



## Optimal Design of Wastewater Treatment Plant Using Adaptive Simulated Annealing

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**ABSTRACT:** This paper deals with the application of Adaptive Simulated Annealing (ASA) for the optimal design of the wastewater treatment plant. The plant consists of a trickling filter, an activated sludge aeration vessel and a secondary clarifier. In this work a successful attempt has been made to use the ASA for optimal design of wastewater treatment plant. ASA based optimal design values have been compared with conventional optimization approaches and has been found to yield the lowest total construction cost of wastewater treatment plant. From this work, it has been found that artificial intelligence based optimization techniques such as adaptive simulated annealing is found to be suitable for the optimal design of wastewater treatment plant. @JASEM

Several conventional and global optimization techniques had been used for the optimal design of varieties of process engineering problems. The conventional optimization techniques are complex, computationally expensive, and not efficient and in most cases, find local optimum values. These drawbacks of conventional techniques are overcome by adapting a universal technique that could solve all possible optimization problems and should have a higher probability to yield the solutions near enough to global optimum value. There has been an increasing interest recently in the use of artificial intelligence based optimization techniques in solving the design problems.

Artificial intelligence based optimization like simulated annealing, genetic algorithm, differential evolution, evolutionary strategies and evolutionary programming have been used as the replacement for conventional optimization techniques because they overcome the limitations of conventional approaches, due to ease of computation

and simplicity in programming (Rao 1996). The objective of this work is to study the efficiency and robustness of the artificial intelligence based optimization technique for the optimal design of wastewater treatment plant. In this work adaptive simulated annealing technique has been implemented for the optimal design of wastewater treatment plant.

*Wastewater Treatment Plant:* Wastewater treatment plant (WTP), considered in this paper, was designed by Mishra et al., 1973 that consists of a trickling filter, an activated sludge aeration vessel and a secondary clarifier as shown in Fig 1.

The principal unit operations in water-treatment plant include coagulation-flocculation, sedimentation and filtration. The performance of each treatment unit affects the efficiency of the subsequent units. Therefore the design decisions should be made with regard to the interactions between various unit operations present in the plant.

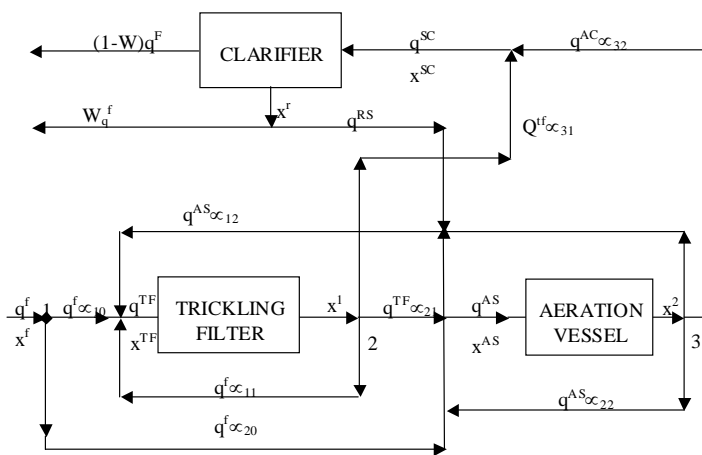


Fig 1 Wastewater Treatment Plant

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**Simulated Annealing:** Simulated Annealing (SA) is an artificial intelligence based optimization technique that resembles the cooling process of molten metals through annealing. At high temperature, atoms in molten metal can move freely with respect to each another, but as the temperature is reduced, the movement of atoms gets restricted. The atoms start to get ordered and finally form the crystals having the minimum possible energy. However, the formation of the crystal mostly depends on the cooling rate. If the temperature is reduced at a very fast rate, the crystalline state may not be achieved at all; instead, the system may end up in a polycrystalline state, which may have a higher energy state than the crystalline state. Therefore, in order to achieve the absolute minimum energy state, the temperature needs to be reduced at a slow rate. Controlling a temperature-like parameter introduced with the concept of the Boltzmann probability distribution simulates the cooling phenomenon. According to the Boltzmann probability distribution, a system in thermal equilibrium at a temperature T has its energy distributed probabilistically according to  $P(E) = \exp(-E/kT)$ , where k is the Boltzmann constant. Simulated annealing is a point-by-point method. The algorithm begins with an initial point and a high temperature T. A second point is created at random in the vicinity of the initial point and the difference in function values ( $\Delta E$ ) at these two points

