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Optimal Design of Squeeze Film Supports for Flexible Rotors

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Assuming central preloading operation below the second bending critical speed and full film lubrication, this paper presents a theoretical model which allows one, with minimum computation, to design squeeze film damped rotors under conditions of high unbalance loading. Closed form expressions are derived for the maximum vibration amplitudes pertaining to optimally damped conditions. The resulting vibration amplitude and transmissibility data of design interest are presented for a wide range of practical operating conditions on a single chart. It can be seen that for a given rotor, the lighter the bearing the more easily one can satisfy design constraints relating to allowable rotor vibration levels and lubricant supply pressure requirements. The data presented are shown to be applicable to a wide variety of rotors, and a recommended procedure for optimal design is outlined.

NOMENCLATURE

a speed parameter $= \omega/\omega_c$

a_1-a_6 unique speeds, defined in Table 1

B bearing parameter $= \frac{\mu R}{2M\omega_r}(L/C)^3$

C radial clearance of bearing

C_1 equivalent damping coefficient $= F_t/(\omega e)$

C_2 non-dimensional damping coefficient $= C_1/(M\omega_r)$

e journal eccentricity

f frequency ratio $= \omega_r/\omega_c$

F_r, F_t fluid film force on journal in radial and transverse directions $= \frac{\mu R L^3 \omega}{C^2} g_{r,t}$

g displacement of rotor geometric centre with respect to static deflection

g_r $\frac{2\varepsilon^2}{(1-\varepsilon^2)^2}$ or 0 for unpressurized or pressurized bearings

g_t $\frac{\pi\varepsilon}{2(1-\varepsilon^2)^{3/2}}$ or $\frac{\pi\varepsilon}{(1-\varepsilon^2)^{3/2}}$ for unpressurized or pressurized bearings

G non-dimensional rotor orbit radius at midspan $= g/C$

G_R non-dimensional rotor orbit radius at midspan of a rigidly supported rotor assuming a damping ratio of 0.02

K_r retainer spring stiffness per bearing station

$2K_s$ rotor stiffness

K_1 equivalent stiffness coefficient $= K_r + F_r/e$

K_2 non-dimensional stiffness coefficient $= K_1/(M\omega_r^2) = K_1/K_r$

L axial width of bearing

$2M$ rotor mass

p minimum supply pressure for full film

P pressure parameter $= pLR/(K_sC)$

R journal radius

T transmissibility $= \sqrt{(F_r + K_r e)^2 + F_t^2}/[(1-\alpha)M\omega^2\rho]$

T_R transmissibility of a rigidly supported rotor

U unbalance parameter $= (1-\alpha)\rho/C$

α mass ratio = fraction of the total mass lumped at the bearing

ε non-dimensional journal orbit radius (eccentricity ratio) $= e/C$

μ absolute viscosity of lubricant at the mean lubricant temperature

ρ unbalance eccentricity

ω rotor speed

ω_c first pin-pin critical speed of rotor $= \sqrt{\frac{K_s}{(1-\alpha)M}}$

ω_r natural frequency of retainer spring with respect to rotor mass $= \sqrt{K_r/M}$

INTRODUCTION

In recent years, sophisticated computer programs have been developed for analyzing bearing systems. Ref. (1) summarizes the capability of such programs to evaluate the effect of damped flexible supports on critical speed location, on unbalance response and on

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stability. Linear theory techniques are used in most of these programs, and linear support damping and stiffness coefficients are needed as input. For those squeeze film dampers where the journal centre rotates in a circular orbit about the bearing centre (e.g. vertical rotors or centrally preloaded rotors), these linear coefficients have been evaluated both theoretically (2-4) and experimentally (5-9). However, owing to their non-linearity, apart from small unbalance loading situations, a knowledge of their value is of use more for stability rather than unbalance response investigations.

The unbalance response of even the simplest flexibly supported rotor, the so-called 'Jeffcott' rotor, is already dependent on a daunting array of system parameters, making parametric response studies tedious and difficult to present in a form useful for design (10). Actually, since the flexibly supported symmetric flexible rotor has two degrees of freedom, a logical design strategy has been to utilize the tuning capabilities of such systems as originally reported in (11) and later extended to flexibly supported 'Jeffcott' rotors (12) and in particular, to squeeze film supported flexible rotors (13,14).

More recently, an approach for optimally damping rotors at any critical speed has been proposed (15) and tested (16). These methods implicitly assume constant support stiffness and damping, and for squeeze film supports, would appear to be restricted to small unbalance loadings. An alternative approach to support design has been suggested in (17), though it is not clear how easily this method can be simply extended to cater for non-linear support forces.

However, unbalance loading, while small initially can increase markedly in service owing to corrosion, blade failure and thermal effects. Since the stiffness and the damping of squeeze film bearings can be highly non-linear, the unbalance response of a system using them can be markedly dependent on rotor unbalance, and can result in bistable operation, characterized by high unbalance transmissibilities and extremely high rotor amplitudes.

The design information in (10) does cater for large unbalance loading but owing to the non-linearity of the system and the large number of parameters affecting the system response, the data is for discrete parameter values only, requiring interpolation for optimal system design. It is the aim of this paper to show how, by making use of frequency response intersection points pertaining to two degree of freedom systems, but also present in response data for non-linear squeeze film supports (10), the design optimization process can be significantly simplified. Indeed, it will be shown that over a wide range of practical conditions, it is feasible to present relevant design data on a single design chart.

