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Optimal Design of Adaptive Type-2 Neuro-Fuzzy Systems: A Review

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Abstract

Type-2 fuzzy logic systems have extensively been applied to various engineering problems, e.g. identification, prediction, control, pattern recognition, etc. in the past two decades, and the results were promising especially in the presence of significant uncertainties in the system. In the design of type-2 fuzzy logic systems, the early applications were realized in a way that both the antecedent and consequent part parameters were chosen by the designer with perhaps some inputs from some experts. Since 2000s, a huge number of papers have been published based on the parameter adaptation of the parameters of type-2 fuzzy logic systems using the training data either online or offline. Consequently, the major challenge was to design these systems in an optimal way in terms of their optimal structure and their corresponding optimal parameter update rules. In this review, the state of the art of the three major classes of optimization methods for the training of type-2 adaptive fuzzy-neuro systems are investigated: derivative-based (computational approaches), derivative-free (heuristic meth-

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ods) and hybrid methods which are the fusion of both the derivative-free and derivative-based methods.
Keywords: Interval type-2 fuzzy logic systems, optimal learning algorithm, hybrid learning, parameter update rules, genetic algorithms, particle swarm optimization.

1. Introduction

Since the inception of the fuzzy set theory in 1965, the mathematical advancements have progressed to exceptionally high standards. A plethora of research has been conducted on fuzzy systems and its implementations in many disciplines. A demanding analysis is required to collect the information on fuzzy logic systems (FLSs) i.e, the theoretical and real-time applications of fuzzy sets available in literature. In this paper, a literature review is conducted through searching of bibliographic databases. The search is limited to four databases: IEEE Xplore, SpringerLink, ScienceDirect and Wiley online library, and to the years 2000-2014, respectively. These four databases are the major publishers in the field of fuzzy logic theory.

In this survey, the term "fuzzy system" was searched initially. The search for this term identified papers in every aspect of the field such as; control system, modeling, design, expert system, knowledge, regression and classification. A total of 98,702 conference and 55,715 journal publications are found using the above term. The search is then refined by the term "type-2 fuzzy", using the query of $\langle$fuzzy system$\rangle$ AND $\langle$type-2 fuzzy$\rangle$, leaving away the publications in type-1 fuzzy logic theory. The annual number of publications for type-1 and type-2 fuzzy logic theory can be seen in Fig. 1a. The large number of publications reported for type-1 fuzzy logic theory is due to the fact that the early introduced type-1 fuzzy logic systems (T1FLSs) have several software packages that simplify the task of researchers. However, a continuous increase in the publication of type-2 FLSs (T2FLSs) can be seen in Fig 1b. The search is again refined by the query $\langle$fuzzy system$\rangle$ AND $\langle$fuzzy learning$\rangle$ in order to pick

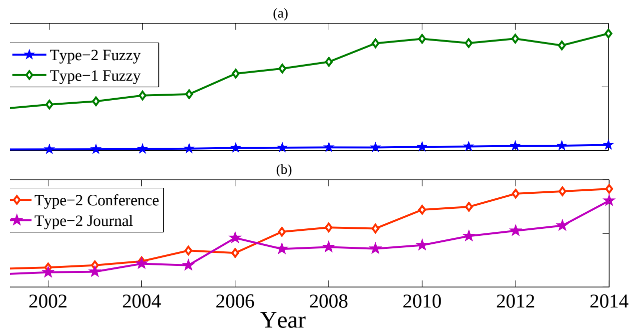

Figure 1: (a) Annual number of publications for type-1 and type-2 fuzzy logic theory, (b) Annual number of conference and journal publications for type2 fuzzy logic theory.

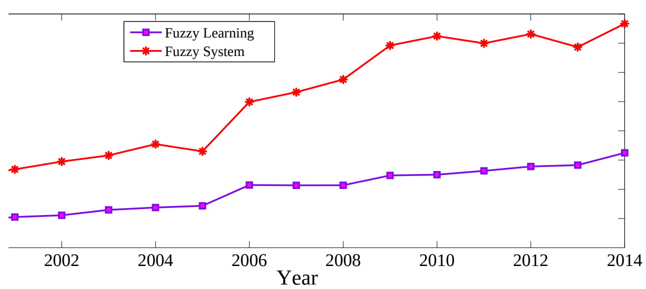

Figure 2: The annual number of publication on fuzzy learning.

the publications on fuzzy learning models only, which is also the main research focus of this review. Figure 2 shows the trend in the number of publications on fuzzy learning which shows the wider interest in adaptive FLSs rather than conventional FLSs in which the parameters are fixed.

"Learning" and "tuning" can be used interchangeably in the design of a FLS. However, the difference between the two is that the former is a process in FLSs where the search does not depend on predefined parameters and automatic design of a FLS starts from the scratch, whereas the later starts the optimization of a FLS with a set of predefined parameters and focuses to find the best set. Different approaches of soft computing can be applied here to enhance the computational and predictive performance of fuzzy systems. Indeed, research has demonstrated that formalizing an issue pertaining human expert knowledge is a difficult and time consuming job. More often than not, it does not even prompt completely fulfilling results. For that reason, a sort of data-driven approach of fuzzy systems is usually beneficial [1].

