OPTIMAL DESIGN OF FIR FREQUENCY-RESPONSE-MASKING FILTERS USING SECOND-ORDER CONE PROGRAMMING

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ABSTRACT

Since Limś 1986 paper on the frequency-response masking (FRM) technique for the design of FIR digital Ýlters with very small transition widths, the analysis and design of FRM Ýlters have been a subject of study. In this paper, a new optimization technique for the design of FRM Ýlters is proposed. Central to the new design method is a sequence of linear updates for the design variables, with each update carried out by second-order cone programming. Design simulations are presented to illustrate the proposed algorithms and to evaluate the design performance.

1. INTRODUCTION

Since the work of [1], the frequency-response masking (FRM) technique for the design of FIR digital Ýlters with very narrow transition bands has been a subject of study [2]ñ[9], [13]. As a result, in many cases it has become the method of choice primarily because of the considerably reduced realization complexity it offers compared with other available options [5][8].

As illustrated in Fig. 1, a *basic* FRM *filter* involves a linear-phase prototype Ýlter $H_a(z)$ up-sampled by M, a pair of linear-phase masking Ýlters $\{H_{ma}(z),\ H_{mc}(z)\}$, and a delay line that, together with the prototype Ýlter, helps form a linear-phase complementary pair $\{H_a,\ H_c\}$ [1]. Given an up-sampling factor, lengths of the subÝlters involved, and passband/stopband edges, the design of a basic FRM Ýlter is usually carried out by *separately* designing the subÝlters [1][5][6]. As such the FRM Ýlter obtained is only suboptimal. In this paper, we present a rather different optimization technique in which the set of Ýlter coefÝcients of *all* subÝlters is treated as a single design vector and an optimal basic FRM Ýlter is designed through a sequence of linear updates for the design variables, with each update carried out in a second-order cone programming (SOCP) framework.

The second issue to be addressed in this paper is the optimal design of FRM Ýlters with *reduced passband group delay*. Linear-phase FIR Ýlters have constant group delay in the entire frequency band, but for a Ýlter with very narrow transition width, the group delay can be exceedingly large, a property not desirable in many applications. For a linear-phase FRM Ýlter with a large up-sampling factor M, its large group delay is dominantly contributed by the prototype Ýlter. Therefore, if the prototype Ýlter has a nonlinear phase response with a reduced passband group delay, say d, and if the delay line (the lower-left block in Fig. 1) is accordingly modiÝed to z^{-dM}, then the Ýlter is expected to have its passband group delay reduced by $M[0.5(N-1)-d]$ where N is the length of the prototype Ýlter. Hence the reduction in group

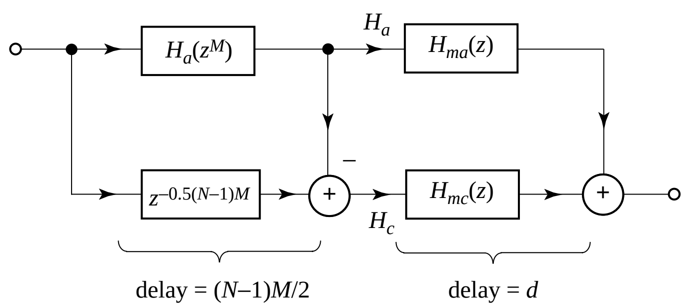

Figure 1: A basic FRM Ýlter structure.

delay can be signiÝcant especially when M is large. In this paper, we pursue this idea and show that, by a joint optimization of the entire set of subÝlters, the prototype as well as masking Ýlters all contribute to minimizing the Ðuctuation in the reduced passband group delay.

2. OPTIMIZATION METHODOLOGY

Let $H_d(\omega)$ be a desired real-valued or complex-valued function of frequency variable ω, and $H(\omega, \boldsymbol{x})$ be a real-valued or complex-valued function of ω, which depends on a real-valued parameter vector $\boldsymbol{x} \in R^{n\times 1}$. We seek to Ýnd a vector $\boldsymbol{x}^*$ that solves the weighted minimax optimization problem

$$\underset{\boldsymbol{x}}{\text{minimize}}\{\underset{\omega\in\Omega}{\text{maximize}}\, W(\omega)|H(\omega,\boldsymbol{x}) - H_d(\omega)|\} \qquad (1)$$