THEORETICAL MODEL

Fig. 1 depicts schematically a flexible rotor running in a rolling element bearing, whose outer race forms the non-rotating journal of a centrally preloaded squeeze film damper support. The following assumptions, justified in (18) pertain to the analysis: (a) the rotor is symmetric; (b) part of the mass may be lumped at the rotor centre with the remainder at the bearing stations; (c) gyroscopic effects are negligible; (d) excitation forces due to rolling

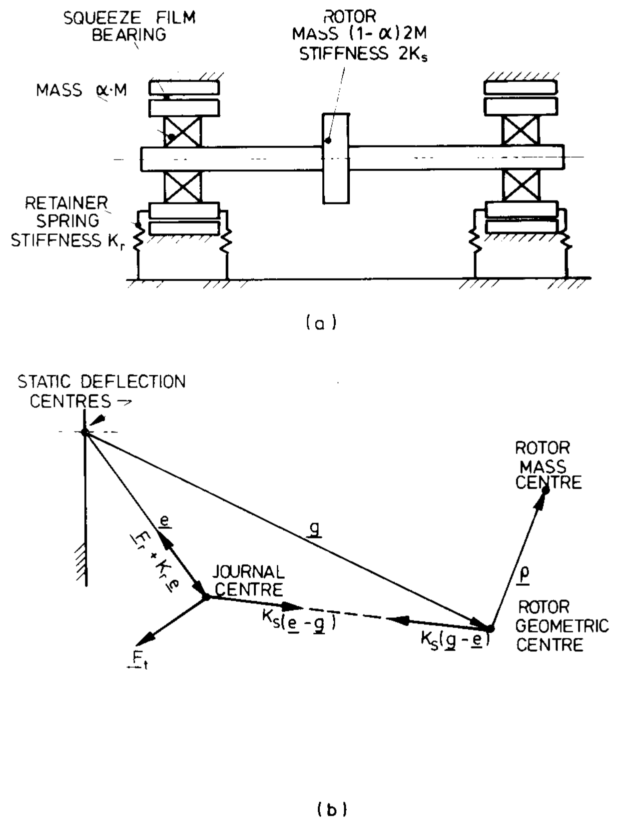

Fig. 1 (a) Schematic diagram of squeeze film mounted single-disc flexible rotor;
(b) Vectorial representation of displacements and forces

element bearings are negligible; (e) the Reynolds equation for constant lubricant properties is applicable; (f) the short bearing approximation is valid; (g) the pressures at the end of the bearings are either atmospheric or sufficiently above atmospheric to ensure full film lubrication; (h) only positive pressures contribute to the fluid film forces; (i) the rotor is centrally preloaded with constant radial support stiffness; (j) all unbalance in the rotor may be concentrated in the lumped mass at the rotor midspan; (k) operation is sufficiently below the second bending pin-pin critical speed to enable its effect to be ignored; (l) any damping or flexibility in the rolling element bearings may be ignored.

Referring to Fig. 1, the equation of motion for the mass αM lumped at each bearing station is given by

$$\alpha M\ddot{\underline{e}} + C_1\dot{\underline{e}} + K_S(\underline{e} - \underline{g}) + K_1\underline{e} = 0 , \qquad (1)$$

where C_1 and K_1 are equivalent damping and stiffness coefficients respectively.

Similarly, for the mass $2(1-\alpha)M$ lumped at the rotor midspan, the equation of motion is given by

$$(1-\alpha)M(\ddot{\underline{g}} + \ddot{\underline{\rho}}) + K_S(\underline{g} - \underline{e}) = 0 . \qquad (2)$$

Assuming that steady state conditions have been reached, with the rotor and journal centres describing synchronous circular orbits about the static deflection line, Eqns. (1) and (2) may be nondimensionalized and solved for the journal orbit radius ε, the rotor orbit radius G and the unbalance transmissibility T as explained in (10). An alternative derivation which leads to expressions for ε, G and T in a form more suited to this paper, is given in Appendix 1, where it is shown that

$$\varepsilon = \frac{Ua^2}{\{[K_2 f^2(1-a^2) + a^2(\alpha a^2 - 1)]^2 + C_2^2 a^2 f^2 (1-a^2)^2\}^{\frac{1}{2}}}, \quad (3)$$

$$G = \frac{Ua^2}{(1-\alpha)|1-a^2|} \left\{ \frac{\left[K_2 - \alpha \frac{a^2}{f^2} + \frac{1-\alpha}{f^2}\right]^2 + C_2^2 a^2/f^2}{\left[K_2 + \frac{a^2}{f^2}\left(\frac{\alpha a^2 - 1}{1-a^2}\right)\right]^2 + C_2^2 a^2/f^2} \right\}, \quad (4)$$

$$T = \frac{1}{|1-a^2|} \left\{ \frac{K_2^2 f^2/a^2 + C_2^2}{\left[K_2 f/a + \frac{a(\alpha a^2 - 1)}{f(1-a^2)}\right]^2 + C_2^2} \right\}^{\frac{1}{2}}. \quad (5)$$

In general, both the nondimensional stiffness coefficient K_2 and the nondimensional damping coefficient C_2 are themselves functions of the journal orbit radius ε and the bearing parameter B. Hence, given the relevant system parameters, viz: the unbalance parameter U, the mass ratio α, the speed paramater a, the frequency ratio f and the bearing parameter B, Eqn. (3) can be solved iteratively for ε, whereupon the corresponding solutions for the rotor orbit radius G and the transmissibility T follow. Since Eqn. (3) is non-linear, more than one solution for ε in the acceptable range $0 < \varepsilon < 1$ may result, depending on the actual values of the system parameters. Equilibrium solutions for Eqns. (3), (4) and (5) over a wide range of the system parameters have been presented in (4,10) where the stability of these solutions has also been investigated. Note that in the case of pressurized supports (i.e. sufficiently high lubricant pressure to ensure a continuous lubricant film throughout the bearing), the fluid film force has no radial component, i.e. $F_r = 0$, so that $K_2 = 1$.

