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Optimal design of a plate with three symmetrical holes under uniaxial tension using boundary elements

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ABSTRACT

This paper presents an effective and practical structural shape optimization method which can be used to optimize the design of mechanical elements. The important features of this method are: (1) it provides an effective computational method for practical problems with implicit cost functions as functions of actual design variables, (2) it reduces maximum stress by reducing the weight of the design, and (3) it is useful in improving failed prototype designs without major design modifications. The example given to illustrate the method is the design of a plate under uniaxial tension.

INTRODUCTION

There have been a great many practical applications in the optimum design of continuum structures whose shapes are defined by their external geometry. These practical applications include the design of the arch dam [1], the torque arm [2], and pressure vessels [3]. In many cases, the principal emphasis is the reduction of stress concentration in order to provide a more uniform stress distribution in the structure. Moreover, in the design of mechanical elements, it is known that the stress concentration can be reduced by removing material from the original design [4].

Typically, in this type of shape optimization for continuum structures problems, the number of actual design variables is small. However, it is difficult to find the direct relation between the design variables and the structure displacements. This in turn makes it difficult to find the gradient information of the shape optimal design problem analytically. Furthermore, due to changes in the external geometry, the models for the stress analysis have to be constantly modified and remeshed to reflect the new design in the optimal design process. Ref. 5 presents a survey on the numerical methods for shape optimization of continuum structures.

The objective of this paper is to present an effective and practical structural shape optimization method which can be used to optimize the design of mechanical elements. The development of this method is based on boundary element analysis and minimization of a quadratic approximation to the maximum stress. Boundary element analysis is selected over the finite element method because the former method is more effective in mesh generation and the results are more accurate for stress concentration problems. To obtain the optimal design solution, a suitable number of observations of stress are made in the design space. Based on the observations, a quadratic surface is fitted via least squares regression. The quadratic surface is then minimized to obtain the optimal design point.

To illustrate the method we begin with a flat plate with a center hole under uniaxial tension. A practical way of minimizing the maximum stress caused by stress concentration is to cut two more symmetric circular holes. The objective of the shape optimization is to determine the location and the radius of these two symmetric holes. The design variables used in optimization are actual design variables: the location and the radius. The cost function is the maximum normal stress of the plate which is also an implicit function of the design variables.

OPTIMIZATION PROCEDURE

The approach taken was to derive a polynomial approximation to maximum stress (henceforth referred to as stress) around a given design and to carry

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out unconstrained minimization of the polynomial by variation of the design variables (location x and radius r of the symmetric hole). In order to do this, a number of simulations of stress were made in the design space at locations selected around the original design using a well known experimental design technique - the orthogonal array search method [6, 7]. A response surface for stress was constructed using a quadratic polynomial which was then minimized. Practical constraints on x and r were considered through designer interaction. A more detailed discussion on the optimization procedure (see Figure 1) now follows.