Let η be an upper bound of $W(\omega)|H(\omega,\boldsymbol{x}) - H_d(\omega)|$ on Ω. As the Ýrst step of the optimization we convert the problem in (1) into a constrained minimization problem

$$\text{minimize} \quad \eta \qquad (2a)$$

$$\text{subject to:}\ \ W(\omega)|H(\omega,\boldsymbol{x}) - H_d(\omega)| \le \eta \ \text{ for } \omega\in\Omega \qquad (2b)$$

Suppose we have a reasonable initial point $\boldsymbol{x}_0$ to start the design, and we are now in the kth iteration. For a nonlinear and smooth $H(\omega,\boldsymbol{x})$ in a vicinity of $\boldsymbol{x}_k$, we can write

$$H(\omega,\boldsymbol{x}_k+\boldsymbol{\delta}) = H(\omega,\boldsymbol{x}_k) + \boldsymbol{g}_k^T(\omega)\boldsymbol{\delta} + o(||\boldsymbol{\delta}||)$$

where $\boldsymbol{g}_k(\omega)$ is the gradient of $H(\omega,\boldsymbol{x})$ with respect to $\boldsymbol{x}$ and evaluated at $\boldsymbol{x}_k$. Hence, provided that $\|\boldsymbol{\delta}\|$ is small, with $\boldsymbol{x}$ =

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$\boldsymbol{x}_k + \boldsymbol{\delta}$ we have

$$W^2(\omega)|H(\omega, \boldsymbol{x}) - H_d(\omega)|^2 \approx [\boldsymbol{g}_{rk}^T(\omega)\boldsymbol{\delta} + e_{rk}(\omega)]^2 + [\boldsymbol{g}_{ik}^T(\omega)\boldsymbol{\delta} + e_{ik}(\omega)]^2 \quad (3)$$

where $\boldsymbol{g}_{rk}(\omega)$ and $\boldsymbol{g}_{ik}(\omega)$ are the real and imaginary parts of $W(\omega)\boldsymbol{g}_k(\omega)$, respectively, and

$$\begin{aligned} e_{rk}(\omega) &= W(\omega)[H_r(\omega, \boldsymbol{x}_k) - H_{rd}(\omega)] \\ e_{ik}(\omega) &= W(\omega)[H_i(\omega, \boldsymbol{x}_k) - H_{id}(\omega)] \end{aligned}$$

with $H_r(\omega, \boldsymbol{x})$, $H_i(\omega, \boldsymbol{x}_k)$, $H_{rd}(\omega)$, and $H_{id}(\omega)$ being the real and imaginary parts of $H(\omega, \boldsymbol{x}_k)$ and $H_d(\omega)$, respectively. From (2) and (3), it follows that an approximate solution of (2) in the kth iteration can be obtained by solving the following problem:

$$\text{minimize} \quad \eta \quad (4a)$$

$$\text{subject to:} \quad [(\boldsymbol{g}_{rk}^T\boldsymbol{\delta} + e_{rk})^2 + (\boldsymbol{g}_{ik}^T\boldsymbol{\delta} + e_{ik})^2]^{1/2} \leq \eta \quad \text{for } \omega \in \Omega \quad (4b)$$

$$\|\boldsymbol{\delta}\| \leq b \quad (4c)$$

where b is a prescribed bound to control the magnitude of $\boldsymbol{\delta}$.

If we treat the upper bound η as an additional design variable and deﬁne an augmented vector as $\boldsymbol{u} = [\eta \ \boldsymbol{\delta}^T]^T$, then the problem in (4) can be formulated as

$$\text{minimize} \quad \boldsymbol{c}^T\boldsymbol{u} \quad (5a)$$

$$\text{subject to:} \quad \|\boldsymbol{G}_k\boldsymbol{u} + \boldsymbol{e}_k\| \leq \boldsymbol{c}^T\boldsymbol{u} \quad \text{for } \omega \in \Omega_d \quad (5b)$$

$$\|\hat{\boldsymbol{I}}\boldsymbol{u}\| \leq b \quad (5c)$$

where

$$\begin{aligned} \boldsymbol{c} &= [1 \ 0 \ \cdots \ 0]^T \\ \boldsymbol{G}_k &= \begin{bmatrix} 0 & \boldsymbol{g}_{rk}^T \\ 0 & \boldsymbol{g}_{ik}^T \end{bmatrix}, \ \boldsymbol{e}_k = \begin{bmatrix} e_{rk} \\ e_{ik} \end{bmatrix} \\ \hat{\boldsymbol{I}} &= [\boldsymbol{0} \ \boldsymbol{I}] \end{aligned}$$

and $\Omega_d = \{\omega_1, \ \ldots, \ \omega_k\} \subset \Omega$ is a set of dense grid points in the frequency bands of interest. Obviously, the problem in (5) is a SOCP problem [10][11].

