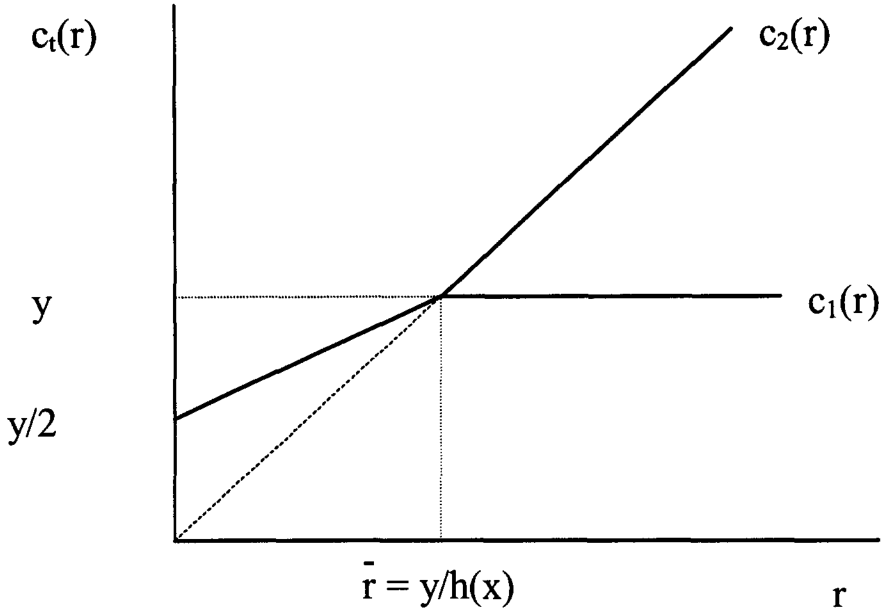
3 Banking

We next consider the risk-sharing that can be achieved through a competitive banking system, in which individual banks purchase assets to provide for the future consumption of depositors. The country is assumed to have a large number (continuum) of banks. Competition among banks leads them to maximize the expected utility of the typical depositor subject to a zero-profit (feasibility) constraint. Agents who live in the country only have access to domestic banks.

Banks are assumed to take deposits from agents at date 0 and offer them a deposit contract specified in real terms promising $d_1 \geq 0$ units of consumption at date 1 and $d_2 \geq 0$ units of consumption at date 2. It is crucial here, as in all the literature on bank runs, that the deposit contract is not explicitly contingent on the returns to the risky assets. When the returns to the risky assets are low, the banks may not be able to meet their commitments to pay out fixed amounts to their depositors. In that case, what the banks do pay out depends on the rules governing the banks' behavior and the

Figure 1

The Efficient Allocation in a Closed Economy



possibility/necessity of liquidating assets. A banking panic may result.

Allen and Gale (1998) show that such banking panics can, in fact, be beneficial when the risky asset is completely illiquid. When the return on the risky asset is low, optimal risk-sharing requires that the consumption of both the early and late consumers be reduced. A banking panic achieves this end. Some of the late consumers join the early consumers in withdrawing their deposits. Given the limited amount of liquidity available for those withdrawing at the first date, the amount each agent receives is smaller the greater is the number of premature withdrawals by late consumers. Panics allow deposit contracts to be de facto contingent on r. The optimality of bank runs in this model depends crucially on the assumption that assets cannot be liquidated prematurely. When liquidation is possible and costly, things are not quite so simple.

Suppose that the risky asset can be liquidated at date 1 and that this premature liquidation is costly. Here we simplify the analysis by assuming that premature liquidation costs are a fixed proportion of the return at maturity.[2] More precisely, if the return on the risky asset is r at date 2, x units of the asset can be liquidated at date 1 for a return of $\gamma r h(x)$, where $0 < \gamma < 1$. Note that we are assuming that all or none of the risky asset is liquidated.

If the costs of liquidation are small, it may sometimes be optimal to use the liquidation technology to provide liquidity, rather than holding the short asset. We rule out this possibility by assuming that

$$u(c_1(r)) + u(c_2(r)) > 2u\left(\frac{y + \gamma r h(x)}{2}\right), \forall r \in [r_0, r_1], \tag{4}$$

where (x, y) is the optimal portfolio from the planner's problem and $(c_1(r), c_2(r))$ is the optimal consumption allocation.

Next, we specify the bankruptcy rules that govern the bank's behavior if it cannot meet its obligations. If it can pay d_1 to depositors demanding withdrawal at date 1 it must do so, even if that means liquidating its holding of the risky asset at a loss; if it cannot pay d_1 to all the depositors demanding withdrawal at date 1, it must liquidate all its assets and pay out the liquidated value to the depositors at date 1. Obviously, in this last case, there will be nothing left for depositors at date 2, so all depositors, whether early or late consumers, will withdraw at date 1. In other words, there will be a run on the bank.

The assets remaining in the bank at date 2 are paid out to the remaining depositors. Hence, it is optimal for the bank to choose d_2 large enough so that nothing is left over after the late consumers have been paid. Since

[2] In Allen and Gale (1998), we show how liquidation values can be endogenized by introducing a market for the risky asset at date 1. The return on the liquidated asset is then determined by the price at which it can be sold at short notice on the asset market.

only premature liquidation is costly, there are no deadweight losses from insolvency at date 2. In what follows we assume without loss of generality that $d_2 \equiv \infty$ and write d in place of d_1.

As a result of these assumptions, there will be a critical value of r at which the bank is just able to avoid a run. To avoid a run, it must be possible to give both early and late consumers d units of consumption. Given (4) it will never be optimal for the bank to choose $d > y$. It would be better for the bank to increase y and avoid the need to liquidate the long asset. Thus, in equilibrium we have $d \leq y$, that is, liquidation only occurs when there is a run.

The consumption of late consumers in the absence of a run is

$$c_2(r) = rh(x) + y - d.$$

Let r^* denote the critical value of r defined by the condition $c_2(r) = d$. Then r^* is implicitly defined by

$$d = r^*h(x) + y - d.$$

For $r \geq r^*$, the early consumers receive d and the late consumers receive

$$c_2(r) = rh(x) + y - d.$$

For $r < r^*$, all consumers receive an equal share of the liquidated value of the assets at date 1:

$$c_1(r) = c_2(r) = \frac{\gamma rh(x) + y}{2}.$$

With these assumptions, the bank's decision problem can be written as follows:

$$\begin{array}{ll} \max & E_R[u(c_1(r)) + u(c_2(r))] \\ \text{s.t.} & x + y \leq 2 \\ & c_1(r) = d, \forall r \geq r^* \\ & c_2(r) = rh(x) + y - d, \forall r \geq r^* \\ & c_1(r) = c_2(r) = \frac{1}{2}(\gamma rh(x) + y), \forall r < r^* \\ & r^* = (2d - y)/h(x). \end{array}$$

Assuming that (4) is satisfied, so the planner does not want to use the liquidation technology at the optimum, we can compare the solution of the planner's problem directly with the solution of the typical banker's problem and conclude that the two are different if there is a positive probability of liquidation.

Proposition 2 *Let* $(x, y, c_1(r), c_2(r))$ *be the solution to the planner's problem and let* $(\hat{x}, \hat{y}, \hat{c}_1(r), \hat{c}_2(r))$ *be the solution to the bank's problem above. If condition (4) is satisfied, the solution to the planner's problem does not require premature liquidation of the long asset. If* $\Pr[r < r^*] > 0$ *then the solution to the bank's problem yields depositors a lower ex ante expected utility than they obtain in the first-best allocation.*

Figure 2 illustrates the allocation provided by a banking system using real deposit contracts. The optimal consumption allocation has the same general form as in Figure 1 with one important difference. When $r < r^*$ there is costly liquidation of the risky asset, resulting in a discontinuity at r^*. The portfolio (x, y) chosen by the bank may also be different. A positive probability of liquidation reduces the marginal returns to investment in the risky asset, so the amount invested by the bank may be lower. It is also possible that the bank will choose $y > d$. This "buffer stock" of the safe asset reduces r^* and hence reduces liquidation costs.

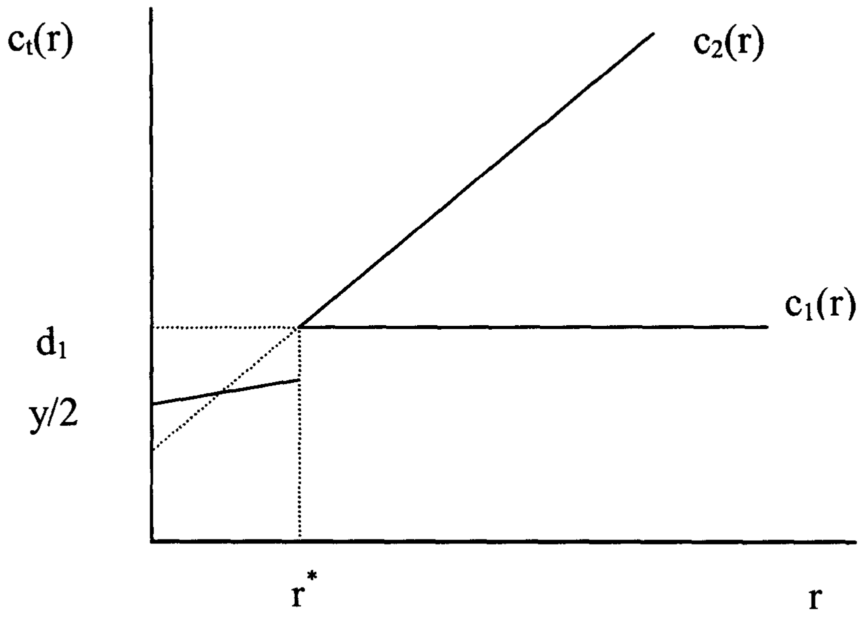
3.1 *Optimal monetary policy*

The inefficiency of equilibrium with bank runs arises from the fact that liquidating the risky assets at date 1 is costly. Costly liquidation can be avoided if the deposit contract is specified in nominal terms and the central bank adopts a monetary policy that has the effect of making the price level contingent on the state of nature. In the previous version of our model, banks are restricted to use a deposit contract that promises a constant amount of *consumption* in every state of nature (except in states where the bank defaults). Now we assume that a deposit contract promises a constant amount of the *domestic currency* in every state of nature. The real value of this deposit contract will depend on the price level and since the price level is contingent on the state of nature, so is the real value of the deposit contract. In short, we have replaced a deposit contract that is noncontingent in real terms with a deposit contract that is noncontingent in nominal terms and contingent in real terms. If the central bank chooses its monetary policy appropriately, that is, if it introduces the appropriate variation in the price level, the banks can use the deposit contract to avoid costly bank runs and achieve efficient risk-sharing.

Formally, a deposit contract promises the depositor D_1 units of money if the depositor withdraws in the middle period and D_2 if the depositor withdraws in the final period. (Nominal amounts are denoted by upper case variables.) As before, there is no loss of generality in assuming that D_2 is chosen large enough that the depositors receive whatever assets the representative bank has left in the final period. In the sequel, we set $D_2 \equiv \infty$ and write D for D_1.

Figure 2

Banking Equilibrium without a Central Bank in a Closed Economy



Let $p_t(r)$ denote the price level at date $t = 1, 2$ when the return on the risky asset is r. In what follows, it simplifies matters to note that

$$p_1(r) = p_2(r) = p(r).$$

This follows from a no-arbitrage argument. Note that when $p_1(r) = p_2(r)$, the return on holding money between date 1 and date 2 is the same as the return on the safe asset. By contrast, if $p_1(r) > p_2(r)$ then banks will only be willing to hold money while if $p_1(r) < p_2(r)$ they will only be willing to hold the short asset (store goods).