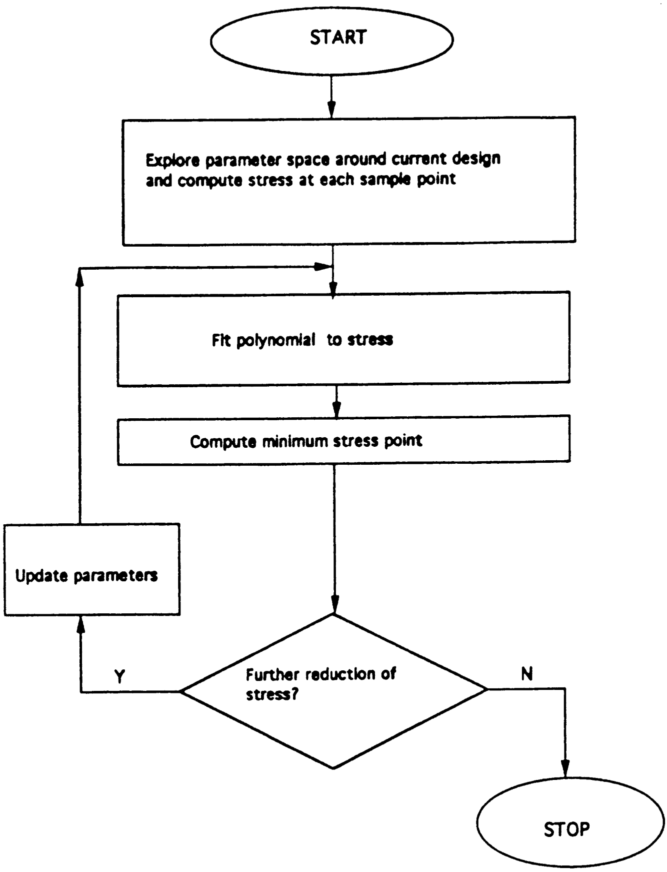

Figure 1. The Flowchart of the Optimization Procedure

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Orthogonal arrays are special matrices used in experimental design [6,7] to assess the effect of individual variables on system performance. Rows of the array correspond to experimental trials while columns correspond to individual variables; each entry in the array represents pre-defined quantization levels of the variable. Columns of an orthogonal array are mutually orthogonal, that is for any pair of columns, all combinations of variables occur an equal number of times. The procedure for the orthogonal array search is as follows: for each design variable the range of permitted values is quantized into a small number of levels (typically 2 - 5). A suitable orthogonal array is then selected from standard tables based on considerations which include the number of independent variables and the number of levels per variable. Finally, the simulation experiment is carried out using settings of variables derived from the orthogonal array. In this design example, an orthogonal array was used to define search points around the original design, and simulations of stress were then carried out at these points.

A second order polynomial approximation S_{fit} to simulate stress S was derived using least squares regression. Assuming a n-dimensional design space of variables $\overline{P} = \{p_1, p_2, \ldots, p_n\}$, the polynomial approximation

$$S_{fit}(\overline{P}) = \overline{P}^T A \overline{P} + B^T \overline{P} + C \qquad (1)$$

is the solution to the following minimization problem,

$$\underset{A,B,C}{Minimize} \quad \Lambda(A, B, C) = \sum_{over\ k} (S_{fit}(\overline{P}^k) - S^k)^2 \qquad (2)$$

where A is an $n \times n$ matrix, B^T is an n-vector and C a constant. The optimum coefficients A^*, B^*, C^*, (which correspond to the minimum of Λ) are found in the standard way [8] by differentiating Λ with respect to A, B, C and equating to zero. Wellness of fit was assessed by computing maximum percent deviation of $S_{fit}(\overline{P})$ from S.

Following derivation of the quadratic approximation S_{fit} to stress, the variable displacement $\Delta\overline{P}_{opt}$ which minimizes maximum stress can be obtained from

$$\Delta\overline{P}_{opt} = \frac{1}{2}A^{-1}B \tag{3}$$

or other optimization methods such as recursive quadratic programming method together with other practical considerations.

SIMULATION/ANALYSIS AND RESULTS

To illustrate the optimization procedures discussed above, an example structure of a flat plate with a center hole under uniaxial tension is studied (Figure 2). A practical way of minimizing the maximum stress caused by stress concentration is to cut two more symmetric circular holes. The objective of the shape optimization is to determine the location and the radius of these two symmetric holes. The design variables used in optimization are actual design variables where x is the distance from the center of the plate to the center of one of the new symmetric holes and r is the radius of the new holes (Figure 3).

For a two-design variables problem with three levels, a suitable orthogonal array was selected and included in Table 1. Based on this orthogonal array, nine simulations are needed to construct the quadratic surface to approximate the maximum stress of the plate. In each simulation, a set of design variables is used to construct the geometric model. Boundary element analysis is then used to perform the stress analysis. A commercially available finite element analysis computer program was first used to carry out the analysis. However, without an advanced automatic mesh generation, it became difficult and time consuming in mesh generation when the holes were close to each other.

The simulation results are included in Table 2. The Young's Modulus of the plate is 1×10^7 psi and the Poisson's ratio is 0.3.

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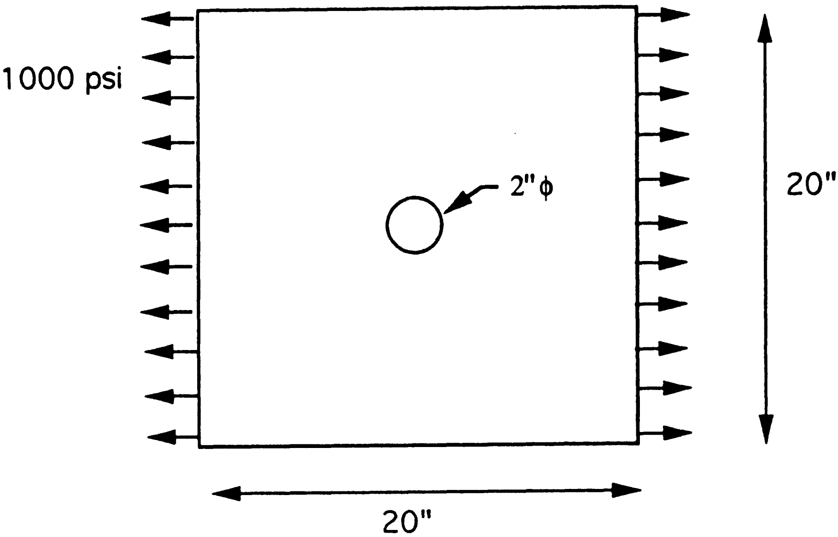