Having solved the problem in (11) for a minimizer

$$\boldsymbol{u}_k^* = \begin{bmatrix} \eta_k^* \\ \boldsymbol{\delta}_k^* \end{bmatrix}$$

vector $\boldsymbol{\delta}_k^*$ is used to update $\boldsymbol{x}_k$ as

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \boldsymbol{\delta}_k^*$$

The iteration continues until $\|\boldsymbol{\delta}_k^*\|$ becomes insigniﬁcant compared to a prescribed tolerance.

3. OPTIMIZATION OF FRM FILTERS: BASIC AND LOW-DELAY STRUCTURES

3.1. Basic FRM Filters

A. Frequency response and its gradient

The reader is referred to the structure in Fig. 1 where all subﬁlters are assumed to have linear-phase responses, and the lengths of the masking ﬁlters are either both even or both odd. The transfer functions of the subﬁlters are denoted by

$$\begin{aligned} H_a(z) &= \sum_{k=0}^{N-1} h_k z^{-k}, \ H_{ma}(z) = \sum_{k=0}^{N_a-1} h_k^{(a)} z^{-k}, \\ H_{mc}(z) &= \sum_{k=0}^{N_c-1} h_k^{(c)} z^{-k} \end{aligned} \quad (6)$$

Without loss of generality, the FRM ﬁlter can be treated as a zero-phase FIR ﬁlter, and the frequency response of the FRM ﬁlter is then given by

$$H(\omega, \boldsymbol{x}) = [\boldsymbol{a}^T\boldsymbol{c}(\omega)][\boldsymbol{a}_a^T\boldsymbol{c}_a(\omega) - \boldsymbol{a}_c^T\boldsymbol{c}_c(\omega)] + \boldsymbol{a}_c^T\boldsymbol{c}_c(\omega) \quad (7)$$

where

$$\boldsymbol{a} = \begin{cases} [h_{(N-1)/2} \ 0.5h_{(N+1)/2} \ \cdots \ 0.5h_{N-1}]^T & \text{if } N \text{ odd} \\ 0.5[h_{N/2} \ \cdots \ h_{N-1}]^T & \text{if } N \text{ even} \end{cases}$$

$$\boldsymbol{c}(\omega) = \begin{cases} [1 \ \cos M\omega \ \cdots \ \cos[(N-1)M\omega/2]^T & \text{if } N \text{ odd} \\ [\cos(M\omega/2) \ \cdots \ \cos[(N-1)M\omega/2]]^T & \text{if } N \text{ even} \end{cases}$$

$$\boldsymbol{a}_a = \begin{cases} [h_{(N_a-1)/2}^{(a)} \ 0.5h_{(N_a+1)/2}^{(a)} \ \cdots \ 0.5h_{N_a-1}^{(a)}]^T & \text{if } N_a \text{ odd} \\ 0.5[h_{N_a/2}^{(a)} \ \cdots h_{N_a-1}^{(a)}]^T & \text{if } N_a \text{ even} \end{cases}$$

$$\boldsymbol{c}_a(\omega) = \begin{cases} [1 \ \cos\omega \ \cdots \ \cos[(N_a-1)\omega/2]^T & \text{if } N_a \text{ odd} \\ [\cos(\omega/2) \ \cdots \ \cos[(N_a-1)\omega/2]]^T & \text{if } N_a \text{ even} \end{cases}$$

$$\boldsymbol{a}_c = \begin{cases} [h_{(N_c-1)/2}^{(c)} \ 0.5h_{(N_c+1)/2}^{(c)} \ \cdots \ 0.5h_{N_c-1}^{(c)}]^T & \text{if } N_c \text{ odd} \\ 0.5[h_{N_c/2}^{(c)} \ \cdots h_{N_c-1}^{(c)}]^T & \text{if } N_c \text{ even} \end{cases}$$

$$\boldsymbol{c}_c(\omega) = \begin{cases} [1 \ \cos\omega \ \cdots \ \cos[(N_c-1)\omega/2]^T & \text{if } N_c \text{ odd} \\ [\cos(\omega/2) \ \cdots \ \cos[(N_c-1)\omega/2]]^T & \text{if } N_c \text{ even} \end{cases}$$

and the design variables are put together as parameter vector

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{a}_a \\ \boldsymbol{a}_c \end{bmatrix}$$

