Transactions on the Built Environment vol 2, © 1993 WIT Press, www.witpress.com, ISSN 1743-3509

Optimal design of reinforced concrete frames

N.C. Das Gupta, V. Thevendran, G.H. Tan
Department of Civil Engineering, National University of Singapore, 10 Kent Ridge Crescent, 0511 Singapore

ABSTRACT

The paper considers the minimum cost design of reinforced concrete frames in accordance with the British standard code of practice, BS8110. Cost in this study includes the cost of concrete, reinforcement and formwork. Design and practical constructional requirements are formulated as optimization constraints. A simple and effective procedure to solve the resulting nonlinear constrained minimization problem is presented. The nonlinear minimization problem has been solved numerically using direct search methods in a microcomputer.

INTRODUCTION

Study in the field of structural optimization has received considerable attention with the advances made in optimization algorithms and computer technology. Krishnamurthi and Munro [1] formulated the problem as a linear programming problem and solved it with the simplex algorithm. Krishnamurthi and Mosi [2] used the concept of suboptimization, solving the nonlinear problem using penalty functions and the variable metric method. Liebman and Khachaturian [3] have solved the RC frame problem using integer variables, but method proposed is difficult for large frames. Choi and Kwak [4] proposed a method, whereby a large database that contains details of sections and reinforcement has been used.

In the present study, the minimum material cost of reinforced concrete frames is considered. The objective of the study is to develop an optimization procedure that is simple and effective and can be easily adapted by designers having access to microcomputers.

The problem basically consists of three parts: namely analysis, design and optimization. Structure is analyzed using normal elastic analysis. The design is performed in accordance with the British Code of Practice BS8110 [5]. Consequently, the optimization problem becomes a constrained nonlinear programming problem, which is transformed into an equivalent unconstrained problem using the exterior point method of the Sequential Unconstrained Minimization Technique (SUMT) developed by Fiacco and McCormick [6]. The time taken for full structural optimization is long for a large structure having a large number of optimization variables. A suboptimization procedure (Kavlie and Moe [7], Krishnamurthi and Mosi [2]) is used. The problem is reduced into a series of two variables, width and depth of each member during the suboptimization stage. The two variable problem has been solved using direct search methods [8,9].

STRUCTURAL ANALYSIS

The technique used for the analysis is the direct stiffness method [10], the fundamental equation of which is

$$\{F\} = [K]\{D\} \tag{1}$$

where $\{F\}$ = the vector of nodal forces, $[K]$ = structural stiffness matrix and $\{D\}$ = the vector of nodal displacements.

STRUCTURAL OPTIMIZATION

The optimization variables in each member are treated as continuous. The general requirements of design are incorporated as design constraints.

Objective function

The objective function is the total cost of concrete, steel and formwork which are taken as S\$110 per m^3, S\$850 per tonne and S\$20 per m^2 respectively.

Design constraints

The design of frames is divided into design of beams and columns.

(a) **Beam design:** The beam is divided into three sections for the purpose of design, the configuration being L/4-L/2-L/4 format where L denotes length of a beam. The maximum stresses in each section are obtained and the section is designed accordingly. For detailing, simplified rules stipulated in clause 3.12.10 (Part 1: BS8110) have been adopted. The design constraints are listed below: (The clauses referred to in subsequent paragraphs are those from BS8110).

(i) The design ultimate moment of resistance, M_u is obtained as stipulated in Clause 3.4.4. A simplified stress block approach is adopted.

(ii) The design shear stress, v, at any cross section should not exceed $0.8\sqrt{f_{cu}}$ or 5 N/mm^2 whichever is less. The shear reinforcement required is as defined in Clause 3.4.5 and as summarized in Table 3.8 (BS8110).

(iii) Deflection may be calculated and then compared with serviceability requirements as given in section 3 of BS8110: Part 2: 1985. But in normal cases the deflection of a beam will not be excessive if it satisfies the specified value of the ratio of span to effective depth. To simplify the design process the latter approach is adopted. The limit of 'span/effective depth' ratio is taken as 26 as stipulated in clause 3.4.6 with requisite multiplication factor considering tension reinforcement and compression factor. In addition the beam is required to satisfy the slenderness requirement as in clause 3.4.1.6.