is calculated. If the second point has a smaller function value, the point is accepted; otherwise the point is accepted with a probability  $\exp(-\Delta E/T)$ . This completes the one iteration of simulated annealing procedure. In the next generation, another point is created at random in neighborhood of current point and Metropolis algorithm is used to accept or reject the point. The algorithm is terminated when a sufficiently small temperature is obtained or a small enough change in function values is found (Deb 1995).

**Adaptive Simulated Annealing:** Adaptive simulated annealing (ASA) maintains all the advantages of standard simulated annealing algorithm along with improvement in speed of convergence. ASA is also known as the very fast simulated reannealing, is a very efficient version of SA. ASA uses only the value of the cost function in the optimization process and is very simple to program (Ingber, 1989). The successful working of ASA depends up on three function, those are generating probability density function, acceptance function, and annealing schedule. These three functions are modified in ASA algorithm to improving the convergent speed. The ASA, however, can employ a very fast annealing schedule, as it has self-adaptation ability to re-scale temperature.

**Optimal Design Of Wastewater Treatment Plant:** The construction cost and operation cost of the trickling filter, is given by the correlation

$$C^{TF} = 25.0 [V^{TF}]^{0.827} + 3650.0 + 1150.0 [q^{TF} \alpha_{11}] \quad (1)$$

Where  $V^{TF}$  is expressed in  $ft^3$  and the last two terms are due to the cost of the recycle pump. The construction cost and operation cost of the aeration vessel is given by the correlation

$$C^{AS} = V^{AS} [175,000.0 + 36,500.0 (V^{AS})^{-0.818}] + 3650.0 + 1150.0 [q^{AS} \alpha_{12}] \quad (2)$$

where  $V^{AS}$  is expressed in million gallons and the last two terms are due to the cost of the pump between the aeration vessel and the trickling filter.

The construction cost and operation cost of the secondary clarifier is given by the correlation

$$C^{SC} = 12,600.0 [A^{SC}] + 5,350 [A^{SC}]^{-0.126} + 3650.0 + 1150.0 [q^{RS}] \quad (3)$$

Totaling the costs of the individual process subsystems and allowing 40% in excess for engineering fees and ancillary costs estimated the total construction cost of the system. Mathematically the total construction cost is given by the equation

$$C^T = 1.4 [C^{TF} + C^{AS} + C^{SC}] \quad (4)$$

*Optimization Problem Formulation:* The objective function for optimal design of wastewater treatment plant is given by

$$\text{Min } C^T = 1.4 [C^{TF} + C^{AS} + C^{SC}] \quad (5)$$

Subject to the equality constraints

$$q^{TF} = (q^f \alpha_{10} + q^{AS} \alpha_{12}) / (1 - \alpha_{11}) \quad (6)$$

$$\alpha_{20} = 1 - \alpha_{10} \quad (7)$$

$$\alpha_{31} = 1 - \alpha_{21} - \alpha_{11} \quad (8)$$

$$\alpha_{32} = (q^{SC} - \alpha_{31} q^{TF}) / q^{AS} \quad (9)$$

$$\alpha_{22} = 1 - \alpha_{32} - \alpha_{12} \quad (10)$$

$$r_1 = q^{AS} (1 - \alpha_{11}) (1 - \alpha_{22}) - q^{AS} \alpha_{12} \alpha_{21} - [(1 - \alpha_{11}) \alpha_{20} + \alpha_{10} \alpha_{21}] q^f \quad (11)$$