Let $(x, y, c_1(\cdot), c_2(\cdot))$ denote the solution to the planner's problem in Section 2 and suppose that at date 0 the representative bank chooses the portfolio (x, y). The central bank determines the price level $p(r)$ by promising to exchange money for goods at a ratio of $p(r)$. Since r is publicly observable, the central bank is able to implement such a policy. The individual banks take $p(r)$ as given. If $p(r)$ is chosen to be inversely proportional to $c_1(r)$, then the banks will choose D so that

$$\frac{D}{p(r)} = c_1(r). \tag{5}$$

For example, we could choose the deposit contract so that $D = y$. The price level that implements the first-best allocation is

$$p(r) = \begin{cases} 1 & \text{for } r \geq \overline{r} \\ \frac{2D}{rh(x)+y} & \text{for } r < \overline{r}. \end{cases}$$

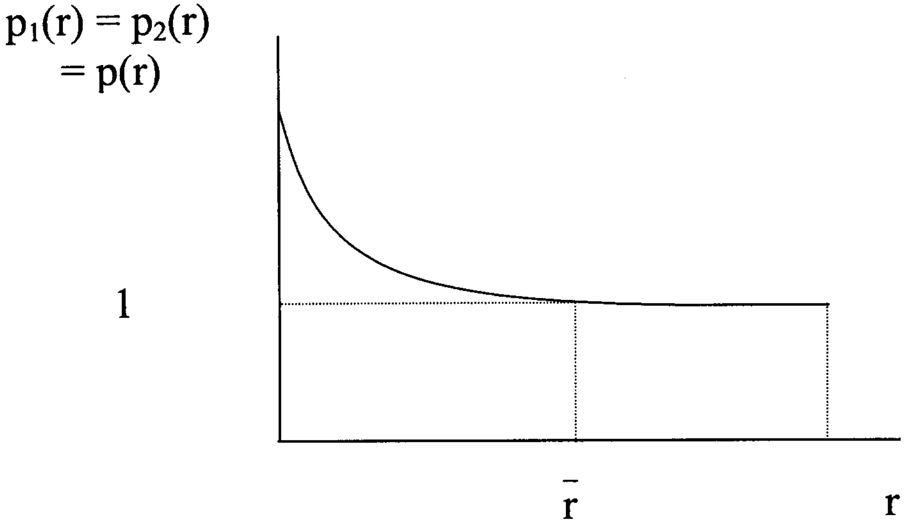
This is illustrated in Figure 3. For $r \geq \overline{r}$, the central bank fixes the price level at 1 by promising to exchange money for goods at this ratio. For $r < \overline{r}$, the central bank sets the price level equal to the ratio of the nominal claims on the banking system $2D$ to the real output from the banking system's assets $rh(x) + y$.

To show that (x, y, D) is optimal for the bank's decision problem, we simply appeal to the fact that $(x, y, c_1(r), c_2(r))$ solves the planner's problem. Thus, there is no better allocation $(x, y, c_1(r), c_2(r))$ satisfying the constraints of the planner's problem. It is easy to show that anything that is feasible for the bank must also satisfy the planner's constraints. Thus, it cannot do better than the solution to the planner's problem.

Proposition 3 *If the central bank chooses the appropriate monetary policy (one that makes the price level contingent on the state of nature) and banks use nominal deposit contracts, the solution to the bank's decision problem implements the first-best allocation.*

Figure 3

The Price Level in a Closed Economy



4 International finance

The closed economy can be "opened" by assuming the existence of an international bond market. The economy is assumed to be small relative to the rest of the world, so the value of foreign currency is fixed in terms of the consumption good and interest rates are also fixed. For simplicity one unit of the foreign currency is normalized so that it purchases one unit of the consumption good. Initially, we assume that short-term bonds are used in the international bond market. The introduction of long-term bonds is considered later. The risk-free return on the short-term bonds is fixed at $\rho \geq 1$. This means that one unit of the good at date t can be exchanged for ρ units at date $t+1$ for $t = 0, 1$. Of course, the risk of default will be reflected in the face value of any debt that is issued by the banks of the small country. We assume that because the country is small, the international bond market is risk neutral in the sense that, when there is a risk of default, the loan is priced so that the expected return is ρ. The banks of the small country can also invest in the international bond market. International bonds now replace the storage technology as the safe asset.

To guarantee there is positive investment in the risky asset, it is necessary that (2) be replaced by

$$E[r]h'(0) > \rho^2.$$

In the closed-economy version of the model, condition (3) ensures that banks invest some of the deposits in the short asset. This condition is no longer imposed.

Access to international capital markets is then potentially valuable for three reasons.

- First, because the return on bonds is lower than the expected return on the risky asset, banks can make a profit for their depositors by borrowing short in the international market and investing the proceeds in the risky asset at date 0.

- Second, it means that when r is high, liquidity can be obtained for early consumers by borrowing in the international market at date 1.

- Third, it may be possible to transfer the small country's asset return risk to lenders in the international bond market.

It is important to note that individuals also have access to the international bond market. In the closed-economy version, we assumed that individuals had access to the storage technology. Here they can buy or sell bonds, that is, they can lend or borrow at the rate ρ.

4.1 *Optimal risk-sharing*

As usual, we start by characterizing the first-best allocation assuming the planner can use contracts which are contingent on r. If a planner in the small country can write state-contingent contracts with the international capital market, he can transfer all risk to the foreigners and give depositors a constant amount of consumption, independent of r, at each date. Let $I(r)$ be the transfer from the international capital market contingent on r. The planner's problem is

$$\begin{array}{ll} \max & E_R[u(c_1(r)) + u(c_2(r))] \\ \text{s.t.} & x + y \leq 2 \\ & c_1(r) + c_2(r)/\rho \leq rh(x)/\rho + \rho y + I(r) \\ & \int_0^\infty I(r)dF = 0 \\ & \rho c_1(r) \leq c_2(r). \end{array}$$

The first constraint is the familiar budget constraint at date 0. The second is the present-value budget constraint covering dates 1 and 2: for each value of r, the present value of consumption must be less than or equal to the present value of asset returns plus the state-contingent transfer from the international capital market. The third constraint ensures that the expected state contingent transfer is zero. The final constraint is the incentive constraint. Since all consumers have access to the international capital market, the late consumers can withdraw $c_1(r)$ at date 1, invest it in the international bond market, and consume $\rho c_1(r)$ at date 2. We assume that early consumers cannot imitate late consumers, borrowing against an anticipated future withdrawal at date 2.

Since consumption must be non-negative, a feasible transfer function $I(r)$ must satisfy

$$rh(x)/\rho + \rho y + I(r) \geq 0.$$

It will be seen that this condition is automatically satisfied by the solution to the planner's problem. The first-order conditions for this problem are

$$u'(c_1(r)) \geq \rho u'(c_2(r)), \forall r,$$

with equality if $\rho c_1(r) < c_2(r)$,

$$\int_0^\infty u'(c_1(r)) \left(\frac{r}{\rho} h'(x) - \rho\right) dF = 0$$

and

$$u'(c_1(r)) = \lambda, \forall r,$$

where λ is the Lagrange multiplier on the third constraint.

The third condition implies that $c_1(r)$ is a constant, independently of r. The following result then follows directly.

Proposition 4 *The incentive-efficient allocation with access to complete, risk-neutral international capital markets has a consumption allocation*

$$(c_1(r), c_2(r)) = (\bar{c}_1, \bar{c}_2)$$

where the ordered pair $(\bar{c}_1, \bar{c}_2)$ solves the problem

$$\begin{array}{ll} \max & u(\bar{c}_1) + u(\bar{c}_2) \\ \text{s.t.} & \bar{c}_1 + \bar{c}_2/\rho = E[r]h(\bar{x})/\rho + \rho(2 - \bar{x}) \\ & \rho\bar{c}_1 \leq \bar{c}_2 \end{array}$$

and the amount $\bar{x} > 0$ invested in the risky asset satisfies

$$E[r]h'(\bar{x}) = \rho^2.$$

Since the international capital market is risk neutral and domestic depositors are risk averse, the optimal allocation imposes no risk on domestic consumers when the planner can enter into state-contingent contracts. The international capital market bears all the risk. The investment in the risky asset equates the expected marginal product to the opportunity cost of funds.

In Section (3) the incentive constraint does not bind. Because the return to the short asset is 1, the first-order condition for an optimal consumption allocation implies that $c_1(r) \leq c_2(r)$ and the incentive constraint is automatically satisfied. When the return on the short asset is $\rho > 1$ the incentive may or may not bind, depending on the curvature of the utility function. The consumption allocation $(\bar{c}_1, \bar{c}_2)$ that solves the planner's problem satisfies a Kuhn-Tucker condition

$$u'(\bar{c}_1) \geq \rho u'(\bar{c}_2)$$

and this holds as an equation when the incentive constraint does not bind, i.e., when $\rho\bar{c}_1 \leq \bar{c}_2$. So whether the incentive constraint binds depends on whether the solution of the first-order condition

$$u'(\bar{c}_1) = \rho u'(\bar{c}_2) \tag{6}$$

is consistent with the incentive constraint. Suppose that $u(\cdot)$ has constant relative risk aversion:

$$u(c) = c^{1-a}/(1-a).$$

Then a solution to the first-order condition (6) satisfies

$$\rho^{\frac{1}{a}}\bar{c}_1 = \bar{c}_2,$$

which implies that $\rho\bar{c}_1 \lesseqgtr \bar{c}_2$ as $a \lesseqgtr 1$. Intuitively, the first-order condition (6) requires the ratio of the marginal utilities $u'(\bar{c}_1)/u'(\bar{c}_2)$ to be ρ, whereas the incentive constraint requires the ratio of consumption $\bar{c}_2/\bar{c}_1$ to be at least ρ. When the marginal utility is elastic, the first-order condition (6) implies that the ratio of consumption $\bar{c}_2/\bar{c}_1$ will be less than ρ, thus violating the incentive constraint. So the incentive constraint binds when marginal utility is elastic and does not bind when marginal utility is inelastic.

4.2 *Domestic currency debt*

We next consider the allocation of resources when the international capital market is a debt market and there is a domestic banking system that issues deposits denominated in the domestic currency. For the moment we assume that domestic banks can issue debt denominated in the domestic currency on the international capital market. A bond issued at date 0 promises one unit of the domestic currency to the holder at date 1. Let q denote the price of one domestic currency bond and let B denote the number of bonds issued at date 0. The benefits of borrowing at a low rate in the international bond market are passed on to the depositors in the form of a more attractive deposit contract $(c_1(r), c_2(r))$. We assume, as before, that competition in the banking sector ensures that each bank seeks to maximize the expected utility of the typical depositor.

For simplicity, we assume that the nominal domestic interest rate is 0. Arbitrage between foreign-currency bonds and domestic currency ensures that

$$p_1(r) = \rho p_2(r) \tag{7}$$

for every value of r, where $p_t(r)$ is the domestic price level at date $t = 1, 2$. In what follows we write $p(r)$ for the price level at date 1 and set $p_2(r) \equiv p(r)/\rho$.[3]

Suppose the representative bank chooses a portfolio (x, y) at date 0, where x is the investment in the risky asset and y is the investment in the safe asset (debt denominated in the international reserve currency). The bank takes in a deposit of 1 from each of the consumers and in return gives each a nominal claim of D_1 at date 1 and D_2 at date 2. As usual, we can assume without loss of generality that D_2 is chosen so large that the late withdrawers get whatever assets are left over at date 2. In what follows, we write D in place of D_1 and assume that $D_2 = \infty$.