(iv) Minimum percentage of reinforcement for a section under tension is 0.13% and that for a section under compression is 0.2% of the gross cross sectional area of concrete. Maximum reinforcement allowable is 4%.

(v) Minimum distance between bars is as stipulated in clause 3.12.11.1. The horizontal distance between bars should not be less than h_{agg} + 5 mm, where h_{agg} is the maximum size of coarse aggregate.

(vi) Maximum clear horizontal spacings between bars is taken as 160 mm as stipulated in clause 3.12.11.2.3.

(vii) From practical consideration, minimum allowable width is taken as 150 mm and the maximum allowable as 800 mm.

(b) Column design: In this study, only short column design is considered. The columns may be designed as braced or unbraced. The steel reinforcement are arranged symmetrically.

(i) Minimum eccentricity is taken as 0.05 times the overall dimension of the column in the plane of bending considered but not more than 20 mm. If the ultimate moment is not a governing factor, the minimum reinforcement of 0.4% of concrete section is provided.

(ii) No check is required when 'moment/axial force' ratio is less than 0.75h, provided that the shear stress does not exceed $0.8\sqrt{f_{cu}}$ or 5 N/mm^2, whichever is less (clause 3.8.4.6).

(iii) Relative lateral deflection in any storey under the characteristic wind load should not exceed h/500, where h is the storey height.

Transactions on the Built Environment vol 2, © 1993 WIT Press, www.witpress.com, ISSN 1743-3509

The total lateral deflection of frame at any storey should not exceed $H/500$, where H is the height at that particular storey.

(iv) Maximum allowable reinforcement should not exceed 6% of the gross cross-sectional area of the concrete.

(v) Minimum depth and width have been taken as 200 mm respectively.

(c) Selection of reinforcement: The allowable bar sizes (in mm) are 13, 16, 20, 25 and 32. Minimum number of bars allowed in each level is 2. The maximum number of layers of reinforcement is 2. If the reinforcement required does not satisfy minimum spacing requirements the congestion constraint is violated.

Methodology

A large frame problem requires tremendous computational effort. In practice, it is convenient to group beams and columns according to their respective sizes. In the suboptimization procedure, beams and columns of same sizes are grouped separately and optimized. Thus the computational time could be greatly reduced. The stresses in member elements are assumed to be constant during each cycle of suboptimization. However, it was found from numerical experimentations that columns tend to become slender, if beams and columns are optimized independently. Therefore, it is required to have some link to provide interaction between beam and column designs. This problem can be overcome by having a slenderness check while optimizing beam elements. The slenderness check can be considered as a global constraint since it is active both in column and beam designs. The resulting problem becomes a constrained nonlinear minimization problem. A constrained minimization problem may be converted into an equivalent unconstrained one using the 'exterior point' method of the SUMT developed by Fiacco and McCormick [6]. Accordingly, a problem of minimizing a function $f(\underline{x})$ subject to m constraints $g_j(\underline{x}) \geq 0$, $j = 1, 2, ..., m$ is solved by considering the problem of minimizing

$$F(\underline{x};r_k) = f(\underline{x}) + r_k \sum_{j=1}^{m} [g_j(\underline{x}) - |g_j(\underline{x})|]^2 \tag{2}$$

over monotonically increasing sequence of r_k. The resulting unconstrained minimization problem can be solved using direct search methods. The direct search methods have been chosen instead of gradient techniques as these methods can be easily programmed, requiring minimal storage which is important in microcomputer applications. Furthermore, it does not require explicit evaluation of any partial derivatives but rely solely on evaluation of the objective function. Practising engineers would particularly favour all these characteristics of the adopted methods. As mentioned earlier, the present problem has been reduced to a series of two variable suboptimization problem and therefore, does not require sophisticated optimization techniques.

Solution procedure

The width and depth of a member section are considered as continuous optimization variables. In order to obtain practical sections which are discrete, optimization is done in three stages. In the first stage, groups of beams and columns are optimized independently. In the second stage, the column sizes are rounded off to practical section sizes. Optimization is then proceeded with groups of beams only as the optimization variables. Finally, in the third stage of optimization the beam sizes are rounded off to the desired practical sizes and the whole frame is reanalysed and checked for any violation of constraints. The optimization procedure is summarized in the flow chart (Figure 1).