Fig. 2 (Fig. 5(b) in (10)) shows the rotor

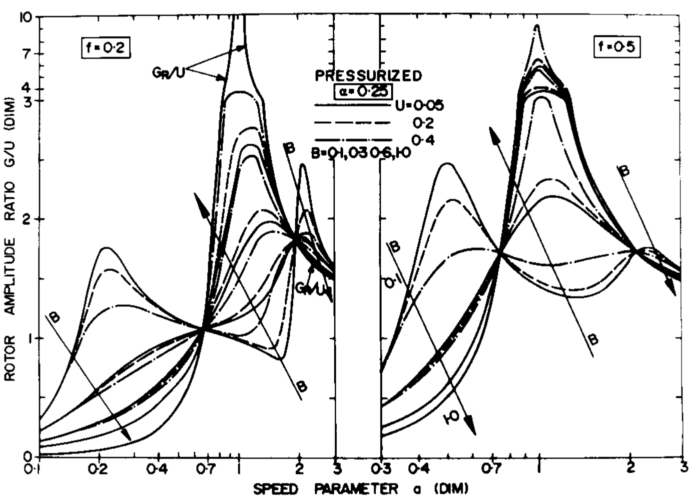

Fig. 2 Amplitude ratio frequency responses for range of B and U: pressurized mount, $\alpha = 0.25$, $f = 0.2$ and 0.5 (from Ref. (10))

amplitude ratio G/U frequency responses for a range of bearing parameter B and unbalance parameter U, for pressurized mounts, mass ratio $\alpha = 0.25$ and frequency ratios $f = 0.2$ and 0.5. As may be seen from Fig. 2 and from Figs. 2 to 7 in (10) for pressurized bearing mounts, for $0 \leq \alpha \leq 0.5$ and for $0.1 \leq f \leq 0.5$, unique points exist on the frequency response plots where all the rotor amplitude ratio frequency response curves intersect.

Also, it may be seen from Figs. 3 to 11 in (4) and Figs. 8 to 10 in (10), that similar unique points exist for the journal amplitude ratio ε/U frequency response plots for unpressurized mounts and for the transmissibility frequency response plots. These points are for a wide range of bearing parameter of $0.1 \leq B \leq 1$ and unbalance parameter $0.05 \leq U \leq 0.40$. At the unique speeds defining these points, the system response is independent of the bearing parameter B, and so is independent of the support damping and in some cases, of the support stiffness.

As is shown in Appendix 2, Eqns. (3) to (5) yield six such unique speeds, viz: a_1 to a_6. These speeds tend to depend on the mass ratio α and the frequency ratio f and are summarized in Table 1, together with the corresponding constant response expressions and an indication as to whether the results are applicable to unpressurized in addition to pressurized bearing supports.

Tuned Systems

For linear stiffness and damping, the existence of the unique speeds a_1 and a_2 has been long documented for two degree of freedom systems (11). For pressurized bearing supports, where the support stiffness is linear, even though the damping is not, one can apply a similar approach to (19) to minimize rotor amplitudes. Thus, for systems operating at a speed above both unique speeds it may be possible to minimize the maximum rotor amplitude ratio G/U_{max}, encountered in attaining the operating speed by suitable choice of the frequency ratio f and the damping C_2. Such a system constitutes an optimally damped system.

Regardless of the damping C_2, G/U_{max} can never be less than the larger of G/U_1 and G/U_2. Now since $a_1 < 1$ and $a_2 > 1$ (see Appendix 2), G/U_1 increases as a_1 increases and G/U_2 decreases as a_2 increases. Further, since both a_1 and a_2 increase as f increases (see Appendix 2), G/U_{max} will be minimal for that value of f for which

$$G/U_1 = G/U_2 . \quad (6)$$

Substituting for a_1 and a_2 into Eqn. (6) one obtains

$$f = \sqrt{\alpha}. \quad (7)$$

Such a choice of f defines a tuned system, whereupon

$$G/U_{max} \geq G/U_1 = G/U_2 = \left[\frac{1+\alpha}{(1-\alpha)^2}\right]^{\frac{1}{2}}, \quad (8)$$

and

$$K_r = \frac{\alpha}{1-\alpha} K_s . \quad (9)$$

Though these results are valid for pressurized bearing systems regardless of unbalance loading, they may also be applied to unpressurized systems provided the unbalance load is sufficiently small. The results are in agreement with (12), which investigates a similar rotor bearing system but assumes linear stiffness and damping supports.

As an illustration of the above, one can see in

	UNIQUE SPEEDS	RESPONSES	UNPRESSURIZED ?
G/U	$a_{1,2} = \left\{\frac{f^2+1}{2\alpha} \mp \left[\left(\frac{f^2+1}{2\alpha}\right)^2 - \frac{f^2}{\alpha} - \frac{1-\alpha}{2\alpha}\right]^{\frac{1}{2}}\right\}^{\frac{1}{2}}$	$G/U_{1,2} = \frac{a_{1,2}^2}{(1-\alpha)\lvert 1-a_{1,2}^2\rvert}$	ONLY IF $U \leqslant 0.05$
T	$a_3 = \alpha^{-\frac{1}{2}}$	$T_3 = \frac{\alpha}{1-\alpha}$	YES
	$a_{4,5} = \left\{\frac{1+2f^2}{2\alpha} \pm \left[\left(\frac{1+2f^2}{2\alpha}\right)^2 - \frac{2f^2}{\alpha}\right]^{\frac{1}{2}}\right\}^{\frac{1}{2}}$	$T_{4,5} = \frac{1}{\lvert a_{4,5}^2 - 1\rvert}$	ONLY IF $U \leqslant 0.05$
ε	$a_6 = 1$	$\varepsilon_6 = \frac{U}{1-\alpha} = \frac{\rho}{C}$	YES

Table 1 Speed and response at the unique points

Fig. 2 for $\alpha = 0.25$ and $f = 0.5$ (i.e. a tuned system) that $G/U_1 = G/U_2 = 1.72$. However, G/U_{max} is generally much greater than 1.72, being apparently dependent on the value of the unbalance parameter U and bearing parameter B. Note that the curve for $U = 0.4$ and $B = 0.1$ suggests that G/U_{max} can be lowered significantly to approach 1.72 by appropriately selecting B for a given unbalance; and that the appropriate value of B is that which satisfies the requirement that at a_1 and a_2

$$\frac{\partial(G/U)}{\partial a} = 0 \,. \tag{10}$$

This approach, originally suggested in (11), has been used successfully in (12,19) for linear systems where analytical expressions for the optimal damping were obtained. Unfortunately, no such expressions are available for non-linear damping, and Eqn. (10) needs to be satisfied numerically. Should appropriate damping be available to satisfy Eqn. (10) at a_1 and a_2, one has an optimally damped system, in that G/U_{max} has been minimized to equal G/U_1 or G/U_2.