Generally, a fuzzy system with learning ability allows its different parameters to be tuned. The dashed arrow crossing the blocks of a T2FLS in Fig. 3 shows the possible components that can be tuned. During the design of a non-adaptive fuzzy system, experts assign linguistic labels to the problem variables by using fuzzy membership functions (MFs). However, they cannot give the precise MFs defining the semantics of these labels. Normally, these values are defined by partitioning the domain of interest. Through discretization, the variables in the domain are partitioned into the equivalent number of intervals that of linguistic labels considered. The process needs to define a uniform fuzzy partition with symmetric and identical shape fuzzy sets. However, this approach generally ends up with a sub-optimal performance of the fuzzy system [1]. In order to address this as a specific end goal, different learning techniques have been reported in literature for the generation of fuzzy set automatically. These techniques include decision tree [2, 3, 4], clustering [5, 6], hybrid models [7, 8, 9] and evolutionary algorithms [10, 11, 12]. The presence of 3D-MF in T2FLS necessitates the adjustment of more parameters than T1FLS, which makes the learning process more complicated [13]. The footprint of uncertainty (FOU) in interval T2FLSs (IT2FLS) can also be tuned to improve the performance in the presence of noise [14].

In general, fuzzy modeling is a system modeling with fuzzy rule based systems (FRBS) that represents a local model which is effectively interpretable and analyzable [15]. When the expert is not available or does not have sufficient information to stipulate the fuzzy rules, then numerical information is utilized to determine these rules. Two distinguished fusions of fuzzy with neural networks (NNs) also known as neuro-fuzzy models [16] and with genetic algorithm known as genetic fuzzy systems [15] have been used to automatically generate the fuzzy rules. FRBS is a universal approximator as it can approximate any function to

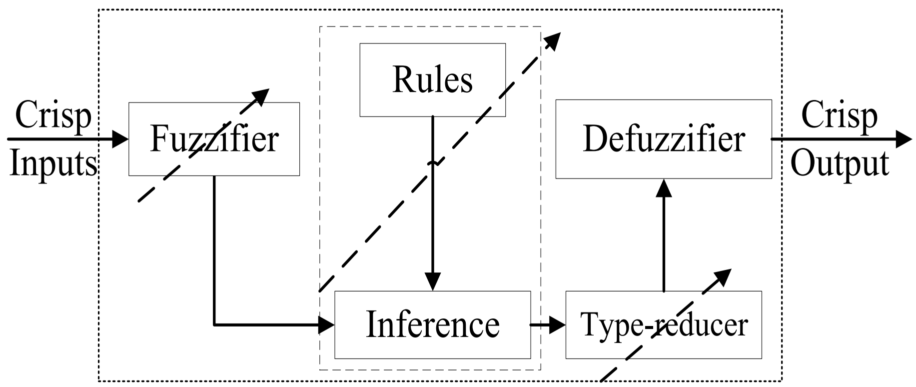

Figure 3: Learning parts of a T2FLS.

the desired degree of accuracy [17, 18]. FRBS is a preferable choice over NNs, as the parameters involved have a real world meaning and consequently, the initial guess parameters can substantially enhance the training algorithm.

The optimization methods for FLSs can be broadly categorized into three methods as shown in Fig 4. In this review, the main motivation is to present the state of the art of the three major classes of optimization methods:

- Derivative-based (computational approaches),
- Derivative-free (heuristic methods),
- Hybrid methods which are the fusion of both the derivative-free and derivative-based methods.

To the best of our knowledge, there is no paper in literature which focusses on the comparison of all the different learning algorithms listed in this survey paper specifically on interval type-2 fuzzy neural networks (T2FNNs). Recently, a book has been published in Elsevier in which some of the methods are compared [19]. We think that the learning control by using T2FNNs will be a hot topic in the near future too. This survey paper, which collects all of the learning methods in literature, will help researchers a lot to see their pros and cons in one paper.

The rest of the paper is organized as follows: The derivative-based optimization algorithms for T2FNN are described in Section 2. The derivative-free optimization algorithms are given in Section 3. Section 4 discusses the hybrid learning algorithms of T2FNN. Some comparison and discussions are given in Section 5.

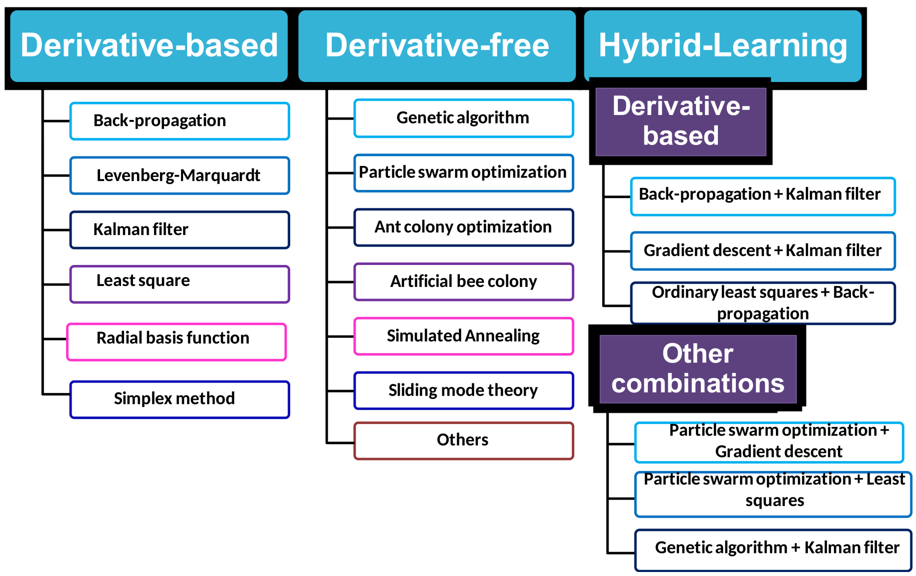

Figure 4: Different learning methods for T2FLS.