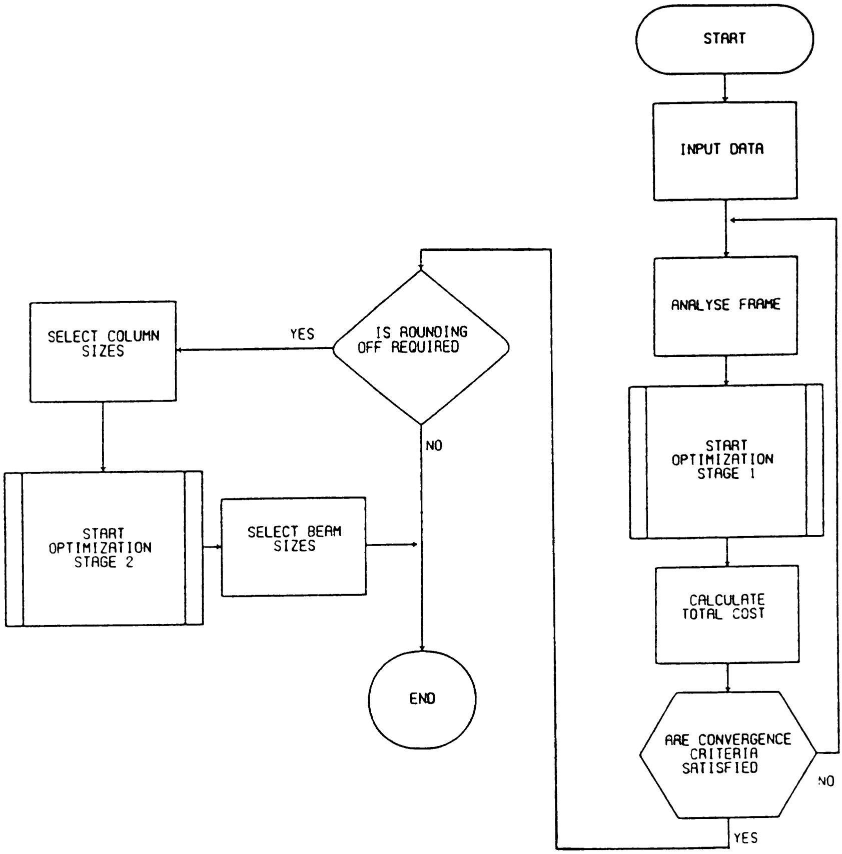

Figure 1 Optimization Procedure

Transactions on the Built Environment vol 2, © 1993 WIT Press, www.witpress.com, ISSN 1743-3509

NUMERICAL EXAMPLES

Three examples are presented in this section. The columns are designed as braced. The modulus of elasticity of concrete is taken as 250 kN/mm^2, concrete cube strength as 30 N/mm^2 and steel yield strength as 460 N/mm^2.

Example 1: Four frames are optimized using optimization of full frame and suboptimization. Figure 2 shows the different frames studied in this example. The details of frame configurations are listed in Table 1. Table 2 also shows the difference between the cost before rounding off and the cost after rounding off.