OPTIMIZED DESIGN CHART

Scope of Design Chart

The paramount design considerations in flexibly supported rotor bearing systems are that for the likely rotor unbalance (i) the unbalance transmissibility at operating speed be low to achieve maximum bearing life and to minimize vibrations and stresses in the support structure, and (ii) the maximum rotor amplitude in attaining the operating speed be minimal to enable tight blade tip clearances to be maintained. The optimized design chart in Fig. 3 is presented to enable preloaded squeeze film dampers to be utilized as flexible supports to ensure minimum rotor vibration amplitudes. The design chart covers a wide range of operating conditions, viz: a mass ratio $\alpha \leqslant 0.5$, for an unbalance parameter $U \leqslant 0.4$ and a speed parameter $a \leqslant 5$. The chart assumes in addition to the assumptions pertaining to the theoretical model, that the bearings are pressurized and the rotor bearing system is tuned. It provides design information on (i) how to ensure that optimal damping can be assured over the speed range, (ii) what the corresponding unbalance transmissibility frequency response will be, (iii) what supply pressures are needed to ensure pressurization, (iv) what the maximum rotor amplitude ratio will be and (v) what the maximum amplitude ratio would be in the absence of flexible damped supports (assuming a damping ratio for the rotor of 0.02).

Optimal Damping

As noted in the above section for tuned systems, a low value of G/U_{max} requires careful consideration of the support damping C_2 which, for squeeze film supports, depends in a complicated manner on the bearing parameter B, the unbalance parameter U, the mass ratio α and the speed parameter a. In no sense can a constant C_2 be maintained while coming up to operating speed. To further complicate matters, the value of U is not strictly defined, as significant deterioration in the level of unbalance over the life of the machine is a definite possibility. Also, without some form of lubricant temperature control, it is often impossible to properly fix the bearing parameter B, which, by virtue of its dependence on the lubricant viscosity, is markedly temperature-dependent.

In view of this, the design strategy proposed here is to admit of possible variations in U and to utilize the temperature dependency of B to ensure that G/U_{max} is minimized. A perusal of the rotor amplitude frequency responses for pressurized bearings in (10) shows that in all cases presented there, G/U decreases as B decreases and as U decreases in the speed range $a_1 < a < a_2$. For $a < a_1$ or $a > a_2$, this pattern is reversed. Hence, minimum G/U may be assured by maintaining a sufficiently low bearing parameter while running between the unique speeds, i.e. for $a_1 < a < a_2$, and by increasing the bearing parameter above prescribed limits for $a \leqslant a_1$ and for $a \geqslant a_2$. The result is given in Fig. 3 wherein B_1 and B_3 are the minimum bearing parameters recommended for the speed ranges $a < a_1$ and $a > a_2$ respectively (obtained by satisfying Eqn. (9) for an assumed unbalance parameter $U = 0.05$); and B_2 is the maximum bearing parameter for the speed range $a_1 \leqslant a \leqslant a_2$ (obtained by ensuring that for $U = 0.4$, the maximum value of G/U in the speed range $a_1 \leqslant a \leqslant a_2$ is less than G/U_1 or G/U_2). Also shown are the unique speeds a_1 and a_2 which determine the switch from B_1 to B_2 and from B_2 to B_3.

Hence, provided B_1, B_2 and B_3 are practically realisable, G/U_{max} will be minimal for all unbalance parameters $U \leqslant 0.4$ (noting that for $U \leqslant 0.05$, the system behaves almost linearly in that G/U tends to be independent of U). Though the strategy has the apparent disadvantage that lubricant temperature control has to be provided while running up to or down from the operating speed, it is expected that some form of temperature control would normally be advisable to control B anyway; and by specifying B_1, B_2 and B_3 as limiting values of B, rather than actual values to be attained, the lubricant temperature control would be simple.