$$r_2 = q^{AS} \alpha_{12} \alpha_{21} + q^{AS} (1 - \alpha_{11}) \alpha_{22} + [(1 - \alpha_{11}) \alpha_{20} + \alpha_{10} \alpha_{21}] q^f \quad (12)$$

$$r = r_1 / r_2 \quad (13)$$

$$q^{RS} = r_s (q^f \alpha_{20} + q^{TF} \alpha_{21} + q^{AS} \alpha_{22}) \quad (14)$$

$$x_1^2 = (q^{TF} x_1^{TF} - q^f \alpha_{10} x_1^f - q^{TF} \alpha_{11} x_1^1) / (q^{AS} \alpha_{12}) \quad (15)$$

$$x_1^{SC} = (\alpha_{31} q^{TF} x_1^1 + q^{AS} \alpha_{32} x_1^2) / (q^{SC}) \quad (16)$$

$$x_1^{AS} = (\alpha_{21} q^{TF} x_1^1 + \alpha_{22} q^{AS} x_1^2 + q^{RS} x_1^{SC} + \alpha_{20} q^f x_1^f) / q^{AS} \quad (17)$$

$$x_2^1 = x_2^{TF} + Y(x_1^{TF} - x_1^1) \quad (18)$$

$$x_2^2 = (q^{TF} x_2^{TF} - q^f \alpha_{10} x_2^f - q^{TF} \alpha_{11} x_2^1) / (q^{AS} \alpha_{12}) \quad (19)$$

$$V^{AS} = q^{AS} (x_1^{AS} - x_1^2) Y(K + x_1^2) / (k_c x_1^2 x_2^2) \quad (20)$$

$$x_2^{AS} = x_2^2 - \frac{V^{AS}}{q^{AS}} \left[ \frac{k_c x_1^2 x_2^2}{K + x_1^2} - k_D x_2^2 \right] \quad (21)$$

$$x_2^r = (q^{AS} x_2^{AS} - \alpha_{21} q^{TF} x_2^1 - \alpha_{22} q^{AS} x_2^2 - \alpha_{20} q^f x_2^f) / q^{RS} \quad (22)$$

$$x_1^3 = x_1^{SC} \quad (23)$$

$$x_2^{SC} = (\alpha_{31} q^{TF} x_2^1 + q^{AS} \alpha_{32} x_2^2) / q^{SC} \quad (24)$$

$$x_2^3 = (Cd - x_1^3) / m \quad (25)$$

$$T^{SC} = -\ln \left[ \frac{x_2^3}{p(x_2^{SC})^\beta} \right] / K_s \quad (26)$$

$$D^{TF} = -20^{n_f} \ln \left( \frac{x_1^1}{x_1^{TF}} \right) / K^{TF} A_p^{TF} \quad (27)$$

$$V^{SC} = T^{SC} \left( 832.67 \times 160.544 q^{SC} \right) / 24 \tag{28}$$

$$A^{SC} = V^{SC} / D^{SC} \tag{29}$$

$$x_1^r = x_1^{SC} \tag{30}$$

$$A^{TF} = 2178 \times q^{TF} \tag{31}$$

$$V^{TF} = A^{TF} \times D^{TF} \tag{32}$$

$$W = (q^{SC} x_2^{SC} - q^{RS} x_2^r - q^f x_2^3) / (q^f x_2^r - q^f x_2^3) \tag{33}$$

$$C^{TF} = 25(V^{TF})^{0.827} + 3,650 + 1,150 q^{TF} \alpha_{11} \tag{34}$$

$$C^{AS} = V^{AS} [1,75,000 + 36,500(V^{AS})^{-0.818}] + 3,650 + 1,150 q^{AS} \alpha_{12} \tag{35}$$

$$C^{SC} = 12.6 A^{SC} + 5,350 \left( \frac{A^{SC}}{1,000} \right)^{-0.126} 3,650 + 1,150 q^{RS} \tag{36}$$

There are 26 design equations and 51 variables of which 16 variables are specified. Thus there are 9 variables free for the optimal design purposes. In this work the variables selected for the optimal design are  $\alpha_{10}$ ,  $\alpha_{11}$ ,  $\alpha_{12}$ ,  $\alpha_{21}$ ,  $x_1^{TF}$ ,  $x_2^{TF}$ ,  $x_1^1$ ,  $q^{AS}$  and  $q^{SC}$ .