The bank borrows qB of the consumption good on the international cap-

[3]With the domestic interest rate normalized to 0, the domestic price level must fall between date 1 and date 2 in order to satisfy the covered interest arbitrage condition (7). A different normalization would imply a different rate of inflation. For example, if we set the domestic interest rate equal to $\rho - 1$, then the domestic inflation rate would be 0.

ital market so the budget constraint at date 0 is

$$x + y \leq 2 + qB. \tag{8}$$

At date 1, r is observed and investors learn whether they are early or late consumers. The bank has promised early withdrawers D units of the domestic currency and the international bondholders B. The real values of these claims are $D/p(r)$ and $B/p(r)$, respectively. Since the bank can borrow and lend at the rate ρ (now that uncertainty has been resolved), the present value budget constraint at dates 1 and 2 can be written as

$$c_1(r) + \frac{c_2(r)}{\rho} + \frac{B}{p(r)} = \frac{rh(x)}{\rho} + \rho y.$$

The bank must also satisfy the incentive constraint

$$\rho c_1(r) \leq c_2(r).$$

Otherwise, late consumers can withdraw D units of currency at date 1 and spend it on $D/p_2(r) = \rho D/p(r) = \rho c_1(r)$ units of goods at date 2, thus increasing their utility. Using the definition of $c_1(r) = D/p(r)$ and the budget constraint, we can rewrite the incentive constraint as

$$\frac{D}{p(r)} \leq \frac{c_2(r)}{\rho} = \frac{rh(x)}{\rho} + \rho y - \frac{D+B}{p(r)}$$

or

$$\frac{2D+B}{p(r)} \leq \frac{rh(x)}{\rho} + \rho y. \tag{9}$$

Conversely, if this constraint is satisfied, then the budget constraint and the incentive constraint can also be satisfied with $c_1(r) = D/p(r)$.

If it is possible to satisfy all of these constraints at date 1, then the bank is solvent and there is no need to liquidate the risky asset. (We are assuming that runs do not occur unnecessarily—if there exists an equilibrium without runs, we assume that such an equilibrium obtains). However, if (9) is not satisfied, then it is impossible to satisfy the budget constraint and the incentive constraint simultaneously. The real value of claims on the bank is greater than the value of its assets and the bank must declare bankruptcy. All assets are liquidated to meet the claims of the domestic depositors and the international bondholders. The liquidated value of the assets is distributed in proportion to the creditors' claims. The depositors each receive a fraction $D/(2D+B)$ of the asset value and the international bondholders receive the rest. Then

$$c_1(r) = c_2(r)/\rho = \frac{D}{2D+B}\left(\gamma rh(x) + y\right).$$

Note that, although early and late consumers receive equal shares of the liquidated assets of the bank, the late consumers can invest in the international bond market, so their share of the liquidated assets yields them a higher consumption level at date 2.

The bank takes prices as given and chooses a portfolio (x, y, B) and a deposit contract D to maximize the expected utility of the typical depositor. Thus, the bank's decision problem is

$$\begin{array}{ll} \max & E_R[u(c_1(r)) + u(c_2(r))] \\ \text{s.t.} & x + y \leq 2 + qB \\ & \rho c_1(r) \leq c_2(r), \forall r, \end{array} \tag{10}$$

where the consumption functions of the early and late consumers are given by the equations

$$c_1(r) = \begin{cases} \frac{D}{p(r)} & \text{if } \frac{2D+B}{p(r)} \leq \frac{rh(x)}{\rho} + \rho y \\ \frac{D}{2D+B}(\gamma r h(x) + y) & \text{otherwise.} \end{cases} \tag{11}$$

and

$$\frac{c_2(r)}{\rho} = \begin{cases} rh(x)/\rho + \rho y - \frac{D+B}{p(r)} & \text{if } \frac{2D+B}{p(r)} \leq \frac{rh(x)}{\rho} + \rho y \\ \frac{D}{2D+B}(\gamma r h(x) + y) & \text{otherwise,} \end{cases} \tag{12}$$

respectively.

4.3 *Optimal exchange-rate policy*

In the closed economy of Section 3.1, the central bank makes the real value of deposits contingent on the state of nature by controlling the price level. Using this contingent deposit contract, the banking system avoids financial crises and achieves optimal risk-sharing between early and late consumers.

In the open economy, the central bank is assumed to control the exchange rate $e(r) \equiv 1/p(r)$ (i.e., foreign currency per unit of domestic currency). The central bank's exchange-rate policy makes the real values of the deposit contract and the domestic-currency bond contingent on the state of nature at date 1. The state-contingent variation in the real value of domestic debt allows the banks to shift risk to the international market. Banks borrow from the international market by issuing domestic-currency bonds and invest the proceeds in foreign-currency bonds. Because of the state-contingent exchange-rate policy, domestic-currency bonds and foreign-currency bonds have different state-contingent real returns. Domestic banks, by holding an optimal portfolio of the two kinds of bonds, share risk between the domestic depositors and the international capital market.

Exactly how optimal risk-sharing is achieved is a complicated story. We begin by exploring the possibilities for sharing risk from the point of view of an individual bank. A single bank, taking as given the central bank's policy and the behavior of other banks, can eliminate risk for its depositors by simultaneously issuing domestic currency debt and buying foreign-currency debt. This cannot be achieved in equilibrium, however, because each bank wants to issue more debt than each of the others. To analyze the equilibrium possibilities for risk-sharing, we assume a bank's access to the international market is artificially constrained. In the artificially constrained equilibrium, banks choose to borrow the maximum amount. As the borrowing constraint is relaxed, the banks increase their borrowing and, in the limit, achieve perfect risk-sharing.

To gain some insight into the structure of equilibrium, consider the following situation. Suppose the representative bank chooses $(x, y, \bar{B}, D)$ and the incentive constraint is binding for all values of r. Rearranging (9) with an equality and using the date 0 budget constraint gives

$$\frac{1}{p(r)} = \frac{rh(x)/\rho + \rho(2 + q\bar{B} - x)}{2D + \bar{B}}. \tag{13}$$

The central bank implements this exchange rate, which is simply the ratio of the representative bank's (real) asset returns to the nominal claims on it.

The equilibrium price q at which domestic-currency bonds promising to repay 1 unit of domestic currency are issued must on average allow the lenders to recoup ρq on each bond. Since there is no risk of default if (13) holds, the fair-pricing condition is

$$q = \frac{1}{\rho} \int_0^\infty \frac{1}{p(r)} dF. \tag{14}$$

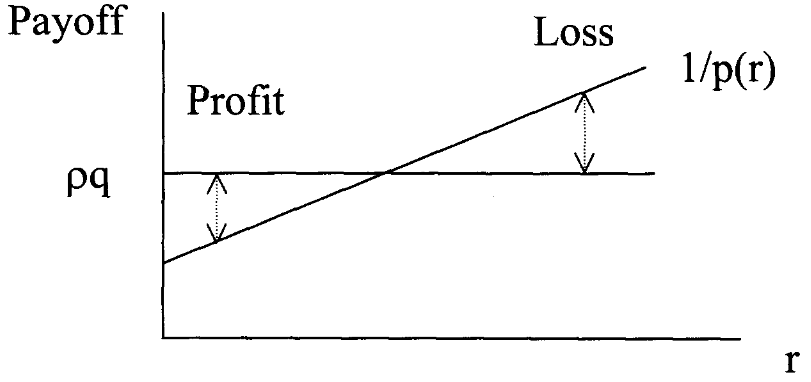
From (11), (12), and (13) each bank choosing $(x, y, \bar{B}, D)$ is able to give its depositors a consumption allocation

$$c_1(r) = \frac{c_2(r)}{\rho} = \frac{D}{2D + \bar{B}} \left(\frac{rh(x)}{\rho} + \rho(2 + q\bar{B} - x) \right).$$

It can be seen that the exchange rate is positively correlated with consumption. This suggests that by issuing domestic-currency bonds and putting the proceeds in foreign-currency bonds, a bank can create a portfolio which is negatively correlated with consumption. This is illustrated in Figure 4. Suppose at date 0 the bank issues a bond promising to pay 1 unit of domestic currency at date 1. This will raise q units of consumption which can be invested in foreign-currency bonds to give ρq units of foreign currency at date 1. At date 1 the bank will owe 1 unit of domestic currency which is equivalent to $1/p(r)$ units of foreign currency. The net payoff on the portfolio is $\rho q - 1/p(r)$. There is a profit when r is low and a loss when r is high. Since bonds are fairly priced the expected payoff on the portfolio is zero and

Figure 4

The Payoff $\rho q - 1/p(r)$ from Issuing Domestic-currency Bonds
and Investing in Foreign-Currency Bonds



its only effect is to transfer funds from high payoff states to low payoff states. Since anything left over at date 2 is consumed by the late consumers, using this portfolio in addition to $(x, y, \bar{B}, D)$ will allow the bank to improve their welfare and hence ex ante expected utility by reducing the variability of their consumption.

In fact since in this particular case $1/p(r)$ and $c_2(r)$ are both linear in r, it is possible for the bank to eliminate all risk in $c_2(r)$. To see this, suppose the bank chooses x and D the same as other banks but borrows B in domestic currency and invests $2 + qB - x$ in foreign bonds. Using (12) and (13) gives

$$\frac{c_2(r)}{\rho} = \frac{D + \bar{B} - B}{2D + \bar{B}}\left(\frac{rh(x)}{\rho} + \rho(2 - x)\right) + \rho q D \frac{2B - \bar{B}}{2D + \bar{B}}$$

By setting

$$B = \bar{B} + D \tag{15}$$

it is possible to eliminate all risk for the late consumers and

$$\frac{c_2(r)}{\rho} = \rho q D = E_R[c_1(r)]$$

where the second equality follows from (11) and (14). Since D is held constant this means that the ex ante expected utility of depositors is raised.

In general, it is not possible for banks collectively to hedge all the risk in this way. First, the incentive-constraint may not be binding for some or all values of r, in which case the real returns to the domestic-currency bonds will not be a linear function of r. In that case, shorting domestic-currency bonds will not provide a perfect hedge for this risk. Secondly, (15) shows that in order to hedge the risk perfectly, each bank has to issue more bonds than the other banks. This alerts us to the fact that existence of equilibrium may be problematical, unless we find some way to limit the issue of domestic currency bonds.

We adopt the following strategy for analyzing "equilibrium" in the limiting case where B becomes very large. We restrict the borrowing of the representative bank so that $B \leq \bar{B}$, where $\bar{B}$ is an exogenously-imposed bound on borrowing in terms of the domestic currency. The representative bank chooses (x, y, B, D) taking the exchange-rate policy of the central bank as given. For the moment, we restrict banks to choose (x, y, B, D) so that runs do not occur. Then the banks' modified decision problem is:

$$\begin{array}{ll} \max & E_R[u(c_1(r)) + u(c_2(r))] \\ \text{s.t.} & B \leq \bar{B} \\ & x + y \leq 2 + qB \\ & c_1(r) = D/p(r) \\ & c_2(r) = rh(x) + \rho^2 y - \rho(D + B)/p(r) \\ & \rho c_1(r) \leq c_2(r). \end{array} \tag{16}$$

This problem has a concave objective function and a convex feasible set for any price function $p(r)$.

Define a *pseudo-equilibrium* to be an array $(x, y, B, D, q, p(\cdot))$ such that (x, y, B, D) solves the problem (16) for the given values of $(q, p(\cdot))$ and q satisfies the fair-pricing condition (14). Condition (14) is the appropriate condition since runs are not allowed in a pseudo-equilibrium. The representative bank is maximizing the expected utility of the investors, as required in an ordinary equilibrium, subject to two additional constraints, one being the limit on domestic-currency borrowing and the other being the no-runs condition. The first of these we can treat as a regulatory requirement for the moment. The no-runs condition will later be shown to be optimal when the borrowing limit $\bar{B}$ is sufficiently large.

First, we note that a pseudo-equilibrium exists for each possible borrowing limit $\bar{B} > 0$.

Proposition 5 *For any value of $\bar{B} > 0$, there exists a pseudo-equilibrium $(x, y, B, D, q, p(\cdot))$ such that $B = \bar{B}$, $D = 1$, and for each value of r the consumption allocation $(c_1(r), c_2(r))$ solves the problem:*

$$\begin{array}{ll} \max & u(c_1(r)) + u(c_2(r)) \\ \text{s.t.} & c_1(r) + c_2(r)/\rho = rh(x)/\rho + \rho(2 + qB - x) - B/p(r) \\ & \rho c_1(r) \leq c_2(r). \end{array}$$

Proof. See the Appendix.□

The pseudo-equilibrium described in Proposition (5) has three special features:

- the consumption allocation satisfies the conditions analogous to those in Proposition (4);
- the nominal value of a deposit is normalized to 1;
- every bank borrows the maximum on the international bond market.