The group delay of the FRM ﬁlter is given by

$$D = \frac{(N-1)M}{2} + d \quad (8)$$

where $d = \max((N_a-1)/2, (N_c-1)/2)$, and the gradient of $H(\omega, \boldsymbol{x})$ with respect to $\boldsymbol{x}$ is given by

$$\boldsymbol{g}(\omega, \boldsymbol{x}) = \begin{bmatrix} y(\omega)\boldsymbol{c}(\omega) \\ [\boldsymbol{a}^T\boldsymbol{c}(\omega)]\boldsymbol{c}_a(\omega) \\ [1 - \boldsymbol{a}^T\boldsymbol{c}(\omega)]\boldsymbol{c}_c(\omega) \end{bmatrix} \quad (9)$$

$y(\omega) = \boldsymbol{a}_a^T\boldsymbol{c}_a(\omega) - \boldsymbol{a}_c^T\boldsymbol{c}_c(\omega)$.

B. Desired frequency response and weighting function

For the sake of presentation clarity, we consider the case of designing a lowpass FRM ﬁlter with up-sampling factor M, normalized passband edge ω_p and stopband edge ω_a. The desired $H_d(\omega)$ in this case becomes

$$H_d(\omega) = \begin{cases} 1 & \text{for } 0 \leq \omega \leq \omega_p \\ 0 & \text{for } \omega_a \leq \omega \leq \pi \end{cases} \quad (10)$$

and a staircase weighting function $W(\omega) = 1$ for $0 \le \omega \le \omega_p$, $W(\omega) = w$ for $\omega_a \le \omega \le \pi$ and $W(\omega) = 0$ elsewhere is chosen, where w is a positive scalar to weigh the stopband relative to the passband.

C. Initial design

Given parameters $M,\ N,\ N_a,\ N_c,\ \omega_p$, and ω_a, a reasonable initial design can be obtained by designing lowpass $H(z),\ H_{ma}(z)$, and $H_{mc}(z)$ as discussed in [1]. It is important to stress that although (as will be demonstrated by simulations shortly) the optimized $H(z)$ does not at all look like a lowpass filter, the initial design prepared here worked flawlessly in a variety of FRM designs we have attempted.

D. Placement of grid points and bound b

Our design practice has indicated that relatively denser grid points should be placed in the regions near the band edges in both passband and stopband so as to avoid using unnecessarily large number of total grid points. We recommend that about 25% of the grid points be placed in the 10% of that band nearest to the band edge.

As expected, the value of bound b in constraint (8c) is taken to be proportional to the dimension of vector $\boldsymbol{x}$, namely, $b = \gamma n$ where n denotes the dimension of $\boldsymbol{x}$ and γ is a constant factor. It was found in our simulations that the norm constraint (8c) worksed effectively when the value of γ was in the range of [0.005, 0.05].

E. A design example

The design is a linear-phase lowpass FRM filer with the same design parameters as the first example in [1], i.e., $N = 45,\ N_a = 41,\ N_c = 33, M = 9,\ \omega_p = 0.6\pi$, and $\omega_a = 0.61\pi$. The weight was set to $w = 1$, bound b in (4c) was set to $b = 0.005n$ ($n = 61$ in this design), and the total number of grids was $K = 900$. In this case the optimization algorithm handles 62 variables with 1862 constraints. With 10 iterations the algorithm converges to an FRM filter with the amplitude response of its subfilters $H_a(z^M),\ H_{ma}(z)$, and $H_{mc}(z)$ shown in Fig. 2a and 2b, respectively, and the amplitude response of the FRM filter and its passband ripples shown in Fig. 2c and 2d, respectively. The maximum passband ripple was found to be 0.0674 dB and the minimum stopband attenuation was 42.25 dB. By comparison, the passband ripple and stopband attenuation of the design in [1] were 0.0896 dB and 40.96 dB, respectively.

As can be seen from Fig. 2, the masking filters $H_{ma}(z)$ and $H_{mc}(z)$ resulted from the joint optimization remain to be lowpass with very similar passband widths, but the optimized prototype filter $H_a(z^M)$ is not at all a lowpass filter. Note that $H_a(z^M)$ has a sharp drop-down precisely at the passband edge (normalized to 0.3 in Fig. 2a).

