# Optimal design of dual-channel subband filter structures for video and image applications

## Conference or Workshop Item

# Optimal Design of Dual-Channel Subband Filter Structures for Video and Image Applications

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*Abstract*— This paper discusses the many and varied design considerations of a 2–D digital subband filter bank implementation for orthogonal sampling structures, which enables the derivation of only two subbands rather than the usual four. In particular, the design trade–off and overall performance of a Quadrature Mirror Filter pair, in combination with a 2–D diagonal prefilter is explored and the ensuing problems of avoiding and compensating for aliasing within the subbands, as well as for the signal reconstruction process at the decoder are highlighted.

## I. INTRODUCTION

SINCE their introduction, quadrature mirror filter (QMF) techniques has been widely applied in one and two dimensional subband coding [1] [2]. The design presented in this paper, is based closely upon the principles of a standard two dimensional subband coding implementation, with the crucial difference that a 2-D diagonal prefilter is additionally employed to suppress all oblique frequencies in the spatial plane. Subjectively, the loss of such information is acceptable, since the probability of diagonal components occurring is generally much less than for either horizontal or vertical frequencies. The spectral effect of diagonal filtering is illustrated in Fig. 1, where it clear that the input signal bandwidth is halved without compromising either horizontal or vertical resolution. By employing a 2-D QMF analysis bank to obtain the lowpass band, the residual now con-

sists of the two highpass wedged-shaped frequency components shown in Fig. 1b. Following subsampling by a factor of two, these spectral components fold back into the same frequency range as the lowpass band, so the requisite bandwidth of the highpass subbands becomes equivalent to that of the lowpass signal (see Fig. 1c) and the total number of subbands required for transmission is reduced to only two.

This implementation is particularly useful, where a low resolution version of the input signal is required such as for example, in compatible applications [3][4]. The drawback however, is that a straightforward hierarchical expansion of the sub-band signals is no longer feasible.

## II. THEORY

### A. Review

Fig. 2 reviews the standard two band analysis/synthesis filter bank for one dimensional signal processing. The input signal $x(n)$ is passed initially through the two analysis filters $H_0(z)$ and $H_1(z)$, which split it into two subbands $y_0(n)$ and $y_1(n)$ respectively, prior to subsampling by a factor of two. The inverse procedure occurs in the synthesis section, with each band firstly being interpolated by a factor of two, before being passing through the synthesis filters $G_0(z)$ and $G_1(z)$, and summing their outputs to form $\hat{x}(n)$. Neglecting any possible coding effects, the signal $\hat{x}(n)$ differs from $x(n)$, due to three principal sources of error; aliasing, amplitude and phase distortion. By choosing the filters as:-

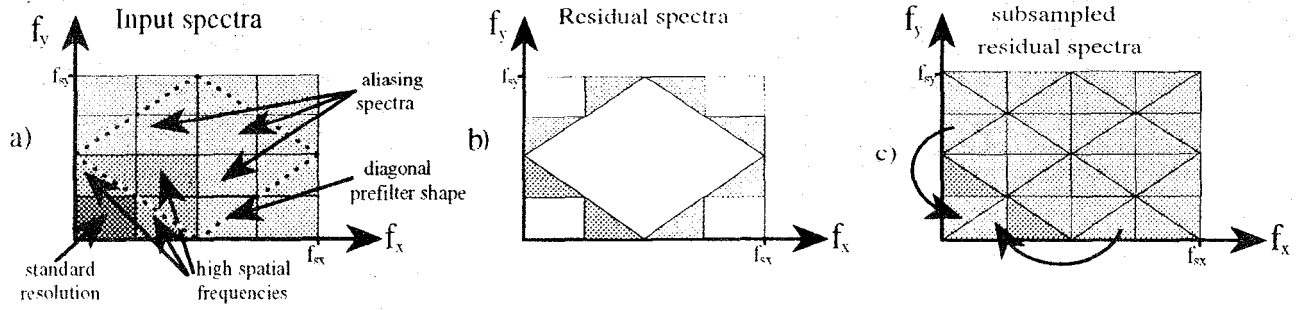$$G_1(z) = -H_0(-z) \qquad H_1(z) = G_0(-z) \qquad (1)$$