The fact that the consumption allocation satisfies necessary conditions for incentive-efficiency, given the other choices of the bank, reflects the way in which prices are chosen, that is, the exchange-rate policy attributed to the central bank. In order for a feasible (incentive-compatible) consumption allocation $(c_1(r), c_2(r))$ to solve the maximization problem in the proposition, the following conditions are necessary and sufficient:

$$u'(c_1(r)) \geq \rho u'(c_2(r)),$$

and

$$u'(c_1(r)) = \rho u'(c_2(r)) \text{ if} \rho c_1(r) < c_2(r).$$

As in Proposition 4, the fact that the incentive constraint may or may not be binding complicates the analysis. Another way of expressing the conditions is to say that

$$c_2(r) = \max\{\rho c_1(r), \varphi(c_1(r))\},$$

where $\varphi(\cdot)$ is defined implicitly by the equation $u'(z) = \rho u'(\varphi(z))$. To ensure that these conditions are satisfied in equilibrium, we choose the price function $p(r)$ so that

$$\max\left\{\rho\frac{D}{p(r)}, \varphi\left(\frac{D}{p(r)}\right)\right\} = rh(x) + \rho^2(2 + q\bar{B} - x) - \rho\frac{1+\bar{B}}{p(r)}. \tag{17}$$

This equation determines the price function $p(r)$ uniquely for any values of x and q.

This policy is not necessarily optimal and it is certainly not the only policy that the central bank could have chosen. We adopt it here because it is salient (suggested by Proposition 4) and because it is consistent with an incentive-efficient outcome in the limit, as the next proposition shows.

Normalizing the face value of the deposit to 1 is equivalent to normalizing prices. It ensures that the nominal constraint $B \leq \bar{B}$ on borrowing is a real constraint (equi-proportionate changes in D, B, and $p(\cdot)$ leave the pseudo-equilibrium conditions unchanged). As a result, the fact that banks borrow the maximum amount $B = \bar{B}$ has real content: banks want to shift the maximum amount of risk to the international market and in fact would like to borrow more if they were allowed to do so.

The next proposition shows that, as the borrowing limit $\bar{B}$ increases, all risk is shifted from the domestic economy to the international capital market.

Proposition 6 *Let $\{\bar{B}^k\}$ be an increasing sequence of bounds such that $\bar{B}^k \to \infty$ and let $\{(x^k, y^k, 1, \bar{B}^k, q^k, p^k(\cdot))\}$ be the corresponding sequence of pseudo-equilibria described in Proposition 5. Then for all values of r,*

$$\begin{aligned}
\bar{p} &= \lim_{k\to\infty} p^k(r), \\
(\bar{c}_1, \bar{c}_2) &= \lim_{k\to\infty} (c_1^k(r), c_2^k(r)), \\
\bar{x} &= \lim_{k\to\infty} x^k,
\end{aligned}$$

where $(\bar{c}_1, \bar{c}_2)$ is the incentive-efficient consumption allocation from Proposition 4 and $\bar{x}$ is the efficient investment in the risky asset.

Proof. See the Appendix. □

The proposition can be illustrated for the case where the incentive constraint binds for all r. In that case, it can be seen from (13) that

$$\bar{p} = \lim_{k \to \infty} p^k(r) = \frac{1}{\rho q}.$$

The change in the exchange-rate policy $e(r) = 1/p(r)$ as $\bar{B}^k \to \infty$ is illustrated in Figure 5.

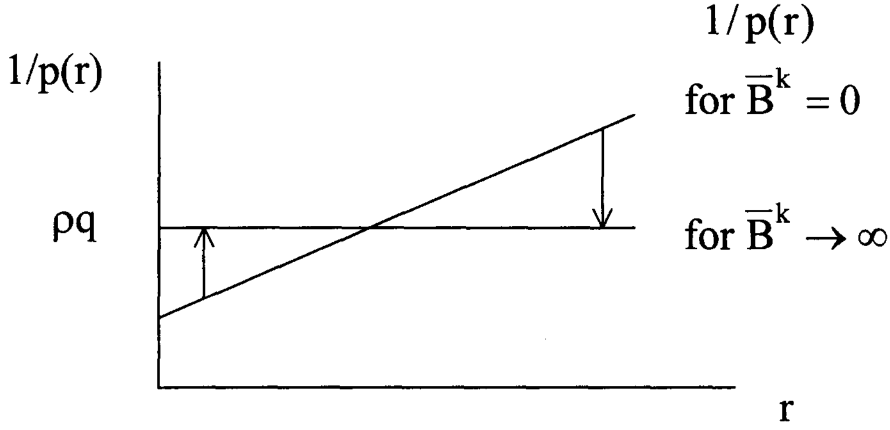
In effect, what is happening is that the individual banks construct portfolios consisting of a large investment in riskless foreign-currency debt y^k and a small investment x^k in the risky asset. Most of this portfolio is "owned" by the foreign bondholders, who hold the outstanding domestic-currency bonds $\bar{B}^k$, so the domestic investors receive a relatively small share of the returns. As a result, they bear a relatively small share of the risk generated by the returns from the risky asset.

The mechanism by which risk is transferred is rather subtle. In the limit, prices are constant at $\bar{p}$ and so the domestic-currency debt is riskless: it pays $1/\bar{p}$ for every r. However, if the banks were to issue a real bond, that is, a bond denominated in the foreign currency, none of the risk could be transferred to the international market. In order to transfer risk, the real returns to the two assets, domestic-currency bonds and foreign-currency bonds, must be different. This requires variability in the exchange rate $e(r) = 1/p(r)$. For each value of k, the early consumers who receive $c_1^k(r) = D/p^k(r)$ bear some risk. As $\bar{B}^k \to \infty$, the degree of exchange-rate variability needed to transfer the risk to the international market shrinks. In the limit, the exchange rate becomes constant and the early consumers bear no risk. There are thus two reasons why borrowing using domestic-currency bonds must be limited. One is to ensure existence of a pseudo-equilibrium and the other is to ensure enough variability in the real value of domestic bonds to transfer risk to the international market.

There is a similarity between this problem and the nonexistence of equilibrium with incomplete markets studied by Hart (1975). Hart gives an example of an economy with two states of nature, two goods, and two assets with returns represented by a fixed basket of goods. When markets are complete, the returns to the two assets in terms of the numeraire are collinear and the assets cannot be used to span the entire commodity space. When markets are incomplete, the returns to the two assets in terms of the numeraire are not collinear and the assets can be used to span the entire commodity space. Thus, markets can neither be complete nor incomplete: an equilibrium does not exist. Placing an arbitrary bound on trades in the two assets can resolve the nonexistence problem. As the bound is relaxed, the returns of the two assets will become more nearly collinear and, in order to span the entire commodity space, the trades in the two assets will grow larger as well. In

Figure 5

The Exchange Rate $1/p(r)$ as $\overline{B}^k$ Increases from 0 to ∞



the limit, even infinite trades in the assets will not suffice to make markets complete because the asset returns have become perfectly collinear. While Hart's example suffices to give substance to the nonexistence problem, we are not aware of any practical application before now.[4]

It has been noted that Hart's example, which relies on an exogenously specified matrix of asset returns, is nongeneric (see Duffie and Shafer (1985, 1986)). The asset returns in our model are endogenous so the issue of genericity does not arise.

The role of the central bank in maintaining an optimal exchange-rate rule is critical. If the central bank did not choose the price function $p^k(r)$ in the manner prescribed, the banks would not necessarily choose $B^k = \bar{B}^k$, and the risk borne by the domestic investors would not necessarily disappear even in the limit as $\bar{B}^k \to \infty$.

So far we have only considered pseudo-equilibria, in which banks are constrained to choose their portfolios and deposit contracts so that runs do not occur. However, for $\bar{B}^k$ sufficiently large, we can show that this is in fact an optimal choice. A pseudo-equilibrium $(x, y, B, D, q, p(\cdot))$ is called an *equilibrium relative to the borrowing constraint* $\bar{B}$ if (x, y, B, D) solves the maximization problem

$$\begin{array}{ll} \max & E_R[u(c_1(r)) + u(c_2(r))] \\ \text{s.t.} & B \leq \bar{B} \\ & x + y \leq 2 + qB \\ & \rho c_1(r) \leq c_2(r), \forall r. \end{array}$$

This is just the maximization problem (10) with the added borrowing constraint $B \leq \bar{B}$.

Proposition 7 *For all k sufficiently large, the pseudo-equilibrium* $(x^k, y^k, \bar{B}^k, 1, q^k, p^k(\cdot))$ *described in Proposition 6 is an equilibrium relative to the bound* $\bar{B}^k$.

Proof. The one condition that needs to be checked is whether the bank will want to violate the no-bankruptcy condition for large k. Violating the no-bankruptcy constraint involves a loss of output and a possible distortion in the allocation of consumption, but may improve risk-sharing. Because the consumption allocations and prices are becoming approximately constant as

[4] Gale (1990) identifies a similar problem in the design of optimal government debt in an overlapping generations economy. In order to achieve optimal intergenerational risk-sharing, it is necessary to make the returns to government securities contingent on the state of nature. While it is possible to get close to the first best by introducing state-contingent government securities, the first-best is unattainable. In order to span the states of nature the returns of government securities have to be unbounded.

$k \to \infty$ (Proposition 4), the gain from risk-sharing vanishes as $k \to \infty$. For k sufficiently large, the costs of bankruptcy must outweigh the benefits.□

It is interesting to note that, when implementing the first-best allocation, the representative bank must simultaneously borrow large amounts in domestic currency and then invest in foreign bonds. This is consistent with the puzzling observation that the volume of trade in foreign exchange is many times the magnitude needed to finance world trade.

4.4 *Long-term versus short-term debt*

We have considered short-term debt and have so far excluded the case of long-term debt. Suppose next that instead of borrowing qB at date 0 and repaying B at date 1 the representative bank borrows $q_L L$ at date 0 and repays L at date 2. The opportunity cost in real terms for lenders in the international bond market between dates 0 and 1 is ρ_{2L}. Thus the counterpart to (14) is

$$q_L = \frac{1}{\rho_{2L}} \int_0^\infty \frac{1}{p_2(r)} dF. \tag{18}$$

The other changes are that the date 0 budget constraint becomes

$$x + y \leq 2 + q_L L$$

and the date 1 budget constraint becomes

$$c_1(r) + \frac{c_2(r)}{\rho} + \frac{L}{\rho p_2(r)} = \frac{rh(x)}{\rho} + \rho y.$$

It is easiest to start by considering the case with a flat yield curve so that

$$\rho_{2L} = \rho^2.$$

Substituting this into (18) and using the fact that $\rho p_2(r) = p(r)$, it can be seen that $q_L = q$. Then the two budget constraints are identical to before, since we can choose $L = B$. Hence it does not matter whether short- or long-term debt is used. This is not very surprising given that all uncertainty is resolved at date 1. It does not matter whether debt is rolled over or repaid at the final date.

If $\rho_{2L} > \rho^2$ so that the yield curve is upward-sloping, then clearly long-term borrowing will be undesirable relative to short-term debt, other things equal, because it is more expensive. Of course, if the yield curve is downward sloping so $\rho_{2L} < \rho^2$, long-term debt will be superior but this is not often the empirically relevant case. This gives the following result.

Proposition 8 *When the yield curve is flat there is no difference between long-term borrowing and short-term borrowing. When it is upward (downward)-sloping, short-term debt is strictly preferred (inferior) to long-term borrowing.*

5 Foreign currency debt

The analysis in the previous section suggests that the combination of a flexible exchange rate and international debt denominated in domestic currency can lead to a first-best allocation of resources. In the advanced industrial countries such as the U.S., U.K., Japan, Germany, and France, it is possible for banks to borrow in the domestic currency and invest in foreign-currency bonds. The results in the previous section may be applicable to these countries. In contrast, in emerging economies foreign debt is usually denominated in dollars (i.e., in real terms) rather than in domestic currency. How can this be understood in the context of the current model? The problem in emerging economies is that large amounts of domestic-currency debt held by foreigners create a temptation for the government to adopt inflationary policies after debt contracts have been signed. This "inflation tax" has the effect of reallocating resources to the government from the domestic depositors and foreign bondholders. The government may be able to return some of these resources to the domestic depositors so that the net effect of such inflation is to expropriate foreign bondholders. If political constraints or the desire to create a reputation for fiscal rectitude limit inflationary policies, then the foreign lenders' expectations may be reflected in a lower interest rate. If political constraints are lax or the desire to form a reputation is low, then the "inflation premium" foreign lenders' demand may be substantial. Also, they will only be willing to lend short term because this reduces their exposure to inflation risk. In extreme cases foreign lenders may not be willing to lend in the country's domestic currency at all. Banks will find it preferable to borrow using debt denominated in a foreign currency, that is, in real terms.