2. Derivative-Based or Gradient Descent-Based Learning Algorithms

The objective of the methods listed in this category is to solve nonlinear optimization problems through an objective function by using derivative information. Some of the derivative-based methods, also known as Gradient-based optimization, are discussed below particularly for T2FLS and IT2FLS.

2.1. Back-Propagation Algorithms

Back-propagation (BP), also known as steepest-descent or gradient descent (GD), algorithm is one of the most popular techniques used to update the parameters a T2FLS [20]. In [21] the mathematical formulation and computational flowcharts for computing the derivatives have been provided that were needed to implement the steepest-descent algorithm to tune the parameters of T2FLSs. In order to adjust the parameters of a T2FLS, this algorithm needs to compute the first derivatives of the objective function with respect to every single parameter. The main body of the paper has focused on IT2FLSs. A challenging task of deriving the derivatives of the BP algorithm was taken for the antecedent and consequent parameters of the IT2FLS. FOU selection was prolonged so as to make the results appropriate to all sorts of FOU. In the last part, the type of MFs for IT2FLS were specified keeping in mind the end goal to finish the computations. Center-of-set type-reduction was replaced by the two end-points of the centroid to reduce the number of design parameters. The IT2FLS designed with GD method is usually used for the benchmark purposes.

A novel structure, T2FNN, was presented by Wang et al. [22] as the fusion of NNs and T2FLS, in order to handle the uncertainty with dynamical optimal learning. A T2FNN consists of a T2 fuzzy linguistic process as the antecedent part, and the two-layer interval NN as the consequent part. In order to simplify the computational process, interval T2FNN was adopted. The training algorithm of the antecedent and consequent parameters in interval T2FNN was derived using a GD. Genetic algorithm was combined with the dynamical optimal training algorithm to determine the optimal spread and learning rate for the antecedent part of the interval T2FNN. The proposed model outperformed T1FNN in several examples. However, the fuzzy rule reordering problem while computing the left and right end points were not appropriately derived with the parameters learning equations. The issue was highlighted and a complete and detailed version of the specific BP equations were derived in [23] to tune both the antecedent and consequent parameters of the interval T2FNN.

2.2. Levenberg-Marquardt Algorithm

Keeping in mind that the performance might be improved if higher derivatives instead of first derivatives are used, Khanesar et al. proposed a T2FNN based on the Levenberg-Marquardt algorithm [24]. The algorithm uses second order derivatives that made the training process faster. A simple method of computation of the Jacoboan matrix which is being the most difficult step in implementing the Levenberg-Marquardt algorithm was also described. A modified version of the novel T2 fuzzy MF with certain values on both ends of the support and the kernel, and some uncertain values on the other values of the support (Elliptic MF) [25] was also proposed. The proposed learning algorithm in T2FNN was utilized for the prediction of a Mackey-Glass time series data. The effectiveness of the proposed algorithm was shown with the benchmark GD algorithm.

The Levenberg-Marquardt algorithm was also utilized by Castillo et al. in [26] for optimizing the parameters of an adaptive IT2FNN. The universal approximation of the IT2FNN was shown based on Stone-Weierstrass theorem as the major contribution of the paper. Simulation results of nonlinear function identification using the proposed IT2FNN for different number of variables with the Mackey-Glass time series data has been presented.

2.3. Kalman Filter-based Algorithm

Khanesar et al. proposed the use of decoupled extended Kalman filter for the optimization of both the parameters of the antecedent and consequent parts of T2FLS [27]. By utilizing the decoupled extended Kalman filter, certain group of parameters had interaction between groups instead of one group that minimized the computational cost. A novel T2 fuzzy MF having certain values on both ends of the support and the kernel, and uncertain values on other parts of the support were taken to benefit the T2FLS. Comparison of the models was conducted with a population based particle swarm optimization and with a first-order GD-based method for the optimization of the antecedent part of T2FLS. The proposed T2FLS structure was tested on different noisy data set, that have illustrated better performance of extended Kalman filter based method over the benchmark models. Moreover, noise reduction characteristics of the novel T2 fuzzy MF was shown in the simulation results.

The impact of inaccurate statistics on the obtained results of a noise-sensitive

Kalman filter was avoided by an adaptive Kalman filter based design of an IT2FLS [28]. Based on the proportion of the actual value of the residual covariance to its theoretical value, the proposed model dynamically adjusted the measurement noise covariance. The adjustment changed the values of the filter to improve the accuracy of the state estimation. The proposed method was validated by conducting extensive simulations with respect to the position estimation of ship. The simulation results were compared with a standard Kalman filter based T2FLS (where adaptive techniques was not utilized) and with an adaptive Kalman filter based T1FLS.

2.4. Least Square Method

A regression model for IT2 fuzzy sets based on the least squares estimation technique was presented by Poleshchuk and Komarov in [29]. Unknown coefficients were assumed to be triangular fuzzy numbers. Aggregation intervals for T1 fuzzy sets were determined whose lower and upper MFs were of IT2 fuzzy sets. These aggregation intervals were called weighted intervals. The IT2 fuzzy MFs for the developed regression models were taken of type piecewise linear functions. The standard deviation, hybrid correlation coefficient, and hybrid standard error of estimates were defined for reliability evaluation.