Fig. 1. a) input spectrum with respect to the four subbands; b) highpass bands with diagonal prefilter; c) decimated highpass signal

aliasing distortion is completely eliminated, regardless of the actual filter response, and once this is achieved, the equation for the output signal $\hat{x}(n)$ reduces to:-

$$\hat{X}(z) = \frac{1}{2}[H_0(z)G_0(z) -$$
$$H_0(-z)G_0(-z)]X(z)$$
$$= T(z)X(z) \qquad (2)$$

where $T(z)$ is referred as the overall transfer function. By substituting $H(z)$ with $e^{-j\omega\frac{N}{2}}|H(z)|$, where $N$ is the filter order, $T(z)$ can be written as:-

$$T(z) = \frac{1}{2}e^{-j\omega N}[|H_0(z)||G_0(z)| -$$
$$(-1)^N|H_0(-z)||G_0(-z)|] \qquad (3)$$

Eq. (3) shows that the system is only really usable for odd filter orders, since for even order filters, $T(e^{j\omega})$ will have a zero at $\omega = \frac{\pi}{2}$ and so amplitude distortion cannot be eliminated. This apparent drawback can be overcome by slightly modifying the subband coding system, by placing an additional delay in the lowpass analysis and highpass synthesis paths. By now choosing the filters as

$$G_1(z) = H_0(-z) \qquad H_1(z) = G_0(-z) \qquad (4)$$

the transfer function becomes:-

$$T(z) = \frac{1}{2}e^{-j\omega N}[|H_0(z)||G_0(z)| +$$
$$(-1)^N|H_0(-z)||G_0(-z)|] \qquad (5)$$

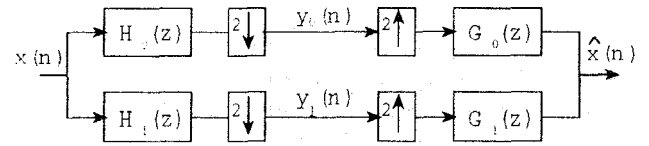To avoid amplitude distortions $T(e^{j\omega})$ must be an all-pass response (an amplitude value equal to



Fig. 2. Simple one dimensional QMF bank

one for all values of $\omega$), which is only possible by using trivial cosine filters[5]. Several design techniques have been developed, [1] [2], to approximate this constraint by applying filters with superior transition bandwidth and passband–stopband characteristics, which would be more efficient for this particular application. To ensure there is no phase distortion within the system, it is clear that if the filters are selected to be symmetric FIR filters, the overall system has a constant group delay. Other design methods [6] guarantee there will be no phase distortions in the reconstituted signal, but necessarily not for the individual subband signals.

The extension of these basics into two dimensions can be taken as a separable product of identical 1-D QMF pairs [7][8]. This means, a 2-D input is initially processed in one dimension generating two subbands, which are then processed in the other direction, so giving a total of four subbands. For reconstruction, the resulting four bands are processed in an inverse order.

B. Dual Channel Subband Filter Bank

As previous alluded, the general case for a separable analysis section with orthogonal subsampling results in four subbands. The block diagram in Fig. 3 shows the analysis / synthesis filter bank
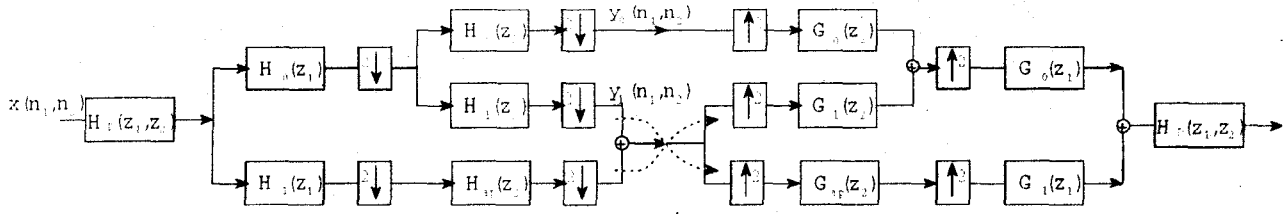
Fig. 3. Block diagram of the dual channel QMF bank approach