Denominating international debt in terms of foreign currency avoids the inflation premium but it introduces other problems. The ability to avoid costly liquidation can be lost and the degree of risk-sharing that can be obtained may also be reduced. The benefits that the central bank can generate are reduced. We start by considering what happens in the absence of a central bank that issues domestic currency and then consider what benefits the central bank can bring.

5.1 *The dollarized economy*

The dollarized economy is essentially a real economy. It is closed apart from access to the international bond market. There is no interaction between the banks. The rate at which each bank can borrow on the international market depends on the amount that it borrows and the portfolio of bonds and risky investments it chooses. Each bank has a distinct contracting problem and each bank's decisions can be analyzed separately from the behavior of other banks. We focus on short-term debt. A result similar to Proposition

(8) concerning long-term debt can also be proved in this context. Since uncertainty is resolved at date 1, there is essentially no difference between short-term and long-term debt except possibly for a different interest rate.

To simplify the analysis, we assume that the random variable R has a two-point support $\{r_L, r_H\}$, where $0 < r_L < r_H < \infty$. Let $0 < \pi_i < 1$ denote the probability that $R = r_i$ for $i = L, H$. The analysis of the representative bank's decision problem can be broken down into three cases.

No default. The bank chooses a consumption allocation $\{c_1(r_i), c_2(r_i)\}$ that is determined by a portfolio (x, y), a level of borrowing b, and a deposit contract d. Because there is no risk of default, the price of the bank's debt is $q = 1/\rho$. The bank's decision problem is

$$\begin{array}{ll} \max & \sum_i \pi_i \{u(c_1(r_i)) + u(c_2(r_i))\} \\ \text{s.t.} & x + y \le 2 + b/\rho \\ & c_1(r_i) = d, i = L, H \\ & c_2(r_i) \le r_i h(x) - \rho(d + b), i = L, H \\ & \rho c_1(r_i) \le c_2(r_i), i = L, H. \end{array}$$

The first three constraints are budget constraints corresponding to dates 0, 1, and 2, respectively and the last is the incentive constraint. Also, notice that if the incentive constraint is satisfied for r_L it is automatically satisfied for r_H.

Since domestic debt and international debt are perfect substitutes, there is no essential loss of generality in assuming that the bank will not simultaneously borrow and lend on the international market. The case in which we are interested is the one in which the bank borrows in the international market ($b > 0$) and does not invest in international bonds ($y = 0$). Here the investment in the domestic risky asset is greater than the endowment and the welfare of depositors is greater than in the closed economy.

The avoidance of default is costly in several ways. First, depositors bear the entire risk of the returns on the long asset. Secondly, there will be an intertemporal distortion because the early consumers do not bear any of the risk and receive a low average consumption. It will be optimal to avoid default in both states when uncertainty is low, the risk aversion of depositors is low, and the costs of premature liquidation are high.

Default in one state. Suppose that bankruptcy occurs only when asset returns are low at r_L. The international bondholders receive the face value of the debt b if asset returns are high and a fraction $\beta \equiv b/(2d + b)$ of the bank's assets if asset returns are low. The maximum amount the bank can borrow at date 0 is the expected present value of this stream:

$$qb = \frac{1}{\rho} \{\pi_L \beta (\gamma r_L h(x) + \rho y) + \pi_H b\}.$$

Substituting this into the first-period budget constraint, we get

$$x + y = 2 + \frac{1}{\rho}\left\{\pi_L \beta \left(\gamma r_L h(x) + \rho y\right) + \pi_H b\right\}.$$

The bank's maximization problem can be written as follows

$$\begin{array}{ll} \max & \sum_i \pi_i \left\{u(c_1(r_i)) + u(c_2(r_i))\right\} \\ \text{s.t.} & x + y \leq 2 + \frac{1}{\rho}\left\{\pi_L \beta \left(\gamma r_L h(x) + \rho y\right) + \pi_H b\right\} \\ & c_1(r_L) = c_2(r_L)/\rho = \beta\left(\gamma r_L h(x) + \rho y\right) \\ & c_1(r_H) = d \\ & c_2(r_H)/\rho \leq r_H h(x)/\rho + \rho y - (d + b) \\ & \rho c_1(r_H) \leq c_2(r_H). \end{array} \tag{19}$$

Unfortunately, this problem turns out to have no solution. The "optimum" requires unbounded values of b and y. Rather than analyze the problem (19) directly, we adopt a two-step strategy. First, we set up an artificial problem in order to define a benchmark consumption allocation. The artificial problem has the same objective function as the original problem (19). The constraints are such that any solution to the original problem is also a solution to the artificial problem. Thus, the solution to the artificial problem (the benchmark consumption allocation), must be at least as good as the solution to the original problem. The second step is to show that the benchmark can be approximated by a choice of (b, d, x, y) that satisfies the feasibility constraints of the original problem (19). As the amount borrowed and invested in the international bond market becomes larger, so $b \to \infty$, $y \to \infty$, and the feasible consumption allocation from the original problem (19) converges to the benchmark. This is the sense in which the "solution" to the original problem (19) is achieved in the limit as $b \to \infty$.

The artificial problem is defined as follows:

$$\begin{array}{ll} \max & \sum_i \pi_i \left\{u(c_1(r_i)) + u(c_2(r_i))\right\} \\ \text{s.t.} & \sum_i \pi_i \left(c_1(r_i) + c_2(r_i)/\rho\right) \leq \rho(2 - x) + \left(\pi_L \gamma r_L + \pi_H r_H/\rho\right) h(x) \\ & c_1(r_L) = c_2(r_L)/\rho \\ & c_1(r_H) \leq c_2(r_H)/\rho \\ & c_1(r_L) \leq c_1(r_H). \end{array} \tag{20}$$

The first constraint is a present value budget constraint in terms of consumption, endowments, and profits. The second and third constraints are incentive constraints. The final constraint is added because the rules for bankruptcy imply that $c_1(r_L) \leq c_1(r_H)$. It is straightforward to check that a solution of (19) must satisfy the constraints of (20). Hence, the solution to (20) must be at least as good as the solution to (19).

Proposition 9 *Suppose that* $(\{c_1(r_i), c_2(r_i)\}, b, d, x, y)$ *satisfies the feasibility constraints associated with the problem (19). Then* $(\{c_1(r_i), c_2(r_i)\}, x)$ *satisfies the feasibility constraints associated with the problem (20).*

Proof. See the Appendix. □

The proof proceeds by showing that the budget constraints for date 1 and date 2 from (19) imply that $(\{c_1(r_i), c_2(r_i)\}, x)$ satisfies the budget constraint for (20). The incentive constraints are the same in both problems and the final constraint in (20) follows from the fact that there is bankruptcy in state r_L. Since the objective functions are the same, the solution to (20) must be at least as good as the solution to (19).

The next step is to characterize the solution to (20) and show that borrowing and lending large amounts allows the benchmark solution to be approximated.

Proposition 10 *Suppose that* $(\{\hat{c}_1(r_i), \hat{c}_2(r_i)\}, \hat{x})$ *is a solution to the artificial problem (20). Then*

$$\hat{c}_1(r_L) = \hat{c}_1(r_H),$$

$$\hat{c}_2(r_L) = \rho\hat{c}_1(r_L)$$

and

$$\hat{c}_2(r_H) = \max\{\rho\hat{c}_1(r_H), \phi(\hat{c}_1(r_H))\},$$

where $\phi(c)$ *is defined implicitly by the equation*

$$(\pi_L + \pi_H)u'(c) + \pi_L\rho u'(\rho c) \equiv (2\pi_L + \pi_H)\rho u'(\phi(c)), \forall c.$$

For any $\varepsilon > 0$ *we can find a feasible choice* $(\{c_1(r_i), c_2(r_i)\}, x, y, b, d)$ *for the original problem (19) such that*

$$\sum_i \pi_i \{u(c_1(r_i)) + u(c_2(r_i))\} \geq \sum_i \pi_i \{u(\hat{c}_1(r_i)) + u(\hat{c}_2(r_i))\} - \varepsilon.$$

Proof. See the Appendix. □

In the previous section the variation in the exchange rate $e(r)$ makes the real value of domestic-currency debt state-contingent and this allows risk to be transferred to the international market. Here even though domestic banks issue bonds denominated in the foreign-reserve currency, the possibility of default makes the real value of domestically issued debt contingent on the state of nature. By issuing risky bonds and investing some of the proceeds in risk-free bonds, domestic banks can shift all of the domestic risk to the international market. Note that it is important for this result that we have two securities (domestic debt and foreign debt) and two states of nature.

As in the previous section, as the amount borrowed and reinvested in the international market increases without bound, the riskiness of the banks' portfolios becomes relatively small. Since the depositors own a progressively smaller amount of it, the risk they bear eventually becomes negligible. Hence there is efficient risk-sharing in the bankrupt state.

Improved risk-sharing comes at the price of default in one state. There is a trade-off between the state-contingency of domestically issued debt and costly liquidation of assets.

Default in both states. Bankruptcy in both states implies that

$$c_1(r_i) = \frac{c_2(r_i)}{\rho} = \frac{1}{2}(1-\beta)\left(\gamma r_i h(x) + \rho y\right) \tag{21}$$

for $i = H, L$ where $\beta \equiv b/(2d+b)$ and the first-period budget constraint can be written

$$x + y \leq 2 + \frac{\beta}{\rho}\left\{\gamma E[r]h(x) + \rho y\right\}. \tag{22}$$

The problem is solved by choosing β, x and y to maximize the usual objective function subject to the budget constraint.

Rearranging the budget constraint (22), we can calculate that

$$(1-\beta)y = 2 - x + \frac{\beta}{\rho}\gamma E[r]h(x).$$

Substituting this into the consumption equations (21) we get

$$c_1(r_i) = \frac{c_2(r_i)}{\rho} = \frac{1}{2}(1-\beta)\gamma r_i h(x) + 2 - x + \frac{\beta}{\rho}\gamma E[r]h(x).$$

From inspection of this equation, it is clear that the expected value of consumption is independent of β, for a fixed value of x. Further, all uncertainty is eliminated as $b \to \infty$ and so $\beta \to 1$. Thus, a risk-averse consumer would strictly prefer increasing β to the limit.

We have shown that the optimum policy for the bank, given that it goes bankrupt in both states, is to eliminate all risk by issuing an unlimited amount of risky debt and investing in an unlimited amount of risk-free debt. Since the bank is bankrupt with probability one, an increase in borrowing and lending does not increase the probability of bankruptcy and hence does not increase liquidation costs.

Bankruptcy and risk elimination. This last argument also applies with a continuum of states. If bankruptcy occurs in every state, then it is optimal to eliminate all risk. The argument is essentially the same as that given above. From the budget constraints, we see that

$$c_1(r) = \frac{c_2(r)}{\rho} = \frac{1}{2}(1-\beta)\gamma r h(x) + 2 - x + \frac{\beta}{\rho}\gamma E[r]h(x).$$

Again, the expected value of consumption is independent of β whereas the variance of consumption converges to 0 as β converges to 1. So once again, conditional on bankruptcy occurring with probability 1, it is optimal to eliminate all the risk by borrowing and lending an unbounded amount in the international market.