### RESULTS AND DISCUSSION

Adaptive Simulated Annealing (ASA) based optimal design value of wastewater treatment plant has been furnished in Table 1. The computation time required to obtain these optimal design values is in the order of 3 s in P-III 500 MHz processor. Table 1 indicates that maximum specific growth rate  $k=2.4$  is the feasible region to operate the wastewater treatment plant from the results of the system operated at various  $k$  values such as 0.48, 2.4 and 4.8. The reason for selecting

$k=2.4$  as the feasible region is that it the plant can be designed and operated at the minimum total construction cost for the waste water treatment plant that including the operating cost. A further decrease in  $k$  to a 0.24, the system acts like an infeasible one. The system was computed with two different regions of  $X_2^r$ , such as unbounded region and bounded value with the maximum of  $1 \times 10^5$ . From both cases we are getting minimum  $C^T$  value for unbounded region with  $k= 2.4 \text{ day}^{-1}$ .

**Table 1** ASA Based Optimal Design Value of Wastewater Treatment Plant

Variables	$X_2^r$ unbounded			$0 < X_2^r < 1 \times 10^5$		
	k=0.48	k=2.4	k=4.8	k=0.48	k=2.4	k=4.8
$\alpha_{10}$	0.021	0.2	0.082	0.019	0.24	0.28
$\alpha_{11}$	0.47	0.36	0.54	0.72	0.37	0.17
$\alpha_{12}$	0.15	0.26	0.23	0.14	0.30	0.34
$\alpha_{21}$	0.53	0.64	0.46	0.27	0.63	0.83
$X_1^{TF}$	23.0	32.0	27.0	2.0	31.0	26.0
$X_2^{TF}$	10000	7500	5600	9000	4900	3100
$X_1^1$	23.0	32.0	27.0	22.0	31.0	26.0
$q^{AS}$	4.4	6.8	3.9	5.1	7.4	9.8
$q^{SC}$	3.0	3.0	3.0	3.1	3.2	3.1
$C_T$	181700	154600	161500	196700	161200	168000

From Table 2 shows the optimal design values of wastewater treatment plant at different initial vector values with the condition of  $X_2^r$  unbounded and  $k= 2.4 \text{ day}^{-1}$ . The minimum total construction cost  $C^T$  of WTP was

found to be \$ 151400 (case-3). The optimal design component values such as depth, cross-sectional area and volumetric flow rates for the equipment present in WTP have been furnished in Table 3 (case-3).

**Table 2** Optimal Design Value of Wastewater Treatment Plant by ASA

Variable	Units	Case-1	Case-2	Case-3	Case-4
$\alpha_{10}$	-	0.029	0.19	0.29	0.2
$\alpha_{11}$	-	0.2	0.44	0.25	0.36
$\alpha_{12}$	-	0.15	0.35	0.20	0.26
$\alpha_{21}$	-	0.8	0.56	0.75	0.64
$X_1^{TF}$	PPM	18.0	30.0	45.0	32.0
$X_2^{TF}$	PPM	10000	7900.0	4600.0	7500.0
$X_1^1$	PPM	18.0	30.0	45.0	32.0
$q^{AS}$	MGD	4.9	5.3	6.4	6.8
$q^{SC}$	MGD	3.0	3.0	3.0	3.0
$C_T$	\$	156300	155900	151400	154600