arrangement for processing only two bands, by including an additional diagonal pre- and postfilter, together with the combination of the two highpass bands to form $y_1(n_1, n_2)$. The reconstituted signal can be written as

$$\hat{X}(z_1, z_2) =$$
$$\frac{1}{4}[T(z_1, z_2) + C_T(z_1, z_2)]X_D(z_1, z_2) +$$
$$\frac{1}{4}[A_1(z_1, z_2) + C_{A1}(z_1, z_2)]X_D(-z_1, z_2) +$$
$$\frac{1}{4}[A_2(z_1, z_2) + C_{A2}(z_1, z_2)]X_D(z_1, -z_2) +$$
$$\frac{1}{4}[A_3(z_1, z_2) + C_{A3}(z_1, z_2)]X_D(-z_1, -z_2) \quad (6)$$

where $T(z_1, z_2)$ and $A_1(z_1, z_2) \cdots A_3(z_1, z_2)$ denote what is usually referred as the transfer function and the alias components respectively. $C_T(z_1, z_2), C_{A_1}(z_1, z_2) \cdots C_{A_3}(z_1, z_2)$ highlight the signal parts, which occur due to the addition of the two subbands to form $y_1(n_1, n_2)$ and consist of vertical high frequency crosstalk components, which because of the subsampling appear as horizontal frequencies and vice versa. The dotted arrows in Fig. 3 illustrate this phenomenom. Choosing the QMF - pairs similar to Eq. (1), the alias term of Eq. (6) will reduce due to alias cancellation, but it is clear, that no overall cancellation can occur, so that apart from the addition of the two subbands, the system is very close to a standard four-band implementation, where the fourth band (the high diagonal frequency components) has been removed. If ideal diagonal pre- and postfiltering is assumed, the frequencies responsible for this aliasing, are suppressed and so these terms are not taken into account as shown in Fig. 4.

The crosstalk signal component $C_T(z_1, z_2)$ of the transfer function leads to additional amplitude
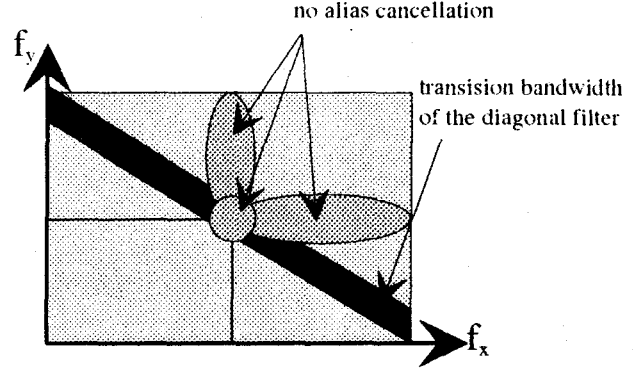


Fig. 4. Two dimensional fequency region where no alias cancellation is possible

distortions depending on the analysis / synthesis filter shape. Fig. 5a provide an example by using cosine filter for $H(z)$ and an ideal diagonal filer response.