Of course, typically it is sub-optimal to have bankruptcy with probability one. If the probability of bankruptcy is less than one, then there is a trade-off between risk-sharing and liquidation costs. Issuing more risky domestic debt to invest in safe international debt improves risk-sharing, but increases the probability of default and hence liquidation costs.

There is one case where bankruptcy always occurs with probability 1 and that is the case where liquidation costs are zero because $\gamma = 1/\rho$. If $\gamma = 1/\rho$ there is no loss of generality in assuming that all assets are liquidated at date 1. Let $\{\bar{c}_1, \bar{c}_2\}$ denote the first-best consumption allocation and let $x = \bar{x}$ be the first-best investment in the risky asset. If the incentive constraint is binding for every value of r, then the first-best can be approximated by setting $\beta \approx 1$.

One point worth noting is that when default does occur with probability one, both the domestic depositors and international bondholders are essentially holding shares in the domestic bank. When bankruptcy is costly, default is something to be avoided. In that case, the use of equity contracts can avoid default while allowing some beneficial risk-sharing.

5.2 *Foreign-currency loans and domestic-currency deposits*

In the dollarized economy, there is no role for the central bank. All contracts are specified in real terms. Even though the possibility of bankruptcy makes domestic-debt contracts risky, there is nothing the central bank can do to alter the probability of bankruptcy or the realized returns on domestic debt.

We turn now to the case where bank deposits are denominated in terms of the domestic currency. Domestic depositors are effectively holding domestic-currency debt, since a bank deposit promises a fixed amount of the domestic currency at each date. International bondholders are holding dollar-denominated debt. In this case, the central bank can alter the real value of the domestic-currency debt, so it can alter the returns received by domestic depositors and indirectly the returns received by international bondholders. For example, by reducing the exchange rate (raising the domestic price level) the central bank reduces the real value of domestic deposits, thus making it easier for the banks to repay the foreign bondholders. In this way, the central bank can prevent some inefficient liquidation. It cannot entirely eliminate the risk of bankruptcy, however. Since the foreign-held debt is denominated in dollars, it may be impossible to repay the foreign debt in full for very low realizations of asset returns, even if the domestic depositors receive nothing.

In addition to reducing inefficient liquidation, the introduction of debt instruments denominated in domestic currency may allow risk-sharing between early and late consumers.

Banks now face two quite different types of creditors in the event of bankruptcy, foreigners who hold reserve-currency debt and domestic depositors who hold domestic-currency debt. It is not obvious what the rule for dividing the assets between the two different classes of creditors should be. We begin by considering an extreme case, in which the foreign debtholders are assumed to have absolute priority. Other possibilities are discussed below.

The analysis of the equilibrium at date 1 is similar to the previous section. Let qb be the amount borrowed at date 0 and b be the amount repaid at date 1. The bank will go bankrupt when the output available is insufficient to pay the foreign debt. Hence r^* is given by

$$\frac{r^* h(x)}{\rho} + \rho y = b. \tag{23}$$

The value of q will be set so that the foreign bondholders obtain their opportunity cost,

$$\int_0^{r^*} (\gamma r h(x) + \rho y) dF + \int_{r^*}^{\infty} b dF = \rho q b. \tag{24}$$

For simplicity we again focus on the case where the incentive constraint $\rho c_1(r) \leq c_2(r)$ binds. For $r < r^*$ the foreign bondholders receive everything at date 1 and the domestic depositors receive nothing. For $r \geq r^*$ the price level is given by the ratio of nominal claims to output when the incentive constraint binds

$$p_1(r) = \rho p_2(r) = \frac{2D}{rh(x)/\rho + \rho(2-x) + (\rho q - 1)b}$$

$$c_1(r) = c_2(r)/\rho = \frac{1}{2}(rh(x)/\rho + \rho(2-x) + (\rho q - 1)b). \tag{25}$$

If the counterpart of (3) is satisfied so that

$$u'(0)\rho^2 > E[u'(rh(2))rh'(2)], \tag{26}$$

an interior solution in the sense that $x < 2$ is assured. Now if $r^* \leq r_0$, then $\rho q = 1$ and the level of b is irrelevant. If $r^* > r_0$, then $\rho q < 1$ and the representative bank's optimal choice given its objective is to maximize the expected utility of the representative consumer involves $b = 0$.

Although there will be no borrowing at date 0 there will of course be borrowing at date 1 to smooth consumption between periods. Apart from that, the outcome is similar to the case where there is no international finance. In particular there will be no bankruptcy or inefficient liquidation.

It can be seen that the introduction of a central bank is a mixed blessing. It does allow risk-sharing between the early and late consumers for all values of r. However, there is no risk-sharing with the international bond market. As demonstrated above, when there is no central bank so all contracts are in foreign-currency terms, there can be risk-sharing with the international bond market. It is therefore not immediate whether a central bank and independent monetary policy are desirable. It will depend on the parameter values. For $\gamma = 1/\rho$, using foreign-currency denominated debt and deposits will be optimal since in that case $r^* = r_1$ and the first-best can be implemented. At the other extreme if γ is very small and $E[r]$ is sufficiently large, a system with a central bank will do better.

It follows from (23) and (25) that $c_1(r^*) = c_2(r^*) = 0$. As more and more of the economy's output goes to pay the foreign debtholders, less is left for domestic depositors and the domestic price level becomes very high. In fact, as $r \to r^*, p_1(r) \to \infty$. For $r < r^*$ the domestic price level is not well-defined. We return to this problem below.

So far, we have assumed that the foreign debtholders have priority in the event of bankruptcy. Given this extreme assumption, it is optimal not to borrow at date 0. With different priorities, this may no longer be the case.

In the (opposite) extreme, where domestic depositors have absolute priority in bankruptcy, the analysis is similar. The main difference will be that in (24) there will be no term for $r < r^*$. As a result the interest rate charged will be higher. Given that the international bond market is risk neutral and depositors are risk averse, the effective transfer from high income states to low income states that a higher interest rate involves will lead to an increase in welfare compared to the case where foreign bondholders receive the liquidation proceeds.

Giving priority to domestic depositors raises a problem, however. We noted above that $c_1(r^*) = c_2(r^*) = 0$. If depositors receive anything from bankruptcy, then there will be an incentive for the bank to declare bankruptcy even though it is in fact solvent. This makes the administration of anything other than full priority to foreign bondholders problematic. However, the lenders can take into account this aspect and adjust the interest rate on the debt appropriately.

In addition to the two extremes of absolute priority, there are many intermediate cases where both parties receive some portion of the liquidated assets. The problem with analyzing these cases is to specify the precise way in which the liquidation proceeds are split between the two groups. One possibility is to rely on ex post bargaining. Another is to base the priority rules on the proportionate claims at date 0. The problem here is that some claims are denominated in dollars and some in the domestic currency. Until the exchange rate is determined, it is not clear what the relative shares of domestic depositors and foreign bondholders should be. If the priority rules are specified as a proportion of the liquidation proceeds, independently of the real values of the respective claims, then the analysis is determinate and can be undertaken as in the case of absolute priority. Clearly, the range of possibilities is large.

6 Policy implications

The events in South East Asia in recent years have sparked a debate about the role of the International Monetary Fund (IMF) in dealing with international crises. One part of the debate revolves around the appropriateness of IMF actions in particular countries (see Corsetti, Pesenti, Roubini (1998a, b, 1999) for a detailed discussion of these issues). Another part of the debate focuses on the broader issue of whether the IMF has a role to play in such crises and if so what the rationale for such intervention is.

There is widespread acceptance of the need for a lender of last resort (LOLR) in a domestic context. Krugman (1998), Fischer (1999), and others have argued by analogy with the domestic role for a LOLR, that the IMF should act as an international LOLR.

At the other end of the spectrum, Friedman (1998) and Schwartz (1998) have argued that when the IMF intervenes it distorts markets and leads to inefficiency. They argue that by bailing out imprudent investors, the IMF encourages lenders to invest without due care and attention. If the lenders knew that the IMF would not intervene, they would take more care to investigate projects and invest only where the risk was justified by the expected return.

A number of authors, such as Sachs (1995) and Feldstein (1998), have taken a middle course, suggesting that the IMF has a role to play, but criticizing many of its actions. Sachs argues for the need for an international bankruptcy court, while Feldstein emphasizes that the IMF's actions should be more closely related to overcoming market failures.

Chari and Kehoe (1999) have suggested that the role of the IMF should be limited to cases where there is a clear problem of collective action. They argue that, in recent international crises, liquidity has been adequately provided by the U.S. Federal Reserve and other major central banks, which suggests that a problem of collective action among the central banks does not exist. Chari and Kehoe do suggest that there are important collective-action problems with regard to creditor coordination. In the domestic context, bankruptcy laws and institutions are designed to overcome these problems but in an international context there is no equivalent. They argue that the IMF has an important role to play as an international bankruptcy court. One of the major activities the IMF currently undertakes is the provision of information about the economic situation in member countries. Chari and Kehoe suggest that this is a valuable function and should continue. Finally, they argue that the IMF could provide a currency to which member countries could peg their exchange rate.

This paper does not resolve the debates about the proper role of the IMF, but it does provide a framework for understanding the different perspectives on the global financial system and the conditions under which each might be valid. In some situations it appears that an international organization has little role to play. In others, however, it may be able to prevent the costly liquidation and contagion associated with financial crises and improve the allocation of resources. Speaking very broadly, we may distinguish two different situations.

- The first case is applicable to advanced industrial economies. These countries have flexible exchange rates and can issue debt denominated in terms of their own domestic currency. The analysis in Section (4) shows that a combination of appropriate exchange-rate policy and borrowing and lending by banks in the international capital market leads to optimal risk-sharing and avoids the costly liquidation associated with bankruptcy.

- The second case is applicable to emerging markets. Here, problems of commitment to financial discipline mean that international lenders are unwilling to buy bonds denominated in the domestic currency. As a result, foreign lending takes the form of dollar loans. In the versions of the model analyzed in Section (5), domestic financial intermediaries issue bonds denominated in the foreign-reserve currency. In this case, banking crises with inefficient liquidation can occur. Given that bankruptcy occurs with positive probability, it may or may not be optimal to eliminate the risk borne by the domestic depositors. Even if it is technically possible, shifting the risk to the international market may increase the probability of bankruptcy and the associated costs of inefficient liquidation.

From the point of view of monetary policy, the difference between these two situations is that, in the former, which we have identified with advanced industrial economies, the domestic central bank controls the supply of the domestic currency and consequently has the ability, together with an optimally functioning international capital market and domestic financial system, to adjust the foreign claims on the domestic economy in a way that promotes risk-sharing and investment and avoids financial crises. In what we identify as the emerging markets case, the central bank has lost much of its control over foreign claims on the economy because they are denominated in the foreign currency.

It is tempting to think that some kind of intervention by the U.S. Federal Reserve or by the IMF could somehow achieve optimal risk-sharing and investment in the case of emerging markets. We have not analyzed this possibility, but the analysis of Section (4) suggests some obstacles. In principle, some combination of the Fed and the IMF could transfer dollars to the emerging markets to prevent inefficient liquidation of banks. However, the political feasibility of making real transfers to foreign countries may be questioned. One thinks here of the opposition in the United States to the Mexican bailout.

Another possibility would be to vary the real value of the dollar to make the real value of the debt issued in emerging markets state-contingent. This would give emerging markets the same opportunity as the advanced industrial economies to avoid inefficient liquidation and transfer risk to the international market. There are several problems with this kind of policy. First, there are many emerging economies and only one reserve currency (many targets, one instrument), so it is not clear that we could replicate the results in Section (4.3) by varying the value of the dollar. Secondly, by using the value of the dollar to support optimal risk-sharing in emerging economies, the Fed would lose the ability to vary the price level for domestic reasons. Concerns about inflation are likely to discourage the Fed from accommodating this policy. Finally, any variations in the real value of the dollar would again imply real transfers between the developed and emerging economies, which are likely to lead to objections ex post. So the prospects of an international agency like the IMF—or a domestic agency like the Fed—playing this particular role may founder on the shoals of political reality.