**Table 3** Optimal Synthesis of Biological Wastewater Treatment System

Component	Trickling Filter (TF)	Aeration Vessel (AS)	Secondary clarifier (SC)
Depth, D, ft	2.5	---	10.0
Cross-Sectional area, A, ft <sup>2</sup>	$2.4 \times 10^{-4}$	---	$4.7 \times 10^3$
Volume, V, ft <sup>3</sup>	0.016	0.057 (MG)	$4.7 \times 10^4$
Volumetric flow rates, q, MGD	$2.9 \times 10^{-7}$	6.4	3.0
Sub total cost, \$	$4.5 \times 10^3$	$3.7 \times 10^4$	$6.7 \times 10^4$

Table 4 Optimal design of Wastewater Treatment Plant: Comparison of ASA with Other Optimization Techniques

Optimization Techniques	$C^T$ , \$	Plant Parameters
Generalized Reduced Gradient	2,43,841.00	$0 < X_2^r < 10^5$ & $k=0.48$
Simplex pattern search	2,15,095.00	$0 < X_2^r < 10^5$ & $k=2.4$
Modified random search	1,55,000.00	$X_2^r$ unbounded & $k=2.4$
	1,66,409.00	$0 < X_2^r < 9.1 \times 10^4$ & $k=2.4$
Hybrid technique of Integrated Control	1,57,902.00	$X_2^r$ unbounded & $k=2.4$
Random Search	1,70,902.00	$0 < X_2^r < 10^5$ & $k=2.4$
Continuous search	1,68,132.00	$0 < X_2^r < 10^5$ & $k=2.4$
Adaptive Simulated Annealing	1,51,400.00	$X_2^r$ unbounded & $k=2.4$
	1,61,200.00	$0 < X_2^r < 10^5$ & $k=2.4$

*Comparison Of Asa Based Optimal Design Values With Other Optimization Techniques:* ASA based optimal design value obtained for the wastewater treatment problem has been compared with the optimization approaches used in the previous investigations. The optimal synthesis of WTP using a simplex pattern approach had been studied (Mishra et al., 1973). The cost obtained 2,15,095 with  $0 < X_2^r < 10^5$  and  $k=2.4$  as the plant operating conditions. The same problem was also solved by using generalized reduced gradient (G.R.G) method (Himmlebalu, 1976). The total construction cost obtained by was \$ 2,43,841 with upper bound constraint on the recycle sludge concentration of  $1 \times 10^5$  PPM. Modified random search was also used for the optimal design of WTP (Wang and Luus, 1977). The total construction cost obtained was \$ 1,55,000 and 1,66,409, with two regions of recycle sludge concentration as unbounded region and maximum of  $9.1 \times 10^4$  respectively. The hybrid technique of

integrated control random search was also used for the design of WTP (Banga and Long, 1981). The total construction cost obtained was \$1,57,902 and 1,70,902, with two regions of recycle sludge concentration as unbounded region and maximum of  $1 \times 10^5$  respectively. The wastewater treatment system synthesis had been also solved by using continuous search method (Nishida and Power, 1983). The total construction cost was found to be \$ 1,68,132, with the recycle sludge concentration as  $1 \times 10^5$  and structural parameter value  $k=2.4$ . Table 4 shows that minimum value of  $C^T$  for the WTP by various optimization techniques and ASA. For most of the cases  $C^T$  minimum is obtained at the condition of  $k=2.4 \text{ day}^{-1}$  and  $X_2^r$  unbounded, ASA results obtained in this work also favors the condition previously established by the other approaches but reaches a new low total construction cost value of \$ 1,51,400 and 1,61,200.00 with the plant parameters of  $X_2^r$  unbounded and  $0 < X_2^r < 10^5$  respectively for  $k=2.4$ .

The value obtained by ASA has been found to be a stable from the sensitivity analysis done for the problem considered in this work.