2.5. Radial Basis Function

The IT2 fuzzy MFs from labeled pattern data and its application to radial basis function networks (RBFN) was presented by Rhee and Choi in [30]. The authors constructed the histogram of the sample data for each labeled class and feature by smoothing the domain of each feature by a symmetric window function (e.g., a triangular function). The vertex of the triangular function was positioned at the first bin of the histogram and the weighted moving average was calculated, then the vertex was moved to the next bin. This procedure was repeated for all of the bins. The histogram was fitted by a 4th degree polynomial function to determine the number and approximate parameter values for a IT2 fuzzy Gaussian MF. T1 fuzzy MFs, which were computed from the centroid of the IT2 fuzzy MFs, were incorporated into the RBFN. The proposed MF assignment was shown to improve the classification performance of the RBFN since the uncertainty of pattern data were desirably controlled by IT2 fuzzy MFs.

A new robust controller based on the integration of a RBFN and an IT2 fuzzy logic controller for robot manipulator actuated by pneumatic artificial muscles was proposed by Amar et al. in [31]. The proposed approach was synthesized for each joint using the sliding mode control (SMC) and named RBFN T2 fuzzy sliding mode control. Avoiding difficult modeling, attenuating the chattering effect of the SMC, reducing the rules number of the fuzzy control, guaranteeing the stability and the robustness of the system, and handling the uncertainties of the system were highlighted as some of the objectives that can be accomplished using this control scheme. The proposed control approach was synthesized and the stability of the robot using this controller was analyzed using Lyapunov theory. The efficiency of the proposed controller was compared with other control technique. The superiority of the proposed controller compared to a RBFN T1 fuzzy SMC was demonstrated from the results. Finally, an experimental study of the proposed approach was presented using 2-DOF robot.

2.6. Simplex Method

A modified IT2 Takagi-Sugeno-Kang (TSK) FLS was proposed by Wang et al. in [32]. First, the T1 TSK FLS was built using subtractive clustering method combined with least square method. The T2 TSK FLS was then obtained from the T1 TSK FLS through unconstrained optimization using the Nelder-Mead Simplex method by varying the parameters of the antecedent and consequent parts. The modified IT2 TSK FLS was applied to a heat exchange process on the equipment CE117 Process Trainer. The efficiency of the proposed Simplex method for IT2 TSK FLS over T1 TSK FLS was demonstrated during experiment.

The T2 fuzzy linear programming problems was solved in [33] by using two-phase simplex method. A new ranking function for T2 fuzzy sets was defined using graded mean integration representation. Original objective function for the fuzzy linear programming was defined during the first phase. The simplex method was employed in phase 2 to find the optimal solution to the original problem. The two phase method, using the proposed new ranking function as linear ranking functions on T2 fuzzy numbers, appeared to be a natural extension of the results for the linear programming problem with the crisp data. The authors suggested that the capabilities offered may be useful for the post optimal analysis.

3. Derivative-free or Gradient free Learning Algorithms

In the conditions that the derivative information is unavailable, unreliable or unfeasible, derivative-free methods are preferred. These methods do not need functional derivative information to search a set of parameters that minimize (or maximize) a given objective function. Rather, they depend solely on repeated evaluation of the objective function [34]. Derivative-free optimization has encountered a restored enthusiasm over the previous decade that has energized another influx of theory and algorithms. Automatic design of T1FLS using such optimization algorithms has become a standard practice. The trend has now transferred to automatic design of T2FLS and IT2FLS using these algorithms. A concise review on some of such optimization algorithms for T2FLS has been done in [35]. Reference [36] presented a comparative study of bio-inspired algorithms applied to the optimization of T1 and T2 fuzzy logic controller (FLC). Below are some of the derivative-free optimization methods that have been utilized for optimizing T2FLS.

3.1. Genetic Algorithm

Genetic algorithm (GA) is an adaptive heuristic search algorithm based on a formalization of natural selection and genetics. The basic principles of GAs were first proposed by John Holland in 1975, inspired by the mechanism of natural selection, where stronger individuals are likely the winners in a competing environment [15]. A population of chromosomes, objective function and stopping criteria are required to be defined in GA. The population then undergoes genetic operation to evolve and the best population is selected based on the objective function. In order to optimize a T2FLS by means of GA, it must be represented as a population of chromosomes. A diverse applications using GAs for T2FLS optimization has been overviewed in [37].

A designing method for a T2FLS using GA was proposed by Park and Kwang in [38]. The positions and the shapes of the MFs and the rules of a T2FLS were determined through the proposed method. T2 fuzzy parameters in the T2FLS were encoded as chromosome. The proposed method was applied to the prediction of a chaotic time-series data and the result of the experiment was shown to demonstrate the performance.

GA for the optimization of a T2FNN was also proposed in [13]. The feature parameters to represent a T2 fuzzy set were determined first, then using these parameters, a T2FNN system was encoded as a chromosome. The real-code GA was then used to optimize the T2FNN antecedent and consequent MFs.

Wu and Tan [39] utilized GA for the design of a T2FLS to control non-linear plants and presented performance evaluation of the interval T2FLC using GA. The paper focused on advancing the understanding of the interval T2FLC. The T2FLC was then compared with another three GA evolved T1FLCs that have different design parameters. The objective was to examine the amount by which the extra degrees of freedom provided by antecedent T2 fuzzy sets were able to improve the control performance. Experimental results showed that better control can be achieved using a T2FLC with fewer fuzzy sets/rules. This implies that a lower trade-off between modeling accuracy and interpretability is one benefit of the T2FLC.

The design methodology of IT2FNN was introduced by Park et al. in [40] to optimize the network using a real-coded GA. The antecedent part was comprised of the fuzzy division of input space and the consequent part of the network was represented by polynomial functions. The parameters of the network were optimized using GA. The proposed network was evaluated with the chaotic

Mackey-Glass time series data and NOx emission process data of gas turbine power plant. Forecasting comparison of IT2FNN with T1FNN proved better performance of the proposed model.