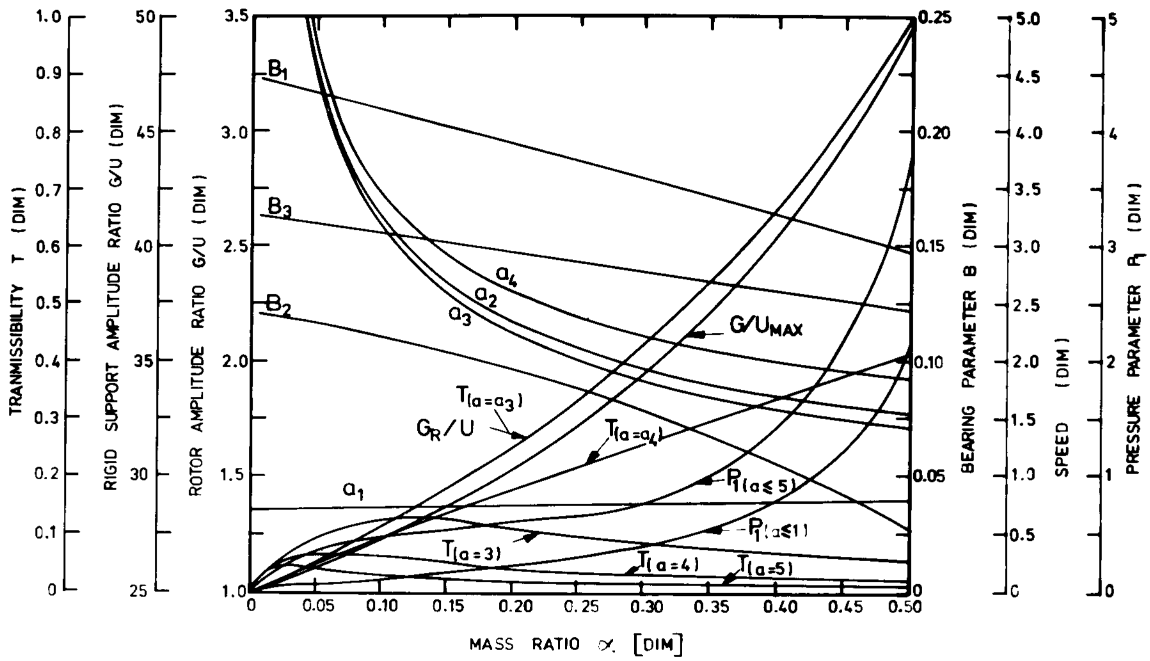

Fig. 3 Optimized design chart for pressurized mount

Also shown in Fig. 3 are the maximum rotor amplitudes G/U_{max} should the optimized damping strategy be adhered to, and the corresponding rigid support maximum amplitude ratio G_R/U. It can be seen that reductions in amplitude ratio by factors of 14 or more are possible for mass ratio $\alpha \leqslant 0.5$.

Unbalance Transmissibilities

Five transmissibility values are shown for speeds in the range $\sqrt{2} \leqslant a \leqslant 5$. In addition to the transmissibilities T_3 and T_4 at the unique speeds a_3 and a_4, the transmissibilities at $a = 3$, $a = 4$ and $a = 5$ are also included. These last three transmissibilities were also found to be insensitive to variation in U for $0.05 \leqslant U \leqslant 0.4$. The unique speed a_5 falls outside the speed range of interest, so T_5 is not included. The transmissibility T at the unique speed $a_6 = 1$, i.e. at the rotor pin-pin critical speed, is unbalance and damping dependent and is best calculated from

$$T = \frac{\sqrt{(K_r e)^2 + F_t^2}}{UMC\omega^2} = \frac{\sqrt{\alpha}}{1-\alpha}\left\{\alpha + \frac{(2\pi B)^2}{[1-(\rho/C)^2]^3}\right\}^{\frac{1}{2}}, \qquad (11)$$

where $e = \rho$. These six values of transmissibility over the speed range $1 \leqslant a \leqslant 5$ should be adequate to approximate the transmissibility frequency response and thereby, to determine the unbalance transmissibility at operating speed. As an aid, the unique speeds a_3 and a_4 are also given in Fig. 3.

Supply Pressure

Implicit to the applicability of the design chart is that the bearing be pressurized. Should the bearing be pressurized at both ends, the minimum required pressure parameter P is given by (21)

$$P = \frac{48\,Baf\,[2(1+24\varepsilon^2)^{\frac{1}{2}} - 8\varepsilon^2 - 2]^{\frac{1}{2}}}{(1-\alpha)[5-(1+24\varepsilon^2)^{\frac{1}{2}}]^3}. \qquad (12)$$

One end pressurization would require four times this value of P.

One can see from Eqn. (12) that for a tuned system, P is a function of B, α, a and ε (which in turn is also a function of U). Hence, assuming an unbalance parameter $U = 0.4$ and a speed range $0 < a \leqslant 5$, P was computed for all values of α in the range $0 < \alpha \leqslant 0.5$ utilizing the appropriate bearing parameters in Fig. 3. The corresponding values of the journal eccentricity ratio ε may be obtained from the solution of Eqn. (3). The resultant values of P are given in Fig. 3.

Regardless of pressurization, bistable operation is not predicted to occur above the first pin-pin critical speed. Thus, should it be desired to remove pressurization once $a > 1$, the required pressure parameter P which assumed a speed range up to $a = 5$ is unduly conservative. Hence, the required pressure parameter P for the more limited speed range of $0 < a \leqslant 1$ has been similarly evaluated and is also given in Fig. 3. Note that in such a case, once pressurization has been removed, the information obtainable from Fig. 3 becomes approximate only. Further, whereas the pressurized system is always stable, the stability of the unpressurized system is not globally guaranteed and needs to be investigated (4).

APPLICABILITY

Mass Ratio Considerations

As may be seen from Fig. 3, the lower the mass ratio α, the lower the maximum amplitude G/U_{max}, the transmissibility T, the required pressure parameter P and the bearing parameter ratio B_2/B_1. The smaller the B_2/B_1 ratio, the smaller the required variation of the lubricant temperature for optimal damping and hence the shorter the delay in heating or cooling the lubricant while coming up to or down from the operating speed.

For heavy rotors, say $M > 20$ kg, it is reasonable to assume that a low mass ratio is attainable. In this case, the unique speed a_2 may be designed to

CASE	FROM REF. (22) $(1-\alpha)M$ [kg]	ω_c [rad/s]	ESTIMATED αM [kg]	α	p [MPa]	μ_1 Pa s $\times 10^{-3}$	μ_2 Pa s $\times 10^{-3}$
1	2.7	1257	0.45	0.14	0.53	14.1	7.0
2	9.1	943	0.9	0.09	0.89	29.7	15.6
3	318	388	1.4	0.004	0.12	43.1	23.5
4	295	388	1.4	0.005	0.17	44.7	24.4
5	239	848	1.4	0.006	0.75	86.8	47.4
6	20.5	785	1.4	0.06	1.20	45.3	24.2
7	0.91	1256	0.37	0.29	0.36	9.4	4.2
8	0.45	3142	0.27	0.38	1.31	11.6	4.4
9	255	293	1.4	0.005	0.08	29.2	15.9

Table 2 Estimated lubricant supply pressure and viscosity requirements for nine cases discussed in Ref. (22), U = 0.4

occur outside the operating speed range and there will be only one intersection point of the amplitude ratio G/U curves. In such a case, it may be beneficial to forgo a tuned system and select a frequency ratio for which $f < \sqrt{\alpha}$ to obtain a further reduction in the unique speed a_1 and hence, in the amplitude ratio G/U_1. In such cases, G/U_{max} and a_1 may be obtained from Table 1 but Fig. 3 is no longer applicable and the appropriate bearing parameters B_1 and B_2 to ensure optimal damping, the corresponding pressure parameter P and the transmissibility T, would have to be evaluated from the non-linear model.