3.2. FRM Filters with Low-Delay

A. Frequency response and its gradient

Since the group delay of an FRM filter is dominantly contributed by the prototype filter, we assume here that the prototype filter is the only filter with a *nonlinear* phase response in the entire FRM filter structure. A FRM filter with passband group delay $D_r = dM + d_1$ is illustrated in Fig. 3, where the prototype filter has the frequency response

$$H_a(e^{jM\omega}) = \sum_{k=0}^{N-1} h_k e^{-jkM\omega} \tag{11}$$

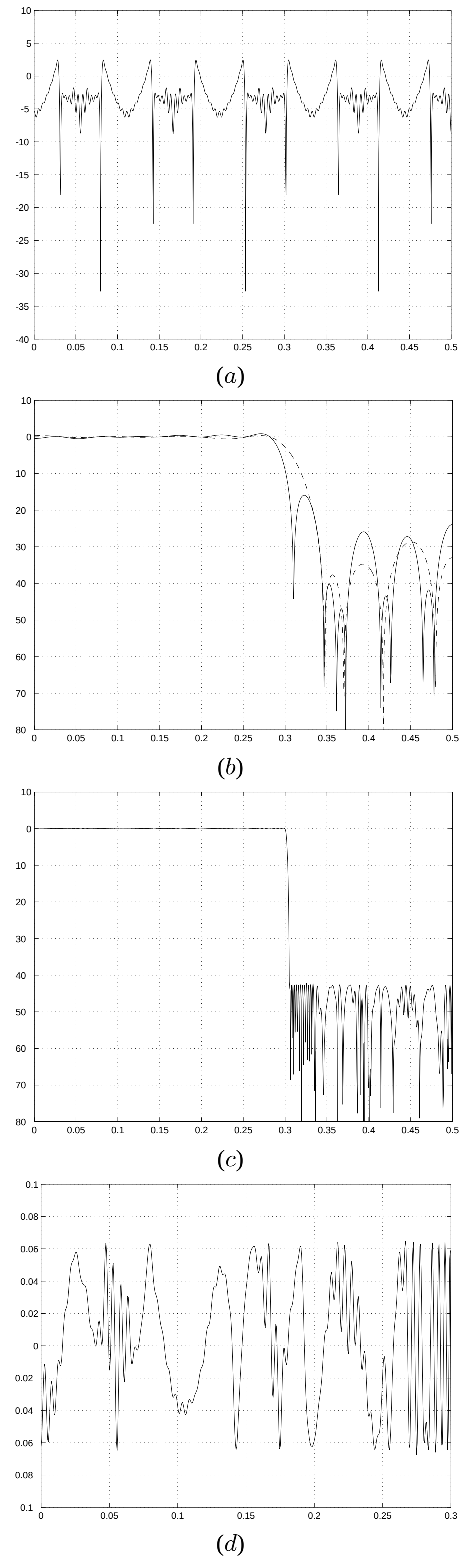

Figure 2: Amplitude responses of (a) prototype filter $H_a(z^9)$; (b) masking filters $H_{ma}(z)$, and $H_{mc}(z)$; (c) FRM filter; and (d) passband ripples of the FRM filter, all in dB.

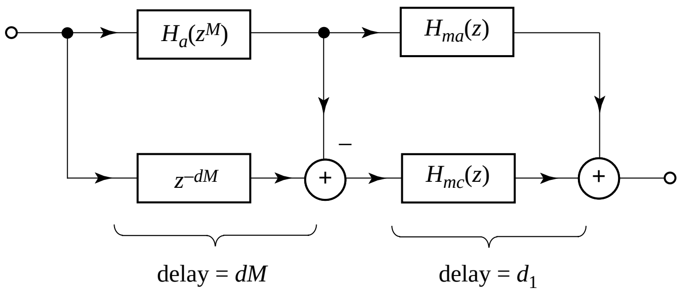

Figure 3: A basic FRM filter with reduced group delay $dM + d_1$.