Figure 2. A Flate Plate with a Center Hole Under Uniaxial Tension

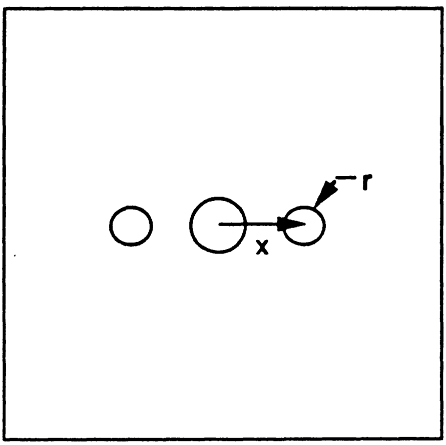

Figure 3. The Design Variables x and r

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Table 1. The Orthogonal Array for Two Variables and Three Levels

Simulation No.	x (in)	r (in)
1	2.5	0.25
2	2.5	0.5
3	2.5	0.75
4	3.0	0.25
5	3.0	0.5
6	3.0	0.75
7	3.5	0.25
8	3.5	0.5
9	3.5	0.75

Table 2. The Simulation Results Based on the Orthogonal Array

Simulation No.	x (in)	r (in)	Analysis Results (psi)
1	2.5	0.25	3032
2	2.5	0.5	2901
3	2.5	0.75	2705
4	3.0	0.25	3030
5	3.0	0.5	2906
6	3.0	0.75	2700
7	3.5	0.25	3034
8	3.5	0.5	2923
9	3.5	0.75	2738

With the simulation results, a quadratic surface was constructed to approximate the maximum stress of the plate and the contour plot of this surface is shown in Figure 4. The comparison between the analysis results and the quadratic approximation results is included in Table 3. Table 3 shows that a

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quadratic surface is a good approximation of the maximum stress as the maximum error is only 0.28%.

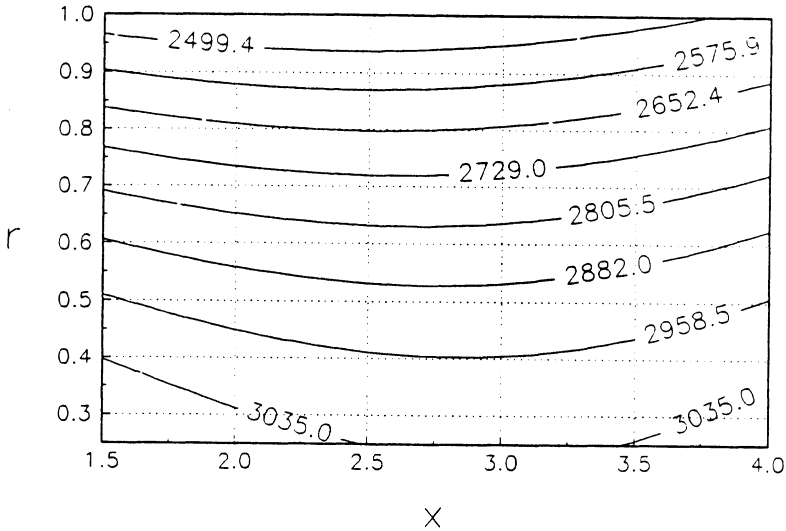

Figure 4. The Contours of the Approximated Stress Surface

Table 3. The Comparison between the Analysis Results and the Approximation Results

x (in)	r (in)	Analysis Results (psi)	Approximation (psi)	Deviation (psi)
2.5	0.25	3032	3033.6	1.6
2.5	0.5	2901	2903.9	2.9
2.5	0.75	2705	2700.5	4.5
3.0	0.25	3030	3025.2	4.8
3.0	0.5	2906	2903.2	2.8
3.0	0.75	2700	2707.6	7.6
3.5	0.25	3034	3037.1	3.1
3.5	0.5	2923	2922.9	0.1
3.5	0.75	2738	2735	3.0

With the quadratic approximation of the maximum stress, the optimal design can be determined by using the recursive quadratic programming method in an optimal design computer program IDESIGN [9]. The optimal design is when $x = 2.373$ in. and $r = 1$ in. The maximum stress of the plate is 2666 psi.

The results of the optimization show a reduction of 12% of the maximum normal stress together with a further volume reduction which is 200% of the original center hole. This method can be applied to plates with multiple holes as well as other mechanical elements design problems.

CONCLUSIONS

A method for the optimal shape design of continuum structures has been presented. An important feature of this method is its application to solving practical problems with implicit cost functions as functions of actual design variables. The number of simulations needed in the example for two design variables with three levels is nine. However, for four design variables with three levels, only nine simulations are needed. Thus, this method becomes more effective as the number of design variables increases.

The design example discussed shows that the maximum stress of the design can be reduced by removing material from the original design. As a result, the weight of the design is also reduced. Therefore, this approach is useful in aerospace and automotive industry as well as in improving existing prototype designs.

With the advances made in parametric modeling [10], the models for boundary elment analysis can be automatically generated and meshed based on the sets of design variables obtained from the orthogonal array. This will lead to a fully automated optimal design process.

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