For light rotors, mass ratios as high as $\alpha = 0.5$ are possible. For such a high α, relatively high transmissibility is predicted at the unique speed a_3, and furthermore, as may be seen from Table 1, this transmissibility cannot be reduced by removing pressurization, being independent of pressurization. Since, at a_3, the transmissibility of the squeeze film damper is predicted to be the same as that of the rigidly supported rotor (10), operation in the vicinity of this speed should be avoided by a careful selection of α.

Lubricant Viscosity and Supply Pressure Considerations

Basic conditions for the design to be practical are that: (a) the supply pressure required to ensure full film is not excessive; (b) the lubricant is an available Newtonian fluid.

The evaluation of the supply pressure and viscosity requirements necessitates knowledge of the rotor bearing system. The following approach is based on data taken from (22) for nine rotors. It is assumed that U = 0.4 and $L/R \approx 0.25$. $L = 10^{-2}$ m is assumed for the lighter rotors ($(1-\alpha)M \leq 20.5$ kg), whereas $L = 2 \times 10^{-2}$ m is assumed for the heavier rotors ($(1-\alpha)M \geq 239$ kg). The supply pressure for two ends pressurization is then

$$p = \frac{K_c C P}{LR} = \frac{(1-\alpha)M\omega_c^2}{400\,L} P \,, \qquad (13)$$

and the lubricant viscosity is given by

$$\mu = \frac{2\sqrt{\alpha}\, M\omega_c B C^3}{RL^3} = 5 \times 10^{-7} \frac{\sqrt{\alpha}\, M\omega_c B}{L} \,. \qquad (14)$$

Due to lack of information in (22), αM values are assumed as shown in Table 2. Assuming an optimally damped system, one can obtain the required limiting values for P, B_1 and B_2 from Fig. 3 and calculate the corresponding supply pressure p and viscosity μ_1 and μ_2 requirements using Eqns. (13) and (14). The results are summarized in Table 2 where it can be seen that the highest pressure, p = 1.31 MPa, is required for Case 8. This supply pressure does not appear to be excessive. The lubricants for all these cases are commercially available (23). Cases 1 to 9 represent a wide variety of rotor bearing configurations, and hence, it is concluded that the design information is practical for a wide range of applications.

Unpressurized Supports

Table 1 shows that at the first bending critical speed, $e = \rho$ is predicted regardless of pressurization. Similarly, the unique speed a_3 for the intersection of transmissibility curves is independent of pressurization. Further, it may be seen from Figs. 2(a) to 7(a) in (10) that for unpressurized bearing mounts and U = 0.05, a good approximation to the unique intersection points is obtained in the rotor amplitude and transmissibility frequency responses. Hence, it is reasonable to presume that the theoretical treatment presented here may be used for an optimal design of an unpressurized rotor bearing system, for which an unbalance greater than $\rho = 0.05C/(1-\alpha)$ is unlikely to occur.

Thus for $U \leq 0.05$, all the unique speeds and responses for pressurized systems, as summarized in Table 1, are applicable to unpressurized systems. The optimized design chart of Fig. 3 is also applicable except for the bearing parameter data for optimal damping for which further investigation is needed. However, the bearing parameter data may be approximated from Figs. 2(a) to 7(a) in (10) and from Fig. 2.12 in (18). These are summarized in Table 3, which should be used in conjunction with Fig. 3 to evaluate the optimal supports design.

MASS RATIO, α	BEARING PARAMETER, B
0	B ⩾ 0.6
0.25	$B_1 > 0.3$ $B_2 < 0.3$ $B_3 > 0.3$
0.5	B ≈ 0.3

Table 3 Bearing parameter design data for unbalance parameter U = 0.05, unpressurized mount and mass ratios α = 0.0, α = 0.25 and α = 0.5

Comparison between Table 3 and Fig. 3 shows that the bearing parameters needed for optimal unpressurized supports are invariably higher than those required for the pressurized ones. The higher bearing parameter is needed to increase the damping reserve of the unpressurized mounts and to ensure a stable operation (4).

For unpressurized rotor bearing systems operating with higher unbalance loading, higher eccentricity ratios and consequently, larger radial film force components are expected in the vicinity of the intersection points. Hence, the approximation $K_2 \approx 1$ is no longer valid, Fig. 3 is not applicable, and detailed design charts should be used for the selection of design parameters (4,10).

DESIGN PROCEDURE

This procedure for the design of squeeze film damped supports is for the usual case where the designer has no control over the rotor dimensions or the operating speed a. However, one can find the rotor stiffness K_s, the pin-pin critical speed ω_c and the mass of the rotor lumped at the centre span using a procedure such as outlined in (13,15). Next, the following steps are suggested:

1 Assess the maximum unbalance eccentricity likely to be encountered, ρ.

2 Select radial clearance C ⩾ ρ.

3 Select bearing mean radius 100 C ⩽ R ⩽ 1000 C, noting that as small an α as practicable is desirable.

4 Select bearing land width L ⩽ R/2.

5 Select rolling element bearing and estimate the mass ratio α, avoiding $\alpha = 1/a^2$ if possible, when α is large.

6 Calculate the retainer spring stiffness, K_r from Eqn. (9).

7 Read the limiting bearing parameters B_1, B_2 and B_3 from Fig. 3.

8 Calculate the oil viscosity from $\mu = 2BM\omega_r C^3/(RL^3)$.

9 Read the pressure parameter P from Fig. 3 and calculate the supply pressure required for two end pressurization from $p = K_s CP/(LR)$ For one end pressurization, use four times this pressure.