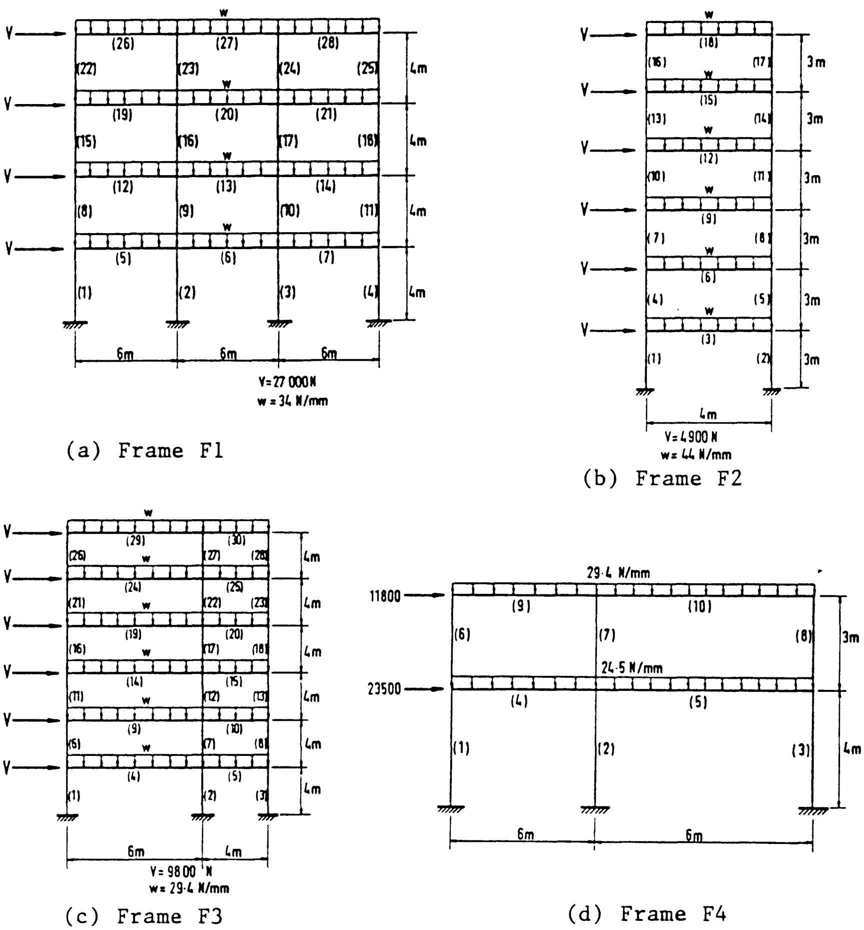

Figure 2 Frames of Example 1

Table 1: Frame configurations

Frame	F1	F2	F3	F4
No. of members	10	18	30	28
No. of joints	9	14	21	20
No. of DOF*	18	36	54	48
No. of variables	10	18	20	24

* DOF - Degrees of freedom.

Table 2: Optimized Cost for Example 1

Frame	F1	F2	F3	F4
Cost before rounding off	\$1117	\$1696	\$3746	\$4338
Cost after rounding off	\$1147	\$1711	\$3795	\$4372
% difference	2.7	0.9	1.3	0.8

Example 2: A three storey frame, shown in Figure 3, is considered. The loadings are as follows:

Load case 1: The frame is subject to dead and live loads only. The factor for live load is 1.6 and the factor for dead load is 1.4.

Load case 2: The frame is subject to dead, live and wind loads. The load factor is 1.2 for all loads.

The unfactored loads are as listed in Table 3.

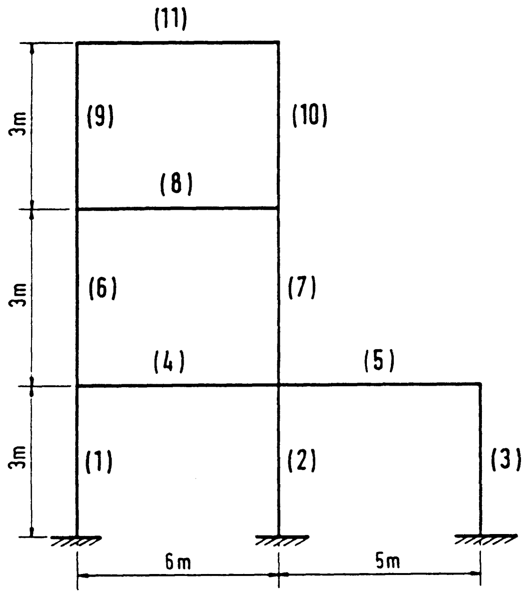

Figure 3 Frame of Example 2

Table 3: Frame loadings - Example 2

Load (kN/m)	Dead	Live	Wind
Level 1	16	10	6
Level 2	16	10	6
Roof	5	3	6

The grouping of members are as follows:

Group 1: 1,6,9. Group 2: 2,7,10. Group 3: 3.
Group 4: 4,5,8. Group 5: 11.