Another aspect of currency crises, not considered in this paper, is the possibility of financial contagion. Our analysis of optimal risk-sharing demonstrates the need for financial linkages between countries. These are the conditions under which the possibility of financial contagion becomes an issue. Allen and Gale (2000) show that contagion allows a shock in one region to propagate throughout a network of interlinked regions. In their model, liquidity is a public good that is subject to a free-rider problem. Contagion can be prevented if the banks in all the countries coordinate to provide a small amount of liquidity. However, each country has an incentive to let the other countries supply the liquidity. The result can be a "meltdown" in which all countries' financial systems are adversely affected and forced to liquidate assets inefficiently. In this kind of situation, there is a role for an agency like the IMF to solve the coordination problem by forcing each country to play its part in providing liquidity.

In summary, the IMF may have an important role to play. Whether it can effectively play this role—and what the optimal policy would be—are subjects for future research.

Proof of Proposition 1

If we ignore the incentive constraint, the risk-sharing problem described in (1) becomes:

$$\begin{array}{ll} \max & E_R[u(c_1(r)) + u(c_2(r))] \\ \text{s.t.} & x + y \leq 2 \\ & c_1(r) \leq y, \forall r \\ & c_2(r) \leq rh(x) + y - c_1(r), \forall r. \end{array} \tag{27}$$

A necessary condition for a solution to (27) is that, for each value of r, the consumption levels $c_1(r)$ and $c_2(r)$ solve the problem

$$\begin{array}{ll} \max & u(c_1(r)) + u(c_2(r)) \\ \text{s.t.} & c_1(r) \leq y \\ & c_2(r) \leq rh(x) + y - c_1(r). \end{array}$$

The necessary Kuhn-Tucker conditions imply that

$$u'(c_1(r)) \geq u'(c_2(r)),$$

with strict equality if $c_1(r) < y$. In other words, $c_1(r) \leq c_2(r)$ so the incentive constraint is satisfied automatically. Thus, a solution to (27) is also a solution to the original problem (1).

The Kuhn-Tucker condition implies that $c_1(r) = c_2(r)$ whenever $c_1(r) < y$, so there are two regimes to be considered. Either $c_1(r) = y$ and (hence) $c_2(r) = rh(x)$ or $c_1(r) = c_2(r) = \frac{1}{2}(rh(x) + y)$. The first case arises if $y \leq rh(x)$, so the optimal consumption allocation must satisfy

$$c_1(r) = c_2(r) = \frac{1}{2}(rh(x) + y) \text{ if } y \geq rh(x),$$

and

$$c_1(r) = y, c_2(r) = rh(x) \text{ if } y \leq rh(x).$$

This allows us to write the risk-sharing problem more compactly as follows:

$$\begin{array}{ll} \max & \int_0^{\bar{r}} 2u\left(\frac{rh(x)+y}{2}\right) dF + \int_{\bar{r}}^{\infty} (u(y) + u(rh(x))) \, dF \\ \text{s.t.} & x + y \leq 2, \end{array}$$

where $\bar{r} \equiv y/h(x)$ is the critical value of R at which the liquidity constraint begins to bind. Note that so far we have not established that the critical value of $\bar{r}$ belongs to the support of R.

It remains to characterize the optimal portfolio. We first rule out two extreme cases. Suppose that $x = 0$. Then it is clear that $c_1(r) = c_2(r) = 1$ and $\bar{r} = \infty$. This will be optimal only if $y = 2$ maximizes

$$u(y/2) + E[u(rh(2-y)) + y/2)],$$

and the first-order condition for this is

$$u'(2/2)/2 + u'(2/2)(\frac{1}{2} - E[r]h'(2-2)) \geq 0,$$

which implies $E[r]h'(0) \leq 1$, contradicting one of our maintained assumptions.

Next suppose that $y = 0$. Then $c_1(r) = 0 \leq c_2(r) = rh(2)$. For this to be an optimal choice, it must be the case that $x = 2$ maximizes

$$u(2-x) + E[u(rh(x))],$$

and the necessary first-order condition for this is

$$u'(0) \leq E[u'(h(r2))rh'(2)],$$

which contradicts another of our maintained assumptions. Thus any optimal portfolio must satisfy $y > 0$ and $x > 0$.

Returning to the compact form of the risk-sharing problem above, we see that necessary conditions for an interior solution are:

$$\int u'(c_1(r))dF = \lambda$$

and

$$\int u'(c_2(r))rh'(x)dF = \lambda,$$

where λ is the Lagrange multiplier of the constraint $x + y = 2$. Under the strict concavity of $u(\cdot)$, these first-order conditions uniquely determine the optimal values of y and x, which in turn determine $\bar{r}$, $c_1(r)$, and $c_2(r)$ through the relationships described above.□

Proof of Proposition 5

Set $B = \bar{B}$ and $D = 1$. From the date 1 budget constraint we have

$$c_1(r) = \frac{1}{p(r)}. \tag{28}$$

Use the date 0 budget constraint $y = 2 + q\bar{B} - x$ to eliminate y from the date 2 budget constraint:

$$c_2(r) = rh(x) + \rho^2 \left(2 + q\bar{B} - x\right) - \rho\left(1 + \bar{B}\right) c_1(r). \tag{29}$$

Thus, consumption at each date is expressed in terms of the parameters x, q, and $c_1(r)$. In order for the consumption allocation $(c_1(r), c_2(r))$ to solve the maximization problem in the proposition, it is necessary and sufficient that $u'(c_1(r)) \geq \rho u'(c_2(r))$, $\rho c_1(r) \leq c_2(r)$, and $u'(c_1(r)) = \rho u'(c_2(r))$ if $\rho c_1(r) < c_2(r)$. Another way of expressing this is to say that

$$c_2(r) = \max\{\rho c_1(r), \varphi(c_1(r))\} \tag{30}$$

where $\varphi(\cdot)$ is defined implicitly by the equation $u'(z) = \rho u'(\varphi(z))$.

Substituting (28) and (29) into (30), we obtain the following:

$$\max\{\rho c_1(r), \varphi(c_1(r))\} = rh(x) + \rho^2(2 + q\bar{B} - x) - \rho\left(1 + \bar{B}\right)c_1(r). \tag{31}$$

This equation determines the consumption function $c_1(r)$ uniquely in terms of x and q. To ensure that these conditions are satisfied in equilibrium, we choose the consumption function $c_1(r)$ to satisfy (31). More precisely, let $\bar{q}$ be some large but arbitrary finite value and

$$K \equiv \{(q,x) \in \mathbf{R}_+ \times \mathbf{R}_+ | \mathbf{q} \leq \bar{\mathbf{q}}, \mathbf{r_0 h(x)} + \rho^2(\mathbf{2} + \mathbf{q}\bar{\mathbf{B}} - \mathbf{x}) \geq \mathbf{0}\}.$$

Lemma 11 *For every $(q,x) \in K$ there exists a function $\Phi(\cdot; q, x) : \mathbf{R}_+ \rightarrow \mathbf{R}_{++}$ such that $\Phi(r; q, x)$ satisfies equation (31) for every value of r. Moreover, Φ is continuous.*

Proof. To see that $\Phi(\cdot; q, r)$ is well-defined, note that φ is an increasing function. Thus, the left-hand-side of (31) is an increasing function of $c_1(r)$. The right-hand-side of (31) is a decreasing function of $c_1(r)$ so there is at most one solution $c_1(r)$ for any pair (q, x). To see that a solution exists, note that both sides are continuous in $c_1(r)$. The left-hand-side approaches 0 as $c_1(r) \rightarrow 0$ and ∞ as $c_1(r) \rightarrow \infty$. The right-hand-side approaches $rh(x) + \rho^2(2 + q\bar{B} - x) \geq 0$ as $c_1(r) \rightarrow 0$ and $-\infty$ as $c_1(r) \rightarrow \infty$. Thus, there must be at least one value $c_1(r)$ that satisfies the equation.

By the same argument, the solution value of $c_1(r)$ must be finite and non-negative. Continuity of Φ follows from the implicit function theorem.□

Construct a mapping from the set K to itself as follows. Given (q, x), the consumption function $c_1(r) = \Phi(r; q, x)$ is well defined, and we can define q' by putting

$$q' = \min\left\{\bar{q}, \int_{r_0}^{r_1} \frac{\Phi(r; q, x)}{\rho} dF\right\}.$$

We choose x' to maximize

$$E\left[u\left(rh(x') + \rho^2(2 + q'\bar{B} - x') - \rho(1 + \bar{B})\Phi(r; q, x)\right)\right]$$

subject to the non-negativity constraint $r_0 h(x') + \rho^2(2 + q'\bar{B} - x') \geq 0$. The set of values of x' that solve the maximization problem is convex and non-empty. Let $Z(q, x) \in K$ denote the set of points (q', x') constructed in this way. Standard arguments suffice to show that Z has a closed graph, so by the Kakutani theorem Z has a fixed point $(q^*, x^*) \in Z(q^*, x^*)$.

We claim that (q^*, x^*) defines the desired pseudo-equilibrium. By construction, the consumption allocations corresponding to (q^*, x^*) solve the maximization problem in the proposition and q^* satisfies the pricing equation (14) as long as $\bar{q}$ is chosen large enough. To see this, it is enough to show that $E[c_1(r)]$ is bounded. From (31) we have

$$\rho c_1(r) = rh(x) + \rho^2(2 + q\bar{B} - x) - \rho\left(1 + \bar{B}\right)c_1(r)$$

and by construction $\rho q \leq E[c_1(r)]$ so taking expectations and substituting

$$E[\rho c_1(r)] \leq E[rh(x) + \rho^2(2 - x) - \rho c_1(r)],$$

or

$$E[2\rho c_1(r)] \leq E[rh(x) + \rho^2(2 - x)].$$

This shows that $E[c_1(r)]$ is bounded independently of $\bar{q}$ and $\bar{B}$, so choosing $\bar{q}$ large enough, we will have $q^* = E[c_1(r)/\rho] < \bar{q}$ at the fixed point.

Since the bank's maximization problem (16) is a convex problem, it is sufficient to show that the Kuhn-Tucker first-order conditions are satisfied. This must be true by construction for all the variables except D and B, since they are chosen optimally. To show that $D = 1$ and $B = \bar{B}$ are optimal for the bank, we have to show that the first-order conditions for the decision problem are satisfied. From (16) we can see that the first-order condition for D is

$$E_R\left[\frac{u'(c_1(r))}{p(r)} - \frac{\rho u'(c_2(r))}{p(r)}\right] \geq 0$$

because we cannot increase D if the incentive constraints bind within the support of R. This condition must be satisfied because we know that $u'(c_1(r)) \geq \rho u'(c_2(r))$ for all r, with strict equality if the incentive constraint is not binding.