10 Calculate the transmissibility T(a = 1) from Eqn. (11) and read from Fig. 3 $T_3(a = a_3)$, $T_4(a = a_4)$, T(a = 3), T(a = 4) and T(a = 5). Using these transmissibilities plot a transmissibility versus speed curve and estimate the transmissibility at the operating speed.

11 Check expected life of selected rolling element bearing. If unsatisfactory, reselect bearing and return to Step 5.

DESIGN EXAMPLE

As an illustration of the suggested design procedure, consider the problem of providing a squeeze film damper support for Case 8 in (22). The rotor mass 2(1 - α)M is given as 0.90 kg, the first pin-pin critical speed ω_c as 3142 rad/s and the operating speed a as 2.0. It is desired to use an optimally damped support.

1 Assess that at worst ρ = 0.045 mm

2 Select C = 0.10 mm

3 Select R = 20 mm

4 Select L = 8 mm

5 Select a ball bearing with 28 mm outer diameter, 12 mm bore diameter and 8 mm land width, with mass of 0.02 kg. Using a 6 mm thick sleeve integral with a squirrel cage type retainer spring, an additional 0.075 kg mass is lumped at the bearing station. For an assumed symmetric rotor, half the rotor mass is 0.45 kg, of which one sixth is assumed to comprise the shaft portion (0.075 kg) and one third of this shaft portion is assumed to be lumped at the bearing station (0.025 kg). Hence, the mass ratio α is estimated as (0.02 + 0.075 + 0.025)/(0.45 + 0.075 + 0.02) = 0.22. The unique speed $a_3 = 1/\sqrt{\alpha} = 2.1$. This is in the vicinity of the operating speed so that the transmissibility T will be around 0.3 regardless of pressurization. Should it be desired to reduce α, a ball bearing with 22 mm outer diameter, 8 mm bore diameter and 7 mm land would have a mass of 0.012 kg and a squirrel cage type retainer spring with 30 mm outside diameter and with an estimated mass of 0.035 kg could be selected, giving α = 0.14 and U = 0.39. From Fig. 3, $a_1 = 0.73$, $a_2 = 2.76$, $a_3 = 2.67$ and $a_4 = 2.99$.

Note that in this case $a_2 = 2.76$ falls outside the speed range, simplifying bearing parameter control.

6 The retainer spring stiffness K_r is given by Eqn. (9) as 6.9×10^5 N/m.

7 From Fig. 3, require that $B_1 \geqslant 0.205$ and $B_2 \leqslant 0.105$.

8 The values of B require lubricant viscosities of μ ⩾ 0.046 Pa s for a ⩽ 0.73 and μ ⩽ 0.024 Pa s for a ⩾ 0.73.

9 From Fig. 3, assuming two ends pressurization, P ≈ 0.55 so that p = 2.0 MPa. Should pressurization be removed beyond the pin-pin critical speed, the supply pressure needed for pressurization up to the critical speed would be obtained as P ≈ 0.15 or p ≈ 0.57 MPa. Being an optimally damped system, from Fig. 3 $G/U_{max} = 1.34$ so the maximum rotor amplitude would be $g_{max} = 0.052$ mm representing a 95% reduction in the maximum rotor amplitude of a rigidly supported rotor of damping ratio 0.02 for which $G_R/U = 26.5$.

10 From Eqn. (11), T(a = 1) is 0.44. From Fig. 3 $T_3 = 0.16$, $T_4 = 0.13$, T(a = 3) = 0.13, T(a = 4) = 0.065. T(a = 5) is not needed for interpolation. Using these values, the transmissibility at operating speed is obtained by interpolation as T = 0.24 resulting in a transmitted force amplitude of 182.5 N under the worst unbalance condition.

11 Under this worst unbalance condtiion, the selected ball bearing would have a life of approximately 210 hours. Whether this is satisfactory depends on the actual operating requirements. A further reduction in the transmitted force at operating speed would actually result if μ < 0.024 Pa s ($B_2 < 0.105$) at the operating speed. Had the original support selection with α = 0.22 been adhered to, there would have resulted a 19% increase in the maximum rotor amplitude and a 22% increase in the transmitted force, indicating the desirability of avoiding operation in the vicinity of the unique speed a_3 even for relatively low values of mass ratio α.

For the design recommended here, depending on the operating temperature, a variety of lubricants would be available, and the required temperature change upon passage through the unique speed a_2 while running up to or down from operating speed is typically around 20°C.

CONCLUSIONS

1 A theoretical model has been presented, allowing for fast design of optimally damped pressurized rotor bearing systems.

2 All necessary design data for such optimally damped systems are presented on a single chart for a wide range of mass ratio and for a relatively high level of unbalance parameter.

3 The data presented are practical for a wide variety of rotors operating under the second bending critical speed. In particular, the design is not constrained in many practical applications by the lubricant supply pressure and viscosity.

4 In conjunction with additional bearing parameter data, the design chart may also be used for the design of optimally damped unpressurized support systems for low unbalance.

5 For operation outside the parameter ranges for which the design chart is intended, the analytical model allows for the desired design data to be obtained.

6 The smaller the mass ratio the lower the rotor amplitude and the supply pressure needed. However, the lower limit of the mass ratio may be dictated by rotor mass, and depending on the operating speed, by transmissibility considerations.

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APPENDIX 1

Under steady state conditions, with the rotor geometric and mass centres, and the journal centre describing synchronous circular orbits about the static deflection line in Fig. 1, one can write

$$\underline{e} = e\exp(i\omega t)\,, \qquad (A1.1)$$

whereupon

$$\dot{\underline{e}} = i\omega\underline{e}\,, \qquad (A1.2)$$

and

$$\ddot{\underline{e}} = -\omega^2\underline{e}\,. \qquad (A1.3)$$

Similar expressions pertain to $\ddot{\underline{g}}$ and $\ddot{\underline{\rho}}$ so that Eqns. (1) and (2) may be written as

$$-K_S\underline{g} + (K_S + K_1 - \alpha M\omega^2 + iC_1\omega)\underline{e} = 0\,, \qquad (A1.4)$$

$$[K_S - (1-\alpha)\omega^2 M]\underline{g} - K_S\underline{e} = (1-\alpha)M\omega^2\underline{\rho}\,. \qquad (A1.5)$$

Elimination of $\underline{g}$ from Eqns. (A1.4) and (A1.5) gives

$$\underline{e} = \frac{(1-\alpha)M\omega^2K_S\rho}{\alpha(1-\alpha)M^2\omega^4-(1-\alpha)MK_1\omega^2-MK_S\omega^2+K_1K_S-i(1-\alpha)MC_1\omega^2} \quad (A1.6)$$

From this equation the magnitude of $\underline{e}$ may be easily obtained, and upon appropriate nondimensionalization, one obtains Eqn. (3) for ε.