The high capability of T2FLSs in combination with the GA for managing the uncertainty issues inherited in the inputs of a computer aided detection (CAD) system classifier was studied by Hosseini et al. in [41]. Additionally, the paper also presented an optimized genetic IT2FLS with Gaussian MFs approach for a multidimensional pattern recognition problem with a high number of inputs. Furthermore, GA was employed for tuning the MFs parameters and FOU. In order to assess the performance, the designed IT2FLS was applied on a lung CAD application for classification of nodules and was compared to a T1FLS. The results revealed that the Genetic IT2FLS classifier outperformed the equivalent T1FLS and was capable of capturing more uncertainties.

An optimization method for the design of T2FLS based on the FOU of the MFs using GA was proposed by Hidalgo et al. in [42]. Three different cases were considered to reduce the complexity problem of searching the parameter space of solutions. T2 fuzzy MFs optimized using GA were considered in different cases for changing the level of uncertainty of the MF so as to achieve the optimal solution at the end. The improvement of the designed method over T1FLS was evidenced on three benchmark problems.

A new system of T2 genetic fuzzy system was proposed by Shukla and Tripathi in [43]. A genetic tuning approach named lateral displacement and expansion/compression in which α and β parameters were calculated to adjust the parameters of IT2 fuzzy MFs. The system considered the interpretability and accuracy features during its design. It was concluded that the proposed tuning approach is interpretable and the experimental results were found satisfactory.

3.2. Particle Swarm Optimization

Particle swarm optimization (PSO) is a population-based stochastic optimization technique developed by Eberhart and Kennedy in 1995 [44]. Inspired by the behavior of a population of moving individual particularly, bird flocking and fish schooling, the PSO looks for the best solution. The agents in PSO are called particles. A function or system must be represented as a particle when using a PSO for optimization. The advantages of using the PSO optimization technique for automating the design process of T2FLS has also been illustrated in [45].

A training method for a T2FLS using PSO was presented by Al-Jaafreh and Al-Jumaily in [46]. T2FLS and PSO were utilized together, the procedure to analyse the problem was explained and finally presented a new method to optimize parameters of the primary MFs of T2FLS using PSO to improve the performance and increase the accuracy of the T2FLS. The proposed optimization method was implemented on mean blood pressure estimation. The heart rate was input to the system using five Gaussian MFs. The PSO was utilized to adjust the parameters of MFs to minimize the difference between the actual and obtained mean blood pressure. A satisfactory performance of the proposed method was observed during the analysis of the results.

A T2FNN optimized using PSO as a reliable on-site partial discharge pattern recognition algorithm was developed by Kim et al. in [47]. T2FNNs exploit T2 fuzzy sets which are robust in the diverse area of intelligence systems. Considering the on-site situation where it is not easy to obtain voltage phases to be used for phase resolved partial discharge analysis, the partial discharge data set measured in the laboratory were artificially changed into data set with shifted voltage phases and added noise in order to test the proposed algorithm. The results obtained by the proposed algorithm were compared with that of conventional NN and the RBFN. The proposed T2FNNs appeared to have better performance when compared to conventional NN.

The design and simulation of T2 fuzzy MFs for the average approximation of an interval of T2FLC was proposed using PSO [48]. In order to reduce the runtime of the algorithm, some points of triangular and trapezoidal T2 fuzzy MFs were considered for modifications using optimization and, the consequent parameters were not altered. Three objective functions namely overshoot, undershoot and steady state error were considered for the performance ability of the T2FLC. The proposed controller was applied on FPGA implementation and the results were compared with the same controller optimized using GA under uncertainty.

3.3. Ant Colony Optimization

Ant colony optimization (ACO), a meta-heuristic algorithm, is motivated by the behavior of ants in discovering paths from their colony to the food source [15]. The technique can be utilized for issues that can be reduced to discovering the superior paths along graphs. By optimizing T2FLS with ACO, it should be represented as one of the paths that the ants can follow in a graph. The advantages of using the ACO optimization techniques for automating T2FLSs were briefly reviewed in [49].

A Reinforcement Self- Organizing IT2FLS with ACO was proposed by Juang et al. in [50]. In order to improve system robustness to noise, the IT2 fuzzy sets were used in the antecedent part whereas ACO was utilized to design the consequent part of each fuzzy rule. The consequent part was selected from a set of candidate actions according to ant pheromone trails. The proposed model was applied to a truck backing control. Comparison of the proposed model was done with reinforcement T1FLS to verify its efficiency and effectiveness. The results of the comparison verified the robustness of the proposed model to noise.

A new reinforcement-learning method using online rule generation and Q-value-aided ACO for an IT2FLS based controller was proposed by Juang et al. in [51]. The antecedent part in the IT2FLS utilized the IT2 fuzzy sets to enhance the controller robustness to noise. The structure and parameters of an IT2FLS were simultaneously designed in the proposed method. An online IT2 rule generation method for the evolution of system structure and flexible partitioning of the input space was proposed. Consequent part parameters in an IT2FLS were designed using Q-values and the reinforcement local-global ACO algorithm. The consequent part was selected from a set of candidate actions according to ant pheromone trails and Q-values, both of which were modified using reinforcement signals. The proposed method was applied to the truck-backing control, magnetic-levitation control, and a chaotic-system control. In order to verify the efficiency and effectiveness of the proposed mode it was compared with other reinforcement-learning methods . Comparisons with a T1FLS verified the robustness property of using an IT2FLS in the presence of noise.