Similarly, the first-order condition for B is

$$E_R\left[u'(c_2(r))\left(\rho^2 q - \frac{\rho}{p(r)}\right)\right] \geq 0$$

because the constraint $B \leq \bar{B}$ is binding. Then substituting the condition for q from (14) gives us

$$E_R\left[u'(c_2(r))\left(E_R\left[\frac{\rho}{p(r)}\right]-\frac{\rho}{p(r)}\right)\right] \geq 0.$$

Since $u'(c_2(r))$ is decreasing in r and $p(r)$ is decreasing in r

$$\begin{aligned} & E_R\left[u'(c_2(r))\left(E_R\left[\frac{\rho}{p(r)}\right]-\frac{\rho}{p(r)}\right)\right] \\ \geq \; & E_R\left[u'(c_2(r))\right] E_R\left(E_R\left[\frac{\rho}{p(r)}\right]-\frac{\rho}{p(r)}\right)=0, \end{aligned}$$

as required.□

Proof of Proposition 6

From equation (31), for a fixed but arbitrary r and for each k,

$$\max\left\{\rho c_1^k(r), \varphi\left(c_1^k(r)\right)\right\} = rh(x^k) + \rho^2(2+q^k\bar{B}^k - x^k) - \rho(1+\bar{B}^k)c_1^k(r_1).$$

The left-hand-side is non-negative, so

$$r_1 h(x^k) + \rho^2(2+q^k\bar{B}^k - x^k) - \rho(1+\bar{B}^k)c_1^k(r_1) \geq 0, \tag{32}$$

for all k. From the pricing equation (14) we know that $\rho q^k = E[c_1^k(r)]$, and (31) implies that $c_1^k(r)$ is nondecreasing in r, so $\rho q^k \leq c_1^k(r_1)$. Then (32) implies that the sequence $\left\{\left(\rho q^k - c_1^k(r_1)\right)\bar{B}^k\right\}$ is bounded below, which implies that

$$\lim_{k\to\infty} \rho q^k - c_1^k(r_1) \to 0. \tag{33}$$

The pricing equation (14) together with (33) implies that $c_1^k(r)$ converges to a constant almost surely. Then $(c^k(r), c_2^k(r))$ converges to $(\bar{c}_1, \bar{c}_2)$, say, almost surely and the first-order condition for x^k

$$E_R\left[u'(c_2^k(r))(rh'(x^k)-\rho^2)\right] \geq 0$$

becomes

$$E_R\left[(rh'(\bar{x})-\rho^2)\right] = 0$$

in the limit (assuming the bankruptcy constraint is not binding because $\bar{c} = \lim_k c_1^k(r)$ is positive).□

Proof of Proposition 9

Using the budget constraints for date 1 and date 2 from (19),

$$\begin{aligned}
&\pi_L \left\{ c_1(r_L) + \frac{c_2(r_L)}{\rho} \right\} + \pi_H \left\{ c_1(r_H) + \frac{c_2(r_H)}{\rho} \right\} \\
\leq\ & \pi_L \left\{ (1-\beta)\left(\gamma r_L h(x) + \rho y\right) \right\} + \pi_H \left\{ r_H h(x)/\rho + \rho y - b \right\} \\
=\ & \pi_L \left(\gamma r_L h(x) + \rho y\right) + \pi_H \left(r_H h(x)/\rho + \rho y\right) - \pi_L \beta \left(\gamma r_L h(x) + \rho y\right) - \pi_H b.
\end{aligned}$$

From the first-period budget constraint in (19),

$$\rho(x + y - 2) = \pi_L \beta \left(\gamma r_L h(x) + \rho y\right) - \pi_H b,$$

so

$$\begin{aligned}
&\pi_L \left\{ c_1(r_L) + \frac{c_2(r_L)}{\rho} \right\} + \pi_H \left\{ c_1(r_H) + \frac{c_2(r_H)}{\rho} \right\} \\
=\ & \pi_L \left(\gamma r_L h(x) + \rho y\right) + \pi_H \left(r_H h(x)/\rho + \rho y\right) - \rho(x + y - 2) \\
=\ & \pi_L \gamma r_L h(x) + \pi_H r_H h(x)/\rho - \rho(x - 2).
\end{aligned}$$

So $(\{c_1(r_i), c_2(r_i)\}, x)$ satisfies the budget constraint in (20).

The incentive constraints are the same in both problems, so it remains to show that $c_1(r_L) \leq c_1(r_H)$. But this follows from the fact that there is bankruptcy in state r_L, which requires that

$$2d + b > \gamma r_L h(x) + \rho y,$$

so

$$c_1(r_L) = \frac{d}{2d + b} \left(\gamma r_L h(x) + \rho y\right) < d = c_1(r_H).$$

Since the objective functions for the two problems are the same and any feasible solution of the original problem is feasible for the artificial problem the solution to the artificial problem must be at least as good as the solution to the original problem.□

Proof of Proposition 10

The first part of the proposition characterizes the consumption allocation $\{\hat{c}_1(r_i), \hat{c}_2(r_i)\}$ that solves (20). Since bankruptcy in state r_L implies $c_2(r_L) = \rho c_1(r_L)$, any solution to the problem (20) must solve

$$\begin{array}{ll}
\max & \pi_L \left(u(c_1(r_L)) + u(\rho c_1(r_L))\right) + \pi_H \left(u(c_1(r_H)) + u(c_2(r_H))\right) \\
\text{s.t.} & 2\pi_L c_1(r_L) + \pi_H \left(c_1(r_H) + c_2(r_H)/\rho\right) \leq \hat{w} \\
& c_1(r_L) \leq c_1(r_H) \leq c_2(r_H)/\rho,
\end{array}$$

where

$$\hat{w} = \pi_L \left(\hat{c}_1(r_L) + \hat{c}_2(r_L)/\rho\right) + \pi_H \left(\hat{c}_1(r_H) + \hat{c}_2(r_H)/\rho\right)$$

is the expected present value of the consumption allocation $\{\hat{c}_1(r_i), \hat{c}_2(r_i)\}$.

Suppose that $c_1(r_L) < c_1(r_H)$. From the first-order conditions,

$$\begin{aligned} u'(c_1(r_L)) + \rho u'(\rho c_1(r_L)) &= \lambda \\ u(c_1(r_H)) &= \lambda + \mu \\ u(c_2(r_H)) &= (\lambda + \mu)/\rho. \end{aligned}$$

But $c_1(r_L) < c_1(r_H)$ implies that $c_2(r_H) \geq \rho c_1(r_H) > \rho c_1(r_L)$, so

$$u'(c_1(r_L)) + \rho u'(\rho c_1(r_L)) > u'(c_1(r_H)) + \rho u'(c_2(r_H)),$$

contradicting the first-order conditions. This shows that $c_1(r_L) = c_1(r_H)$, as claimed.

Then any solution to the problem (20) must solve

$$\begin{array}{ll} \max & (\pi_L + \pi_H)u(c_1(r_L)) + \pi_L u(\rho c_1(r_L)) + \pi_H u(c_2(r_H)) \\ \text{s.t.} & (2\pi_L + \pi_H)c_1(r_L) + \pi_H c_2(r_H)/\rho \leq \hat{w} \\ & c_1(r_H) \leq c_2(r_H)/\rho. \end{array}$$

The first-order conditions are

$$\begin{aligned} (\pi_L + \pi_H)u'(c_1(r_L)) + \pi_L \rho u'(\rho c_1(r_L)) &\geq \lambda(2\pi_L + \pi_H) \\ u'(c_2(r_H)) &\leq \lambda/\rho \end{aligned}$$

with strict equality if $\hat{c}_1(r_H) < \rho \hat{c}_2(r_H)$. On the one hand, if the incentive constraint is binding, i.e., $\hat{c}_2(r_H) = \rho \hat{c}_1(r_H) = \rho \hat{c}_1(r_L)$, then

$$\pi_L \rho u'(\rho \hat{c}_1(r_L)) \leq \lambda \pi_L.$$

Substituting this into the first-order conditions implies that

$$u'(c_1(r_L)) \geq \lambda \geq \rho u'(c_2(r_H)).$$

On the other hand, if the incentive constraint is not binding then

$$(\pi_L + \pi_H)u'(c_1(r_L)) + \pi_L \rho u'(\rho c_1(r_L)) = \rho u'(c_2(r_H))(2\pi_L + \pi_H).$$

In either case,

$$\hat{c}_2(r_H) = \max\{\rho \hat{c}_1(r_H), \phi(\hat{c}_1(r_H))\},$$

as claimed.

The second part of the proposition states that the consumption allocation in the benchmark problem (20) can be approximated by a feasible allocation in the original problem (19). Let $x = \hat{x}$. There are two cases to consider.

Case 1. If the incentive constraint is binding in the state r_H, then the bondholders and depositors receive a constant fraction of the total value of the bank's assets in each state. Let β denote the fraction going to the bondholders. Then from the first-period budget constraint, we can calculate that

$$\hat{x} + y = 2 + \frac{\beta}{\rho}\left\{\pi_L \gamma r_L h(\hat{x}) + \frac{\pi_H r_H}{\rho} h(\hat{x}) + \rho y\right\}$$

or

$$y(1-\beta) = 2 - \hat{x} + \frac{\beta}{\rho}\left\{\pi_L \gamma r_L + \frac{\pi_H r_H}{\rho}\right\} h(\hat{x}).$$

So for each value of β we can calculate a feasible value of y. Then set

$$\begin{aligned} c_1(r_H) &= \frac{c_2(r_H)}{\rho} = d = \frac{(1-\beta)}{2}\left\{\frac{r_H h(\hat{x})}{\rho} + \rho y\right\}, \\ c_1(r_L) &= \frac{c_2(r_L)}{\rho} = \frac{d}{2d+b}\left\{\gamma r_L h(\hat{x}) + \rho y\right\} \end{aligned}$$

and

$$b = \beta\left\{\frac{r_H h(\hat{x})}{\rho} + \rho y\right\}$$

and we have determined a feasible set of choices for each value of β. Now let $\beta \to 1$ and observe that $(2d+b)$ equals $r_H h(\hat{x})/\rho + \rho y$ so

$$\begin{aligned} \frac{c_1(r_L)}{c_1(r_H)} &= \frac{\gamma r_L h(\hat{x}) + \rho y}{2d+b} \\ &= \frac{\gamma r_L h(\hat{x}) + \rho y}{r_H h(\hat{x})/\rho + \rho y} \end{aligned}$$

converges to 1 as $y \to \infty$. Then the budget constraint ensures that $\{c_1(r_i), c_2(r_i)\} \to \{\hat{c}_1(r_i), \hat{c}_2(r_i)\}$ as $b \to \infty$.

Case 2. In the case where the incentive constraint is not binding in state $R = r_H$, put $x = \hat{x}$, $d = \hat{c}_1(r_H)$ and choose b arbitrarily. Then, as before, the first-period budget constraint tells us that

$$y = 2 - x + \frac{1}{\rho}\left\{\pi_L \frac{b}{2d+b}\left(\gamma r_L h(\hat{x}) + \rho y\right) + \pi_H b\right\}$$

or

$$y = \left(1 - \frac{\pi_L b}{2d+b}\right)^{-1}\left\{(2-x) + \frac{1}{\rho}\left(\pi_L \frac{b}{2d+b}\gamma r_L h(\hat{x}) + \pi_H b\right)\right\}.$$

This shows that y is uniquely determined by our choice of x, b, and d. Then the consumption allocation is determined by the budget constraints at dates

1 and 2:

$$\begin{aligned} c_1(r_H) &= d \\ c_2(r_H) &= r_H h(x) + \rho^2 y - \rho(b+d) \\ c_1(r_L) &= c_2(r_L)/\rho = \frac{d}{2d+b}\left(\gamma r_L h(x) + \rho y\right). \end{aligned}$$

As $b \to \infty$ we have $y \to \infty$ and $b/(2d+b) \to 1$. From the first-period budget constraint, we have

$$\begin{aligned} \frac{y}{b} &= \left(1 - \frac{\pi_L b}{2d+b}\right)^{-1} \left\{ \frac{(2-x)}{b} + \frac{1}{\rho}\left(\pi_L \frac{1}{2d+b}\gamma r_L h(\hat{x}) + \pi_H\right)\right\} \\ &\to (1-\pi_L)^{-1}\left\{\frac{1}{\rho}(\pi_H)\right\} = \frac{1}{\rho}. \end{aligned}$$

Thus, from the definition of the consumption allocation above,

$$\begin{aligned} \frac{c_1(r_L)}{c_1(r_H)} &= \frac{1}{2d+b}\left(\gamma r_L h(x) + \rho y\right) \\ &\to 1. \end{aligned}$$

Thus, in the limit as $b \to \infty$, the consumption of early consumers is equalized between the two states, and then the budget constraint implies that $c_2(r_H) \to \hat{c}_2(r_H)$. Thus, $\{c_1(r_i), c_2(r_i)\} \to \{\hat{c}_1(r_i), \hat{c}_2(r_i)\}$ as $b \to \infty$ and this in turn implies that for sufficiently large values of b the consumption level $c_2(r_H)$ defined above is non-negative and satisfies the incentive constraint.

This completes the proof that $\{\hat{c}_1(r_i), \hat{c}_2(r_i)\}$ can be approximated by some feasible choice of $(\{c_1(r_i), c_2(r_i)\}, b, d, x, y)$ in the original problem (19).□

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