Considering the vertical bandsplitting by using even filter orders, rather than odd orders for the horizontal and vertical dimension, the vertical filters have to be chosen, similar to Eq. (4), so that $G_1(z_2) = H(-z_2)$, with the result, that the crosstalk term $C_T(z_1, z_2)$ will be cancelled and $T(z_1, z_2)$ reduces to

$$T(z_1, z_2) = e^{-j(\omega_1 N_1 + \omega_2 N_2)}[|H(z_1)|^2 |H(z_2)|^2 +$$
$$(-1)^{N_2}|H(z_1)|^2 |H(-z_2)|^2 -$$
$$(-1)^{N_1}|H(-z_1)|^2 |H(z_2)|^2] \quad (7)$$

### III. FILTER DESIGN

The implementation of the 2-D QMF bandsplitting process is relatively straightforward requiring the separation of the filter design in each of the two dimensions. The lowpass / highpass pair (H0(z) / H1(z) in Fig.2) operate initially in one dimension, to generate two subbands, before being subsequently applied in the other, so
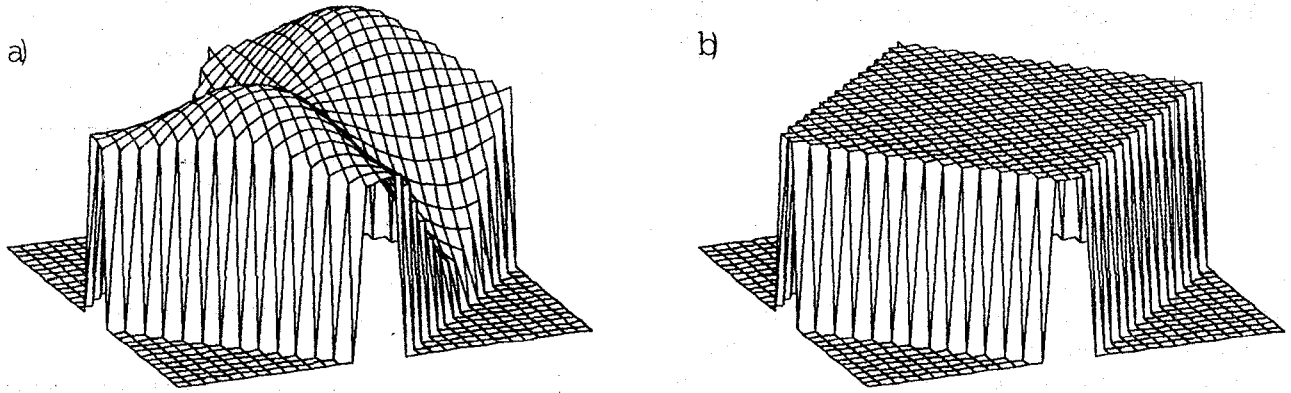
Fig. 5. Transfer function $T(z_1, z_2)$ a) with crosstalk distortion and b) with crosstalk cancellation

the design process is reduced to a one dimensional procedure, for which several realisations are available [5][9]. The non-ideal transition bandwidth of the diagonal filter, implies an overlapping of the two highpass subbands after subsampling. It is clear, that perfect reconstruction is not achievable, because this second source of crosstalk, $C_{A_1}(z_1, z_2) \cdots C_{A_3}(z_1, z_2)$, can neither be compensated for nor cancelled, so the overall diagonal filter design has to balance a trade off between the filter's complexity and crosstalk effects.

## IV. RESULTS

Several QMF pairs have been developed to evaluate their subjective performance in combination with the diagonal pre and post filter. The filter order of the QMF pairs is not so critical in relation to the overall subjective quality, where as the transition bandwidth of the diagonal prefilter effects the performance of the system significantly. It is for this reason that a high order for the encoder is proposed, such as a 31 by 31 tap response, while the design requirement of the diagonal postfilter can be somewhat relaxed, so a lower filter order can be utilised.

## V. CONCLUSIONS

This paper has explored the various design considerations for implementing an optimal dual-channel subband filter arrangement. The main sources of error due to aliasing and crosstalk have been highlighted and it is shown how by judicous selection of the various filter responses, the three

main sources of distortion can be eliminated. The function and performance of the diagonal filter was also analysed and the requisite modifications to the QMF filter pair outlined, in view of the fact that alias cancellation is not feasible.

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