Optimization of the MFs of an IT2FLC using ACO and PSO for an autonomous wheeled mobile robot were presented by Castillo et al. in [36]. Statistical comparison of the optimization model was examined in detail with one another and with a GA based IT2FLS, so as to determine the best optimization method for this specific mechanical autonomy issue. During comparison, it was observed that both PSO and ACO had the capacity to beat GAs for this specific application. However, in a comparison between ACO and PSO, the best results were accomplished with ACO. In this case, the authors concluded that ACO is the most appropriate optimization algorithm for this robotic problem.

A T2FLS with a defuzzifier block determined through ACO as an optimal intelligent controller was proposed by Rezoug et al. in [52]. The optimized T2FLC was exploited under an unmanned aerial vehicle. The performance of the ACO based T2FLC was compared with a PSO based T2FLC applied to Birotor helicopter system. The superiority and the effectiveness of the proposed method was illustrated over the PSO based T2FLC and a classical T2FLC cases.

3.4. Artificial Bee Colony

The artificial bee colony optimization (ABC) or (BCO) is also a metaheuristic algorithm and is inspired by the foraging behavior of honeybees [53]. A bee in BCO represents an agent; and a FLS or FLC must be represented as a bee to optimize it using BCO. A new optimization technique for T1 and T2FLCs using the BCO was presented by Amador-Angulo and Castillo in [54]. The collective intelligent behavior that bees have for the solution of optimization problems was analyzed for T1 and T2FLCs. The optimization of the MF parameters of T1 and T2FLC was made using BCO and was applied to a benchmark problem of water tank controller. The fuzzy controllers were analyzed with different variants of the design. Better result was obtained when noise was applied in a T2FLC.

3.5. Simulated Annealing

An optimized design of IT2FLS was presented using simulated annealing (SA) by Almaraashi et al. in [55]. The parameters of the antecedent and the consequent parts of the IT2FLS were optimized using SA by minimizing the objective function. The optimized model was then applied to predict the Mackey-Glass time series by searching for the best configuration of the IT2FLS. By using an adaptive step size for each input during Markov chain, the SA reduced the computation time of IT2FLS. The results of the proposed methodology were compared to that of a T2FLS.

A general T2FLS was designed using SA algorithm with the aid of an IT2FLS [56]. The focus of the proposed was to reduce the computations needed to get the best FOU using IT2FLS. The proposed methodology consists of three stages, i.e., designing of IT2FLS using SA, conversion of IT2 fuzzy set into symmetrical general T2 fuzzy set and then learning of FOU of general T2FLS using SA. The methodology was applied to four benchmark problems. The outcomes demonstrated that the conversion process conveyed a decent estimate to the IT2FLS outputs with little misfortunes in accuracies and reduces the computations.

3.6. Sliding Mode Theory

SMC theory-based learning algorithms are also extensively used to train T2FNNs [57, 58]. A SMC theory-based learning algorithm was proposed to upgrade the rules for both the premise and consequent parts of a T2FNNs [57] as an extension to its type-1 counterpart [59]. The algorithm also tuned the sharing of the lower and upper MFs of the T2FNN to deal with the varying uncertainties in the rule base of a T2FLS. Besides, the learning rate of the system was updated during the online training. The stability of the proposed learning algorithm has been verified by using an appropriate Lyapunov function.

Faster convergence speed of the proposed algorithm had been demonstrated over the existing methods. The work has then been extended to type-2 fuzzy wavelet neural networks in [60].

3.7. Others

The most influential fuzzy rules in the design of a T2FLS were determined with two novel indices for T2 fuzzy rule ranking presented by Zhou et al. in [61]. These indices were named R-values and c-values of fuzzy rules separately. The estimation of the rank for the singular value decomposition and QR factorization with column pivoting algorithm was avoided by obtaining the R-values of T2 fuzzy rules that were obtained by applying QR decomposition. In order to perform the rule reduction, the c-values of T2 fuzzy rules were suggested to rank rules based on the effects of rule consequents. Experimental results on a signal recovery problem had shown that by using the proposed indices the most influential T2 fuzzy rules were identified and the parsimonious T2FLS was constructed effectively with satisfactory performance.

IT2FLSs were optimized with two types of tabu search (TS) by Almaraashi and Hedar in [62]. The best configuration of the IT2FLS parameters was sought through TS. Directed TS, that uses pattern search to control TS moves, and short-term TS with IT2FLS were utilized and applied to a classification issue of two benchmark data sets. The focus of the paper was to improve the structure and lessen the computation time of IT2FLSs utilizing an intelligent directed search instead of a random search. The directed TS-based IT2FLS outperformed the default TS-based IT2FLS by a noticeable difference during comparison . This perception uncovered the significance of utilizing a guided search moves as opposed to utilizing a randomized search direction in IT2FLS.

An IT2FLS was designed with the help of coevolutionary approach by Hostos et al. in [63]. The number of MFs were kept fixed while that of rules were kept vary to inspect the performance of the IT2FLS. The evolutionary algorithm utilized these parameters to acquire an IT2FLS. The interpretability of the model was satisfied by setting up a constrained fuzzy partition for every input so as the coevolution process look for the best MFs within a constrained distribution. A T1FLS was designed with the same parameters as T2FLS for comparison purposes. Simulation results on a Mackey-Glass time series prediction proved the capability of the proposed IT2FLS in achieving better results on the interest of few generations. However, the approach needed a greater computational burden.

The novel application of Big Bang-Big Crunch optimization approach to optimize the antecedent parameters of the IT2 fuzzy PID controllers in a cascade control structure was presented in [64]. The Big Bang-Big Crunch was employed to tune the parameters of the IT2FLC as its computational cost is low and convergence speed is high. The proposed IT2 fuzzy PID was compared with its T1 fuzzy PID and conventional PID controller counterparts that were also optimized using Big Bang-Big Crunch method. The results illustrated that the proposed IT2 fuzzy PID greatly enhanced the control performance even in the presence of uncertainties and disturbances over other models.