The total number of optimization variables is 10. The detailed results are summarized in Tables 4 and 5. The final optimized cost of frame is $1220.

Transactions on the Built Environment vol 2, © 1993 WIT Press, www.witpress.com, ISSN 1743-3509

Table 4: Results of Example 2 - Details of columns

Member:	1	
Dimensions (mm):	Breadth : 200	Depth : 250
A_s (mm^2):	670	
Rebar:	6T13	
Member:	2	
Dimensions (mm):	Breadth : 200	Depth : 225
A_s (mm^2):	180	
Rebar:	4T13	
Member:	3	
Dimensions (mm):	Breadth : 200	Depth : 250
A_s (mm^2):	1020	
Rebar:	4T20	
Member:	6	
Dimensions (mm):	Breadth : 200	Depth : 250
A_s (mm^2):	1004	
Rebar:	4T20	
Member:	7	
Dimensions (mm):	Breadth : 200	Depth : 225
A_s (mm^2):	1502	
Rebar:	4T25	
Member:	9	
Dimensions (mm):	Breadth : 200	Depth : 250
A_s (mm^2):	1228	
Rebar:	4T25	
Member:	10	
Dimensions (mm):	Breadth : 200	Depth : 225
A_s (mm^2):	1496	
Rebar:	4T25	

Transactions on the Built Environment vol 2, © 1993 WIT Press, www.witpress.com, ISSN 1743-3509

Table 5: Results of Example 2 - Details of beams

Member:	4		
Dimensions (mm):	Breadth : 175 mm	Depth : 325	
Location:	L. support	Mid-span	R. support
Top A_s (mm^2):	982	322	1610
Rebar(1):	2T25	2T16	2T25
Rebar(2):	----	----	2T20
Bottom As: (mm^2):	295	982	982
Rebar(1):	2T16	2T25	2T25
Member:	5		
Dimensions (mm):	Breadth : 175 *mm*	Depth : 325	
Location:	L. support	Mid-span	R. support
Top A_s (mm^2):	1257	251	982
Rebar(1):	2T20	2T13	2T25
Rebar(2):	2T20	----	----
Bottom A_s (mm^2):	628	628	189
Rebar(1):	2T20	2T20	2T13
Member:	8		
Dimensions (mm):	Breadth : 175 mm	Depth : 325	
Location:	L. support	Mid-span	R. support
Top A_s (mm^2):	1257	277	1384
Rebar(1):	2T20	2T16	2T25
Rebar(2):	----	----	----
Bottom A_s (mm^2):	628	982	982
Rebar(1):	2T20	2T25	2T25
Member:	11		
Dimensions (mm):	Breadth : 150 mm	Depth : 325	
Location:	L. support	Mid-span	R. support
Top A_s (mm^2):	402	106	531
Rebar(1):	2T16	2T13	2T13
Rebar(2):	----	----	----
Bottom A_s: (mm^2)	98	265	98
Rebar(1):	2T13	2T13	2T13

DISCUSSION

Optimization of several reinforced concrete frames has been studied. The design requirements have been formulated as constraints in the optimization problems. The entire procedure has been programmed for use in a 80486-33 microcomputer.

The optimized sections obtained satisfy all constraints listed, including practicality considerations. Congestion of bars is checked to ensure that required reinforcement in the optimized section is practical. Other constraints may be added depending on the requirements of the designer. The sections are rounded off to the next multiple of 25 *mm*. The cost differences resulted by rounding off are generally less than 1.5% as shown in Table 2. This is because of the refinement carried out by introducing a second stage of optimization. Figure 4 shows computational time using full-frame optimization and suboptimization procedures. It can be seen for large frames or when the number of optimization variables is high, suboptimization proves to be a more effective method. For instance, the frame F4 requires 306 seconds using full frame optimization and 77 seconds using suboptimization. A saving of 75% in the computation time is obtained.