The frequency response of the FRM filter can be expressed as

$$e^{-jD_r\omega} H(\omega, \boldsymbol{x}) \tag{12a}$$

with

$$\begin{aligned} H(\omega,\boldsymbol{x}) &= \tilde{H}_a(\omega)[\boldsymbol{a}_a^T \boldsymbol{c}_a(\omega) - \boldsymbol{a}_c^T \boldsymbol{c}_c(\omega)] + \boldsymbol{a}_c^T \boldsymbol{c}_c(\omega) && \text{(12b)} \\ \tilde{H}_a(\omega) &= \boldsymbol{h}^T[\boldsymbol{c}(\omega) + j\boldsymbol{s}(\omega)] && \text{(12c)} \\ \boldsymbol{h} &= [h_0 \ \ h_1 \ \ \ldots \ \ h_{N-1}]^T && \text{(12d)} \\ \boldsymbol{c}(\omega) &= [\cos Md\omega \ \ \cos M(d-1)\omega \ \ \cdots \ \ \cos M(d-N+1)\omega]^T && \text{(12e)} \\ \boldsymbol{s}(\omega) &= [\sin Md\omega \ \ \sin M(d-1)\omega \ \ \cdots \ \ \sin M(d-N+1)\omega]^T && \text{(12f)} \end{aligned}$$

and $\boldsymbol{a}_a$, $\boldsymbol{a}_c$, $\boldsymbol{c}_a(\omega)$, and $\boldsymbol{c}_c(\omega)$ defined in (7). Note that because of $\tilde{H}_a(\omega)$ in (12c), $H(\omega, \boldsymbol{x})$ in (12b) is a complex-valued function, where parameter vector $\boldsymbol{x}$ is defined as

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{h} \\ \boldsymbol{a}_a \\ \boldsymbol{a}_c \end{bmatrix}$$

Now if the value of d is strictly less than $(N-1)/2$, then the first factor in (12a) with $D_r = dM + d_1$ represents a reduced group delay provided that the second factor in (12a), $H(\omega, \boldsymbol{x})$, best approximates the zero-phase desired frequency response $H_d(\omega)$ in (10).

The gradient of $H(\omega, \boldsymbol{x})$ is also complex-valued and is given by $\boldsymbol{g}(\omega, \boldsymbol{x}) = \boldsymbol{g}_r(\omega, \boldsymbol{x}) + j\boldsymbol{g}_i(\omega, \boldsymbol{x})$ where

$$\boldsymbol{g}_r(\omega, \boldsymbol{x}) = \begin{bmatrix} y(\omega)\boldsymbol{c}(\omega) \\ [\boldsymbol{h}^T \boldsymbol{c}(\omega)]\boldsymbol{c}_a(\omega) \\ [1 - \boldsymbol{h}^T \boldsymbol{c}(\omega)]\boldsymbol{c}_c(\omega) \end{bmatrix} \tag{13a}$$

$$\boldsymbol{g}_i(\omega, \boldsymbol{x}) = \begin{bmatrix} y(\omega)\boldsymbol{s}(\omega) \\ [\boldsymbol{h}^T \boldsymbol{s}(\omega)]\boldsymbol{c}_a(\omega) \\ -[\boldsymbol{h}^T \boldsymbol{s}(\omega)]\boldsymbol{c}_c(\omega) \end{bmatrix} \tag{13b}$$

$$y(\omega) = \boldsymbol{a}_a^T \boldsymbol{c}_a(\omega) - \boldsymbol{a}_c^T \boldsymbol{c}_c(\omega) \tag{13c}$$

B. Initial design

The initial design of the masking filters $H_{ma}(z)$ and $H_{mc}(z)$ remains the same as in [1]. As well, one can use the formulas there to predict the passband and stopband edges θ and ϕ. However, at this point one needs a lowpass filter $H_a(z)$ with $\omega_p = \theta$, $\omega_a = \phi$ and a reduced passband group delay d (strictly less than $(N-1)/2$). A reasonably good initial $H_a(z)$ is the weighted least-squares solution that minimizes

$$\int_0^{\pi} W(\omega)|H_a(e^{j\omega}) - \tilde{H}_d(\omega)|^2 \, d\omega \tag{14}$$

where

$$W(\omega) = \begin{cases} 1 & \text{for } \omega \in [0, \ \theta) \\ w & \text{for } \omega \in [\phi, \ \pi] \\ 0 & \text{elsewhere} \end{cases}$$

and

$$\tilde{H}_d(\omega) = \begin{cases} e^{-jd\omega} & \text{for } \omega \in [0, \ \theta) \\ 0 & \text{for } \omega \in [\phi, \ \pi] \text{ and elsewhere} \end{cases}$$

It can be shown that the objective function in (14) is a strictly convex quadratic function with a Toeplitz type Hessian matrix. Consequently, the least-square solution can be computed efficiently by solving a Toeplitz system of linear equations [12].

Computer simulations have shown that the proposed method works as expected. Design examples, however, are omitted here due to space limitation.

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