A fuzzy edge detector based on the Sobel technique and IT2FLS was optimized using cuckoo search and GA with the aim to determine the optimal antecedent parameters of the IT2FLS [65]. The goal of using IT2FLS in edge detection methods was to provide them with the ability to handle uncertainty in processing real world images. Simulation results revealed that using an optimal IT2FLS in conjunction with the Sobel technique provides a powerful edge detection method that outperformed its T1 counterparts and the pure original Sobel technique.

4. Hybrid Learning Algorithms

A combination of two or more models in a single model is known as a hybrid model. Hybrid models are becoming increasingly popular due to their synergy in performance. Hybrid learning algorithm are likewise a mix of more than one learning algorithms used in designing the optimized models to improve performance of the models. These algorithms may be of the same type i.e., derivative-based or derivative-free or may be a combination of both.

4.1. Derivative-based Hybrid Learning Algorithms

In the work of Castro et al. [66] the issue of dealing with uncertain information was suggested with the development of new methods. Three IT2FNN models as an integration of IT2 TSK FLS and adaptive NN, with hybrid learning algorithms were proposed to solve the issue. GD and GD with adaptive learning rate were used as a hybrid learning algorithm. Keeping in mind the end goal to fuzzify the antecedents and consequents rules of an IT2 TSK FLS; IT2FNN was utilizes at the antecedents layer and IT1FNN at the consequents layer. Experimental were conducted with a non-linear identification in control system and prediction of a noisy Mackey-Glass time serried data. During the comparative analysis of the optimized IT2FNN and an adaptive neuro-fuzzy inference system, IT2FNN was demonstrated as a proficient mechanism for modeling real-world problems.

In the work of Mendez et al. a hybrid learning algorithm based on recursive Kalman filter and BP was presented for IT2 TSK FLS [67]. The consequent parameters were tuned using recursive Kalman filter during the forward pass and antecedent parameters were tuned using BP algorithm. The IT2 TSK FLS with hybrid learning algorithm was implemented for temperature prediction of the transfer bar at hot strip mill. Comparison of the proposed model was done with the existing models in literature. Better performance of the model was demonstrated with the hybrid learning algorithm than the individual techniques when used alone for the same data sets.

In the work of Lin et al. [68] a TSK-based self-evolving compensatory IT2FNN was proposed for system modeling and noise cancellation problems. The proposed model utilized T2 fuzzy set in a FNN to handle the uncertainties associated with information or data in the knowledge base. The antecedent part of each compensatory fuzzy rule was an IT2 fuzzy set in the proposed model, where compensatory-based fuzzy reasoning utilized adaptive fuzzy operation of a neural fuzzy system to make the FLS effective and adaptive, and the consequent part was of the TSK type. The TSK-type consequent part was a linear combination of exogenous input variables. Initially, the rule base in the proposed model was empty. All rules were derived according to online T2 fuzzy clustering. For parameter learning, the consequent part parameters were tuned by a variable-expansive Kalman filter algorithm to the reinforce parameter learning ability. The antecedent T2 fuzzy sets and compensatory weights were learnt by a GD algorithm to improve the learning performance. Performance of the proposed model for identification was validated and compared with several T1 and T2FNNs. Simulation results have shown that the proposed approach produced smaller errors and converges more quickly.

In the work of Mendez et al. [69] a hybrid learning algorithm of orthogonal least-square (OLS) and BP method was used to tune the consequent and antecedent parameters of an interval singleton T2 TSK FLS, respectively. The proposed hybrid learning algorithm altered the parameters of IT2FLS adaptively. The model was compared with three other models with hybrid learning mechanism and the four models were applied to an industrial application. The proposed hybrid OLS-BP algorithm for IT2 TSK FLS outperformed the rest of the models.

4.2. Other Combinations of Hybrid Learning Algorithms

In the work of Juang and Tsao [70] a self-evolving IT2FNN with online structure and parameter learning was proposed. In this model, the antecedent part parameters were IT2 fuzzy set and the consequent part parameters were of TSK type. The online clustering method was utilized initially to generate the fuzzy rules. The consequent part parameters were then tuned using the rule-ordered Kalman filter algorithm. The antecedent parts parameters were learnt through GA. The proposed self-evolving IT2FNN model was applied to simulations on nonlinear plant modeling, adaptive noise cancellation and chaotic signal prediction. Better performance of the self-evolving IT2FNN was verified in comparison with T1FLS and T2FLS.

In the work of Jeng et al [71] a novel T2 TSK NN that utilizes general

T2 fuzzy set, was proposed for function approximation. The type reduction, structure identification, and parameter estimation were recognized as issues in developing a general T2FNN. The issue of type reduction was solved by utilizing the idea of α-cuts that decomposed a general T2 fuzzy set into IT2 fuzzy set. The issue of structure identification was settled by combining the incremental similarity based fuzzy clustering and linear least squares regression. The fuzzy rules were then extracted from these clusters and regressors. The last issue of the antecedent and consequent parameters identification of general T2FNN was solved using a hybrid learning algorithm of PSO and recursive least squares. Two simulation experiments were conducted to check the performance of the proposed model. Performance of the general T2FNN was compared with that of T2FNN and IT2FNN. Robust performance of the general T2FNN was observed against outliers than the other models.