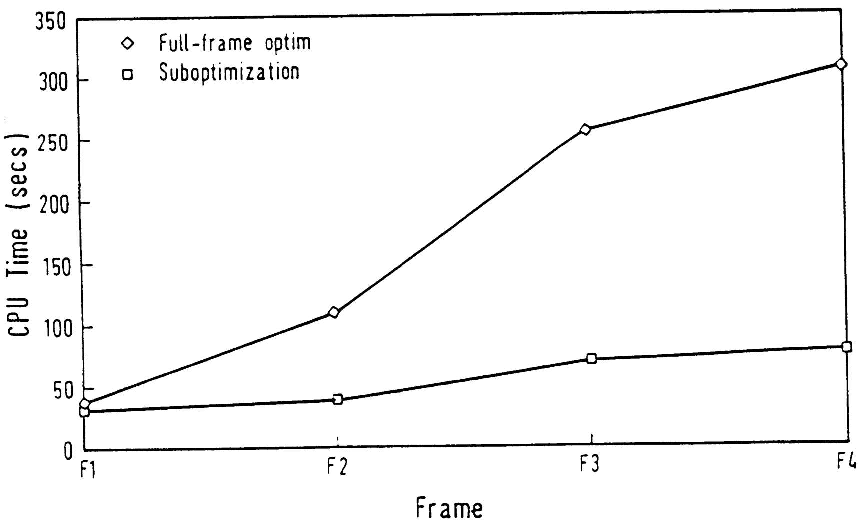

Figure 4 Comparison of CPU Time

Example 2 illustrates the design of a frame under multiple loading condition. This is important as practical designs normally require the structures to be considered under different load cases to obtain the most critical design. The example considers only two load cases for simplicity.

The detailed results of example 2 are shown in Table 2. The optimized dimensions and reinforcement required are shown. The reinforcement is adjusted in accordance with simplified rules of clause 3.12.10 (Part 1 BS8110). A possible configuration of reinforcement bars is also shown as part of the results. However, the final outlay would depend on the designer's discretion. The bar sizes used in this study are as listed in 3.3 (c). However, this can also be adjusted according to the availability of materials. The cost using different groupings is given in Table 4. The cost decreases as the number of groupings increases. But with the increase in number of groupings, the complexity of construction would increase. Therefore, a compromise, chosen by the designer based on his requirement, should be adopted.

CONCLUSION

A simple and efficient algorithm has been introduced. Automation of design can be achieved by incorporating the algorithm. The procedure can be easily implemented in a microcomputer by engineers. Design constraints according to designer's requirement can be included as required without much difficulty. The time taken for optimization has been found to be reasonable even for large frame structures.

ACKNOWLEDGEMENTS

The authors wish to acknowledge that the study has been facilitated by a Research Grant (No. RP890649) from the National University of Singapore.

REFERENCES

1. Munro, J., C.S. Krishnamurthy and C.W. Yu. Optimal design of reinforced frames. The Struct Engr., 50(7), 259-264, July (1972).
2. Krishnamurthy, C.S. and D.R. Mosi. Optimal design of reinforced concrete frames based on inelastic analysis. Engineering Optimization, 5, 151-167, 1981.

Transactions on the Built Environment vol 2, © 1993 WIT Press, www.witpress.com, ISSN 1743-3509

3. Liebman, J.S. and N. Khachaturian. Discrete structural optimization. J. Struct. Engrg., ASCE, 107(11), 2177-2197, 1981.
4. Choi, C.K. and H.G. Kwak. Optimum RC member design with predetermined discrete sections. J. Struct. Engrg., ASCE, 116(10), 2634-2655, October(1990).
5. British Standard Institution. Code of practice for design and construction. BS 8110 : Part 1 : 1985.
6. Fiacco, A.V. and G.P. McCormick. Nonlinear Programming: Sequential Unconstrained Minimization Techniques. John Wiley, New York, 1968.
7. Kavlie, D. and J. Moe. Automated design for framed structures. Proc. ASCE, 97(STI), 33-62, January (1971).
8. Jacoby, S.L.S., J.S. Kowalik, and J.T. Pizzo. Iterative methods for non-linear optimization problems. Prentice Hall, New Jersey, 1972.
9. Gill, P.E. and W. Murray. Numerical methods for constrained optimization. Academic Press, London, 1978.
10. Ghali A. and A.M. Neville. Structural Analysis. Chapman and Hall, London, 2nd ed., 1978.