Similarly, one can eliminate $\underline{e}$ from Eqns. (A1.4) and (A1.5) to yield Eqn. (4) for $\bar{G}$. Finally, substitution for e in the expression for transmissibility and nondimensionalizing yields, after some algebraic manipulation, Eqn. (5) for T.

APPENDIX 2

Rotor Amplitude Ratio

From Eqn. (4) G/U is independent of the support damping when

$$\left[K_2 - \alpha\,\frac{a^2}{f^2} + \frac{1-\alpha}{f^2}\right]^2 = \left[K_2 + \frac{a^2}{f^2}\left(\frac{\alpha a^2 - 1}{1 - a^2}\right)\right]^2 . \quad (A2.1)$$

Note that since $0 \leqslant \alpha < 1$ and f is assumed to be nonzero, this condition cannot be satisfied at $a = 1$. Nor can it be satisfied if K_2 is not constant except for $\alpha = 1$. However, for pressurized bearing supports, i.e. with $K_2 = 1$, Eqn. (A2.1) is satisfied at the two speeds, a_1 and a_2, where

$$a_{1,2} = \left\{\frac{f^2+1}{2\alpha} \mp \left[\left(\frac{f^2+1}{2\alpha}\right)^2 - \frac{f^2}{\alpha} - \frac{1-\alpha}{2\alpha}\right]^{\frac{1}{2}}\right\}^{\frac{1}{2}} . \quad (A2.2)$$

The corresponding constant rotor amplitude ratios at these speeds are then given by

$$G/U_{1,2} = \frac{a_{1,2}^2}{(1-\alpha)|1 - a_{1,2}^2|} . \quad (A2.3)$$

Eqns. (A2.2) and (A2.3) may also be used to predict the unique points for unpressurized bearing supports provided the unbalance parameter is sufficiently small so that $K_2 \approx 1$.

It may be noted that for all $0 \leqslant \alpha < 1$, Eqn. (A2.2) requires that a_1 and a_2 be on either side of the pin-pin critical speed, i.e. that $a_1 < 1$ and $a_2 > 1$. Thus, $a_1 > 1$ would require that

$$1 < \frac{f^2+1}{2\alpha} - \left[\left(\frac{f^2+1}{2\alpha}\right)^2 - \frac{f^2}{\alpha} - \frac{1-\alpha}{2\alpha}\right]^{\frac{1}{2}} . \quad (A2.4)$$

This inequality can only be satisfied if $\alpha > 1$. Similarly, it may be shown that $a_2 < 1$ requires $\alpha > 1$. Hence, G/U_1 increases as a_1 increases and G/U_2 increases as a_2 decreases. Further, a rearrangement of Eqn. (A2.2) yields

$$f^2 = \frac{\alpha a_{1,2}^4 - a_{1,2}^2 + (1-\alpha)/2}{a_{1,2}^2 - 1} . \quad (A2.5)$$

Differentiation of Eqn. (A2.5) with respect to $a_{1,2}$ gives

$$2f\,\frac{df}{da_{1,2}} = 2\alpha a_{1,2} + \frac{a_{1,2}(1-\alpha)}{(1 - a_{1,2}^2)^2} . \quad (A2.6)$$

Hence, for $0 \leqslant \alpha \leqslant 1$, $\frac{df}{da_{1,2}} > 0$ so that both a_1 and a_2 increase as f increases.

Unbalance Transmissibility

From Eqn. (5) it can be seen that T is independent of the support stiffness and damping whenever

$$\frac{\alpha a^2 - 1}{1 - a^2} = 0 . \quad (A2.7)$$

Eqn. (A2.7) is satisfied at the unique speed a_4 given by

$$a_3 = 1/\sqrt{\alpha} , \quad (A2.8)$$

and the corresponding constant transmissibility is given by

$$T_3 = \frac{\alpha}{1-\alpha} . \quad (A2.9)$$

For pressurized bearings, T will also be independent of support damping at the two speeds a_4 and a_5 where

$$a_{4,5} = \left\{\frac{1+2f^2}{2\alpha} \pm \left[\left(\frac{1+2f^2}{2\alpha}\right)^2 - \frac{2f^2}{\alpha}\right]^{\frac{1}{2}}\right\}^{\frac{1}{2}} . \quad (A2.10)$$

The corresponding constant transmissibilities at these speeds are then given by

$$T_{4,5} = \frac{1}{|a_{4,5}^2 - 1|} . \quad (A2.11)$$

Again, Eqns. (A2.10) and (A2.11) may also be used to predict unique points for unpressurized bearings provided the unbalance parameter is sufficiently small so that $K_2 \approx 1$.

Journal Amplitude Ratio

From Eqn. (3), ε/U is independent of the support stiffness and damping when $a = 1$; i.e. the pin-pin critical speed corresponds to the unique speed a_6. The corresponding constant journal amplitude ratio is then given by

$$\varepsilon/U_6 = \frac{1}{1-\alpha} . \quad (A2.12)$$

Note that this implies that at the pin-pin critical speed, $e = \rho$, i.e. that the journal eccentricitty must equal the mass eccentricity, as had already been noted in (20). Physically, this means that squeeze film dampers necessarily lock up once the mass eccentricity equals or exceeds the bearing radial clearance.