**Conclusion:** The wastewater treatment plant considered in this study had been solved by using conventional optimization techniques in previous investigations including the simplex search, modified random search algorithm, generalized reduced gradient method, and a hybrid technique of using integrated control random search. But these techniques have their own limitations like requirement of complex mathematical analysis, difficulties in the problem formulation. In this paper, a successful attempt has been made to apply the adaptive simulated annealing technique for the optimal design of wastewater treatment plant. The results have been compared with conventional optimization approaches to establish the superiority of ASA. From this work artificial intelligence based optimization technique such as adaptive simulated annealing is found to be suitable for the optimal design of wastewater treatment plant and ASA based methodology can also be extended for the real time optimization problems.

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## Nomenclature

- $A^{TF}$  -Cross-sectional area of trickling filter,  $ft^2$
- $A^{SC}$  -Cross-sectional area of secondary clarifier,  $ft^2$
- $A_p^{TF}$  -Specific surface area of trickling filter packing,  $25.0 ft^2/ft^2$
- $C_d$  -Effluent discharge criterion, 20.0 ppm
- $C^{AS}$  -Construction cost of aeration vessel, \$
- $C^{SC}$  -Construction cost of secondary clarifier, \$
- $C^T$  -Construction cost of total system, \$
- $C^{TF}$  - Construction cost of trickling filter, \$
- $D^{SC}$  -Depth of secondary clarifier, 10.0 ft
- $D^{TF}$  -Depth of trickling filter, ft
- J -Objective function associated with total system
- k -Maximum specific growth rate of cells,  $0.002hr^{-1}$
- $K_s$  -Specific constant for secondary clarifier, 0.74
- $K^{TF}$  -Specific constant for trickling filter, 0.0259
- m-Conversion coefficient, ppm BOD/ ppm sludge dry weight, 0.5
- $n_r$  -Specific constant for tickling filter, 0.656
- p-Specific constant for secondary clarifier, 2.1
- $q^{AS}$  -Volumetric flow rate to activated sludge aeration vessel, MGD.
- $q^f$  -Volumetric Flow rate to influent wastewater, 3.0 MGD.
- $q^{RS}$  -Volumetric flow rate of return sludge to sludge Aeration vessel, MGD.
- $q^{SC}$  -Volumetric flow rate to secondary clarifier, MGD
- $q^{TF}$  -Volumetric Flow rate to trickling filter, MGD
- r-Sludge recycle ratio, dimensionless.
- $T^{SC}$  -Secondary clarifier residence time, hr.
- $V^{AS}$  -Aeration vessel volume, MG.

$V^{SC}$  -Secondary clarifier volume, ft<sup>3</sup>.

$V^{TF}$  -Trickling filter volume, ft<sup>3</sup>

w-Sludge wasting ratio, dimensionless

$x_1^1$  -Concentration of organic nutrients at trickling filter ppm.

$x_1^2$  -Concentration of organic nutrients at aeration vessel exit, ppm

$x_1^3$  -Concentration of organic nutrients in effluent, ppm

$x_1^{AS}$  -Concentration of organic nutrients at aeration vessel.

$x_1^f$  -Concentration of organic nutrients in influent waste water, ppm.

$x_1^r$  -Concentration of organic nutrients in sludge recycle stream, ppm.

$x_1^{SC}$  -Concentration of organic nutrients at secondary clarifier entrance, ppm.

$x_1^{TF}$  -Concentration of organic nutrients at trickling filter entrance, ppm.

$x_2^1$  -Concentration of cells at trickling filter exit, ppm.

$x_2^2$  -Concentration of cells at aeration vessel exit, ppm.

$x_2^3$  -Concentration of cells in effluent, ppm.

$x_2^{AS}$  -Concentration of cells at aeration vessel entrance, ppm.

$x_2^f$  -Concentration of cells in influent wastewater, ppm.

$x_2^r$  -Concentration of cells in sludge recycle stream, ppm.

$x_2^{SC}$  -Concentration of cells at secondary clarifier entrance, ppm.

$x_2^{TF}$  -Concentration of cells at trickling filter entrance, ppm

Y-Yield constant 0.5