In the work of Khanesar et al. [25] a hybrid method consisting of PSO and GD algorithms was utilized to optimize the parameters of a T2FLS. A diamond-shaped T2 fuzzy MF was introduced as a novel method of MF for T2FLS. The proposed method was then tested on the prediction of a noisy Mackey-Glass time series data. The performance of the model was compared with existing T2FLS. The simulation results has shown that the T2FLS with hybrid learning algorithm and novel MF outperformed the other models.

In the work of Yeh et al. [72] a hybrid learning algorithm incorporating PSO and least-square estimation was presented for T2FNN. The structure of a T2FNN was identified using a self-constructing fuzzy clustering method. The antecedent and the consequent parameters of T2FNN were optimized using PSO and least-square estimation, respectively. Comparison of the proposed model was done with two existing methods in literatures. The effectiveness of the proposed methodology was shown through several experiments.

A PSO based integrated functional link IT2FLS was presented for the prediction of stock market indices [73]. An integrated model of TSK model that employs T2 fuzzy sets in the antecedent parts and the outputs from the functional link artificial NN in the consequent parts was designed. The parameters of the hybrid model were optimized with BP and PSO independently. Forecasting ability of the proposed model was compared with T1FLS and local linear wavelet NN optimized with BP and PSO. Better performance of the proposed model for stock market indices forecasting was observed over other designed models.

In the work of Adisak and Phayung [74] a hybrid heuristic algorithm using PSO and GAs for parameter optimization of IT2FLS was proposed. The proposed system was then utilized for two benchmark data sets of classification problem. Comparison of the model based on proposed hybrid algorithm was done with the existing classifiers in literature. The proposed method was able to minimize the rule-base and linguistic variable, and produced an accurate classification at 95% with the Iris data set and 98.71 with the Wisconsin Breast Cancer data set.

In the work of Long and Meesad [75] an optimal design of IT2 TSK FLS was proposed using a hybrid algorithm. A hybrid of chaos firefly algorithm and GA was utilized to determine the optimal parameters of MFs and consequents parameters of the IT2 TSK FLS. The structure and number of fuzzy rules were determined through a fuzzy c-means clustering algorithm. The optimal design of IT2 TSK FLS was employed to predict sea water level in short-term and long-term horizontal. The performance of the hybrid algorithm of IT2 TSK FLS was compared with GA and firefly algorithm based optimal designs of IT2 TSK FLS. The hybrid algorithm for IT2 TSK FLS outperformed both the GA and firefly algorithm for sea water level prediction problem.

5. Comparisons and discussions

In this section, the goal is to compare and contrast the aforementioned optimization algorithms for the training of IT2FLS/IT2FNN. Undoubtedly, each training method has its own pros and cons. We believe that a deep knowledge about the advantages and disadvantages of the training methods makes it possible to decide on an appropriate optimization method based on the problem to be solved.

The derivative-based methods, which are also called the computational methods, need some partial derivatives to be computed in order to update the parameters of the T2FLS. The use of the derivatives of the output of the system with respect to its parameters gives a mathematical moving direction for the parameters of T2FLS. The parameters of the T2FLS may either appear linearly or nonlinearly in its output. The derivatives of the output of the fuzzy system with respect to the parameters which appear linearly in the output can be easily calculated. Moreover, least square, recursive least square and Kalman filter and its variants are proven be optimal estimators for these parameters. However, the calculation of the partial derivative of the output of T2FLS with respect to the parameters of the antecedent part is difficult and does not have any explicit form. In addition, none of the computational methods are optimal for updating the parameters of the antecedent part. Entrapment in local minima is another disadvantage of these methods. Using computational methods, we may face stability issues as well. For instance, in gradient method too large learning rate may cause divergence. In Kalman filter, the covariance matrix may result in divergence and so on. There are also some other disadvantages which are not common for all computational methods and are restricted to one or two algorithms. For example, in extended Kalman filter the size of covariance matrix is very large when it is used to train the parameters of T2FLS. In Levenberg-Marquardt algorithm, the inverse of a large size matrix is required in each step. In summary, when the parameter search space is too big, these methods start suffering from matrix manipulations.

The derivative-free or heuristic optimization methods are another class of optimization methods which have been successfully applied to the optimal design of T2FLS. The main advantage of these methods is that they are easy to implement and no mathematical update rule is needed to find the next step in the adaptation process of the parameters. Moreover, since they benefit from multiple initial points, the possibility of entrapment of these algorithms in local minima is much less than computational methods. However, since these algorithms are random optimization methods the update process is totally random and even if the values of the parameters are near their optimum values there is no guarantee that in the next step the error becomes less. Another drawback of these algorithms is that they necessitate the huge number of the evaluation of the T2FLS which is generally very slow and time-consuming and generally are recommended for offline problems. Memory requirements may also be another disadvantage of these methods.

The hybrid training methods benefit from both the heuristic and derivative based methods. The consequent part parameters appear linearly in the output of the FLS that is why the derivative based methods to optimize the parameters of the consequent part are easy to implement. Moreover, these methods benefit from some mathematics and usually converge much faster than random optimization methods and hence they are a preferable choice for the training of these parameters. In addition, some of derivative based methods e.g. Kalman filter and recursive least square are proven to be optimal for the parameters which appear linearly in the output of the FLS. However for the optimization of the premise part parameters since they appear nonlinearly in the output, it is quite probable that these parameters trap in local minima and random optimization techniques may be more preferable choices in these cases. In this way, the hybrid learning algorithms benefit from the strong capability of heuristic methods to search the whole space and the mathematics behind the computational methods which boosts the optimization and lessen the probability of searching inappropriate areas.

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